Spectrum of reflected light by self-focusing of light in a laser plasma

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The spectrum of the radiation reflected by a laser-produced plasma is considered. In this situation, self-focusing occurs and a region of low density (caviton) is formed. It is shown that the process leads to a considerable broadening of the spectrum on the "red" side, and to the appearance of a line structure in the spectrum. The results can explain data for the reflected light spectrum [L. M. Gorbunov *et al.*, FIAN Preprint No. 126 (1979)] as being due to the nonstationary self-focusing of light in a laser-produced plasma that has recently been observed [V. L. Artsimovich *et al.*, FIAN Preprint No. 252 (1981); Sov. Phys. Doklady **27**, 618 (1982)].

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One of the most important phenomena determining the interaction of intense laser radiation with matter is self-focusing[1–3]. Recently, a great deal of attention has been devoted to the study of this phenomenon in a laser-produced plasma.[14] Data on the spatial structure of the radiated x rays[4] and harmonics,[8] interferometry of the plasma corona,[6] schlieren photography,[13] and the scattering of a test light beam[9] all indicate that self-focusing is produced in a laser beam. The temporal change of regions of scattering of radiation at the laser frequency was studied in Ref. 12. The measurements were made in the plane of the target perpendicular to the direction of the incident beam; these showed that the scattering region develops near the surface of the target and then moves into the more rarefied plasma, where it disappears. This process lasts 200–300 ps and is repeated several times in the course of the laser pulse. The authors of Ref. 12 connected the generation of such moving regions of increased scattering with the nonstationary self-focusing of the light in the plasma corona. The physical picture of the phenomenon that they disussed is the following. In the initial stage of the laser pulse, the self-focusing (which is connected in the plasma either with thermal or strictive nonlinearty) appears in the more dense plasma. At just that point the critical power is minimal and the growth rate of the self-focusing of the instability is maximal. In this case the plasma density in the focal region decreases, either because of local heating (thermal nonlinearity) or because of pondermotive forces (strictive nonlinearity) and a caviton is formed.[1)] A redistribution of the density then takes place even in the more rarefied portions the plasma corona. As a result, the focus is shifted toward the laser beam and disappears in a sufficiently rarefied plasma. The estimates given in Ref. 12 corroborate the possibility of such a process.

Studies were carried out still earlier in this same group on the spectra of radiation reflected from a laser-produced plasma.[16] They showed that the spectrum changes strongly with the passage of time. Broadenings that fluctuated with a characteristic time of 200–300 ps were observed basically on the "red" side. Here a line structure is traced in the broadened spectrum.

It is shown in the present work that the observed features of the spectrum of reflected radiation[16] can be explained by the self-focussing of light in a laser plasma, which is discussed in Ref. 12.

As has been remarked, upon development of self-focusing in a plasma, a region of lowered density (a caviton) is formed. The nonstationarity of the plasma corona that is associated with this determines the dependence of the phase of the reflected radiation on the time and by the same token affects its spectrum. As a result, a broadening of the spectrum develops during the formation of the focus, on the red side, and a line structure appears.

The phenomenon that we are discussing is similar to the phase self-modulation of light in condensed media.[2] The difference lies in the fact that the density changes with time in the plasma (linear permittivity) while in condensed media, the intensity of the incident radiation undergoes such a change (nonlinear permitivity).[2)]

It should be noted that the process of self-focusing is itself not considered in the present work. Only the corresponding change of the plasma density with time is modeled. Here it is assumed that in the initial stage the caviton evolves according to an exponential law, as usually happens in the formation of a self-focused instability.[3] Later, its growth is showed and then stopped. As calculation shows, the law of formation of the caviton has no essential value for the qualitative explanation of the observed features in the spectrum of the reflected radiation (red wing, line structure). For quantitative agreement of the calcuation with experiment, both the size of the caviton and the characteristic times of its evolution are important. For estimates of these quantities, we have used the experimental data of Ref. 12, which made it possible to obtain rather good agreement with the results of measurement of the spectrum in Ref. 16.

1. PHASE OF THE REFLECTED WAVE

We consider the normal incidence of laser radiation with frequency ω_0 on an inhomogeneous plasma, the electron concentration N of which depends on the coordinate x. We assume that the point of reflection is fixed and we choose it to be the coordinate origin ($x = 0$). Then the difference between the phases of the incident and reflected waves outside the plasma ($x = L$) is equal to

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$$\varphi(t)=2\frac{\omega_0}{c}\int_0^L dx\left(1-\frac{N(x,t)}{N_c}\right)^{1/2}+\psi, \qquad (1)$$

where ψ is a constant phase shift that arises at the point of reflection, $N^c = m\omega_0^2/4\pi e^2$ is the critical concentration of the electrons. Equation (1) is valid in the approximation of geometric optics ($N < N_c$). A more general expression for the quantity φ was recently discussed in Ref. 19.

We now consider the following simple model of evolution of the plasma corona. We shall assume that at the instant of time $t = 0$ the plasma density depends linearly on the x coordinate in the range $L > x > l$ and is constant at $l > x > 0$ (Fig. 1). At the point of reflection of the wave, the density rises jumpwise above the critical value. We further assume that in the range $l > x > 0$ the plasma density, while remaining constant with respect to the coordinate, decays with time according to a law determined by a certain function $F(t)$:

$$N(x,t)/N_c=1-(l/L)F^2(t),\ 0<x<l,$$
$$N(x,t)/N_c=1-x/L,\ l<x<L.$$

Then, according to Eq. (1), we obtain

$$\varphi(t)=2\frac{\omega_0}{c}\left\{\frac{2}{3}L-\frac{l^{3/2}}{L^{1/2}}\left(\frac{2}{3}-F(t)\right)\right\}+\psi. \qquad (2)$$

We choose the law of falloff of the concentration and formation of the caviton such that initially the process develops exponentially, and is then stopped:

$$F(t)=A+(1+a)(1-A)/(ae^{\gamma t}+1), \qquad (3)$$

where the constant $A > 1$ determines the value of the function as $t\to\infty$, characterizes the rate of growth at the beginning of the process, the time $t_0 = (1/\gamma)\ln(1/a)$ of transition from exponential to the slower rate of change of the function is expressed in terms of a constant $a \ll 1$ (Fig. 2).

In the model under discussion, we have considered only the process of formation of a single caviton in the self-focusing and its motion in a more rarefied plasma is not considered. This is done, on the one hand, for simplification of the calculations, and on the other because we can think that the changes occuring in the denser portions of the plasma will have a strong influence on the spectrum of the reflected radiation

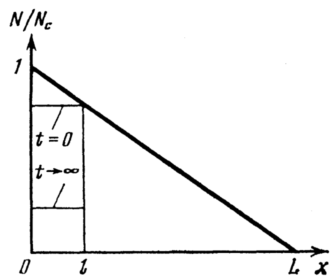

FIG. 1. Plasma concentration N/N_c vs coordinate for the instants of time $t = 0$ and $t\to\infty$.

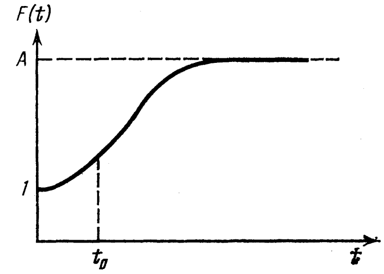

FIG. 2. Curve of the function $F(t)$.

2. APPROXIMATION OF THE INSTANTANEOUS FREQUENCY

One of the simplest and most widely used approximations in the analysis of spectra in the case of a slow change in the phase is the approximation of instantaneous frequency.[18,19] In this approximation, it is assumed that at each instant of time the radiation has a single frequency (i.e., it is monochromatic) which differs from the frequency of the incident radiation by an amount $\Delta\omega = -d\varphi(t)/dt$. According to Eqs. (2) and (3), we get

$$\Delta\omega=-2\frac{\omega_0}{c}\frac{l^{3/2}}{L^{1/2}}\frac{dF}{dt}=-\Delta\omega_m\frac{4ae^{\gamma t}}{(1+ae^{\gamma t})^2} \qquad (4)$$
$$\Delta\omega_m=\frac{(1+a)(A-1)}{2}\gamma\frac{\omega_0 l^{3/2}}{cL^{1/2}}.$$

It follows from Eq. (4) that $\Delta\omega < 0$ and the frequency of the radiation reflected from the plasma is lower than the frequency of the incident radiation (the red shift). The frequency shift in the course of formation of the caviton reaches a maximum at $t = t_0$ and then falls off (Fig. 3). Here each value of the frequency (except the narrow range $|\Delta\omega|/\Delta\omega_m < 4a/(1 + a)^2 \ll 1$) repeats itself twice with a certain time delay.

3. SPECTRUM OF THE REFLECTED RADIATION

For the study of the spectral composition of the radiation, we take the Fourier expansion:

$$E(\omega)=RE_0\int_0^\infty dte^{i\omega t}\cos[\omega_0 t-\varphi(t)],$$

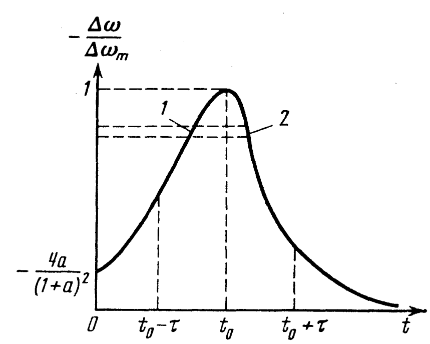

FIG. 3. The change of frequency shift of scattered radiation ($\Delta\omega/\Delta\omega_m$) in time.

where E_0 is the amplitude of the incident wave, R is the reflection coefficient. We have assumed that the laser radiation is turned on at the instant $t = 0$. The spectral density of the square of the electric field intensity is equal to

$$|E(\omega)|^2=|B(\omega)|^2+|B(-\omega)|^2+B(\omega)B(-\omega)+B^*(\omega)B^*(-\omega), \tag{5}$$

where $\Delta\Omega = \omega - \omega_0$,

$$B(\omega)=\frac{RE_0}{2}\int_0^\infty dt \exp[i\Delta\Omega+i\varphi(t)]. \tag{6}$$

Using Eqs. (2) and (3), we write out Eq. (6) in the form

$$B(\omega)=\frac{RE_0}{2\gamma}Ie^{i\varphi_0},\quad I=\int_a^\infty \frac{dx}{x}e^{i\sigma g(x)},$$

$$\varphi_0=\psi+\frac{4\omega_0 L}{3c}\left[1-\left(\frac{l}{L}\right)^{3/2}\left(1-\frac{3A}{2}\right)\right]-\frac{\Delta\Omega}{\gamma}\ln a,$$

$$\sigma=4\frac{\Delta\omega_m}{\gamma},\qquad g(x)=\beta\ln x-\frac{1}{1+x},\qquad \beta=\frac{\Delta\Omega}{4\Delta\omega_m}.$$

The quantity in the argument of the exponential of the integral I has the value

$$\sigma=(1+a)(A-1)\omega_0 l^{3/2}/2cL^{1/2},$$

which is much greater than unity. This allows us to use the method of stationary phase for the estimate of the integral. According to this method, the principle contribution to the integral is made by the regions of integration in which the function $g(x)$ changes most slowly and $g'(x) = 0$. In our case, there are two points of stationary phase

$$x_{1,2}=-\left(1+\frac{1}{2\beta}\right)\pm\left[\frac{1}{\beta}\left(1+\frac{1}{4\beta}\right)\right]^{1/2}.$$

The location of these points is shown in Fig. 4 as a function of the parameter β. It is seen that only in the case $0>\beta> -1/4$ are the points of stationary phase real and positive and, consequently, are located in the region of integration. This interval of change of β corresponds to the values $\Delta\Omega$ which arose earlier in the approximation of instantaneous frequency. Outside this range of values, the integral I is significantly less. Therefore, the spectral density $|E(\omega)|^2$ is also maximal in the range of frequencies $0>\Delta\Omega> -\Delta\omega_m$ and in significantly less for large frequency shifts ($|B(\omega)|\gg|B(-\omega)|$. Taking this circumstance into account and using the general relations for the estimate of the integral,[20] we obtain

$$f(\Delta\Omega)=\frac{|E(\omega)|^2\Delta\omega_m^2}{R^2E_0^2}\approx\frac{\pi\Delta\omega_m}{\gamma(\Delta\omega_m/|\Delta\Omega|-1)^{1/2}}$$
$$\times\left\{1+\sin\left[\frac{8\Delta\omega_m}{3\gamma}\left(\frac{\Delta\omega_m}{|\Delta\Omega|}-1\right)^{3/2}\right]\right\};$$
$$\left(\frac{4\gamma}{\Delta\omega_m}\right)^{1/3}<\left(\frac{\Delta\omega_m}{|\Delta\Omega|}-1\right)^{1/2}<1, \tag{7}$$

$$f(\Delta\Omega)=\frac{|E(\omega)|^2\Delta\omega_m^2}{R^2E_0^2}\approx\frac{[\Gamma(1/3)]^2\Delta\omega_m^{4/3}}{18^{2/3}\gamma^{4/3}};\qquad \Delta\Omega=-\Delta\omega_m,$$

$$f(\Delta\Omega)=\frac{|E(\omega)|^2\Delta\omega_m^2}{R^2E_0^2}\approx\frac{\pi\Delta\omega_m^2}{\gamma|\Delta\Omega|}\left\{1+\sin\left(\frac{\Delta\omega_m}{\gamma}\left[1-2\frac{|\Delta\Omega|}{\Delta\omega_m}\right.\right.\right.$$
$$\left.\left.\left.\times\left(1-\ln\frac{|\Delta\Omega|}{\Delta\omega_m}\right)\right]\right)\right\};\qquad \Delta\Omega<-\gamma;\ -4a\Delta\omega_m.$$

Thus, the spectral density of the square of the electric field intensity of the reflected radiation has the form of a set of discrete lines (Fig. 5). The height of the lines increases as $|\Delta\Omega|\to\Delta\omega_m$ and $|\Delta\Omega|\to 0$. From Eqs. (7), we can find the location of the maxima of the intensity in the spectrum. Thus, at $|\Delta\Omega|\sim\Delta\omega_m$, the maxima are achieved at the values

$$|\Delta\Omega|_n\approx\frac{\Delta\omega_m}{1+(3\pi\gamma n/4\Delta\omega_m)^{2/3}}, \tag{8}$$

where n is the number of the maximum. If the quantity n is not too large, such that the second term in the denominator of Eq. (8) is smaller than unity, then we can obtain an expression for the frequency interval between the maxima in the spectrum:

$$\delta\Omega_n=|\Delta\Omega|_n-|\Delta\Omega|_{n+1}\approx\Delta\omega_m\frac{2}{3n^{1/3}}\left(\frac{3\pi\gamma}{4\Delta\omega_m}\right)^{2/3}. \tag{9}$$

The interval between the maxima in the spectrum at $|\Delta\Omega|<\Delta\omega_m$ can be found in similar fashion:

$$\delta\Omega\approx 2\pi\gamma/[1+\ln(\Delta\omega_m/\gamma)]. \tag{10}$$

4. DISCUSSION OF THE EXPERIMENTS

Diffraction gratings are ordinarily used in spectral measurements. Figure 6 shows the simplest scheme of experi-

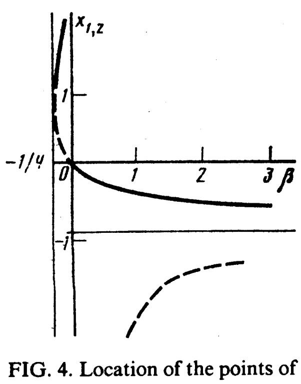

FIG. 4. Location of the points of stationary phase x_1 (solid line) and x_2 (broken line) vs parameter $\beta = \Delta\Omega/4\Delta\omega_m$.

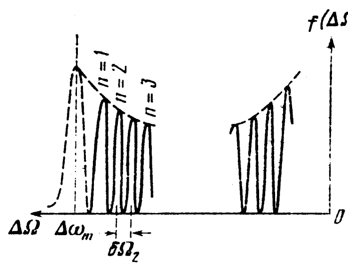

FIG. 5. The spectral density of the square of the electric field intensity of the reflected wave $f(\Delta\Omega)$.

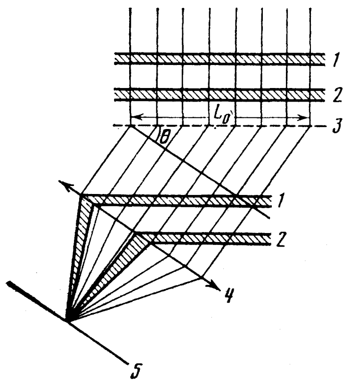

FIG. 6. The scheme of performance of spectral apparatus: 1, 2 section of reflected radiation with equal instant frequency, 3—diffraction grating, 4—lens, 5—screen.

ment. Radiation with a certain frequency passes through the diffraction grating at an angle θ and is focused on the screen by a lens. The spectrum assumes a steady state at a certain time $\tau = L_0 \sin\theta / c$, where L_0 is the minimal of three dimensions of the grating, of the beam, and of the lens. During the time τ the plane front incident on the grating moves along the lens and there is an image of it on the screen.

If the frequency of the radiation passing through the diffraction grating changes so slowly that we can neglect its change within the time τ, then an image at virtually a single frequency will be present on the screen at each instant. Thus, the instantaneous frequency approximation corresponds to the limit $\tau|d \ln \Delta\omega/dt| \ll 1$.

On the other hand, if the time τ is large in comparison with the time of frequency change, then all frequencies will be present at the lens at the same time. As a result of the interference of the images originating from the different parts of the lens, a spectral decomposition if formed at the screen. This case corresponds to the limit $\tau|d \ln \Delta\omega/dt| \gg 1$.

In Refs. 12, 16, and 21, the time τ was approximately equal to 200 ps and was of the same order as the characteristic time of development of the self focusing. Therefore, the spectral decomposition was carried out experimentally not over an infinite time interval, but over the interval of time τ and was more sharply observed at greater frequency shifts $\Delta\Omega \sim \Delta\omega_m$ at the edge of the spectrum (see Refs. 16 and 21). Actually, at the radiation originating in the time interval from $t_0 - \tau/2$ to $t_0 + \rho/2$ (Fig. 3), passing through the diffraction grating, will be present simultaneously at the lens (Fig. 6) and can creat an interference picture on the screen. In the course of this time interval, the scattered radiation with the nearest frequency values in generated twice (see the regions 1 and 3 in Fig. 3) and these are collected at one point on the screen (the cross-hatched regions 1 and 2 in Fig. 6). Here, at certain frequencies, the phase difference is such that intensity maxima are produced in the interference at the screen and minima are produced for other frequencies. In other words, under the experimental conditions[11,8] one could observed decomposition (Fig. 5) only at $\Delta\Omega \sim \Delta\omega_m$ in the interval $\delta\Delta\Omega \sim (\tau\gamma/2)^2 \Delta\omega_m$.

We carry out a comparison of the experimental data with the results of calculation. The self-focusing process can be connected both with the striction and the thermal mechanisms. For the quantity γ we use the growth rate of the striction self-focusing instability $\gamma = \omega_{Li} v_E/\sqrt{2}c$, where ω_{Li} is the Langmuir frequency of the ions, v_E is the velocity of oscillation of the electrons in the field of the pump wave. In correspondence with the definition of the quantity $\Delta\omega_m$, we obtain

$$\frac{\Delta\omega_m}{\omega_0} \approx \frac{A-1}{2}\left(\frac{l}{L}\right)^{3/2} \frac{\omega_{Li} v_E L}{\sqrt{2}c^2}.$$

According to Ref. 12, the size of the plasma corona L reaches $300\,\mu$ while the size of the caviton $l \sim 50\,\mu$. If we assume that the minimum density in the caviton amounts to 0.3 N_e ($A = 2$), then, under the experimental conditions of Refs. 16 and 21 we obtain $\Delta\lambda_m = \lambda_0(\Delta\omega_m/\omega_0) \approx 60$ Å ($\lambda_0 = 10^4$ Å). A broadening $\Delta\lambda_m \sim 40 - 70$ Å is usually observed, which is in excellent agreement with the estimates given.

The ratio

$$\frac{\gamma}{\Delta\omega_m} = \frac{2c}{(A-1)L\omega_0}\left(\frac{L}{l}\right)^{3/2}$$

amounted to to 1.4×10^2 in the experiment. According to Eq. (9), at $n = 3$, the separation of the maxima in the spectral intensity is equal to $\delta\lambda_3 = \delta\Omega_3\lambda_0/\omega_0 \approx 2$ Å. This quantity was equal to 1–3 Å in the experiments of Refs. 16 and 21.

5. CONCLUSION

The effect of self-focusing on the spectrum of radiation from a laser plasma evidently depends significantly on the experimental conditions. Plane targets were used in Refs. 9, 16 and 21, and the size of the focal spot was rather small ($\sim 30\,\mu$). Essentially, self-focusing of the ray as a whole takes place and a single caviton is formed. For a larger focal spot ($\sim$ 100–200 μ) or in the case of irradiation of a spherical target, a filamentation of the beam is observed in the plasma[4–14] and, as can be expected, several cavitons are formed. There is small likelihood that these cavitons vary quite identically. Therefore, a phase scatter arises in the reflected radiation passing through different cavitons and this leads to a smearing of the interference picture.

However, the considered spectrum can arise not only in the case of reflection from the plasma of a powerful laser radiation, which determines the self-focusing process, but also in the reflection of a weak probing signal. Therefore, in the case of simultaneous formation of several cavitons it is possible to separate and investigate only one of them with the help of probing radiation.

It should be noted that not only the process of self-focusing leads to a broadening of the spectra of reflected radiation and the formation of a line structure [the observation of line structure in the spectrum has been reported earlier in a number of researches (see, for example, Ref. 22)]. Other nonstationary processes in the plasma corona also lead to this result. (These nonstationary processes include the motion of the critical density, the change of the absorption with time, the increase or decrease in the size of the corona.). Systematic investigations of these questions are lacking at the present time, although they are basic to the creation of new diagnostics of the laser plasma.

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[1)]The formation of a density dip in a plasma in the case of self-focusing was essentially considered back in Ref. 15.

[2)]The problem of phase self-modulation of light in a laser plasma because of the time dependence of the permittivity has been discussed in Ref. 17.

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