Beta decay of polarized nuclei in the field of an intense electromagnetic wave

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It is shown that the total decay probability depends substantially on the external field only via the quantum parameters $\chi = E/H_c$ and $\lambda = \hbar\omega/mc^2$, where E and ω are the amplitude and frequency of the wave and $H_c = m^2c^3/e\hbar$. The field corrections are exceedingly small ($\lesssim 10^{-12}$–10^{-14}) even at the highest presently attainable laser-field intensities. In the case of a low energy release ($\varepsilon_0 - 1 \ll 1$) the quantum parameters assume a different form, but for decays of real nuclei these corrections remain small as before. At high wave-quantum energies ($\lambda \gg 1$) the decay time can decrease appreciably. Account is taken of the effects due to the polarization of the initial state of the nucleus and to circular polarization of the wave field. In particular, in the region $\lambda \gg 1$ the estimate $\sigma^+/\sigma^- \approx 1/6$ is valid for the ratio of the total cross sections of the β-decay processes induced by photons having different helicities.

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1. INTRODUCTION

Elementary-particle decays due to interaction with an electromagnetic field are being actively studied of late. The decays of particles in the field of an intense electromagnetic wave (EMW) were also investigated earlier.[1–3] The calculation method was based on the usual decay description,[4,5] wherein the electronic states are exact solutions of the Dirac equation in an external magnetic field (the Volkov function) (see, e.g., Refs. 6 and 7).

The interest in the investigation of nuclear β decay in the field of an EMW can be attributed to the fact that whereas nuclear matrix elements are hardly affected by external actions, the EMW field might influence substantially the phase space of the charged particles produced in the reaction.[8,9] Recent papers[10–12] discuss the possibility of experimental verification of the action of an intense laser field on the course of nuclear β decay, and indicate that the β decay of tritium is expected to increase by 10^4 times at the maximum presently attainable laser-beam intensities. A similar conclusion, that the decay rate can be considerably increased, was drawn earlier in Ref. 9.

It must be noted that a consistent use of the exact Volkov equations is made difficult by the need for analyzing rather complicated expressions typical of multiparameter problems. It was shown in Refs. 13 and 14, in the limit of a constant crossed field, that an external action of β decay manifest itself actually only in the quantum corrections to the probability of the process in the absence of a field. From the physical point of view, the results of Refs. 9–12 contradict this limiting case.

It is clear from the foregoing that it is of interest to analyze adequately the expression for the total probability of β decay in the field of an EMW of arbitrary intensity, and to consider various limiting cases from a unified viewpoint. We note also that it is important to investigate polarization effects in the external field of an EMW, for this reveals a number of specific features of space parity violation in weak interactions.[3,13]

On the basis of results obtained by us earlier,[13–16] we consider here the influence of an EMW field on the probability of β decay at different energy outputs $\varepsilon_0 = \Delta/m$, where $\Delta = M(z,N) - M(z+1, N-1)$ is the difference between the nuclear masses (we use here and elsewhere a system of units in which $\hbar = c = 1$). We take into account also effects due to polarization of the initial state of the nucleus and to circular polarization of the EMW field. These phenomena are first described using the so-called Maijer functions,[17] with the aid of which asymptotic expressions are obtained for the probability and are valid in a wid range of the wave-intensity parameters.

$$\xi = eE/m\omega \ll 1/\lambda, \quad \lambda = \omega/m \ll 1. \tag{1}$$

The results obtained here (see Sec. 3) point to a qualitative difference between the estimates of the total probability of the β decay in the field of an intense EMW compared with the results of Refs. 9–12. The fundamental difference from the estimates of Refs. 9–12 is that at typical laser frequency an increase of the beam intensity by even several orders of magnitude will not influence noticeably the total decay probability at the hitherto attained maximum values. Thus, for laser fields of intensity $E \approx 10^9$–10^{10} V/cm and at a wave quantum energy $\omega \sim 1$–10 eV, the field correction to the neutron β-decay probability amounts to $\sim 10^{-12}$–10^{-14}. In the case of tritium decay with low energy release, this correction is higher, of the order or 10^{-8}–10^{-10}, but it is obvious that in this case it is likewise small, as before.

In Sec. 4 we investigate also the high-frequency limit with a certain restriction on the wave-intensity parameter:

$$\xi^2 \ll \lambda^2(1+\xi^2)\,[(\varepsilon_0+\lambda)^2 - 1 - \xi^2]^{-1}. \tag{2}$$

Of particular interest here is the region $\lambda \gg \varepsilon_0 > 1$, when the decay rate can be determined completely only by the parameters of the EMW field.

The dependence of the probability of decay in the wave field on the circular polarization of the wave, which seems quite natural at first glance, manifests itself in fact in complicated fashion. Thus, whereas in the region defined by the condition (1) this dependence is practically nonexistent, at $\lambda \gg 1$ ($\ln 2\lambda \gg 1$) the cross sections for induced β decay are in a ratio 1:6 for photons with different helicities. The nature of other correlations—of the photon momentum with the nuclear spin and of the nuclear spin with the electron momentum—is similarly not obvious.

From our results for arbitrary values of the energy release ε_0 it follows that the field parameters are substantially different in decays that take place with relativistic and nonrelativistic energies. In the region $\varepsilon_0 - 1 \ll 1$ the influence of the field is thus determined by the quantities

$$\xi^* = (eE/m\omega)(\varepsilon_0^2-1)^{-1/2}, \quad \lambda^* = \lambda(\varepsilon_0^2-1)^{-1},$$
$$\chi^* = \xi^*\lambda^* = \chi(\varepsilon_0^2-1)^{-3/2},$$

which differ from ξ, λ, and χ in that the characteristic length in them is not the Compton electron wavelength of the relativistic region but the de Broglie wavelength of the nonrelativistic electron.

2. β-DECAY PROBABILITY OF A POLARIZED NUCLEUS

If we confine ourselves to the usual *V-A* variant of the weak interaction in the lowest order in the weak-interaction constant G and with exact account taken of the interaction of the electron with the field of an intense EMW (the state of the nucleus, owing to its large mass and to the low energy release in β decay in the wave field, is considered in the nonrelativistic approximation), we can obtain for the probability of the β decay of a polarized nucleus (spin $\frac{1}{2}$) in the field of a circularly polarized wave ($g = -1$ and $+1$ for right- and left-hand polarization) the following expression[13]

$$\frac{W}{W_0} = \int_{t_{\text{н}}}^{\infty} dt\,(t^2-1-\xi^2)^{1/2} \int_0^{\pi} d\vartheta_0 \sin\vartheta_0 \sum_{l=\nu}^{\infty} (t-\varepsilon_0-\lambda l)^2$$
$$\times\left\{ tJ_l^2(z) - \mathscr{L}_l + \frac{2\alpha_0(1-\alpha_0)}{1+3\alpha_0^2} s_n [(t^2-1-\xi^2)^{1/2}\cos\vartheta_0 J_l^2(z) - \mathscr{L}_l] \right\}, \tag{3}$$

where

$$\mathscr{L}_l = \lambda l J_l^2(z) - (\xi^2/4\tau)[J_{l+1}^2(z) + J_{l-1}^2(z) - 2J_l^2(z)]$$
$$-g\lambda(z - \xi^2 l/\tau\lambda z) J_l(z) J_l'(z),$$
$$W_0 = (G^2 m^5/4\pi^3)(1+3\alpha_0^2), \tag{4}$$

α_0 is the ratio of the axial and vector constants in the *V-A* model, $s_n = \pm 1$ corresponds to different orientations of the nuclear spin relative to the wave propagation direction. The components of the average electron momentum in the wave are connected with the momentum components p_n by the relation $q_\mu = p_\mu + \xi^2 m k_\mu / 2\tau\omega$, where k_μ is the wave 4-vector, $q_\perp = q \sin\vartheta_0$, J_l is a Bessel function with argument $z = \xi q_\perp/\tau\omega$, $t_0 = (1+\xi^2)^{1/2}$, $\tau = (p_0 - p_3)/m$, $t = q_0/m$, $q_0 = p_0 + \xi^2 m/2\tau$ is the average energy of the electron motion in the wave, $\nu = (t-\varepsilon_0)/\lambda$, and ϑ_0 is the angle of emission of an electron with average momentum q_μ and is reckoned from the wave-propagation direction.

Simple calculations make it possible to express the individual sums of the squared Bessel functions with index l in the range $0 < \nu \leqslant l < \infty$ and of the power function l^k with k equal to 0, 1, 2, and 3, in terms of the Meijer G-functions.[17] For example, the most complicated function S_3^ν is the form

$$S_3^\nu = \frac{z^3}{4\sqrt{\pi}} \frac{1}{2\pi i} \int ds \left(\frac{1}{z^2}\right)^{-s} \frac{\Gamma(s+1)\Gamma(\nu-s-1/2)}{\Gamma(s+5/2)\Gamma(s+\nu+1/2)}$$
$$\times\left[\frac{\nu^2}{\nu-s-3/2} + \frac{(\nu-1)^2}{\nu+s+1/2} + \frac{2\nu-1}{s}\right] = \sum_{l=\nu}^{\infty} l^3 J_l^2(z). \tag{5}$$

The remaining functions with $k = 0, 1, 2$, are expressed similarly (see Refs. 15 and 16). The change of the angle variable

$$\cos\vartheta_0 = (\cos\vartheta + \beta)/(1+\beta\cos\vartheta)$$

enables us to write the argument of the Bessel function in the form

$$z = z_0 \sin\vartheta, \quad z_0 = z_{max} = p\beta t/\lambda,$$

where $p = \xi(1+\xi^2)^{-1/2}$, and $\beta = (1 - t_0^2/t^2)^{1/2}$ is the average electron velocity in the wave.

Integration with respect to the angles, for example by representing the G function in the form of a contour integral and using the formula

$$\int_0^{\pi} d\vartheta \sin\vartheta\, z^{2s+k} (1+\beta\cos\vartheta)^{-2} = 2z_0^{2s+k}\,\Gamma\left(s+\frac{k}{2}+1\right)$$
$$\times\frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(1+s_1)\Gamma(3/2+s_1)\Gamma(-s_1)}{\Gamma(s+3/2+k/2+s_1)} (-\beta^2)^{s_1} ds_1, \tag{6}$$

where $-1 < \gamma < 0$ and $\Gamma(s)$ is the Euler gamma function, we can express the typical structural parts of (4) in the form of a Mellin-Barnes double integral. Since the different functions $Sk(z)$ with different indices $k = 0, 1, 2, 3$ have the same structure, we present here by way of example the expression for

$$S_0^\nu = \frac{1}{\sqrt{\pi}}\left(\frac{1}{2\pi i}\right)^2 \int ds\, z_0^{2s} \int ds_1 (-\beta^2)^{s_1}$$
$$\times\frac{\Gamma(s, s+1/2, \nu-s)\Gamma(1+s_1)\Gamma(3/2+s_1)\Gamma(-s_1)}{\Gamma(s+1)\Gamma(\nu+s)\Gamma(s+3/2+s_1)}. \tag{7}$$

3. REGION OF LOW FREQUENCIES $\omega \ll m$. QUASICLASSICAL LIMIT

Without dwelling on the procedural changes, which require a separate investigation, we calculate here integrals of the type (7) in the region defined by condition (1). A distinguishing feature of this region is that an individual wave quantum has here a very low energy compared with the electron rest energy ($\omega \ll m$), and only by assuming that the wave is intense enough can one hope that the EMW field would still be capable of substantially affecting the decay. In fact, when the intensity ξ of the wave satisfies the condition (1), the contribution made to the total probability by the partial processes with large indices l may turn out to be not small, and it follows thus from the conservation law that the an appreciable energy can be drawn from the wave.

It is easy to verify that at the usual laser frequencies, corresponding to a wave quantum energy of the order of 1 eV, the effective value of l is $l_{\text{eff}} = 10^6$. Using the properties of the functions $S_k(z)$, which are analogous in their essential features to the properties of Bessel functions (see Refs. 15 and 16), it can also be shown that at $l \gg 1$ the main contribution is made at $z \sim l$, and by the same token actually $z_0 \gg 1$. It must be noted, however, that estimates of the total rate of β decay in the field of an EMW, based on the first terms of the expansion of the function $S_k^\nu(z)$ in powers of the parameter ν/z (Refs. 9–12), are untenable, since an adequate analysis of the dependence of the probability of the process on the wave intensity ξ requires that the dependence of $S_\nu(z)$ on ν/z be used in its exact form.[15,16]

If s_1 is fixed, the poles of the integrand in (7) on the complex s plane are in the general case the points

$$s=-n, \quad s=-{}^1/_2-n, \quad s=\nu+n; \quad n=0, 1, \ldots,$$

with the integration contour drawn such that the "left" and "right" sets of poles be separated from each other. Recognizing that in the essential region we have $z_0 \gg 1$, we obtain the asymptotic form of the functions $S_\nu(z)$ in this limit. To this end, we close the integration coutour in (7) on the left and obtain a series in inverse powers of z_0. If we retain in the resultant expression all terms of the form $(\nu/z_0)^p$, where $p = 0, 1, 2, \ldots$, which are of the order of unity in the considered region, and discard the terms $\sim 1/z_0 \ll 1$, we can obtain (7)

$$S_0^* = \frac{1}{\sqrt{\pi}} \frac{1}{2\pi i} \int ds_1 (-\beta^2)^{s_1} \Big\{ \sqrt{\pi}\Gamma(1+s_1)\Gamma(-s_1) - 2\Gamma\left(\frac{3}{2}+s_1\right) \sum_{k=0}^{\infty} x^{2k+1} \frac{\Gamma(k-s_1)}{(1+2k)k!} \Big\}, \tag{8}$$

where $x = \nu/z_0$.

After a number of simple transformations of (8) we obtain ultimately an expression for S_0^* in terms of elementary functions:

$$S_0^* = (1-\beta^2)^{-1}[1-x/A], \quad A=[1-\beta^2(1-x^2)]^{1/2}. \tag{9}$$

It must be noted that the unaccounted-for terms of the order of $1/z_0$ are proportional to Planck's constant $\hbar$ and are thus quantal. Conversely, the terms containing ν/z as a ratio are classical. Making the same transformation for the remaining functions $S_k^*(z)$, $k = 1, 2, 3$, we can verify that all are similar in form:

$$S_1^*(z) = \left(\frac{z_0}{\beta^2}\right)\left(\frac{1}{A}-1\right), \tag{10}$$

$$S_2^*(z) = \left(\frac{z_0}{\beta}\right)^2\left(\frac{x}{A}-1-\frac{1}{\beta}\ln\frac{\beta x+A}{1+\beta}\right), \tag{11}$$

$$S_3^*(z) = \frac{z_0^3}{\beta^2}\left(\frac{x^2}{A}-\frac{2A}{\beta^2}+\frac{2}{\beta^2}-1\right). \tag{12}$$

Some differences, connected with the calculation of the functions at the spin term ($\sim s_n = \pm 1$), consist in the fact that when integrating with respect to the angle ϑ it is necessary to use here the more general formula

$$\int_0^\pi \frac{z^{2s+k}\sin\vartheta\, d\vartheta}{(1+\beta\cos\vartheta)^m} = \frac{\sqrt{\pi} z_0^{2s+k}\Gamma(s+k/2+1)}{\Gamma(s+k/2+{}^3/_2)} {}_2F_1(m/2, m/2+{}^1/_2; s+k/2+{}^3/_2, \beta^2),$$

where ${}_2F_1$ is the Gauss hypergeometric function; but here, too, the final answer can be expressed in terms of elementary functions.

Thus, for the spectral distribution of the probability of β decay of a polarized nucleus in the field of an intense EMW we have in the principal term of the expansion in $\lambda \ll 1$

$$\frac{W}{W_0} = \frac{\xi^3}{(1+\xi^2)^{1/2}} \int_{t_H}^{t_B} dt(t^2-1-\xi^2)\Big\{ x\left(3+\frac{\varepsilon_0}{b}+2k_n\right) \cdot \left[\frac{x}{A}-1-\frac{1}{\beta}\ln\frac{\beta x+A}{1+\beta}\right] + x^3\left(1+\frac{\varepsilon_0}{b}\right)\left(1-\frac{x}{A}\right)\left(\frac{1}{\beta^2}-1\right)^{-1}$$
$$-x^2\left(\frac{1}{A}-1\right)\left(3+2\frac{\varepsilon_0}{b}+k_n\right)-(1+k_n)\left[\frac{x^2}{A}+\frac{2}{\beta^2}(1-A)-1\right] + \frac{k_n}{2a}(t^2-1-\xi^2)$$
$$\times\Big[\left(\frac{x\beta}{A}\right)^3(1-x^2)-\frac{2x}{A^3}\left(\frac{2}{\beta^2}(1-A)A^2-(1-\beta^2)(1-x^2)\right) + \left(1-\frac{3}{\beta^2}\right)\ln\frac{x\beta+A}{1+\beta}-\frac{3}{\beta}\left(1-\frac{x}{A}\right)-\frac{x\beta}{A}\left(1-\frac{x^2}{A^2}\right)\Big]\Big\},$$
$$k_n = \frac{2\alpha_0(1-\alpha_0)}{1+3\alpha_0^2}s_n, \quad a=\lambda z_0, \quad b=\nu\lambda. \tag{13}$$

At $\nu > 0$, the requirement that the functions $S_k(z)$ be positive establishes the limits of the integration over the spectrum:

$$t_{1,2} = (1+\xi^2)[\varepsilon_0 \mp p(\varepsilon_0^2-1)^{1/2}]. \tag{14}$$

At $\nu < 0$ ($t_1 < \varepsilon_0$) the lower limit corresponds to a minimum energy of the electron in the field of the wave: $t_l = t_0$. If we average in (13) over the nuclear spin s_n, we obtain the result of Refs. 15 and 16, where we considered the β decay of an unpolarized nucleus in the field of an EMW. We note that the dependence of the decay rate on the wave polarization ($g = \pm 1$) is contained this case in the discarded quite small quantum terms ($\sim \lambda^3$).

The spectral curves constructed in accord with Eq. (13) and shown in Fig. 1 demonstrate that the dependence of the spectral decay probability on the wave intensity is quite substantial, for according to (14) the spectrum of the admissible electron energies shifts into the relativistic region with increasing wave intensity. As shown first in Refs. 15 and 16, in the case of an unpolarized nucleus the total decay rate (the area under curve 2 and Fig. 1) is nonetheless independent of the wave intensity ξ and coincides with the total decay rate of the nucleus in the absence of a field.

We shall show here that the range of validity of this result can be estimated by starting from the expression for the total β decay probability with allowance for the succeeding terms of the expansion in λ (the field quantum corrections). By a calculation using the method indicated above it is possible to verify that the first correction to the decay probability in the absence of a field is determined only by the maximum amplitude of the wave field and by the energy release ε_0:

$$W/W_0 = \Phi_0(\varepsilon_0) + (E/H_c)^2\Phi_1(\varepsilon_0), \tag{15}$$

where

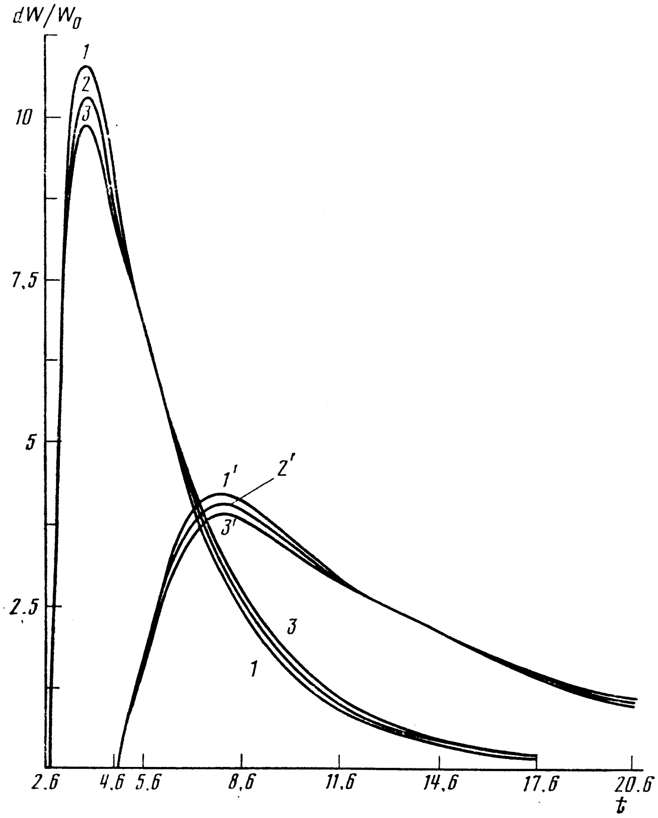

FIG. 1. Spectral distribution of the differential β-decay probability of a polarized neutron $\varepsilon_0 = 2.6$, $k_n = [2\alpha_0(1-\alpha_0)/(1+3\alpha_0^2)]s_n$ in the field of a wave of intensity $\xi = 2.4$ (curves 1–3) and $\xi = 4$ (curves 1'–3') at $\lambda \ll 1$: 1, 1' – $s_n = -1$; 3, 3' – $s_n = +1$.

$$\Phi_0(\varepsilon_0) = {}^1/_2\varepsilon_0 \ln[\varepsilon_0 + (\varepsilon_0^2-1)^{1/2}] + (\varepsilon_0^2-1)^{1/2}(2\varepsilon_0^4 - 9\varepsilon_0^2 - 8)/30,$$

$$\Phi_1(\varepsilon_0) = {}^2/_3\varepsilon_0(1+k_n)\ln[\varepsilon_0 + (\varepsilon_0^2-1)^{1/2}] - ({}^1/_2 + {}^2/_3 k_n)(\varepsilon_0^2-1)^{1/2}.$$

It must be emphasized that the result (15) was obtained without any additional asumptions concerning the wave intensity ξ, and is thus valid in the entire region (1).

We are now able to state that no contradiction whatever can or does exist between the considered region (1) and the investigated case of a constant crossed field, since Eq. (15) agrees fully with an analogous result[13,14] presented earlier in the form of a numerical coefficient for the concrete value of the energy release in neutron decay ($\varepsilon_0 = 2.6$). Obviously, in the nonrelativistic and ultrarelativistic limits one can write in accord with (15)

$$W/W_0 = {}^4/_{105}(\varepsilon_0^2-1)^{7/2}[1 + {}^{35}/_8(E/H_c^*)^2], \quad \varepsilon_0 - 1 \ll 1,$$

$$W/W_0 = {}^1/_{15}\varepsilon_0^5\{1 + 15(E/H_c)^2\varepsilon_0^{-4}[{}^2/_3(1+k_n)\ln 2\varepsilon_0 - ({}^1/_2 + {}^2/_3 k_n)]\},$$

$$\varepsilon_0 \gg 1, \tag{16}$$

where $H_c^* = m^2(\varepsilon_0^2-1)^{3/2}/e$, is the field that performs, over the electron de Broglie wavelength, a work of the order of the maximum kinetic energy released in the case of nonrelativistic decay. In this case Eq. (15) remains valid under a more stringent restriction on the possible amplitudes of the EMW field, namely $E \ll H_c^*$; it can be easily seen, however, that this restriction is not decisive for real decays. Nonetheless, in the hypothetical case $\varepsilon_0 \to 1$ the strength of the electromagnetic field capable of substantially altering the "decay constant" can become very small.

It is quite interesting that the spectral curves in Fig. 1, labeled 1 and 3, demonstrate also that the decay probability depends on the orientation of the nuclear spin $s_n = \pm 1$. As seen from the figure, the largest difference is observed in the region of the maximum, but there is also another region where curves 1, 2, and 3 are distinguishable. The latter region is broader and curves 1 and 3 change places in it relative to 2 compared with the region of the maximum. It is not at all difficult to verify that the total area under curves 1, 2, and 3 remains constant, offering by the same token evidence that the total decay rate is independent of the orientation of the nuclear spin in the considered limit.

4. REGION OF HIGH FREQUENCIES $\omega \gtrsim m$. QUASIQUANTUM LIMIT[1)]

The indicated laws governing nuclear β decay in the presence of the field of an intense EMW are substantially altered in the parameter range defined above by condition (2). Indeed, if the condition (2) is satisfied, we have $z_0 \ll 1$ and the main contribution to the total decay probability consists of the partial contributions with $l = 0$ and ± 1. Retaining in expression (3) for the probability the terms of order not higher than second in ξ, we obtain

$$W/W_0 = \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3, \tag{17}$$

$$\mathcal{I}_1 = (1+\xi^2)\int_{t_0}^{\varepsilon_0}\frac{dt}{t^2}\int_0^\pi \frac{\sin\vartheta\, d\vartheta}{(1+\beta\cos\vartheta)^2}$$

$$\times (t^2-t_0^2)^{1/2}(t-\varepsilon_0)^2\left\{\left[t + k_n(t^2-t_0^2)^{1/2}\frac{\cos\vartheta+\beta}{1+\beta\cos\vartheta}\right]\left(1-\frac{z^2}{2}\right)\right.$$

$$\left. + (1+k_n)\left(-\frac{\xi^2}{2\tau} - g\lambda\frac{z^2}{2}\right)\right\}, \tag{18}$$

$$\left.\begin{matrix}\mathcal{I}_2\\ \mathcal{I}_3\end{matrix}\right\} = (1+\xi^2)\int_{t_0}^{\varepsilon_0\pm\lambda}\frac{dt}{t^2}\int_0^\pi\frac{\sin\vartheta\, d\vartheta}{(1+\beta\cos\vartheta)^2}(t^2-t_0^2)^{1/2}.$$

$$\times (t-\varepsilon_0\mp\lambda)^2\left[\frac{z^2}{4}(t\mp\lambda+g\lambda) + \frac{\xi^2}{4\tau}(1\mp g)\right]. \tag{19}$$

Expression (17)–(19) determine the spectral-angular distribution of the decay probability with allowance for the terms $\sim\xi^2$. In this limit one can have, besides the decays that are allowed without participation of the external field ($\varepsilon_0 > 1$), also induced nuclear β-decay processes due to absorption of a wave-quantum energy ($\varepsilon_0 < 1$).

Integrating with respect to angle, we can obtain from (17)–(19) the spectral distribution of the decay probability in the form

$$W/W_0 = I_1 + I_2 + I_3, \tag{20}$$

$$I_1 = \int_{t_0}^{\varepsilon_0} dt\,(t-\varepsilon_0)^2\left\{2(t^2-t_0^2)^{1/2}[(1+a_1)t + g\lambda b_1]\right.$$

$$\left. - [tb_1(t+g\lambda) + 2f]\ln\frac{1+\beta}{1-\beta}\right\}; \tag{21}$$

$$\left.\begin{matrix} I_2 \\ I_3 \end{matrix}\right\} = \int_{t_0}^{\varepsilon_0 \pm \lambda} dt (t-\varepsilon_0 \mp \lambda)^2 \Big\{ (t^2-t_0^2)^{1/2} [-a_1 t \pm \lambda b_1 (1 \mp g)] + \left[\frac{1}{2} t b_1 (t \mp \lambda (1 \mp g)) + f \mp g f_1 \right] \ln \frac{1+\beta}{1-\beta} \Big\}, \quad \binom{22}{23}$$

where

$$a_1 = (\xi/\lambda)^2 (1+3k_n/2); \quad b_1 = (\xi/\lambda)^2 (1+k_n);$$
$$f = (\xi/2)^2 (1+k_n+k_n/\lambda^2); \quad f_1 = (\xi/2)^2 (1+k_n).$$

With increasing parameter λ, the contribution of the induced process (22) (upper sign) due to absorption of one wave quantum increases against the background of the contributions (21) and (23) that correspond to a zero-photon decay with wave-intensity corrections for the increase of the electron mass in the EMW field, and also for the induced emission into the wave, besides, the usual β-decay products (23), of a quantum indentical with the wave quanta. In the case of a forbidden decay in the absence of a field ($\varepsilon_0 < 1$) only the contribution (22) remains in Eq. (20), and the parameter λ is then subject to the threshold condition

$$\lambda > t_0 - \varepsilon_0. \quad (24)$$

It is interesting that at $\lambda \gg |\varepsilon_0| > 1$ a situtation in possible wherein the fast growth of the contribution (22) in this region

$$I_2 \approx {}^1/_{30} \xi^2 \lambda^3 \ln \lambda \quad (25)$$

can lead to

$$I_2 \gg I_1,$$

so that in this case the total decay probability is determined fully only by the parameters of the EMW field. It is most important that in this case the time of the allowed and forbidden decays are practically equal, and the induced nuclear β decay proceeds under these conditions independently of the magnitude and sign of the energy-release parameter ε_0 in the absence of an external field.

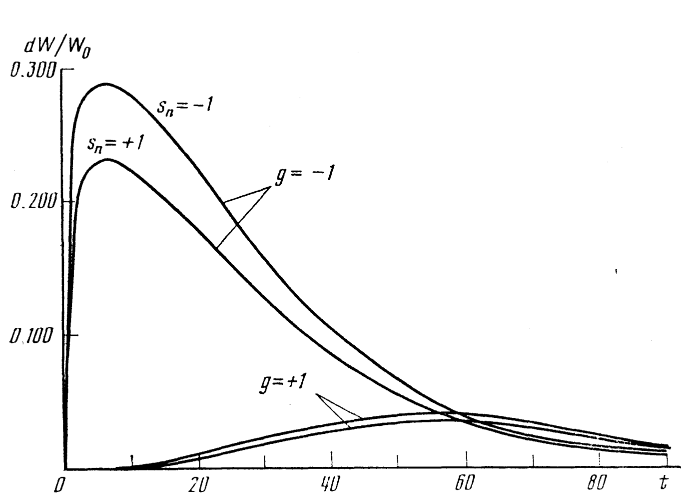

FIG. 2. Spectral distribution of the decay probability of the ^3H nucleus ($\varepsilon_0 = 1.036$, spin $\frac{1}{2}$) vs the orientation of the nuclear spin $s_n = \pm 1$ and the wave-photon circular polarization $g = \pm 1$ at $\xi = 10^{-3}$ and $\lambda = 10^2$.

To describe the induced nuclear β decay it is possible to introduce, besides the probability and time of the decay, also the total cross section, which takes at $\lambda \gg \varepsilon_0$ the form

$$\sigma = \frac{G^2 m^4}{30\pi^2 \alpha} r_0^2 \lambda^2 \Big\{ (1+k_n) \Big[\Big[1 + \frac{5}{2}(1-g) \Big] \times \ln 2\lambda - \frac{95}{24}(1-g) - \frac{137}{60} \Big] + \frac{1}{2} \Big\},$$
$$r_0 = e^2/m, \quad \alpha = e^2 = {}^1/_{137}. \quad (26)$$

It is most instructive that the cross section (26) is independent of the wave intensity ξ. Expression (26) also illustrates clearly that when the quantum effects play a major role ($\lambda \gg \varepsilon_0$) the course of the nuclear β decay of a polarized nucleus is substantially modified compared with decay in the absence of a field or with the low-frequency limit of the action of an intense EMW. Am important role is played in the quasiquantum limit also by polarization effects, which have a direct bearing on the manifestation of spatial parity nonconservation in weak interactions under the action of an external electromagnetic field (see Fig. 2).

It follows from (26) that in the logarithmic approximation the cross section of the induced nuclear β decay produced by photons with different helicities are in a ratio 1:6. Weaker, but likewise a manifestation of spatial violation, is the dependence of (26) on the nuclear spin orientation relative to the wave-propagation direction.

5. DISCUSSION OF RESULTS

The foregoing analysis of the β-decay probability of a polarized nucleus in the field of an intense EMW of circular polarization allows us to make a number of concluding remarks.

1. The approach developed, based on the use of Meijer G-functions to obtain asymptotic expressions, at $\lambda \ll 1$, for the functions that determine the total probability of the process, enable us to write down estimates expressed in terms of elementary functions and valid in a wide range of the EMW intensity parameter (classical limit). We were able to show by this method that the use of the first terms of the expansion of the functions $S_k^\nu(z)$ in terms of the parameter ν/z do not provide a well-grounded estimate of the total decay rate.

2. We call attention to the fact that, after integration over the spectrum, Eq. (13) for the total probability of the β decay of a nucleus polarized along the wave propagation direction agrees, with good accuracy [see (16)], with the probability of the analogous decay in the absence of an external field. A characteristic feature of this result is that the integration eliminates both the dependence of the decay rate on the EMW intensity and the influence of the nuclear spin orientation. It can be seen from (13) that the spectral distribution of the decay electrons in the wave contains a part that is invariant to the wave propagation direction and to the reversal of the sign of the particle velocity β, as well as a part that depends on the projection of the axial vector on this direction.

It follows therefore that the spectral probability (13) contains a P-odd term whose presence can be interpreted as a manifestation of the effect of P-parity violation under the specific conditions of the action of the field of an intense EMW on the β decay of a polarized nucleus. It can thus be established that the spatial-parity violation due to V-A weak interaction that takes place in the EMW field manifests itself also after integration over all the electron emission angles. A peculiarity of this phenomenon, namely that parity nonconservation in the field of an EMW can be described by an integral angular characteristic, is in our opinion most intersting and lends itself apparently to experimental observation by laser techniques.

3. It must be emphasized that in the derivation of (13) we have assumed satisfaction of the condition $\varepsilon_0 > 1$, i.e., that the decay is energy-allowed also in the absence of an external EMW field. Using (13) it can be seen that as $\varepsilon_0 \to 1$ the total decay rate decrease to zero like $(\varepsilon_0^2 - 1)^{7/2}$. If one starts with Eq. (3), it is easy to verify, by a reasoning similar to that in Sec. 3, that in region (1) the total decay probability at $\varepsilon_0 < 1$ remains zero. It can thus be concluded that when the energy of an individual wave quantum is low ($\omega \ll m$) in a rather wide range [see (1)] the change of the EMW intensity will not affect the rate of the allowed decay, and stable nuclei remain stable under such conditions.

4. When condition (2) is satisfied, the situation can change radically. Thus, if the energy of the wave quantum remains sufficient to conserve the energy balance in one-quantum reactions also at $\varepsilon_0 < 1$:

$$\lambda > (1+\xi^2)^{1/2} - \varepsilon_0,$$

induced nuclear β decays that are forbidden in the absence of an external field become possible. Their probability is proportional to the square of the wave intensity parameter ξ, whose value is subject to the restriction from condition (2). It can be easily seen that at $\lambda \gg |\varepsilon_0| > 1$ the restriction on ξ becomes weaker: $\xi \ll 1$. The region of large photon energies $\lambda \gg |\varepsilon_0| > 1$ is remarkable also because in this case one can observe a rapid growth of the probability of the induced decay with increasing λ:

$$W/W_0 \sim \xi^2 \lambda^3 \ln \lambda,$$

and this can lead to equalization, in order of magnitude, of the total times of the induced nuclear β decays that are allowed and forbidden in the absence of external action. Great interest attaches in our opinion also to the study of the polarization effects in this region (see Sec. 4). It must be noted that the upper bound on the parameter λ is a consequence of the considered nonrelativistic nuclear-energy limit in the initial and final states, as well as of the unitary limit. We note also that the most suitable radiation sources with which one can check at present the results of Sec. 4 may be the "photon factories" based on large accelerators and charged-particle storage rings.

[1)]In a pure quantum description of an external electromagnetic field the intensities of this field should be quantum operators rather than specified functions of the coordinates and time. In the present approach the EMW field is regarded as classical, but the conservation laws contain the values of the index l together with the quantity $\hbar\omega$. This serves as the basis for the quasiquantum interpretation, to which we adhere here, of the considered phenomena.

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