Determination of the baryon mass and baryon resonances from the quantum-chromodynamics sum rule. Strange baryons

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The mass differences in the baryon octet with $J^P = \frac{1}{2}^+$, decuplet with $J^P = \frac{3}{2}^+$, and octet with $J^P = \frac{3}{2}^-$ are calculated on the basis of the quantum-chromodynamics (QCD) sum rules. The mass differences are expressed in terms of two QCD parameters: the mass of the current strange quark and the value of the quark condensate $\langle 0|\bar{s}s|0\rangle$. When these parameters are suitably chosen, all the mass differences agree well with experiment.

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1. INTRODUCTION

We continue in this paper an earlier study[1] of the quantum chromodynamics (QCD) sum rules for nonstrange baryons. We investigate here the mass difference in the baryon multiplets whose nonstrange terms were considered in Ref. 1.

Mass splitting in baryon multiplets has attracted interest for a long time. Gell-Mann and Okubo obtained, within the framework of broken SU(3) symmetry, equations that agree well with experiment, but contain phenomenological parameters that must be determined from experiment for each multiplet. It is therefore of interest to investigate the same question from the viewpoint of the QCD sum rules proposed by Shifman, Vainshtein, and Zakharov,[2] in order to determine the masses of the strange baryons in terms of the mass of the strange quark and the properties of the QCD vacuum. Such calculations were performed in Ref. 3 for the nucleon octet and decuplet of the Δ isobar, and it was shown that the masses of the strange baryons are determined not only by the mass of the strange quark, but also by the difference between the vacuum mean values: $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{u}u|0\rangle \neq 0$. Although the analysis in Ref. 3 has demonstrated the possibility in principle of calculating the mass differences in baryon multiplets on the basis of QCD sum rules, it was nevertheless not fully convincing, and its result were not accurate enough, since no account of the continuum contribution was taken in these calculations. This gap is filled in the present paper. In addition, the higher power-law corrections and the perturbation-theory corrections are taken into account in the principal logarithmic approximation. All this has made it possible to improve substantially the accuracy of the calculations and to obtain reliable results of the baryon-mass splittings. Calculation of the baryon-mass splitting is of interest also from another viewpoint. It makes it possible to determine from the experimentally known mass differences two parameters of the theory, namely the mass of the strange quark and the difference $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{u}u|0\rangle$ of the mean values, and to verify the self-consistency of the entire approach.

2. NUCLEON OCTET, $J^P = \frac{1}{2}^+$

In this section we describe, with the nucleon octet as the example, a method of calculating the mass differences between members of one and the same multiplet. The nucleon mass can be calculated by using the following baryon current:

$$\eta = \varepsilon^{abc}[(u^a C d^b) u^c - (u^a C \gamma_5 d^b)\gamma_5 u^c], \quad (1)$$

which differs from the current $\eta_{1\mu}$ in Ref. 1 by a factor γ_μ. Here a, b, and c are the color indices of the quarks, ϵ^{abc} is an antisymmetric tensor, μ is the Lorentz index, $C = -C^T$ is the charge-conjugation matrix, and u and d are the u- and d-quark operators.

To determine the masses Σ and Ξ we need the currents with their quantum numbers. These currents can be easily obtained by SU(3)-transformation of expression (1):

$$\begin{aligned} \eta^{\Sigma^+} &= \varepsilon^{abc}[(u^a C s^b) u^c - (u^a C \gamma_5 s^b)\gamma_5 u^c], \\ \eta^{\Xi} &= \varepsilon^{abc}[(s^a C u^b) s^c - (s^a C \gamma_5 u^b)\gamma_5 s^c], \end{aligned} \quad (2)$$

where s is the strange-quark operator.

Just as in Refs. 1 and 3, the sum rules are obtained by considering the polarization operator $\Pi(q)$ of the quark currents (1) or (2), by representing the structure functions of the polarization operator in the form of dispersion relations in q^2, and by applying Borel transformations to these dispersion relations. Referring the reader to our preceding paper[1] for details, we deal here only with the new aspects that arise when account is taken of the mass of the strange quark and of the nonzero difference between the mean values $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{q}q|0\rangle, q = u,d$. Just as in Ref. 1, we assume the u and d quarks to have zero mass and that $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$ by virtue of isotopic invariance. We take into account in the calculations the same power-law corrections as in Ref. 1, i.e, the same set of Feynman diagrams. The contribution of the gluon corrections is calculated in a fixed-point gauge.[4]

We introduce the parameters that characterize the symmetry breaking in the vacuum mean values

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$$f=\frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle}-1,\qquad f_g=\frac{\langle 0|\bar{s}\sigma_{\mu\nu}\lambda^n sG_{\mu\nu}{}^n|0\rangle}{\langle 0|\bar{u}\sigma_{\mu\nu}\lambda^n uG_{\mu\nu}{}^n|0\rangle}-1. \tag{3}$$

The polarization operation, and hence the baryon masses, will be calculated in an approximation linear in m_s, f, and f_g. In the phenomenological approach this corresponds to breaking of the SU(3) symmetry on account of the $(\bar{3},3)$ component of the octet in the Hamiltonian, $H=H_0+H_3^3$. The hypothesis that the baryon mass splittings in multiplets are described in the linear approximation by the term H_3^3 is the basis of the Gell-Mann–Okubo mass formulas. The success of these formulas is by the same token a justification of our assumption.

In the approximation linear in m_s, the strange-quark propagator is given in the x-representation by

$$-iS_F(x)=i\hat{x}/2\pi^2x^4-m_s/4\pi^2x^2, \tag{4}$$

and the terms proportional to m_s in the vacuum mean values of the product of the quark fields are of the form

$$\langle 0|:s_\alpha{}^a(x),\bar{s}_\beta{}^b(0):|0\rangle=i\frac{m_s}{48}\delta^{ab}\hat{x}_{\alpha\beta}\Big[\langle 0|\bar{s}s|0\rangle$$
$$+\frac{x^2}{3.2^3}\langle 0|g_s\bar{s}G_{\mu\nu}{}^n(\lambda^n/2)\sigma_{\mu\nu}s|0\rangle+\ldots\Big]+O(m_s{}^2). \tag{5}$$

The terms written out explicitly in (5) are only those proportional to m_s, while those independent of m_s are given by Eq. (7.1).

The imaginary part of the polarization operator, expressed in terms of the physical states, is approximated by the sum of the contribution of the lowest state (in our case N, Λ, Σ, or Ξ) and of the continuum, i.e.,

$$\operatorname{Im}\Pi(q)=\pi\beta_R{}^2(\hat{q}-m_R)\delta(q^2-m_R{}^2)$$
$$+\pi\theta(q^2-W_1{}^2)\hat{q}\operatorname{Im}\Pi_1(q^2)+\pi\theta(q^2-W_2{}^2)\operatorname{Im}\Pi_2(q^2), \tag{6}$$

where β_R is the residue [of the amplitudes of creation from vacuum by currents (1) and (2)]:

$$\langle 0|\eta|R,\ J^P={}^1/_2{}^+\rangle=\beta_R\gamma_5 v(q), \tag{7}$$

W_1 and W_2 are the thresholds of the continuum for structures with odd and even number of γ matrices ($\hat{q}$ and 1), $\operatorname{Im}\pi_1(q^2)$ and $\operatorname{Im}\pi_2(q^2)$ are the imaginary parts of the polarization operators for these structures.

The sum rules considered in Ref. 1 for the nucleon can be written in the form

$$\hat{q}:\ I_1(M,W_1)=\tilde{\beta}_N{}^2\exp(-m_N{}^2/M^2),$$
$$1:\ I_2(M,W_2)=\tilde{\beta}_N{}^2m_N\exp(-m_N{}^2/M^2), \tag{8}$$

where $\tilde{\beta}_N=(2\pi)^2\beta_N$, m_N is the nucleon mass, and the contribution of the continuum is transferred to the left-hand side. On the left of each sum rule is indicated the structure to which it corresponds. In the case of hyperons ($Y=\Lambda,\Sigma,\Xi$) allowance for the corrections that break the SU(3) symmetry leads in the approximation linear in m_s, f, and f_g to replacement of the sum rules (8) by

$$\hat{q}:\ I_1(M,W_1)-\delta_{1Y}(M,m_s,f,f_g,\delta W_{1Y})=\tilde{\beta}_Y{}^2\exp(-m_Y{}^2/M^2),$$
$$1:\ I_2(M,W_2)+\delta_{2Y}(M,m_s,f,f_g,\delta W_{2Y}) \tag{9}$$
$$=\tilde{\beta}_Y{}^2m_Y\exp(-m_Y{}^2/M^2),$$

where I_1 and I_2 are the same functions as in (8); β_Y is the residue of the hyperon in the current η^Y; δ_1 and δ_2 contain terms proportional to m_s, f, and f_g, as well as increments δW_Y connected with variation of the continuums:

$$W_{1Y}=W_1+\delta W_{1Y},\quad W_{2Y}=W_2+\delta W_{2Y}$$

(in the approximation linear in δW_Y). The fact that the thresholds of the continua are different for different terms of the SU(3) multiplets is physically obvious, and allowance for this effect is absolutely necessary. This introduces into the calculations additional parameters that must be determined from the very same sum rules. Naturally, this lowers the accuracy. Some control can be provided by the "reasonableness," discussed below, of the obtained δW_Y. To decrease the number of parameters, we assume hereafter that $\delta W_{1Y}=\delta W_{2Y}=\delta W_Y$.

Using (8) and (9) we easily obtain the following equations that are valid in the approximation linear in $m_s, f, f_g, \delta W_Y$ and $\delta\tilde{\beta}_Y{}^2=\tilde{\beta}_Y{}^2-\tilde{\beta}_N{}^2$.

$$m_Y-m_N=[\delta_{2Y}(M)+\delta_{1Y}(M)m_N]/\tilde{\beta}_N{}^2\exp(-m_N{}^2/M^2). \tag{10}$$

It must be noted that Eq. (10) is valid only in the M region where the sum rules (8) and (9) are satisfied. This region Ω is known[1)] from Ref. 1: $\Omega=(M\colon 0.9\,\text{GeV}\leqslant M\leqslant 1.2\,\text{GeV})$. Also known from Ref. 1 are the mass and residue of the nucleon and the thresholds of the continua.

The plan of action is the following. Calculation of the corrections to the polarization operator in QCD yields $\delta_{1Y}(M)$ and $\delta_{2Y}(M)$ in the form of linear functions of m_s, f, f_g, and δW_Y. Substitution of $\delta_{1Y}(M)$ and $\delta_{2Y}(M)$ in (10) leads to analogous expressions for m_Y-m_N:

$$m_Y-m_N=a_Y(M)m_s+b_Y(M)f+c_Y(M)f_g+d_Y(M)\delta W_Y$$
$$=K_{Y-N}(M) \tag{11}$$

with known coefficients $a_Y(M)$, $b_Y(M)$, $c_Y(M)$, $d_Y(M)$. The left-hand side of (11) does not depend on the Borel parameter M, while the right, generally speaking, does. It is therefore reasonable to determine the unknown parameters δW_Y from the requirement that the dependence of the right-hand side of (11) on M be if possible weaker in the region $\Omega\,(M_1\leqslant M\leqslant M_2)$. We can assume formally that δW_Y are linear functions of m_s, f, and f_g. To ensure a weak $K_{Y-N}(M)$ dependence in the interval $M_1\leqslant M\leqslant M_2$ it suffices then to stipulate

$$K_{Y-N}(M_1)=K_{Y-N}(M_2) \tag{12}$$

and determine δW_Y from this condition. Substituting δW_Y in (11), we obtain relations that express δm_{Y-N} in terms of the three QCD parameters m_s, f, and f_g. The problem is thereby reduced to finding those values of m_s, f, and f_g which describe in best fashion the entire aggregate of the experi-

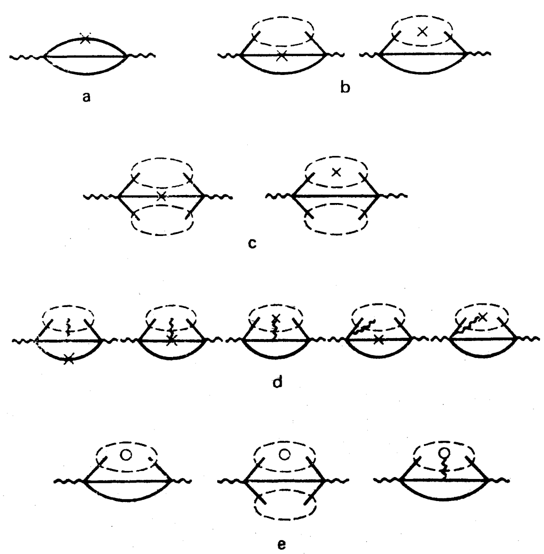

FIG. 1. Feynman diagrams for the calculation of the corrections to the polarization operator. Free ends correspond to emission of a soft quark or gluon into the vacuum. Allowance for the mass of the quark and for the factor f (or f_g) is designated by × and O, respectively.

mental data on the mass splittings in the nucleon octet $J^P = \frac{1}{2}^+ (m_\Sigma - m_N,\quad m_\Xi - m_N)$, decuplet $J^P = \frac{3}{2}^+ (m_{\Sigma^*} - m_\Delta)$, and octet $J^P = \frac{3}{2}^- (m_{\Sigma^{**}} - m_{N^*},\ m_{\Xi^{**}} - m_{N^*})$. It will be clearly seen from the answers that the results depend very little on f_g, so that actually all the mass splittings are determined by the two quantities m_s and f.

The polarization-operator corrections proportional to m_s are calculated by substituting in the diagrams of Fig. 1 the propagators and the vacuum mean values in accord with Eqs. (4) and (5). The terms proportional to f and f_g can be easily taken into account by noting that in the polarization-operator terms with even number of γ matrices the mean values over the vacuum should be replaced by $\langle 0|\bar{s}s|0\rangle$ in the case Σ and by $\langle 0|\bar{u}u|0\rangle$ in the case Ξ, while in the terms with odd number of γ matrices they should be replaced by $\langle 0|\bar{u}u|0\rangle^2$ in the case of Σ and by $\langle 0|\bar{s}s|0\rangle^2$ in the case of Ξ.

The result of the calculation is:

$$\begin{aligned}
\delta\Pi^a &= \frac{m_s}{(2\pi)^4} q^4 \ln\left(-\frac{q^2}{\Lambda^2}\right)\cdot\frac{1}{8}(\cdot 0),\\
\delta\Pi^b &= -\frac{m_s}{(2\pi)^2}\hat{q}\langle 0|\bar{u}u|0\rangle \ln\left(-\frac{q^2}{\Lambda^2}\right)\cdot\frac{1}{4}(\cdot 0),\\
\delta\Pi^c &= \frac{m_s}{q^2}\langle 0|\bar{u}u|0\rangle^2\cdot\frac{1}{3}\left(\cdot\frac{2}{3}\right),\\
\delta\Pi^d &= -\frac{m_s}{q^2} m_0^2\langle 0|\bar{u}u|0\rangle\hat{q}\cdot\frac{1}{24}\left(\cdot\frac{1}{12}\right),\\
\delta\Pi^e &= -f\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} q^2\ln\left(-\frac{q^2}{\Lambda^2}\right)\cdot\frac{1}{4}(\cdot 0),\\
\delta\Pi^f &= -f\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\hat{q}\cdot 0\left(\cdot\frac{1}{3}\right),\qquad \delta\Pi^g = 0,
\end{aligned}\tag{13}$$

where the letters in the polarization operators designate the diagrams that must be used in the calculations; factors without brackets correspond to the case Σ, and those in brackets to Ξ;

$$m_0^2\langle 0|\bar{u}u|0\rangle = g_s\langle 0|\bar{u}G_{\mu\nu}^n(\lambda^n/2)\sigma_{\mu\nu}u|0\rangle.$$

The corrections $\delta_{1\Sigma}(M)$ and $\delta_{2\Sigma}(M)$ are found to be

$$\begin{aligned}
\delta_{1\Sigma}(M) &= m_s[{}^1/_4 aM^2L^{4/9}E_2(W_1, M) + {}^1/_{24} m_0^2 a]\\
&\quad - {}^1/_8\delta W_\Sigma L^{4/9} W_1 \exp(-W_1^2/M^2)[W_1^4 + {}^1/_2 b],\\
\delta_{2\Sigma}(M) &= m_s[{}^1/_4 M^6 E_6(W_2, M)\\
&\quad + {}^1/_3 a^2 L^{8/9}] + {}^1/_4 faM^4L^{8/9}E_4(W_2, M)\\
&\quad + {}^1/_2\delta W_\Sigma a W_2^3 L^{8/9}\exp(-W_2^2/M^2),
\end{aligned}\tag{14}$$

where $L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$, Λ is the strong-interaction constant contained in the definition of the QCD effective charge, $\alpha_s = {}^4/_9\pi\ln^{-1}(-q^2/\Lambda^2)$ (in the case of three flavors); μ is the normalization point of the operator expansion,

$$\begin{aligned}
E_2(W, M) &= 1-\exp(-W^2/M^2),\quad E_4(W, M) = 1-\exp(-W^2/M^2)(1+W^2/M^2),\\
E_6(W, M) &= 1-\exp(-W^2/M^2)(1+W^2/M^2+W^4/2M^4),
\end{aligned}\tag{15}$$

$$a = -(2\pi)^2\langle 0|\bar{u}u|0\rangle = 0.546\ \text{GeV}^3,$$

$$b = (2\pi)^2\langle 0|(\alpha_s/\pi)G^2|0\rangle = 0.5\ \text{GeV}^4,$$

$$m_0^2 = 0.8\ \text{GeV}^2,\ \Lambda = 150\ \text{MeV},\ \mu = 0.5\ \text{GeV}.$$

Equations (14) are written in the principal logarithmic approximation with allowance for the anomalous dimensionalities of the currents (1) and (2) ($d = -2/9$), of the operators $\bar{\psi}\psi$ ($d = -\frac{4}{9}$), $(\alpha_s/\pi)G_{\mu\nu}^nG_{\mu\nu}^n$ ($d = 0$), and of the quark mass m_s ($d = 4/9$).

Substituting (14) in (10) and using the values obtained in Ref. 1 for the nucleon mass $m_N = 1$ GeV, its residue in the quark current (1) $\tilde{\beta}_N^2 = 0.43$ GeV^6, and the continuum thresholds $W_1 = W_2 = 1.5$ GeV, we arrive at the following numerical expressions for the quantity $K_Y(M)$ defined in (11), in the case of the Σ hyperon:

$$\begin{aligned}
K_\Sigma(M_1=0.9) &= 2.82m_s + 0.78f + 0.06\delta W_\Sigma,\\
K_\Sigma(M_2=1.2) &= 2.48m_s + 0.99f + 0.23\delta W_\Sigma.
\end{aligned}\tag{15a}$$

All the quantities in (15) and in the equations that follows are in GeV. Eliminating δW_Σ from (15) in accord with (12), we obtain the following expression for the Σ-hyperon and nucleon mass difference:

$$m_\Sigma - m_N = 2.94m_s + 0.71f.\tag{16}$$

Proceeding in similar fashion, we have in the case of the Ξ hyperon

$$\begin{aligned}
\delta_{1\Xi}(M) &= {}^1/_{12} m_s m_0^2 a - {}^1/_3 fa^2L^{4/9}\\
&\quad - {}^1/_8\delta W_\Xi W_1 L^{4/9}\exp(-W_1^2/M^2)[W_1^4 + b/2],\\
\delta_{2\Xi}(M) &= {}^1/_2 m_s L^{8/9}a^2 + {}^1/_2 aW_2^3L^{8/9}\exp(-W_2^2/M^2)\delta W_\Xi.
\end{aligned}\tag{17}$$

From expressions (10) and (17) follow the numerical equations

$$K_\Xi(M_1=0.9)=1.98m_s-1.34f+0.06\delta W_\Xi,$$
$$K_\Xi(M_2=1.2)=1.29m_s-0.94f+0.23\delta W_\Xi. \tag{18}$$

Eliminating δW_Ξ from (18) we arrive at

$$m_\Xi-m_N=2.22m_s-1.48f. \tag{19}$$

It can be seen from a comparison of (16) and (19) that for the Ξ hyperon to be heavier than Σ we must have $f<0$ and $|f|\sim m_s$ (in GeV units). This conclusion will be confirmed below when other sum rules are considered.

The formula of Gell-Mann and Okubo for a baryon octet enables us to obtain from (16) and (19) the mass difference between the Λ hyperon and the nucleon

$$m_\Lambda-m_N={}^1/_3[2(m_\Xi-m_N)-(m_\Sigma-m_N)]=0.5m_s-1.22f. \tag{20}$$

Comparing (16), (19), and (20) with the experimental mass splittings in the nucleon octet: $(\delta m_{\Lambda-N})_{\rm exp}=0.165$ GeV, $(\delta m_{\Sigma-N})_{\rm exp}=0.250$ GeV, $(\delta m_{\Xi-N})_{\rm exp}=0.380$ GeV, we can determine the values of the strange-quark mass (at the normalization point $\mu=0.5$ GeV) and of the parameter f:

$$m_s=108\ \text{MeV},\quad f=-0.096, \tag{21}$$

as well as the changes of the continuum thresholds

$$\delta W_\Lambda=230\ \text{MeV},\ \delta W_\Sigma=340\ \text{MeV},\ \delta W_\Xi=520\ \text{MeV}. \tag{22}$$

The values δW_Y seem reasonable in the sense that they agree to within 100–150 MeV with the mass differences $(m_{Y^*}-m_Y)-(m_{N^*}-m_N)$, where Y^* and N^* are the lower resonant states with the same quantum numbers as Y and the nucleon (except for the parity, see Ref. 5). Even if (22) contain an appreciable error, this error has little effect on the values of m_s and f, since δW_Y enters in (16) and (18) with small coefficients. The strange-quark mass was found to be somewhat smaller than the usually assumed $m_s=150$ MeV (Ref. 6). It makes sense therefore to ascertain whether this standard value of m_s can be obtained by taking the uncertainties of δW_Y into account or by foregoing exact satisfaction of equality (12). Analysis shows that by using only these two circumstances it is possible to raise m_s to ≈ 130 MeV (in which case $f=-0.13$), but not to higher values.

3. DECUPLET, $J^P=3/2^+$

In this section we consider, following the method described in Sec. 2, the mass splittings in the decuplet $J^P=3/2^+$. Since we have the relation

$$m_{\Sigma^*}-m_\Delta=m_{\Xi^*}-m_{\Sigma^*}=m_\Omega-m_{\Xi^*}, \tag{23}$$

only the difference $m_{\Sigma^*}-m_\Delta$ need be found to describe the baryon decuplet.

In Refs. 1 and 3 the mass of the Δ isobar was determined by using the current

$$\eta_\mu^\Delta=\varepsilon^{abc}(u^aC\gamma_\mu u^b)u^c. \tag{24}$$

From (24) it is easy to find the following current with quantum numbers of Σ^*:

$$\eta_\mu^{\Sigma^*}=3^{-1/2}\varepsilon^{abc}[2(u^aC\gamma_\mu s^b)u^c+(u^aC\gamma_\mu u^b)s^c]. \tag{25}$$

We shall use the current (25) to calculate $m_{\Sigma^*}-m_\Delta$ by the method described above. The corrections to the polarization operator take in our approximation the form

$$\Pi_{\mu\nu}^a=\frac{m_s}{4(2\pi)^4}q^4\ln\left(-\frac{q^2}{\Lambda^2}\right)\times\left\{g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{2}{3}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^b=-\frac{m_s}{(2\pi)^2}\langle 0|\bar{u}u|0\rangle\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu}\hat{q}-\frac{3}{8}\gamma_\mu\gamma_\nu\hat{q}+\frac{3}{8}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)-\frac{1}{8}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)\right]-\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^c=\frac{2}{3}m_s\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\times\left\{g_{\mu\nu}-\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{2}{3}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^d=\frac{1}{2}m_s\frac{m_0^2\langle 0|\bar{u}u|0\rangle}{(2\pi)^2q^2}\{g_{\mu\nu}\hat{q}-\ldots\},$$

$$\Pi_{\mu\nu}^e=-\frac{4}{9}f\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}q^2\ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu}-\frac{5}{16}\gamma_\mu\gamma_\nu+\frac{1}{4}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{1}{2}\frac{q_\mu q_\nu}{q^2}\right\},$$

$$\Pi_{\mu\nu}^f=\frac{8}{9}f\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\times\left\{g_{\mu\nu}\hat{q}-\frac{3}{8}\gamma_\mu\gamma_\nu\hat{q}+\frac{3}{8}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)-\frac{1}{8}(q_\nu\gamma_\mu+q_\mu\gamma_\nu)\right\},$$

$$\Pi_{\mu\nu}^g=\frac{2}{9}f_g\frac{m_0^2\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu}-\frac{1}{4}\gamma_\mu\gamma_\nu\right]+\frac{1}{2}\frac{q_\nu\gamma_\mu-q_\mu\gamma_\nu}{q^2}\hat{q}-\frac{q_\mu q_\nu}{q^2}\right\}. \tag{26}$$

It was shown in Ref. 3 that the sum rules for the structures $g_{\mu\nu}\hat{q}$ and $g_{\mu\nu}$ receive contributions only from resonances with spin 3/2. By applying the procedure developed above to these structures we obtain for the quantities $\delta_{1\Sigma^*}(M)$ and $\delta_{2\Sigma^*}(M)$, defined by formulas similar to (9),

$$\delta_{1\Sigma^*}(M)=m_s[{}^1/_2am_0^2L^{-16/27}-aM^2L^{-4/27}E_2(W_1,M)]-{}^8/_9fa^2L^{20/27}-{}^1/_{10}\delta W_{\Sigma^*}W_1L^{-4/27}\exp(-W_1^2/M^2)[W_1^4-{}^{25}/_{18}b],$$

$$\delta_{2\Sigma^*}(M)=m_s[{}^1/_2M^6E_6(W_2,M)L^{-16/27}+{}^2/_3a^2L^{8/27}]+{}^4/_9faM^4L^{8/9}E_4(W_2,M)+\delta W_{\Sigma^*}aW_2\exp(-W_2^2/M^2)[{}^8/_3W_2^2L^{8/27}-{}^4/_3m_0^2L^{-4/27}]-{}^2/_9f_gam_0^2M^2L^{-16/27}E_2(W_2,M). \tag{27}$$

Expressions (27) take into account the anomalous dimensionality of the currents (24) and (25), equal to $+2/27$. Expressions (27) must be substituted in an equation similar to (10):

$$m_{\Sigma^*}-m_\Delta=[\delta_{2\Sigma^*}(M)+\delta_{1\Sigma^*}(M)m_\Delta]/\tilde{\lambda}_\Delta^2\exp(-m_\Delta^2/M^2), \tag{28}$$

where m_Δ and λ_Δ are the mass and the residues of the isobar Δ.

According to Ref. 1, $\Omega=(M$: 1.1 GeV$\leqslant M\leqslant$1.4 GeV,

$m_\Delta = 1.35$ GeV, $\lambda_\Delta{}^2 = 2.5$ GeV^6, $W_1 = 2.1$ GeV, $W_2 = 2.2$ GeV. As a result we arrive at the following numerical expressions:

$$K_\Sigma^*(1.1) = 0.25m_s - 0.24f - 0.15f_g + 0.78\delta W_\Sigma^*,$$
$$K_\Sigma^*(1.4) = 0.42m_s + 0.23f - 0.12f_g + 1.07\delta W_\Sigma^*. \quad (29)$$

Eliminating δW_{Σ^*} from (28), we have

$$m_{\Sigma^*} - m_\Delta = -0.21m_s - 1.5f - 0.23f_g. \quad (30)$$

For the value of m_s and f determined above (see (21)) the mass splitting in the decuplet is found to be

$$m_\Sigma^* - m_\Delta = 0.124 - 0.23f_g. \quad (31)$$

Compared with the experimental value, $m_{\Sigma^*} - m_\Delta = 0.150$ GeV. It is natural to assume that f_g is of the same order as f, i.e., as the scale of the usual breaking of SU(3) symmetry, $|f_g| \sim 0.1$. The second term of (31) is then negligible and (31) agrees well with experiment.

The contribution of the continuum to the mass splitting in the decuplet is considerably larger than to the mass splitting in the nucleon octet [cf. (15) and (18) with (29)], so that the ensuring uncertainty is also larger. However, the continuum-threshold change obtained from (29) is quite reasonable:

$$\delta W_\Sigma^* = -0.59m_s - 1.62f - 0.1f_g = 0.090 - 0.1f_g \quad (32)$$

at the same values of m_s and f. (In the experiment 5 the first excited states with quantum numbers of Σ^* differ little in mass from those with quantum numbers of Δ.)

4. BARYON OCTET, $J^P = 3/2^-$

Using the current

$$\eta_{2\mu} = \varepsilon^{abc}[(u^a C\sigma_{\rho\lambda} d^b)\sigma_{\rho\lambda}\gamma_\mu u^c - (u^a C\sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu d^c] \quad (33)$$

we obtained in Ref. 1 the values of the mass, of the residue of the resonance N^* ($J^P = 3/2^-$, $E = 1/2$) and the continuum thresholds:

$$M_{N^*} = 1.6 \text{ GeV}, \ \bar\lambda_{N^*}{}^2 = 60 \text{ GeV}^6, \ W_1 = 2.6 \text{ GeV}, \ W_2 = 2.2 \text{ GeV}. \quad (34)$$

The baryon currents belonging to the same octet as for (32), with the quantum numbers of Σ^{**} and Ξ^{**}, are of the form

$$\eta_{2\mu}^{\Sigma^{**}} = \varepsilon^{abc}[(u^a C\sigma_{\rho\lambda} s^b)\sigma_{\rho\lambda}\gamma_\mu u^c - (u^a C\sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu s^c],$$
$$\eta_{2\mu}^{\Xi^{**}} = \varepsilon^{abc}[(s^a C\sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu s^c - (s^a C\sigma_{\rho\lambda} s^b)\sigma_{\rho\lambda}\gamma_\mu u^c]. \quad (35)$$

The corrections to the polarization operator for these currents, calculated in accord with the diagrams of Fig. 1, are:

$$\delta\Pi_{\mu\nu}^a = \frac{m_s}{(2\pi)^4} q^4 \ln\left(-\frac{q^2}{\Lambda^2}\right)\left\{g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat q - \frac{2}{3}\frac{q_\mu q_\nu}{q^2}\right\} \cdot 3(\cdot 12),$$

$$\delta\Pi_{\mu\nu}^b = m_s \frac{\langle 0|\bar u u|0\rangle}{(2\pi)^2} \times\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu}\hat q - \frac{3}{8}\gamma_\mu\gamma_\nu\hat q + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu)\right] - \frac{1}{5}\left(\frac{1}{2}\right)\frac{q_\mu q_\nu}{q^2}\hat q\right\} 40(\cdot 32),$$

$$\delta\Pi_{\mu\nu}^c = \frac{m_s}{q^2}\langle 0|\bar u u|0\rangle^2\{g_{\mu\nu} - \ldots\}\cdot 16(\cdot 32),$$

$$\delta\Pi_{\mu\nu}^d = -\frac{m_s}{(2\pi)^2}\frac{m_0{}^2\langle 0|\bar u u|0\rangle}{q^2}\{g_{\mu\nu}\hat q - \ldots\}\cdot\frac{73}{3}\left(\cdot\frac{59}{3}\right),$$

$$\delta\Pi_{\mu\nu}^e = -f\frac{\langle 0|\bar u u|0\rangle}{(2\pi)^2} q^2 \ln\left(-\frac{q^2}{\Lambda^2}\right)\{g_{\mu\nu} - \ldots\}\cdot\frac{16}{3}\left(\cdot\left(-\frac{64}{3}\right)\right),$$

$$\delta\Pi_{\mu\nu}^f = -f\frac{\langle 0|\bar u u|0\rangle^2}{q^2}\{g_{\mu\nu}\hat q - \ldots\}\cdot\frac{64}{3}\left(\cdot\left(-\frac{32}{3}\right)\right),$$

$$\delta\Pi_{\mu\nu}^g = f_g\frac{m_0{}^2\langle 0|\bar u u|0\rangle}{(2\pi)^2} \times\left\{\ln\left(-\frac{q^2}{\Lambda^2}\right)\left[g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\right] + \ldots\right\}\frac{8}{3}\left(\cdot\left(-\frac{32}{3}\right)\right), \quad (36)$$

where the factors without the brackets correspond to the case Σ^{**}, and those in the brackets to Ξ^{**}. To simplify the notation, only the terms proportional to $g_{\mu\nu}\hat q$ and $g_{\mu\nu}$ were retained in some of the expressions of (36).

Just as in the decuplet case, we consider in this section the sum rules for the structures $g_{\mu\nu}$ and $g_{\mu\nu}\hat q$.

From (36) follow expressions for $\delta_1(M)$ and $\delta_2(M)$ in the cases Σ^{**} and Ξ^{**}:

$$\delta_{1\Sigma^{**}}(M) = m_s(40aM^2L^{-4/27}E_2(W_1, M) - {}^{73}/_3 am_0{}^2L^{-16/27}) + {}^{64}/_3 a^2 fL^{20/27} - \delta W_{\Sigma^{**}}W_1L^{-4/27}\exp(-W_1{}^2/M^2)({}^{24}/_5 W_1{}^4 - {}^4/_3 b),$$

$$\delta_{2\Sigma^{**}}(M) = m_s(-6M^6L^{-16/27}E_6(W_2, M) - 16a^2L^{8/27}) + {}^8/_3 f_g am_0{}^2M^2E_2(W_2, M)L^{-4/27} - {}^{16}/_3 faM^4E_4(W_2, M)L^{8/27} + \delta W_{\Sigma^{**}}W_2\exp(-W_2{}^2/M^2)(32aW_2{}^2L^{8/27} - 16am_0{}^2L^{-4/27}),$$

$$\delta_{1\Xi^{**}}(M) = m_s(32aM^2E_2(W_1, M)L^{-4/27} - {}^{59}/_3 m_0{}^2aL^{-16/27}) - {}^{32}/_3 fa^2L^{20/27} - \delta W_{\Xi^{**}}W_1\exp(-W_1{}^2/M^2)L^{-4/27}({}^{24}/_5 W_1{}^4 - {}^4/_3 b),$$

$$\delta_{2\Xi^{**}}(M) = m_s(24M^6L^{-16/27}E_6(W_2, M) - 32a^2L^{8/27}) + {}^{64}/_3 faM^4L^{8/27}E_4(W_2\ M) - {}^{32}/_3 f_g am_0{}^2M^2L^{-4/27}E_2(W_2, M) + \delta W_{\Xi^{**}}W_2\exp(-W_2{}^2/M^2)(32aW_2{}^2L^{8/27} - 16am_0{}^2L^{-4/27}). \quad (37)$$

Account is taken in Eqs. (37) of the anomalous dimensionality $+2/27$ of the current $\eta_{2\mu}$.

Relations (11) at the points $M = 1.5$ and 1.8 GeV, which belong according to Ref. 1 to Ω, now take the form

$$K_\Sigma^{**}(1.5) = 1.73m_s + 0.28f + 0.11f_g - 0.85\delta W_\Sigma^{**},$$
$$K_\Sigma^{**}(1.8) = 1.36m_s + 0.03f + 0.1f_g - 1.15\delta W_\Sigma^{**},$$
$$K_\Xi^{**}(1.5) = 5.23m_s + 1.91f - 0.44f_g - 0.85\delta W_\Xi^{**},$$
$$K_\Xi^{**}(1.8) = 5.56m_s + 2.13f - 0.39f_g - 1.15\delta W_\Xi^{**}, \quad (38)$$

so that from the requirement $\delta m(M) = \text{const}$ for $M \in \Omega$ we obtain the mass formulas

$$m_{\Sigma^{**}} - m_N^* = 2.77m_s + f + 0.1f_g,$$
$$m_{\Xi^{**}} - m_N^* = 4.3m_s + 1.3f - 0.5f_g,$$
$$m_{\Lambda^{**}} - m_N^* = 1.94m_s + 0.53f - 0.37f_g. \quad (39)$$

The last formula in (35) was obtained from the first two by using the Gell-Mann–Okubo formula for the octet.

Substituting in (39) the values of m_s and f [Eq. (21)], we obtain

$$m_{\Sigma}^{**}-m_{N}^{*}=0.2+0.1f_g,\quad m_{\Xi}^{**}-m_{N}^{*}=0.34-0.5f_g,$$
$$m_{\Lambda}^{**}-m_{N}^{*}=0.16-0.37f_g. \tag{40}$$

The experimental identification of the hyperon resonances in the octet $J^P = 3/2^-$ is not quite unique. There exist two resonances with $J^P = 3/2^-$ and with the quantum numbers of the Λ hyperon: Λ **(1520) and Δ **(1690). The most likely Σ ** is the sufficiently well established resonance Σ **(1670), so that $m_{\Sigma^{**}} - m_{N^*} = 150$ MeV if $m_{N^*} = 1520$ MeV. The experimental situation for Ξ ** is not completely determined, and most likely $m_{\Xi^{**}} = 1820$ MeV. Then $m_{\Sigma^{**}} - m_{N^*} = 300$ MeV. With N *, Σ **, and Ξ ** so identified, the Gell-Mann–Okubo mass formula corresponds to the resonance Λ **(1690). Then $m_{\Lambda^{**}} - m_{N^*} = 170$ MeV. It can be seen that relations (40) agree with experiment within the limits of the expected accuracy ($\sim 30\%$).

5. DISCUSSION OF RESULTS

The investigation of the sum rules for strange baryons yielded formulas for the mass differences of ordinary and strange baryons: Eqs. (16), (19), and (20) for the nucleon octet, Eq. (30) for the Δ decuplet, and Eq. (39) for the octet with $J^P = 3/2^-$. These formulas were derived using the results of Ref. 1, particularly for the residues β_N^2, λ_Δ^2, and $\lambda_{N^{*}}^2$. β_N^2 and λ_Δ^2 were determined accurate to 20–30%, and the accuracy of $\lambda_{N^*}^2$ is worse and is characterized by the factor $2^{\pm 1}$. In relation (10), which contains the residues, the errors of the latter are offset to a considerable degree by the factor $\exp(-m_{N^*}^2/M^2)$ in the denominator, as well as by the numerator. Taking this into account, as well as other sources of errors, we estimate the accuracy of our calculation of the mass splitting at 30% (and possibly somewhat worse in the case of the octet $J^P = 3/2^-$). Stipulating a 30% agreement between the relations obtained and experiment, we can determine the region of admissible values of m_s and f. This is easiest to do by representing the regions in questions on a plot with coordinates m_s and f. To get rid of the dependence on f_g in the case of the decuplet and of the octet $J_p = 3/2^-$, we include the changes due to f_g in the error, assuming $|f_g| < 0.2$. As a result we obtain the diagram of Fig. 2, from which it can be seen that our analysis calls for the following values of m_s and f:

$$m_s=0.1\pm0.02 \text{ GeV},\quad f=-0.11\pm0.05 \tag{41}$$

the value of ms is given for the normalization point $\mu = 0.5$ GeV).

We note that the upper bound on the strange-quark mass is due to the use of the sum rules for the octet $J_p = 3/2^-$, specifically for Σ **. The accuracy of the relations for this octet, however, is lower than for the other considered multiplets (see the preceding section). It is therefore appropriate to estimate m_s without taking the mass formulas for Σ ** into account. In this case

$$m_s=0.105\pm0.030 \text{ GeV}. \tag{42}$$

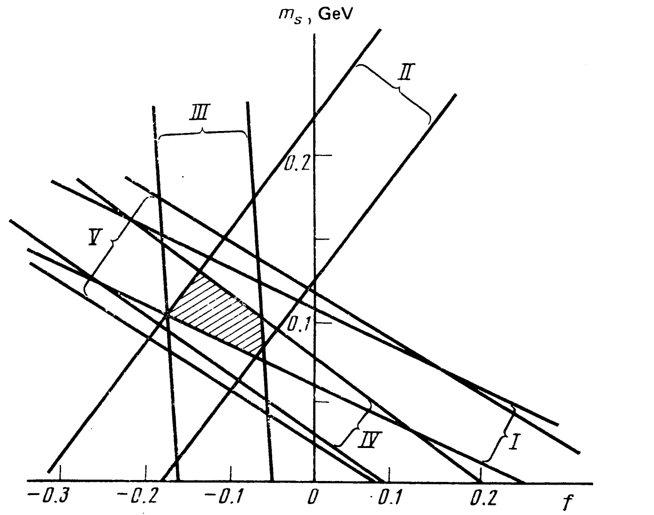

FIG. 2. Regions of admissible values of f and m_s, determined by the sum rules for $m_\Sigma - m_N$ (I); $m_\Xi - m_N$ (II); $m_{\Sigma^*} - m_\Delta$ (III); $m_{\Sigma^{**}} - m_{N^*}$ (IV); $m_{\Xi^{**}} - m_{N^*}$ (V)

The best agreement with the entire set of experimental data takes place at $m_s = 0.105$ GeV, $f = -0.11$, and $f_g = 0.02$, while the theoretical baryon mass differences are

$$m_\Sigma-m_N=0.23,\quad m_\Xi-m_N=0.4,\quad m_\Lambda-m_N=0.19,$$
$$m_\Sigma^{*}-m_\Delta=0.14,$$
$$m_\Xi^{**}-m_N^{*}=0.3,\quad m_\Sigma^{**}-m_N^{*}=0.18,\quad m_\Lambda^{**}-m_N^{*}=0.14. \tag{43}$$

These values differ little from those which follow from the data on the nucleon octet [Eq. (21)]. Summarizing, we repeat that the QCD sum rules for baryons provide a consistent description, which agrees with experiment, of the mass splittings in the lower baryon multiplets. Only two QCE parameters are used in such a description, the bare mass of the strange quark and the quantity f defined in (3) and characterizing the difference between the vacuum condensates of the strange and ordinary quarks. The values obtained for these quantities are given by expression (41). We note that our analysis has made it possible to determine f for the first time. The mass of the current strange quark turned out to be somewhat smaller than the usually assumed [6] $m_s = 150$ MeV. At the present accuracy of the mass-splitting calculation, however, it is difficult to say whether this discrepancy is real.

[a] Unlike in Ref. 1, Ω in the present paper is the region over M on which the sum rules for nonstrange baryons are satisfied. This region can be easily found from the diagrams of Ref. 1.

[1] V. M. Belyaev and B. L. Ioffe, Zh. Eksp. Teor. Fiz. **83**, 876 (1982) [Sov. Phys. JETP **56**, 493 (1982)].
[2] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
[3] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); Errata, Nucl. Phys. **B191**, 591 (1981).
[4] A. V. Smilga, Yad. Fiz. **35**, 473 (1982) [Sov. J. Nucl. Phys. **35**, 271 (1982)].
[5] Particle Data Group, "Review of Particle Properties," Rev. Mod. Phys. **52**, No. 2 (1980).
[6] S. Weinberg, Festschrift for I. I. Rabi, L. Motz, ed., Academy of Sciences, New York, 1977.

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