Theory of localized superconductivity

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A theoretical description of the phenomenon, discovered by Khaĭkin and Khlyustikov [JETP Lett. **33**, 158 (1981); **34**, 198 (1981)] in tin crystals, of localized superconductivity (LS) near a twinning plane (TP) is presented. The behavior of LS in a magnetic field is investigated. It is shown that, as the temperature is raised, the phase transition in a type-I superconductor located in a nonzero field changes its character: it becomes a second-order transition in the vicinity of the temperature at which the LS sets in. The first-order normal-phase–LS transition curve and the normal-phase supercooling field are found. The effect of the dimensions of the twinning-plane region and that of the distance between parallel twinning planes on the temperature at which LS sets in are considered. The results obtained allow us to explain the available experimental data.

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Recently, Khaĭkin and Khlyustikov[1,2] discovered the appearance of localized superconductivity (LS) near the twinning plane (TP) in tin crystals. The LS sets in at a temperature T_c somewhat higher than the critical temperature T_{c0} of the superconducting transition of the bulk metal (in tin $T_{c0} = 3.72$ K and $T_c = T_{c0} + 0.04$ K).

To describe the LS phenomenon, two of us (A. I. and L. N.)[3] proposed for the superconductor a model with an electron-phonon interaction constant λ having a peak near an infinite TP in a region with thickness $d \ll \xi_0$, where $\xi_0 = 0.18 v_F/T_{c0}$ is the superconducting coherence length. A similar model was independently considered by Nabutovskiĭ and Shapiro.[4] In Ref. 3 the upper critical magnetic field H_{c2} for a second-order transition into the LS state and the temperature dependence of the diamagnetic moment are determined. At the same time, the transition into the LS state in a nonzero field actually occurs as a first-order transition,[1,2] and the experimentally measured temperature dependence of the diamagnetic moment turns out to be significantly stronger than the dependence obtained in Ref. 3. Furthermore, as investigations with the aid of an electron microscope show,[5] the TP in tin is not homogeneous, but consists of regular TP sections separated by dislocated regions.

In view of this, in the present paper we shall investigate the character of the phase transition in a nonzero field, find the critical field for the first-order transition, and consider the effect of the inhomogeneity of the TP on the properties of the LS. In the process, we shall, for completeness of exposition, also give the results presented in Ref. 3, and directly used in the present paper.

I. CRITICAL TEMPERATURE OF THE TRANSITION INTO THE STATE OF LOCALIZED SUPERCONDUCTIVITY

1. The infinite twinning plane

We assume that the Cooper-pairing constant λ has a magnitude greater than the corresponding constant λ_0 for the bulk metal in a region of the order of several interatomic distances in the vicinity of a TP, i.e., that $\lambda(x) = \lambda_0 + \lambda_1(x)$, where $\lambda_1(x) > 0$ for $|x| < d/2$. The increase in λ near a TP can be related, in particular, to the contribution of the two-dimensional phonons to the electron-phonon interaction, or to the special nature of the electronic spectrum near a TP.[1] Let us note that the specific mechanism responsible for the increase in λ is unimportant for our analysis below: it is only essential that the increase in λ occur in a narrow region $d \ll \xi_0$.

The transition temperature is determined from the solution to an integral equation for $\psi(x) = \Delta(x)/\lambda(x)$. This equation has the form:

$$\psi(x) = \lambda_0 \int K(\mathbf{r}-\mathbf{r}')\psi(x')d\mathbf{r}' + \int K(\mathbf{r}-\mathbf{r}')\lambda_1(x')\psi(x')d\mathbf{r}', \tag{1}$$

where K is the superconducting kernel (see, for example, Ref. 6).

Solving (1) by going over to momentum representation with allowance for the condition $d \ll \xi_0$, we find the increase in the critical temperature of the LS as compared to the critical temperature of the bulk superconductor[3,4]:

$$\tau_0 = (T_c - T_{c0})/T_{c0} = 12\bar{\lambda}_1^2 d^2 T_{c0}^2/\lambda_0^4 v_F^2 \sim (10/\lambda_0^2)(\bar{\lambda}_1 d/\lambda_0 \xi_0)^2, \tag{2}$$

where

$$\bar{\lambda}_1 d = \int_{-d/2}^{d/2} \lambda_1(x)\,dx.$$

In a dirty superconductor with a mean free path $l \ll \xi_0$, τ_0 has a higher value because of the smaller correlation length [an additional factor $\xi_0/l \gg 1$ appears on the right-hand side of (2)]. The increase in τ_0 may also be caused by the anomalies in the electronic spectrum near the TP that make it difficult for an electron to go into the interior of the sample[1]; their effect also leads to a decrease in the correlation length near the TP.

The expression (2) for τ_0 contains the quantities λ_1 and d, whose direct determination is difficult, but, as will be seen below, the important role is played only by the quantity τ_0, which can be measured experimentally; it can be considered to be a phenomenological parameter of the theory.

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Let us note that, qualitatively, the decrease of τ_0 with increasing ξ_0, which follows from the expression (2), is clearly observed in experiment: according to Ref. 7, in tin $\tau_0 \sim 10^{-2}$ for $\xi_0 \approx 3.2\times10^3$ Å, whereas in thallium $\tau_0 \sim 8\times10^{-4}$ for $\xi_0 \approx 4.2\times10^3$ Å, and in aluminum, in which $\xi_0 \approx 16\times10^3$ Å, the LS effect is not observed.

A small magnitude of τ_0 leads to a situation in which the characteristic dimension of the LS region is large compared to the correlation length, since

$$\xi(\tau_0)=0.74\xi_0/\sqrt{\tau_0}\gg\xi_0.$$

In view of this, we can use for the description of the LS the Ginsburg-Landau (GL) functional, which in our case has the form

$$F=\int\left\{\frac{(\mathbf{B}-\mathbf{H})^2}{8\pi}+\frac{1}{4m}\left|\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right|^2+a\psi^2+\frac{b}{2}\psi^4-\gamma\delta(x)\psi^2\right\}d^3r, \quad (3)$$

where

$$a=\frac{\tau}{\eta},\quad b=\frac{1}{N\eta},\quad \tau=\frac{T-T_{c0}}{T_{c0}}\quad \gamma=\left(\frac{\tau_0}{\eta m}\right)^{1/2},$$

$\mathbf{H}$ is the external field, N is the electron concentration, and $\eta = 7\zeta(3)\varepsilon_F/6(\pi T_{c0})^2$ in the case of a pure superconductor.[8] What makes the functional (3) different from the usual one is the presence of the term $-\gamma\psi^2\delta(x)$, which describes the δ-function increase in T_c near the $x=0$ plane. Notice that a similar functional has been used before[9] to describe magnetic systems of finite dimensions near the Curie point. In this section we shall be interested in LS in the absence of a field; the GL equation then has the form

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial\mathbf{r}^2}+\frac{1}{\eta}\frac{T_{c0}-T}{T_{c0}}-b|\psi|^2\right]\psi=-\gamma\delta(x)\psi, \quad (4)$$

and it is possible to find its exact solution

$$\psi(x)=\frac{[8N(\tau_0-\tau)]^{1/2}\exp[-|x|/\xi(\tau)]}{\{1+(\tau_0/\tau)^{1/2}+[1-(\tau_0/\tau)^{1/2}]\exp[-2|x|/\xi(\tau)]\}^{1/2}}, \quad (5)$$

where $\xi^2(\tau) = \eta/4m\tau = 0.55\xi_0^2/\tau$. In the temperature range from T_c to T_{c0} the characteristic scale of the decrease of ψ is determined by the quantity $\xi(\tau_0)$. But if near T_c this decrease is exponential with attenuation distance $\xi(\tau_0)$, as the temperature is lowered down to T_{c0}, the law of decrease goes over into a power law:

$$\psi(x)\propto[1+x^2/\xi^2(\tau_0)]^{-1/2}.$$

2. Effect of the finiteness of the dimension of the twinning plane on the critical temperature for the onset of localized superconductivity

As has already been noted, the TP in tin is made up of a set of regular sections of different dimensions, separated by regions with dislocations. In view of this, there arises the question of the critical temperature for the onset of LS near a TP section of finite dimension.

The shape of the TP section does not (unless it is filamentary) have a significant effect on the transition temperature; therefore, we shall for simplicity assume the TP to be a circle of radius $R\gg\xi_0$ and denote the corresponding transition temperature by T_R. To determine T_R, it is sufficient to solve the linearized equation (4) with the potential $\gamma\delta(x)$ replaced by the potential $\gamma\delta(x)\theta(R-|\boldsymbol{\rho}|)$, where $\mathbf{r}=(x,\boldsymbol{\rho})$ and $\theta(x)=1$ for $x>0$ and 0 for $x<0$. The indicated equation is equivalent to the Schrödinger equation and the transition temperature is determined by the lowest energy level $E_0 = -2\tau_R/\eta$, where $\tau_R=(T_R-T_{c0})/T_{c0}$. To facilitate the analysis of the solution, let us replace the potential $-\gamma\delta(x)\theta(R-|\rho|)$ by the elliptic well $(\gamma/2q)\tilde\theta(\rho,x)$, where $\tilde\theta(\mathbf{r})=1$ if the point $\mathbf{r}$ lies inside the ellipsoid $x^2/q^2+\rho^2/R^2=1$, and $\tilde\theta(\mathbf{r})=0$ otherwise and $q\ll\xi_0$. Setting $\rho'=(q/R)\rho$, and subsequently dropping the primes, we arrive at the equation

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial x^2}-\frac{1}{2m_\rho}\frac{\partial^2}{\partial\rho^2}-\frac{\gamma}{q}\theta(q-|r|)\right]\psi=-E_c\psi, \quad (6)$$

where $m_\rho = mR^2/q^2\gg m$. Solving (6) with the use of the adiabatic approximation, i.e., separating the fast motion in x and the slow motion in the yz plane, we finally have

$$\tau_R=\tau_0(\xi(\tau_0)/R)^2\exp[-8\xi^2(\tau_0)/R^2],\ R\ll\xi(\tau_0),$$
$$\tau_R=\tau_0[1-2\xi(\tau_0)/R],\ R\gg\xi(\tau_0). \quad (7)$$

Thus, the critical temperature for the onset of LS depends essentially on the dimensions of the TP: τ_R rapidly goes to zero for the sections of the TP that have dimension $R<\xi(\tau_0)$. The result obtained indicates that, if the TP in tin consists of sections for which the range of variation of the quantity $R/\xi(\tau_0)$ is broad, then the transition into the LS state will be substantially smeared with respect to temperature. The effect of this circumstance on the characteristics of the LS is discussed below. Notice that the analysis pertained to an isolated TP section. For large sections with $R\gtrsim\xi(\tau_0)$ located at distances greater than $\xi(\tau_0)$ from each other, the effect of the interaction with neighboring sections can be neglected (since $\psi(\mathbf{r})$ decays over distances $\sim\xi(\tau_0)$ from the edge of the TP in the $\boldsymbol{\rho}$ plane), and our approximation is justified.

3. Localized superconductivity in a plate of finite thickness; closely spaced parallel twinning planes

The localized superconductivity sets in at temperatures very close to the critical temperature T_{c0} for the onset of superconductivity in the bulk metal; this is caused by the departure of superconducting electrons from the region around the TP, i.e., essentially by a proximity effect. The effect in question may be weaker in films of finite thickness, and then the T_c will be higher.

Let us consider a film of thickness L. Let the x axis be perpendicular to the film, with $-L/2<x<L/2$; the TP is, as before, located in the $x=0$ plane (in this section we assume the TP to be infinite). To determine the transition temperature T_L, we must solve the linearized equation (4) with the boundary conditions $\psi'(\pm L/2)=0$. As a result, we find

$$\tau_L=4\tau_0\xi(\tau_0)/L,\quad \xi_0\ll L\ll\xi(T_0),$$
$$\tau_L=\tau_0[1+4\exp(-L/2\xi(\tau_0))],\ L\gg\xi(\tau_0), \quad (8)$$

where $\tau_L=(T_L-T_{c0})/T_{c0}$. The dependence $\tau_L(L)$ is fully depicted by the curve 2 in Fig. 1.

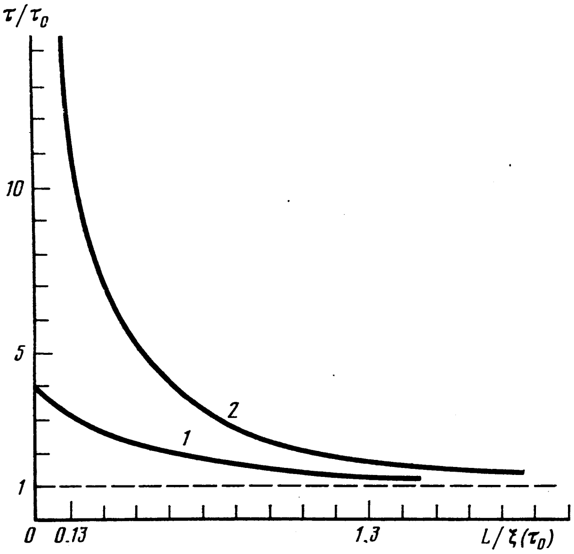

FIG. 1. Dependence of the critical temperature for the onset of LS on the distance L between two parallel TP (curve 1) and on the period L of an infinite series of TP, as well as on the thickness L of a film containing one TP (curve 2).

Experimentally, it is more convenient to observe the LS in closely spaced parallel TP. In the case of two parallel TP we arrive at the $\tau_L(L)$ dependence depicted by the curve 1 in Fig. 1, where L is the distance between the TP. If we have a periodic series of TP with period L, then, solving the linearized equation (4) with a potential in the form of a periodic set of δ functions, we find a $\tau_L(L)$ dependence coinciding with (8) (the curve 2 in Fig. 1). Notice that the $\tau_L(L)$ dependences depicted in Fig. 1 are valid for $L \gg \xi_0$; we cannot use the GL equations in the opposite case. As analysis of the integral equation (1) shows, for $L \ll \xi_0$, the transition temperature for a periodic series of TP will be determined by the mean value $\langle \lambda \rangle = (\lambda_1 d + \lambda_0 L)/L$ of the electron-phonon interaction constant (see Ref. 10), which in the case of tin should give rise to a T_c significantly higher than T_{c0}.

II. EFFECT OF MAGNETIC FIELD ON LOCALIZED SUPERCONDUCTIVITY

1. The field H_{c2} for the production of a superconducting nucleus

To determine the field H_{c2} for LS near an infinite TP, we must consider the linearized GL equation (4), where the field is introduced in the normal fashion:

$$\partial/\partial \mathbf{r} \rightarrow \partial/\partial \mathbf{r} - (2ie/c)\mathbf{A},$$

A being the vector potential of the external field. In the case of an external field perpendicular to the twinning plane, the calculation is entirely similar to the standard method of determining H_{c2} (Ref. 8), and yields

$$H_{c2\perp}(T) = 0.29 \frac{\Phi_0}{\xi_0^2} \frac{(T_c - T)}{T_{c0}},$$

where $\Phi_0 = \pi c\hbar/e$ is the flux quantum.

In the case of a field parallel to the TP, the problem actually reduces to the level-shift problem in a one-dimensional δ-function well under the influence of the oscillator potential produced by the magnetic field. It is clear that for weak fields it is possible to use perturbation theory, and the field can be considered to be weak if $\hbar\omega_c \ll -E_0 = 2\tau_0/\eta$, where $\omega_c = eH/mc$. Thus, perturbation theory can be used for fields $H \ll \Phi_0\tau_0/\xi_0^2$. For weak fields we can neglect the field-induced change in the form of the wave function $\psi(x)$. Carrying out the perturbation-theory calculation, we find that near the critical point T_c for the onset of LS

$$H_{c2\parallel}(\tau) = 0.42\tau_0^{1/2}(\Phi_0/\xi_0^2)(\tau_0 - \tau)^{1/2} \qquad (\tau_0 - \tau \ll \tau_0). \tag{9}$$

The determination of $H_{c2\parallel}$ in the temperature region where $\tau_0 - \tau$ is not small requires numerical computations, since perturbation theory is not applicable here. Figure 2 shows a plot of the function $H_{c2\parallel}(T)$; the value of $H_{c2}(\tau = 0) \approx 0.5\Phi_0\tau_0/\xi_0^2$.

2. The screening of a weak parallel field

Knowing the behavior of the order parameter $\psi(x)$, and solving the equation for the field

$$A'' = (8\pi e^2/m)\psi^2(x)A$$

with the boundary conditions $A(x) = Hx$ for $x \rightarrow \pm\infty$, where H is the field at points far from the TP, we can determine the character of the screening of a weak magnetic field in our system. An analytical solution can be obtained only in the region $(\tau - \tau_0)/\tau_0 \ll 1$, where $\psi(x) \propto \exp[-|x|/\xi(\tau)]$. In this case

$$B(-x) = B(x) = Hk\exp(-|x|/\xi)[K_1(u) + K_0(k)I_1(u)/I_0(k)], \tag{10}$$

$$u = k\exp(-|x|/\xi), \qquad k = 3\xi_0(\tau_0 - \tau)^{1/2}/\lambda_L(0)(\tau^{1/2} + \tau_0^{1/2}),$$

where $\lambda_L(0)$ is the London penetration depth at $T = 0$, while I and K are the modified Bessel functions. In the case $k \gg 1$ the field screening near $x = 0$ is practically complete; the screening is weak at small k values:

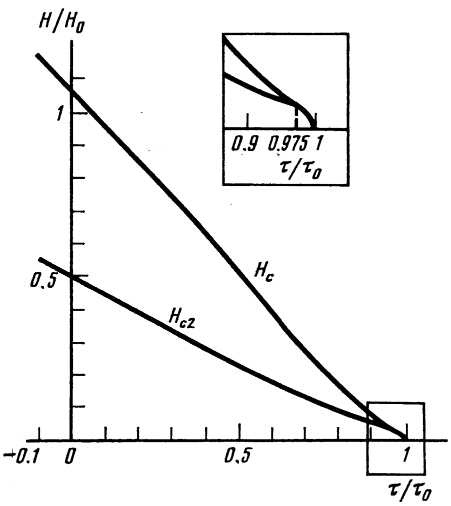

FIG. 2. Phase diagram of the localized superconductivity of the TP in tin for a parallel magnetic field (the field is in units of $H_0 = \Phi_0\tau_0/\xi_0^2$). H_c is the critical field for the first-order transition and H_{c2} is the supercooling field. The inset shows the region around the tricritical point.

$$B(0)=H(\pi k/2)^{1/2}e^{-k},\quad k\gg 1,$$
$$B(x)=H[1-0.24k^2(1+2|x|/\xi)\exp(-2|x|/\xi)],\quad k\ll 1. \quad (11)$$

The diamagnetic moment per unit area is

$$-M=\int\frac{H-B}{4\pi}dx=\begin{cases}\xi(\tau)Hk^2/8\pi, & k\ll 1,\\ \xi(\tau)H[\ln(k/2)+0.57]/2\pi, & k\gg 1.\end{cases} \quad (12)$$

We have already noted above that the temperature dependence of the moment (12), obtained for the infinite plane, is nearly a power-law dependence, whereas an exponential decrease of the moment is observed in experiment[1] as the temperature is raised from T_{c0}. The causes of this discrepancy will be discussed below.

3. Nature of the transition in a magnetic field

In this section we investigate the character of the transition into the LS state in a parallel constant external magnetic field. As has already been noted, the analysis can be performed within the framework of the GL functional (3).

In order to determine the nature of the transition, let us eliminate the field $B(x)$ from (3) and obtain an expansion of the free energy in powers of $\psi^2(0)$; then, as usual, a negative coefficient attached to $\psi^4(0)$ [for $H=H_{c2}(t)$] will indicate a first order transition. Of greatest interest is the behavior of type-I superconductors: it is precisely in their case that the nature of the transition can change under the action of a field. Analytically, it is possible to consider only the case of a superconductor that is of the first type in the extreme, i.e., a superconductor with $\varkappa\ll 1$, where $\varkappa\propto\lambda_L(0)/\xi_0$ is the Ginzburg–Landau parameter (the case $\varkappa\ll 1$ is precisely the case that is realized in tin).

For $\varkappa\ll 1$, as will be seen from the results of the present section, the tricritical point τ_t differs from the critical temperature for the onset of LS in zero field only by a quantity of the order of the parameter $\varkappa^2$, i.e., $\tau_0-\tau_t\ll\tau_0$. This means that (see the expression of (9)) the transition field is weak $H_{c2}(\tau_t)\ll\Phi_0\tau_0/\xi_0^2$, and we can neglect the effect of the field on the form of $\psi(x)$. At the transition point τ_t the first-order transition field $H_c(\tau_t)$ coincides with $H_{c2}(\tau_t)$.

Thus, we can eliminate $B(x)$ from the functional (3), using the results obtained in Subsec. 2 of this section for the screening of a weak field, i.e., the expression (11) [in this case the amplitude $\psi^2(0)$ should, of course, not contain the temperature]. Finally, eliminating $B(x)$, and performing the integration over x in (3), we find that the coefficient b' attached to $\psi^4(0)$ has the form

$$b'\propto(1-H^2/H_0^2),\quad H_0^2=0.23\Phi_0^2\tau_0^2\lambda_L^2(0)/\xi_0^6. \quad (13)$$

Substituting the upper critical field $H_{c2}(\tau)$, (9), into (13), we find the following expression for the tricritical transition temperature from the condition $b'=0$:

$$\tau_t=(T_c-T_t)/(T_c-T_{c0})=1.5(\lambda_L^2(0)/\xi_0^2)\tau_0. \quad (14)$$

As follows from (14), all the assumptions underlying our analysis are fulfilled in the $\varkappa\ll 1$ case.

We can conclude that there exists in the (H,T) plane in the case of a type-I superconductor a tricritical point $[H_{c2}(\tau_t),\tau_t]$ at which the character of the transition changes (see Fig. 2). Qualitatively, this circumstance is explained by the fact that the effective screening distance in the case of LS increases as $\tau\to\tau_0$, since $\lambda_{\rm eff}^2\propto\lambda_L^2(0)/(\tau_0-\tau)$, whereas the correlation length ξ depends weakly on the temperature $\xi^2\propto\xi^2(\tau_0)\propto\xi_0^2/\tau_0$, and the transition changes its character when $\lambda_{\rm eff}\propto\xi$, i.e., when $\tau_0-\tau_t\propto\tau_0\lambda_L^2(0)/\xi_0^2$.

Thus, a characteristic of LS is the certain presence of a second-order phase transition region. In the case of type-II superconductors the transition into the LS state in a magnetic field is of second order everywhere, while in the case of type-I superconductors with $\varkappa\ll 1$ the transition is of second order only in a narrow neighborhood of T_c.

At temperatures $T<T_c$ the transition is of first order, and the curve $H_c(T)$ of this transition lies above the curve $H_{c2}(T)$, which in this case has the meaning of a supercooling line for the normal phase.

4. The transition field H_c for a type-I superconductor

The problem of the shape of the curve $H_c(T)$ can be solved for superconductors that are of the first type in the extreme, i.e., superconductors with $\varkappa\ll 1$. In this case, as noted above, the effective screening distance $\lambda_{\rm eff}\ll\xi(\tau_0)$ everywhere except in a narrow range of $\tau\propto(\lambda_L/\xi_0)^2$ values around T_c. This circumstance indicates that the field decreases sharply at the LS boundary, practically not penetrating into the interior. Therefore, we can assume that the LS exists in a region $-L<x<L$ (the dimension $2L$ of which is itself to be determined) where there is no field and the order parameter is equal to zero at the boundary: $\psi(\pm L)=0$. In this case we neglect the contribution to the energy (3) from the field-decay region; the calculation here is, as usual,[8] accurate to within terms of the order of $\varkappa^{1/2}$.

Introducing the dimensionless quantities[8]:

$$\bar{x}=\frac{x}{\lambda_L},\quad \Psi=\psi\left(\frac{b}{|a|}\right)^{1/2},$$

$$\lambda_L^2=\frac{mb}{8\pi e^2|a|},\quad \bar{A}=\frac{A}{H\lambda_L},\quad \varkappa^2=\frac{m^2b}{2\pi e^2}$$

we can write the approximate expression for the free energy (3) in the form

$$F=\frac{2a^2\lambda_L}{b}\int_0^\infty d\bar{x}\left\{\frac{2}{\varkappa^2}(\Psi')^2+\alpha\bar{A}'(\bar{A}'-1)\right\}-\gamma\frac{|a|}{b}\Psi^2(0), \quad (15)$$

where $\alpha(H)=(H^2/4\pi)(b/a^2)$, and F is the difference between the free energies of the superconducting and normal phases. The first integral of the GL equations has the form

$$(2/\varkappa^2)(\Psi')^2+\alpha|\bar{A}'|^2-\Psi^2(2+\alpha\bar{A}^2)-\Psi^4=\alpha. \quad (16)$$

Neglecting the contribution to the free energy resulting from the penetration of the field into the LS region, we can write

$$F=\frac{2a^2\lambda_L}{b}\int\limits_0^{\bar{L}} d\bar{x}\,\frac{2\bar{\psi}'^2}{\varkappa^2}-\gamma\frac{|a|}{b}\bar{\psi}^2(0),\qquad \bar{L}=L/\lambda_L. \qquad (15')$$

Using the first integral (16), we have for the region $-L<x<L$ (here and below we drop the bars over the letters):

$$[\psi^2+\psi^4/2+\alpha/2]^{-1/2}d\psi=-\varkappa dx.$$

Going over in (15′) from integration over x to integration over ψ, and minizing with respect to $\psi(0)$, we finally obtain

$$2(\tau_0/\tau)\psi^2(0)-2\psi^2(0)-\psi^4(0)=\alpha(H), \qquad (17')$$

$$F=2\sqrt{2}\frac{\lambda_L a^2}{b\varkappa}\int\limits_0^{\psi(0)}[\alpha+y^4+2y^2]^{1/2}dy-\gamma\frac{|a|}{b}\psi^2(0); \qquad (17'')$$

the dimension of the LS region is then given by the integral

$$\frac{L}{L_0}=\int\limits_0^{\psi(0)}\frac{dy}{[\alpha+2y^2+y^4]^{1/2}},\qquad L_0=\sqrt{2}\frac{\lambda_L}{\varkappa}. \qquad (18)$$

The relations (17) and (18) allow us, in principle, to solve the problem completely: from the condition $F=0$ and from (17″) we find the quantity $\psi(0)$, from (17′) we determine the transition field $H=H_c$, and from (18) we get the dimension L of the LS region. The characteristic dimension of the LS region is, as can be seen from (18), of the order of $\xi(\tau_0)$. Carrying out the requisite numerical computations, we find the dependence $H_c(T)$ shown in Fig. 2. Notice that our approximation is inapplicable in the vicinity of T_c (see above). The quantity H_c at $T=T_{c0}$ is given by $H_c(\tau=0)\approx 2.2\tau_0(N/\eta)^{1/2}$, and the ratio

$$H_{c2}(\tau=0)/H_c(\tau=0)=4.2\lambda_L(0)/\xi_0. \qquad (19)$$

The field dependence of the diamagnetic moment is shown in Fig. 3 for different temperatures.

5. The critical current

The method of computing the critical current in thin films with a constant value of the order-parameter amplitude is sufficiently extensively expounded in the literature. An interesting property of localized superconductivity is the fact that the method is suitable for determining the critical current in the case of a coordinate-dependent order parameter as well, though only in a rather narrow neighborhood of the critical point T_c.

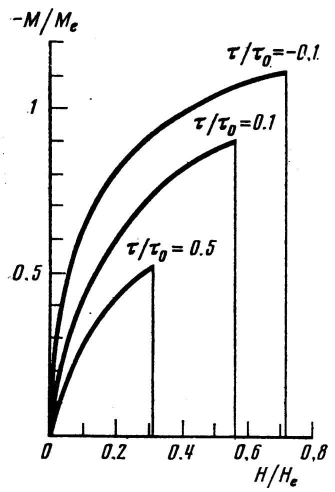

FIG. 3. Dependence of the diamagnetic moment $M=HL(H)$, in units of $M_e=(2\tau_0\pi N/m)^{1/2}$, $H_c=2\tau_0(\pi N/\eta)^{1/2}$, on the magnetic field at different temperatures for the infinite twinning plane.

Let us consider the superconducting current flowing along the twinning plane; the Y axis is oriented along the current and the X axis is, as always, perpendicular to the TP. The coordinate dependences of the order parameter and the current have the following form

$$\psi=|\psi(x)|\exp(i\varphi(y)),\qquad j=ec_0|\psi(x)|^2/m$$

where $c_0=\partial\varphi/\partial y=\text{const}$, on account of the condition $\operatorname{div}\mathbf{j}=0$. We can, taking into consideration the complexity of the order parameter, easily transform the GL equation (4) into the form

$$\left[\frac{1}{4m}\frac{\partial^2}{\partial x^2}-\frac{\tau_1}{\eta}-b|\psi|^2\right]|\psi|=-\gamma\delta(x)|\psi|,$$

where $\tau_1=\tau+c_0^2\eta/4m$. In the present equation we have dropped the terms containing the magnetic field of the current. The criterion for the validity of this approximation will be discussed below.

The solution to the equation will be the solution (5) with τ replaced by τ_1. Consequently, we have

$$|\psi|^2(x=0)=\psi_0^2=2N(\tau_0-\tau_1),\qquad j(x=0)=j_0=ec_0\psi_0^2/m$$

whence we find that

$$j_0=(2e^2/m\eta N)^{1/2}\psi_0^2(2N(\tau_0-\tau)-\psi_0^2)^{1/2}.$$

It is easy to see from the present formula that the critical value of the current amplitude is attained at a value of ψ_0^2 equal to $4N(\tau_0-\tau)/3$.

Let us give the dependence of the total critical current through a unit length in the TP:

$$I_c=8Ne(\tau_0-\tau)^{3/2}/3m((3\tau_0)^{1/2}+(2\tau+\tau_0)^{1/2}).$$

The condition for the applicability of the computational method used was assumed to be the weakness of the magnetic field of the current. Determining the maximum value of the field intensity from the equation

$$\partial B/\partial x=-4\pi j(x),\qquad B(0)=0,$$

and requiring the fulfillment of the condition $\hbar\omega_c < -E_0=(2\tau_0+\eta c_0^2/2m)/\eta$, we obtain the region of applicability of the relations derived:

$$(\tau_0-\tau)/\tau_0\ll\varkappa^{4/3}.$$

In this region of temperatures the LS is equivalent to a thin film with $\lambda \gg d$ ($d\sim\xi(\tau_0)$, $\lambda\sim|\psi|^{-2}$), therefore, the orbital effects, which are of the order of d/λ, can be neglected. We can approximately estimate the I_c for other temperatures, choosing the minimum current from the critical current computed above without allowance for the field produced by it and the current I_H generated by the critical field $H_c(\tau)$. On this basis we obtain the estimate

$$I_c\sim I_H\sim H_c,\quad\text{for}\quad(\tau_0-\tau)/\tau_0>\varkappa^2.$$

III. CONCLUSIONS; DISCUSSION OF THE EXPERIMENTAL SITUATION

The above-proposed theory of LS near an infinite TP essentially contains only one "free" parameter (the quantity τ_0) that depends on the details of the electron-phonon interaction and the characteristics of the electronic spectrum near the TP. The remaining important characteristics $\lambda_L(0)$ and ξ_0, which describe the behavior of LS in a magnetic field, are entirely determined by the bulk metal, and are well known. Thus, there is the possibility of a direct experimental verification of the theory. The presently available experimental data[1,2] allow us to carry out such a comparison. We must, however, bear in mind the inhomogeneous nature of the TP in tin (it is precisely for tin crystals, which we shall have in mind below, that we have the most complete data[1,2,11]). The presence of dimensionally different TP sections leads to a situation in which each such section has its own critical fields $H_c(R)$ and $H_{c2}(R)$, which increase with increasing R, i.e., the greater the dimension R, the higher they are.

In a strong field all the sections are in the normal state. As the field is decreased to a value below the H_c value, we can expect successive transitions into the LS state of the large, and then the smaller, TP sections. For the large sections [the characteristic dimension of which is large compared to $\xi(\tau_0)$], the critical fields practically coincide with the critical fields for the infinite plane. But on account of the fact that for tin ($\varkappa \approx 0.13$) the transition in a nonzero field is a first-order transition, the activation energy barrier for the superconducting nucleus must be overcome, and there is observed a normal-phase "supercooling" in field terms. Figure 4 shows a typical experimental[1] $M(H)$ curve. In a nonzero field the normal state will be preserved until the spontaneous production of superconducting nuclei begins. It is important to note that the field H_{c2} of the infinite TP will be the supercooling field, since the larger TP sections have the highest H_{c2} values, which coincides with $H_{c2}(\infty)$.

Thus, the field in which a jump occurs in the moment M_D (Fig. 4) is the field H_{c2} for the infinite TP. Let us point out an interesting circumstance: the irregular character of the TP does not prevent the experimental determination of the quantity $H_{c2}(\infty)$. As to the magnitude of the jump in the moment M_D, it depends essentially on the size distribution of the TP sections.

As the external magnetic field intensity is increased from zero, the LS is gradually suppressed, beginning with the small TP sections. These transitions occur as first-order transitions when the field attains the $H_c(R)$ values. The "superheating" in field terms of the localized superconductivity is possible in principle as a result of the existence of an activation energy for the normal nucleation center. But in the case of LS the normal phase is always present near the TP (bulk metal), and there will not be superheating: the transition will occur precisely in the H_c field (our attention was drawn to this fact by M. S. Khaĭkin and I. N. Khlyustikov). The moment should undergo a jump upon the destruction of the LS in the $H_c(R)$ field, but since only a small part of the TP goes over into the normal state in the field in question, the resulting $M(H)$ curve will be smooth. The vanishing of the moment indicates the attainment by the field of the value $H_c(\infty)$: the superconductivity is destroyed in all the TP regions. The absence of a jump at $H = H_c(\infty)$ indicates a relatively small number of large TP regions.

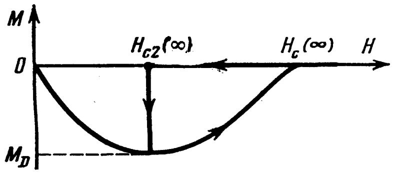

FIG. 4. Typical field dependence of the diamagnetic moment M as experimentally observed in tin.[1]

Thus, the inhomogeneity of the TP does not prevent the experimental determination of the quantities H_c and H_{c2} for the infinite TP. The slopes dH_{c2}/dT and dH_c/dT at $\tau = 0$ and the ratio $H_c(0)/H_{c2}(0)$ are universal τ_0-independent quantities. Using the parameter values $\xi_0 = 3.2\times 10^{-5}$ cm and $\varkappa \approx 0.13$ obtained for tin, we find that $dH_{c2}/dT = 25$ G/K, $dH_c/dT = 55$ G/K, and $H_c(0)/H_{c2}(0) \approx 2$. The experimental data[11] are as follows: $dH_{c2}/dT \approx 40$ G/K, $dH_c/dT \approx 120$ G/K, and $H_c(0)/H_{c2}(0) \approx 3$, which are in quite good agreement with the theory, considering the indeterminacy in the field orientation with respect to the TP and the tensor character of ξ_0.

The results reported in Ref. 11 also indicate, in accord with the predictions made in Subsec. 3 of Sec. II, the existence in tin of a tricritical point where the H_c and H_{c2} curves meet. It would be of interest to experimentally investigate the region around T_t, but the small values of the corresponding fields and moments in tin make such an investigation extremely difficult. A suitable object might be a superconductor with $\lambda_L(0) \lesssim \xi_0$, for which $\tau_t \sim \tau_0$.

A significantly higher τ_0 value is experimentally observed[10] in the case of two close TP's. From the appropriate dependence in Fig. 1 we find that a TP spacing L of the order of 10^4 Å corresponds to a doubling to τ_0. This value agrees with Kirzhnits and Maksimov's independent estimate for L.[11]

The experimentally observed[1] rapid exponential decrease of the moment with increasing temperature also needs to be explained, since the theory predicts a significantly weaker dependence (see Subsec. II.2). The fact that the TP in tin in highly inhomogeneous allows a natural explanation of this discrepancy. Indeed, as shown in Subsec. I.2, the transition temperature depends strongly on the dimensions of the TP. The number of TP sections with LS decreases with increasing temperature, which is the main factor in the decrease of the moment. In this case it is, of course, necessary that the TP sections be sufficiently far from each other [at distances large compared to $\xi(\tau_0)$]. This condition is fulfilled for the samples investigated by Khaĭkin and Khlyustikov.[1,2] Indeed, it follows from the data presented in Ref. 1 that the absolute magnitude of the diamagnetic moment is much smaller than what we can expect in the case of an infinite TP: according to Ref. 1, at $\tau = 0$ the moment per unit area is in order of magnitude equal to $\xi_0 H/4\pi$, whereas for the infinite TP it is, according to Subsec. II.4, equal to $\xi(\tau_0)H/4\pi$. This indicates that the mean dimension of the TP sections is smaller than the distance between them.

If the size distribution of the TP sections is such that the mean dimension is smaller than $\xi(\tau_0)$, then the decrease of the moment with temperature will be determined by the asymptotic form of the distribution function for large TP dimensions. As an illustration, let us consider the following model for a TP. We shall assume that the TP consists of small circles of radius $R<\xi(\tau_0)$ randomly distributed over the $x=0$ plane. Let the mean distance between the circles be much greater than $\xi(\tau_0)$, i.e., let their concentration c be low: $cR^2 \ll 1$. We are interested in TP regions with dimensions $r \gg R$. They are produced through the joining of $n \gtrsim r^2/R^2$ small circles. The probability that a circle belongs to a cluster with dimension $r \gg R$ is given by the Poisson distribution:

$$\frac{1}{\sqrt{2\pi}}\frac{R}{r}\exp\left(-\frac{r^2}{R^2}\ln\frac{1}{cR^2}\right).$$

This exponential dependence on the dimension r leads to an exponential dependence of the LS area, and, hence, of the moment, on the temperature. Using the dependence (7) for τ_r in the case $r>\xi(\tau_0)$, we arrive at the expression

$$M\propto cR^2\frac{R}{\xi(\tau_0)}\frac{T_c-T}{T_c-T_{c0}}\exp\left[-\frac{\xi^2(\tau_0)}{R^2}\left(\frac{T_c-T_{c0}}{T_c-T}\right)^2\ln\frac{1}{cR^2}\right],$$

which is valid for $T_c-T \ll T_c-T_{c0}$. It is also clear that there will be a sharp exponential decrease in the region $T_c-T \lesssim T_c-T_{c0}$.

Thus, the inhomogeneous character of the TP qualitatively explains the experimentally observed rapid decrease of the moment with increasing temperature. But it is not possible to carry out a quantitative comparison with the experimental $M(T)$ curve, since the size distribution of the TP sections is not known.

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