Ion stopping in a degenerate electron gas

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The polarization losses of the energy of a fast ion in a degenerate electron gas are calculated for different values of the Born parameter $\alpha = e^2/\hbar v_F$, where v_F is the electron Fermi velocity. In the case of an ideal electron gas ($\alpha \ll 1$) the energy loss is calculated analytically. It is shown, in particular, that stopping of high-velocity ions produced as a result of nuclear rections in surface layers of neutron stars (in the region of the strong degeneracy of the electrons), a substantial increase of the secondary-reaction rates can take place. This in turn can influence the character of the burster activity of the neutron stars. The energy loss calculated at the value $\alpha \sim 1$ of interest in metal physics agrees well with the experimentally measured proton losses in a number of metals (Be, Al) at proton energies in the region of the maximum loss and lower.

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§1. INTRODUCTION

It is known that fast ions moving in a plasma are stopped mainly by interaction with electrons. The polarization energy loss of a nonrelativistic ion per unit length of path is determined by the expression (see, e.g., Ref. 1)

$$\frac{dE}{dx} = \frac{Z^2e^2}{2\pi^2 V}\int d\mathbf{k}\frac{\omega}{k^2}\frac{\operatorname{Im}\varepsilon(\omega,k)}{|\varepsilon(\omega,k)|^2}, \tag{1}$$

where $\hbar\mathbf{k}$ and $\hbar\omega$ are the momentum and energy transferred from the ion to the plasma electrons in the elementary interaction act, $\varepsilon(\omega, k)$ is the longitudinal dielectric constant of the plasma, in which the frequency ω must be set equal to $\mathbf{k}\cdot\mathbf{V}$, and $\mathbf{V}$ and Ze are the velocity and charge of the ions.

Generalizing the results obtained in Ref. 2 for a nondegenerate electron gas to include the case of arbitrary degeneracy of the electrons, it is convenient to represent the polarization loss (1) in the form

$$\frac{dE}{dx} = -\frac{4\pi Z^2e^4}{m_e V^2}n_e(V)\Lambda(V), \tag{1a}$$

where $n_e(V)$ is the density of the number of plasma electrons with velocities $v \leqslant V$, while $\Lambda(V)$ is a slowly varying function of V and has the meaning of the Coulomb logarithm (see Refs. 1 and 2, as well as §3 below), which in fact must be determined from the calculation.

The present paper is devoted to calculation of polarization losses in a strongly degenerate electron plasma. This problem is of interest, in particular, for astrophysical plasma (see §3), in which the characteristic values of the Born parameter $\alpha \equiv e^2/\hbar v_F \ll 1$ (v_F is the Fermi velocity of the electrons), as well as for metal plasma (§4), where $\alpha \sim 1$.

Fermi and Teller,[3] Larkin,[4] and Ritchie[5] calculated the energy loss at $V \ll v_F$ in the case $\alpha \ll 1$. In addition, Larkin[4] and Ritchie[5] obtained an expression for the polarization loss in the high-velocity limit. This expression is independent of the degree of degeneracy of the electron gas and is applicable to a degenerate gas at $V \gg v_F$. Numerical calculations of the polarization energy loss in a degenerate gas at arbitrary V, for several values of α, were carried out by Lindhard and Winther,[6] and also by Ferrel and Ritchie.[7] Yavlinskiĭ[8] published recently detailed calculations[8] for the electron gas of metals ($\alpha \geqslant 0.5$). At $V > v_F$, however, his calculations are not accurate, and, in particular, do not agree with the results of Refs. 4–7, inasmuch as he did not take into account the energy lost to excitation of Langmuir plasmons.

In this paper we calculate dE/dx numerically, with allowance for plasmon excitation, in a wide α interval of physical interest. At $\alpha \ll 1$ (for an astrophysical plasma) a simple analytic expression was also obtained for dE/dx. At $\alpha \sim 1$ the calculation results agreed well with the experimental data on proton energy loss in a number of metals, and allowance for the excitation of the plasmons improves noticeably the agreement in the velocity region $V \gtrsim v_F$.

§2. GENERAL RELATIONS

Recognizing that $\operatorname{Im}\varepsilon(\omega, k)$ is an odd and $\operatorname{Re}\varepsilon(\omega, k)$ is an even function of ω, it is possible to change from integration over all $\mathbf{k}$ in (1) to integration over the region in which $\omega < 0$. The longitudinal dielectric constant of a fully degenerate electron gas was calculated by Lindhard[9] and takes at $\omega < 0$ the form

$$\varepsilon = 1 + \frac{\alpha}{\pi\xi^2}(A + iB), \tag{2}$$

where

$$A = \frac{1}{2} + \frac{1-(\Omega+\xi)^2}{8\xi}\ln\left|\frac{\Omega+\xi+1}{\Omega+\xi-1}\right| - \frac{1-(\Omega-\xi)^2}{8\xi}\ln\left|\frac{\Omega-\xi+1}{\Omega-\xi-1}\right|;$$

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$$B=\begin{cases} \frac{\pi}{2}\Omega & \text{at } \Omega+\xi\leq 1 \\ & \text{(region I)}, \\ \pi\frac{1-(\Omega-\xi)^2}{8\xi} & \text{at } |1-\Omega|<\xi<1+\Omega \\ & \text{(region II)}, \\ 0 \text{ at } \xi\leq\Omega-1 \text{ (region III)} & \\ & \text{and } \xi\geq 1+\Omega \text{ (region IV)}. \end{cases} \quad (3)$$

Here $\alpha = e^2/\hbar v_F$, $\xi = \hbar k/2m_e v_F$ and $\Omega = |\omega|/kv_F$. By virtue of (3) the imaginary part of the dielectric constant differs from zero for the values of ξ and Ω in regions I and II (Fig. 1). In regions III and IV the phase velocities of the electromagnetic oscillations ω/k exceed v_F. Since the Fermi distribution is steplike, there are no electrons capable of absorbing these oscillations, so that $\text{Im}\varepsilon = 0$.

The polarization losses of the ion energy (1a) in a degenerate electron gas take the form

$$\frac{dE}{dx} = -\frac{4\pi Z^2 e^4 n_e}{m_e V^2}\Lambda(u)\cdot\begin{cases} u^3 & \text{at } u<1, \\ 1 & \text{at } u\geq 1, \end{cases} \quad (4)$$

where $u\equiv V/v_F$, n_e is the plasma-electron density and $\Lambda(V)\equiv\Lambda(u)$ is the Coulomb logarithm. We note that the quantity $\Lambda(u)$ is connected with the friction coefficient $g(u,\alpha)$ introduced in Ref. 8 by the relations $\Lambda(u\leq 1) = g/2$ and $\Lambda(u>1) = gu^3/2$. We note also that in place of α one frequently uses the parameter $r_s = (4\pi n_e a^3/3)^{-1/3} = (9\pi/4)^{1/3}\alpha$ (a is the Bohr radius).

Substituting (2) in (1) we obtain

$$\Lambda(u) = \frac{6}{\alpha}\int_0^u d\Omega\int_0^{1+\Omega} d\xi\,\Omega\xi\frac{\text{Im}\,\varepsilon(\Omega,\xi)}{|\varepsilon(\Omega,\xi)|^2}\cdot\begin{cases} u^{-3} & \text{at } u<1, \\ 1 & \text{at } u\geq 1. \end{cases} \quad (5)$$

At $u\leq 1$ the integral in (5) breaks up into two integrals over the regions I and II (Fig. 1), in which $\text{Im}\,\varepsilon\neq 0$. At $u>1$, however, besides regions I and II, it is necessary to take into account[4-7] the region III, which corresponds to values $\xi < \Omega - 1$ (Fig. 1). The point is that in this region is located the curve $\Omega = \Omega(\xi)$ of the poles of the dielectric constant, on which $\text{Re}\varepsilon(\Omega,\xi) = 0$. Therefore, although $\text{Im}\,\varepsilon(\Omega,\xi) = 0$ in region III, the intergral (5) over this region still differs from zero and easily reduces to an integral along the $\Omega(\xi)$ curve with the aid of the relation

$$\frac{\text{Im}\,\varepsilon(\Omega,\xi)}{|\varepsilon(\Omega,\xi)|^2}\to\frac{1}{\pi}\delta[\text{Re}\,\varepsilon(\Omega,\xi)]. \quad (6)$$

From the physical viewpoint the contribution of region III describes the energy lost to collisionless excitation of Langmuir plasmons by the Landau mechanism (see, e.g., Refs. 4 and 5). This process begins to operate at $u>1$, inasmuch as under this condition the velocity V of the polarization electron cloud around the stopping ion becomes larger than v_F, and it becomes possible to excite the weakly damped plasmons. It must be noted that the curve of the poles of $\Omega(\xi)$ in the region III has a weakly pronounced minimum $\Omega(\xi) = \Omega_0(\xi_0)$ that lies in the immediate vicinity of the boundary $\Omega = \xi + 1$ of the regions II and III (Fig. 1). Strictly speaking, plasmon generation becomes possible at a value $u = \Omega_0$, which increases with increasing α.

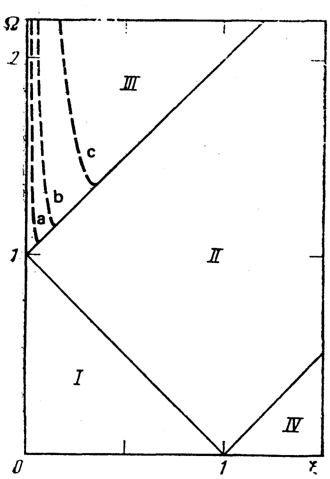

FIG. 1. Regions of values of dimensionless arguments ξ and Ω of the dielectric constant ε in which $\text{Im}\,\varepsilon$ has a different form (3). In region III, the dashed lines are the curves $\Omega(\xi)$ of the poles of the dielectric constant at values $\alpha = 0.01$ (a), 0.1 (b), and 1 (c).

§3. POLARIZATION LOSSES IN AN IDEAL ELECTRON GAS ($\alpha\ll 1$)

The calculation of the Coulomb logarithm $\Lambda(u)$ becomes greatly simplified in the case of an ideal electron gas, for which the Born parameter $\alpha\ll 1$. In this case the screening momentum $k_{TF} = 2k_F(\alpha/\pi)^{1/2}$ (the reciprocal Thomas-Fermi static-screening radius) is much less than the quantum momentum $2k_F = 2m_e v_F/\hbar$. It suffices then to use in (5) the dielectric constant (2), (3) in the limit $\xi\ll 1$:

$$A = 1-\frac{\Omega}{2}\ln\left|\frac{\Omega+1}{\Omega-1}\right|,\quad B(\Omega<1) = \frac{\pi}{2}\Omega,$$
$$B(\Omega\geq 1) = 0. \quad (7)$$

Substituting (7) in (5) and discarding small terms $\sim\alpha$, we obtain at $\alpha\ll 1$ and $u<1$

$$\Lambda(u) = \ln\left[\left(\frac{\pi}{\alpha}\right)^{1/2}(1-u)\right] + \frac{u^3+1}{2u^3}\ln\frac{1+u}{1-u} - \frac{1}{u^2} - \frac{1}{3} - \frac{1}{2}C(u), \quad (8)$$

where

$$C(u) = \frac{3}{u^3}\int_0^u d\Omega\,\Omega^2\left[\frac{1}{2}\ln(A^2+B^2) + \frac{A}{B}\left(\frac{\pi}{2} - \text{arctg}\frac{A}{B}\right)\right]. \quad (9)$$

The quantity $C(u)$ can be calculated analytically for two values of u: $C(0) = 1$ and $C(1) = 8/3 - \ln 12 = 0.182$. For intermediate u, the calculation is performed numerically. With an error less than 1%, the results are described by the simple interpolation formula

$$C(u) = 0.182 + 0.818(1-u^2)^{1/2} + 0.170u^3(1-u^2)^2. \quad (9a)$$

The integral (5) is calculated analytically to the end at $\alpha\ll 1$ and $u>1$:

$$\Lambda(u)=\Lambda_e(u)+\Lambda_{pl}(u)$$
$$=\ln\left[\left(\frac{3\pi}{\alpha}\right)^{1/2}(u^2-1)\right]+u^3\ln\frac{u+1}{u-1}-2u^2-\frac{2}{3}. \quad (10)$$

Here Λ_c and $\Lambda_{\rm pl}$ denote the components of Λ obtained upon integration in (5) over the regions I + II and III, respectively. We present also an expression for $\Lambda_{\rm pl}(u)$

$$\Lambda_{pl}(u)=\frac{u^3-1}{2}\ln\frac{u+1}{u-1}+\ln\frac{u+1}{2}-u^2+1. \quad (11)$$

In the limiting cases we obtain from (8)–(10)

$$\Lambda=\ln(2k_F/k_{TF})-{}^1/_2 \quad \text{at} \quad u\ll 1, \quad (12)$$

$$\Lambda=\ln(8\sqrt{3}\,k_F/k_{TF})-{}^8/_3 \quad \text{at} \quad u=1, \quad (13)$$

$$\Lambda=\ln(2m_eV^2/\hbar\omega_{pe}) \quad \text{at} \quad u\gg 1, \quad (14)$$

where $k_{TF}=v_F^{-1}\sqrt{3}\omega_{pe}=(4\alpha/\pi)^{1/2}k_F$ is the screening momentum (see above) and ω_{pe} is the electron plasma frequency. The asymptotic relations (12) and (14) coincide with the well known asymptotic relations in Refs. 4 and 5.

The principal terms in expressions (8), (10), and (12)–(14) for $\Lambda(u)$ are the large logarithms. Confining ourselves to these terms, we can write down approximately $\Lambda(u)\approx\ln(\rho_{\max}/\rho_{\min})$, where $\rho_{\max}$ and $\rho_{\min}$ are the maximum and minimum impact parameters of the interaction of the ions with electrons, and have a simple meaning. Namely, the parameter $\rho_{\min}\sim\hbar[2m_e(v_F+V)]^{-1}$ is determined by the maximum momentum transferred in electron collision, and varies in the range from $\rho_{\min}\approx(2k_F)^{-1}$ at $u\ll 1$ to $\rho_{\min}\approx\hbar/2m_eV$ at $u\gg 1$. The parameter $\rho_{\max}$ at $u\lesssim 1$ is determined by the screening radius of the plasma: $\rho_{\max}\sim k_{TF}^{-1}$, and at $u>1$ it is determined by the largest wavelengths of the plasmons excited by the ion: $\rho_{\max}\sim V/\omega_{pe}$. In this case we have in (10) $\Lambda_{pl}\approx\ln(\rho_{\max}k_{TF})$, $\Lambda_c\approx\ln(\rho_{\min}k_{TF})^{-1}$. Therefore at $u>1$ in the impact-parameter interval $\rho_{\min}\lesssim\rho\lesssim k_{TF}^{-1}$ the ion energy loss is due to collisions with electrons, while in the interval $k_{TF}^{-1}\lesssim\rho\lesssim\rho_{\max}$ it is due to plasmon excitation.

Equations (8) and (10) give a kink on the energy-loss curve at $u=1$. This kink is due to the use of the simplified ($\xi\ll 1$) dielectric constant (7) in the calculation of $\Lambda(u)$ and becomes smoothed out when the exact dielectric constant (2) and (3) is used, see Fig. 2.

We note that the Coulomb logarithm for a degenerate plasma depends on V qualitatively in the same manner as the Coulomb logarithm that enters in the expression for the energy loss in a nondegenerate plasma,[2] in which $\rho_{\min}$ is determined by the de Broglie wavelength of the particles. To establish a correspondence with the case of a nondegenerate plasma, the velocities v_F must be matched to the thermal velocity of the electrons $v_{Te}=(T/m_e)^{1/2}$, while the Thomas-Fermi radius k_{TF}^{-1} must be matched to the Debye radius $r_{De}=v_{Te}/\omega_{pe}$. Not only the Coulomb logarithms but the very expressions for the polarization losses in a degenerate and nondegenerate plasma are quite similar. At $V\gg v_F$ and $V\gg v_{Te}$ these expressions coincide completely,[4,5] and at $V\ll v_F$ and $V\ll v_{Te}$, after making the substitution $v_F\rightleftarrows v_{Te}$, they differ only by a numerical coefficient. In a nondegenerate plasma,

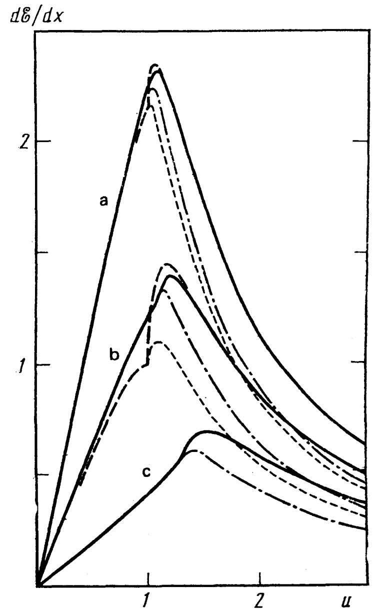

FIG. 2. Energy loss in dimensionless units $d\mathscr{E}/dx=3\pi\alpha a^3m_e(2Z\hbar)^{-2}dE/dx$ ($a\equiv\hbar^2/me^2$) as a function of u at values $\alpha=0.01$ (a), 0.1 (b) and 1 (c). Solid curves—numerical calculations using the quantum dielectric constant (2), (3); dash-dot curves—numerical calculations with allowance for the plasmon excitation. The energy losses calculated from the analytic formulas (8)–(11) are also shown for $\alpha=0.01$ and 0.1, with (dashed curves) and without (short dashes) allowance for the plasmon excitation.

in contrast to a degenerate one, the plasmon excitation process has no threshold and sets in smoothly with increasing V at $V\gtrsim v_{Te}$. In a degenerate plasma, however, where the excitation is turned on at a fully defined ion velocity $V=\Omega_0v_F$ (see above), it does not lead to the appearance of any noticeable singularity in the energy-loss curve (if the exact dielectric constant is used in the calculations, see Fig. 2).

Earlier calculations of the polarization losses at arbitrary u and $\alpha\lesssim 0.3$ were performed by Lindhard and Winther.[6] They did not obtain, however, the simple analytic formulas (8)–(11), although Eq. (10) can in fact be derived by using expressions (19′) and (20) of their paper. Our results agree with the numerical results of Ref. 6, but cover a larger interval of α values of interest for applications (see below). Numerical calculation of the energy loss at $\alpha\ll 1$ were also made by Dar *et al.*[10] The values of dE/dx indicated in Fig. 2 of their paper, however, are too high by approximately an order of magnitude and seem to be in error.

The results are of interest for astrophysical conditions, particularly for the study of the activity of exploding x-ray sources. According to contemporary notions, x-ray bursts are due to nuclear combustion of matter in the surface layers of neutron stars, which are contained in tight binary systems (see, e.g., Ref. 11). Depending on the rate of accretion and on the parameters of the neutron stars, the combustion can take place at different depths, and in particular, at a depth where the electron gas is strongly degenerate. Consider, e.g., two successive reactions of the proton cycle ^{2}D(p, γ)^{3}He, ^{3}He(^{3}He, $2p$) ^{4}He, which is effective at not very high temperatures T. In the first reaction, high-energy ^{3}He ions are produced. The rate of the reaction is[12]

$$I = \frac{n_H n_D C}{T_9^{2/3}} \exp\left[-3\left(\frac{E(\mathrm{H, D})}{4T}\right)^{1/3}\right],$$

$$E(1,2) \approx \frac{Z_1^2 Z_2^2 A_1 A_2}{A_1 + A_2} \ \mathrm{MeV}, \qquad (15)$$

where n_i and A_i are the density and atomic number of the ions, $E(1,2)$ is the Gamow energy, $C \approx 0.4 \cdot 10^{-20}$ cm$^3 \cdot$sec^{-1}, and $T_9 \equiv T/10^9$ K. As a result of creation of ^{3}He ions, their distribution function at sufficiently high energies $E > E_*$ will differ from Maxwellian (see, e.g., Ref. 13). The limiting energy E_* for a degenerate electron plasma can be easily estimated on the basis of the results for a nondegenerate plasma[13] and the aforementioned similarity of the expressions for the energy losses in degenerate and nondegenerate cases. An estimate yields

$$E_* \approx T \ln (n_{^3\mathrm{He}} \nu / I), \quad \nu \sim n_i m_e^2 Z_i^2 e^4 (m_i n_e \hbar^3)^{-1} \sim 10^{13} X_{^3\mathrm{He}} \ \mathrm{sec}^{-1},$$

where ν is the frequency of the collisions of the ^{3}He ions with the electrons with energy loss at $u < 1$; $X_{^3\mathrm{He}}$ is the mass content of these ions. If the distribution of the ^{3}He ions were Maxwellian, the main contribution to the succeeding reaction would be made by ions with energy $E_0 \approx T\,[E\,(^3\mathrm{He}, {}^3\mathrm{He})/4T]^{1/3}$. The energy ratio is

$$\frac{E_*}{E_0} = 3\left[\frac{E(\mathrm{H, D})}{E(^3\mathrm{He}, {}^3\mathrm{He})}\right]^{1/3} + \left[\frac{4T}{E(^3\mathrm{He}, {}^3\mathrm{He})}\right]^{1/3} \ln \frac{\nu n_{^3\mathrm{He}} T_9^{2/3}}{C n_H n_D}. \qquad (16)$$

At a temperature $T \approx 10^7$ K we have

$$E_*/E_0 = 0.91 + 0.05 \ln [10^8 X^2_{^3\mathrm{He}} / \rho X_D].$$

Therefore at a matter density $\rho \sim 10^5$ g/cm^3 ($n_e \sim 10^{29}$ cm^{-3}), where the electron gas is strongly degenerate and ideal ($\alpha \sim 10^{-2}$), the energy $E_* \approx E_0$ in a wide range of values of $X_{^3\mathrm{He}}$ and X_D, i.e., the customarily employed expression for the rate of the reaction ^{3}He (^{3}He, $2p$)^{4}He, obtained under the assumption of a Maxwellian distribution of the ions, no longer holds. This situation obtains apparently also for a number of other secondary reactions. A consistent calculation can be carried out in such cases on the basis of the results obtained above and should yield substantially higher reaction rates. This in turn can influence the conditions for the ignition of the matter in x ray bursts in the surface layers of neutron stars.

The results are of interest also for a plasma compressed to densities $n_e \sim 10^{26}$ cm^{-3}, with a temperature not exceeding 0.3 keV (Ref. 14). Under these conditions the electron gas becomes degenerate, the parameter $\alpha \sim 0.1$, and the polarization losses of the ions are well described by Eqs. (8)–(11).

§4. POLARIZATION LOSSES IN METALS

In the study of polarization losses in metals, the case of interest is $\alpha \sim 1$. For arbitrary u the energy losses in this case were calculated numerically. The results are listed in the table. The analysis becomes simpler in the limits of large and small u.

At $u \ll 1$ and at arbitrary α, the integral with respect to Ω in (5) can be evaluated, after which the calculation reduces to the determination of the single integral:

$$\Lambda = \int_0^1 \frac{d\xi}{\xi}\left[1 + \frac{\alpha}{2\pi\xi^2}\left(1 - \frac{1-\xi^2}{2\xi}\ln\frac{1+\xi}{1-\xi}\right)\right]^{-2}. \qquad (17)$$

Thus, in this case Λ is independent of u. As is well known (see Refs. 3–8), the energy loss is then $dE/dx \propto u$. At $\alpha \sim 1$ the integral (17) was calculated numerically; the obtained values of Λ practically coincide with the values in the table at

TABLE I. Dependence of the Coulomb logarithm Λ on the dimensionless velocity $u = V/v_F$ for different values of the parameter α.

u	α														
	0.01	0.05	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0
0.1	2.38	1.60	1.28	0.972	0.693	0.547	0.453	0.386	0.335	0.296	0.264	0.237	0.215	0.173	0.142
0.3	2.37	1.59	1.27	0.969	0.694	0.550	0.458	0.392	0.342	0.303	0.272	0.246	0.224	0.182	0.151
0.5	2.35	1.58	1.26	0.961	0.692	0.552	0.462	0.398	0.350	0.312	0.281	0.255	0.234	0.192	0.162
0.7	2.31	1.55	1.24	0.947	0.687	0.553	0.466	0.405	0.358	0.321	0.291	0.266	0.244	0.203	0.173
0.9	2.25	1.50	1.20	0.931	0.681	0.553	0.469	0.410	0.365	0.329	0.300	0.276	0.255	0.213	0.184
1.0	2.20	1.47	1.19	0.919	0.678	0.553	0.470	0.414	0.368	0.333	0.306	0.280	0.260	0.219	0.189
1.2	3.19	2.36	2.00	1.60	1.16	0.954	0.820	0.727	0.651	0.592	0.545	0.503	0.468	0.398	0.347
1.4	3.71	2.89	2.55	2.19	1.81	1.59	1.42	1.28	1.09	0.990	0.907	0.837	0.781	0.669	0.586
1.6	4.08	3.27	2.93	2.58	2.21	2.01	1.86	1.74	1.64	1.55	1.46	1.40	1.35	1.20	1.07
1.8	4.39	3.57	3.23	2.88	2.52	2.32	2.17	2.06	1.97	1.89	1.82	1.75	1.70	1.57	1.47
2.0	4.64	3.82	3.49	3.14	2.78	2.58	2.44	2.33	2.23	2.15	2.08	2.02	1.98	1.85	1.75
2.5	5.15	4.33	3.50	3.65	3.29	3.10	2.95	2.84	2.75	2.67	2.61	2.54	2.50	2.38	2.28
3.0	5.55	4.73	4.40	4.05	3.69	3.50	3.35	3.24	3.15	3.07	3.01	2.94	2.90	2.78	2.69
5.0	6.62	5.80	5.46	5.11	4.76	4.56	4.41	4.31	4.22	4.14	4.07	4.01	3.97	3.85	3.76
10.0	8.03	7.19	6.88	6.52	6.16	5.96	5.82	5.71	5.62	5.55	5.48	5.41	5.38	5.25	5.16

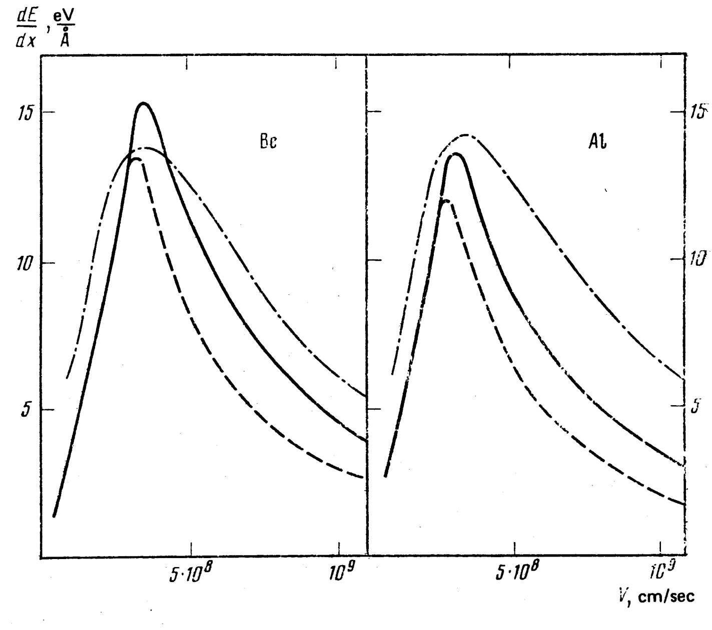

FIG. 3. Proton energy losses in beryllium and aluminum. The dash-dot curves are plots of the interpolation formulas given in Ref. 15 and agree well with the experimental results. Solid lines—theoretical calculation, dashed—calculation without allowance for plasmon excitation. The electron effective mass is assumed equal to the free-electron mass.

$u = 0.1$. We note that at $\alpha \ll 1$ the integral (17) can be easily calculated and yields (12).

At $u \gg 1$ and at arbitrary α we obtain from (5) the well known[4,5] expression (14), which does not depend on the degree of degeneracy of the electron gas.

We note that at $\alpha \sim 1$ the polarization energy loss depends on u qualitatively in the same manner as at $\alpha \ll 1$ (§3). The most significant difference between these cases is that at $\alpha \sim 1$ the static screening momentum k_{TF} becomes of the order of k_F. We note also that with increasing α the maximum of the polarization loss shifts towards larger u (Fig. 2).

Good agreement between the calculated polarization losses and the experimentally measured proton energy losses[15] is obtained also for beryllium ($\alpha \approx 1.0$, $v_F \approx 2.2 \cdot 10^8$ cm/sec, Fig. 3). For aluminum ($\alpha \approx 1.1$, $v_F \approx 2.0 \cdot 10^8$ cm/sec, Fig. 3) agreement with the data on the stopping of protons at energies in the region of the maximum of energy losses and lower is also obtained. At $u > 2$ the polarization losses in aluminum become lower than the total losses, apparently as a result of the appearance of losses to ionization.

Our calculations for metals agree with the earlier less complete calculations of Lindhard and Winther,[6] as well as with those of Ferrel and Ritchie.[7] They agree also with the recent detailed calculations,[8] but only at values $u \leqslant 1$. At $u > 1$ the polarization loss in Ref. 8 is underestimated, inasmuch as no account is taken there of the loss to excitation of the plasmons (see §2). As seen from Fig. 3, at $u > 1$ the loss to excitation of plasmons makes a noticeable contribution, and allowance for it improves considerably the agreement with experiment at proton energies in the region of the maximum of the energy losses and higher.

The calculations performed for copper, gold, and lead are in worse agreement with the experimental data on proton stopping[15] [apparently because of failure to take into account the ionization losses, and also because the expression (2) and (3) for the dielectric constant is less applicable to these metals]. In the case of lead, there is good agreement with the experimental data on proton stopping, but only at $u < 1.5$. When the calculation results are applied to stopping of heavier particles it is necessary to take into account the effective charge of the heavy particle (see, e.g., Ref. 16.)

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