Contribution to the nonlinear theory of magnetostatic spin waves

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We investigate the nonlinear properties of nonexchange magnetostatic spin waves (MSW) produced in a ferromagnetic film by four-magnon self-action processes. Equations for the evolution of the envelope are derived in the weak nonlinearity and thickness inhomogeneity approximation for all three types of waves in the film-surface waves and forward and backward bulk MSW. The stability of the nonlinear MSW to longitudinal and transverse perturbations is investigated. The threshold powers for the onset of self-modulation and self-channeling of the considered MSW is investigated.

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The nonlinear properties of magnetostatic spin waves (MSW) propagating in a magnetic film (wave numbers $k \sim 10\text{–}10^3$ cm^{-1}) are of interest from two points of view. First, the MSW have rather low exciting-signal thresholds compared with the optic acoustic waves at which nonlinear wave processes set in.[1] Second, the study of the nonlinear properties is of practical importance in view of the prospects of using MSW in microwave electronics.[2]

1. We shall single out among the nonlinear processes the effects connected with self-action of the wave, when its nonlinear properties are due to the change of its own amplitude. Some features of nonlinear effects of this type were investigated earlier for propagating surface MSW.[3] In the present paper we consider the nonlinear properties of all three types of MSW that propagate in a film, namely surface waves and forward and backward bulk waves.

We start with the Landau-Lifshitz and magnetostatics equations[1)]:

$$\text{rot}\,\mathbf{h}=0, \quad \text{div}\,(\mathbf{h}+4\pi\mathbf{m})=0, \quad \dot{\mathbf{m}}=-\gamma[\mathbf{m}\times\mathbf{h}], \tag{1}$$

where **h** is the magnetic field, **m** is the magnetic moment, and γ is the gyromagnetic ratio. Solution of these equations in the linear approximation with the magnetostatic boundary conditions $h_\parallel$, $h_\perp + 4\pi m_\perp = \text{const}$ ($h_\parallel$, $h_\perp$, and $m_\perp$ are respectively the tangential component and the normal components of the magnetic field and of the magnetization) makes it possible to determine the structure of the MSW. We choose the coordinate frame such that its origin is at the midpoint of the film and the Z axis is perpendicular to the film plane. We consider for the sake of argument a magnetic film, normally magnetized (to saturation), when $H \parallel Z$. In this case forward bulk MSW propagate in the film[4] and their spectrum is isotropic in the XY plane. The solution of Eqs. (1) that are linearized in terms of the wave amplitude can be represented in the form[2)]:

$$\Psi = \frac{4\pi M}{k\chi_2} \times \varphi \exp[i(\mathbf{kr}-\omega t)] \begin{cases} \cos k_1 d \exp[-k(|z|-d/2)], & |z|>d/2, \\ \cos k_1 z, & |z|<d/2, \end{cases} \tag{2}$$

where φ is the dimensionless amplitude of the wave of the potential Ψ ($\nabla\Psi = \mathbf{h}$), and **r** is the radius vector in the plane of the film,

$$\mathbf{k}=(k_x, k_y, 0), \quad k_1=k[-(1+\chi_1)]^{1/2}, \quad \chi_1=\omega_H\omega_M/(\omega_H^2-\omega^2),$$
$$\chi_2=\omega\omega_M/(\omega_H^2-\omega^2), \quad \omega_H=\gamma(H-4\pi M_z), \quad \omega_M=\gamma 4\pi M_z.$$

Knowing the potential Ψ we can determine from Eqs. 1 the magnetization

$$4\pi\begin{pmatrix} m_x \\ m_y \end{pmatrix} = ik\Psi \begin{pmatrix} \chi_1 \\ -i\chi_2 \end{pmatrix}. \tag{3}$$

The dispersion equation for forward bulk MSW is of the form

$$2+\chi_1+2[-(1+\chi_1)]^{1/2}\,\text{ctg}\,kd[-(1+\chi_1)]^{1/2}=0. \tag{4}$$

This and the other dispersion equations will be represented below in the form $G(k,\omega)=0$.

To analyze the nonlinear MSW we use an approach based on the ideas of geometric optics, known in the literature as the method of "envelopes."[5,6] According to this method, an approximate solution of the nonlinear equations (1) is sought likewise in the form (2), but the complex amplitude φ is assumed to be a function of the coordinate and of the time, i.e., $\varphi = \varphi(\mathbf{r},t)$ and to vary slowly over a distance of the order of the period of the fundamental harmonic. It is assumed that the higher harmonics due to the nonlinearity are small. The presence of dispersion prevents in the general case the growth of the higher harmonics (at least at not too large amplitudes of the fundamental). The role of the nonlinearity manifests itself in a dependence of the frequency or of the phase velocity on the wave amplitude, i.e., the nonlinear dispersion equation takes the form

$$G(k, \omega, |\varphi|)=0. \tag{5}$$

The dispersion relation (5) enables us to write down a nonlinear parabolic equation for the function $\varphi(\mathbf{r},t)$. This is done in the following manner (for details see Refs. 5 and 6). Let ω_0 and k_0 be the frequency and the wave vector of the fundamental. We expand the function $G(k,\omega,|\varphi|)$ in a series in $\Delta k = k - k_0$, $\Delta\omega = \omega - \omega_0$, and $|\varphi|$:

$$G=G(k_0,\omega_0)+\frac{\partial G}{\partial\omega}\Delta\omega+\frac{\partial G}{\partial k_i}\Delta k_i+\frac{1}{2}\frac{\partial^2 G}{\partial k_i \partial k_j}\Delta k_i\Delta k_j + \frac{1}{2}\frac{\partial^2 G}{\partial\omega^2}\Delta\omega^2+\frac{\partial^2 G}{\partial\omega\partial k_i}\Delta\omega\Delta k_i+\frac{\partial G}{\partial|\varphi|^2}|\varphi|^2=0.$$

The expansion coefficients are taken at $\Delta\,\mathbf{k},\Delta\omega,|\varphi|=0$. Replacing the deviations of the frequency and of the wave vec-

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tor by the operators $\Delta\omega = i\partial/\partial t$ and $\Delta \mathbf{k} = -i(\partial/\partial x, \partial/\partial y, 0)$ and recognizing that $G(k_0,\omega_0) = 0$, we obtain from the last relation a nonlinear parabolic equation for[3] φ:

$$i\frac{\partial\varphi}{\partial t}+iv_g\frac{\partial\varphi}{\partial x}+\frac{1}{2}\beta_{ij}\frac{\partial^2\varphi}{\partial x_i\partial x_j}-\alpha|\varphi|^2\varphi=0. \quad (6)$$

In the derivation of (6) it was assumed also that $k_0 = (k_0,0,0)$ and $G = G(\omega,k_x,k_y,|\varphi|^2)$. In addition, the substitution $\partial/\partial t \to -v_g\,\partial/\partial x$. was made in the terms with the second derivative. Using the dispersion relations for the corresponding MSW, we easily obtain the coefficient β_{ij} of (6). The coefficient α in (6) can also be easily determined by calculating the dependence of the frequency of the homogeneous ferromagnetic resonance for the corresponding film-magnetization geometry, a procedure justified as $k \to 0$.

2. Before we proceed to the investigation of concrete cases, let us dwell briefly on the limitations of the employed method as applied to MSW. To this end we must discuss the effects of nonlinear interaction of MSW with other harmonics (multimagnon processes). Wave propagation in a nonlinear medium is accompanied by excitation of other harmonics. Their influence on the propagation on the fundamental wave reduces to the appearance, in the right-hand side of (6), of a driving force proportional to the product of the amplitudes that take part in the interaction of the waves) magnons). At sufficiently low amplitude of the fundamental wave the analysis is usually confined to three- and four-magnon processes. In three-magnon processes the dependence of the aforementioned driving force on the time and on the coordinates is of the form $\exp\{i[\mathbf{k}_1 \pm \mathbf{k}_2)\mathbf{r} - (\omega_1 \pm \omega_2)t]\}$. One can therefore expect the response of the system to this small deriving foce to be insignificant so long as there is no resonance, i.e., $\mathbf{k}_1 \pm \mathbf{k}_2 \neq \mathbf{k}_0)$, $\omega_1 \pm \omega_2 \neq \omega_0$. Under resonance conditions energy is transferred from the fundamental mode to others that interact with it: $(\mathbf{k}_0,\omega_0) \to (\mathbf{k}_1,\omega_1),(\mathbf{k}_2,\omega_2)$. The presence of such magnon decay (or coalescence processes) depends on the type of the MSW, on its frequency, and on the applied external magnetic field.

The resonance regions for three-magnon processes in a magnetic film were determined in Refs. 1 and 7. Obviously, these processes can be the cause of the instability of the MSW. Three-magnon processes were studied in detail by Suhl[8] and in later papers,[9,10] in connection with the problem of the instability of homogeneous magnetization precession in a ferromagnet. A calculation similar to that in Ref. 8 leads to the following expression for the threshold amplitude of the homogeneous precession in the case of tangential magnetization of the film:

$$\left|\frac{m_y}{M}-i\frac{\omega}{2\omega_H}\frac{m_x}{M}\right|^2=\frac{4\omega^2(\delta\omega)^2}{(\omega^2-4\omega_H^2)(4\omega_0^2-\omega^2)}, \quad (7)$$

where $\omega_0 = [\omega_H(\omega_H + \omega_M)]^{1/2}$, $\omega_H = \gamma H$, and $\delta\omega$ is the linewidth of the excited spin wave of frequency $\omega(k) = \omega/2$ $(\omega_H < \omega(k) < \omega_0)$. Recognizing that as $k \to 0$ and $\omega \to \omega_0$ the magnetization in the MSW is connected with its amplitude in the following manner [see (2) and (3)]:

$$\begin{pmatrix} m_x \\ m_y \end{pmatrix}=M\varphi\begin{pmatrix} i\omega_H/\omega \\ 1 \end{pmatrix},$$

we obtain from (7) an approximate expression for the MSW threshold amplitude

$$|\varphi|^2={}^{16}/_9\,\omega^2(\delta\omega)^2/(\omega^2-4\omega_H^2)(4\omega_0^2-\omega^2). \quad (8)$$

The corresponding estimate of the threshold power will be obtained below.

As for four-magnon processes, they can be divided into two types.[11] The first, "diagonal," describe accurate to third-order terms the "self-action" of the wave, which leads to a dependence of the frequency or of the phase velocity on the wave amplitude. They are taken into account in Eq. (6). The "nondiagonal" terms describe the changes of the phase velocities as a result of the mutual influence of the pair of waves—an effect hardly ever investigated for MSW. We shall neglect this effect.

3. We consider a normally magnetized film. The structure of the wave takes in this case the form (2), and the dispersion relation is determined by Eq. (4). Recognizing that, at small deviation of the magnetization from equilibrium,

$$M_z\approx M_0\left(1-\frac{(m_x^2+m_y^2)}{2M_0^2}\right),$$

we obtain with the aid of (2) and (3) as $k \to 0$

$$\omega_M=\gamma 4\pi M_z\to\omega_M{}^0(1-|\varphi|^2),\quad \omega_H=\gamma(H-4\pi M_z)\to\omega_H{}^0+\omega_M{}^0|\varphi|^2$$

as $k \to 0$. After substituting the last expressions in the dispersion relation (4) we shall see that the latter specifies implicitly the frequency dependence $\omega = \omega(k,|\varphi|^2)$. For this dependence, the equation that describes the evolution of the wave envelope is of the form

$$i\left(\frac{\partial\varphi}{\partial t}+v_g\frac{\partial\varphi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\omega}{\partial k^2}\frac{\partial^2\varphi}{\partial x^2}+\frac{v_g}{2k}\frac{\partial^2\varphi}{\partial y^2}-\frac{\partial\omega}{\partial|\varphi|^2}|\varphi|^2\varphi=0, \quad (9)$$

$$v_g=\left.\frac{\partial\omega}{\partial k}\right|_{k,\varphi=0}=\frac{\omega_M d}{4},\quad \left.\frac{\partial^2\omega}{\partial k^2}\right|_{k,\varphi=0}=-\frac{\omega_M{}^2d^2}{16\omega_H}\left(1+\frac{8}{3}\frac{\omega_H}{\omega_M}\right),$$

$$\left.\frac{\partial\omega}{\partial|\varphi|^2}\right|_{k,\varphi=0}=\omega_M. \quad (10)$$

In the derivation of (9) it was assumed that the wave-front propagation direction coincides with the X axis. The stationary solution of (9)—a wave of constant amplitude—is stable to longitudinal perturbations if $\partial\omega/\partial|\varphi|^2)/(\partial^2\omega/\partial k^2) > 0$ (see Ref. 6). It follows therefore from (10) that direct bulk MSW are unstable to longitudinal perturbations. At the same time the conditions of stability to transverse perturbations of the wave front, $(k/v_g)\partial\omega/\partial|\varphi|^2 > 0$ are satisfied for the wave. Equation (9) takes at $\partial^2\varphi/\partial y^2 = 0$ the form of a nonlinear Schrödinger equation, which is solved in principle by the inverse scattering problem method.[2,3] In this method the solution of the nonlinear equation is connected with the solution of the linear spectral problem

$$\begin{aligned}\frac{\partial\psi_1}{\partial x}&=i\lambda\psi_1+i\varphi\left(-\frac{\partial\omega/\partial|\varphi|^2}{\partial^2\omega/\partial k_\parallel^2}\right)^{1/2}\psi_2,\\ \frac{\partial\psi_2}{\partial x}&=-i\lambda\psi_2+i\varphi\left(-\frac{\partial\omega/\partial|\varphi|^2}{\partial^2\omega/\partial k_\parallel^2}\right)^{1/2}\psi_2.\end{aligned} \quad (11)$$

These equations describe scattering by a potential $\varphi(x,t)$ that is a solution of the initial nonlinear Schrödinger equation.

The time evolution of the scattering data is uniquely determined from the data on the scattering by the initial potential. By reconstructing the scattering potential for an arbitrary instant of time one can solve the Cauchy problem for the initial nonlinear equation. To reconstruct the potential it is necessary to solve a system of integral equations that is analytically solvable only for particular cases of "nonreflecting" potentials.[13] The soliton solution (a solitary wave propagating without damping and without change of shape and amplitude) corresponds to the nonreflecting spectral-problem potential containing one eigenievel λ_1. Since the number of eigenlevels λ_n is conserved in time because the linear spectral problem corresponds to a nonlinear equation, it follows that knowing the number of eigenvalues λ_n of the system (11), which correspond to an initial potential having the form of the initial perturbation $\varphi(x,0)$, we can predict the number of solitary waves into which the initial pulse will break up as $t \to \infty$. In particular, an initial pulse of constant amplitude φ_0 and duration τ (length $b_\parallel = v_g \tau$) is completely scattered as a result of dispersion if the amplitude is lower than the critical value defined by the condition

$$(\varphi_0{}^0 b_\parallel)^2 = \frac{\pi^2}{4} \left| \frac{\partial^2\omega/\partial k_\parallel{}^2}{\partial\omega/\partial|\varphi|^2} \right|. \tag{12}$$

This follows from the fact that the first eigenvalue λ_1 in the spectral problem (11) for the rectangular potential appears at $\varphi_0 > \varphi_0^0$.

The power of a forward bulk MSW is connected with its amplitude as follows:

$$P = L \int_{-\infty}^{+\infty} \frac{dz}{8\pi}(E_y H_z^*) = \frac{L\omega}{8\pi} \int_{-\infty}^{+\infty} \left| \frac{\partial\Psi}{\partial z} \right|^2 dz \underset{k\to 0}{\longrightarrow} \frac{\pi}{2} d^2 L M^2 \omega_H \varphi_0{}^2, \tag{13}$$

where L is the width of the MSW beam. In the derivation of (13) we used the equation

$$\mathrm{rot}\,\mathbf{E} = -\frac{i\omega}{c}(\mathbf{h} + 4\pi\mathbf{m}),$$

and we used the potential (2) to determine $\mathbf{h}$ and $\mathbf{m}$. From (10), (12), and (13) we obtain the threshold value of the wave power:

$$P_{\rm thr} = \frac{\pi^3}{8} \frac{L d^2 M^2}{\omega_M \tau^2} \left(1 + \frac{8}{3} \frac{\omega_H}{\omega_M}\right). \tag{14}$$

At $P > P_{\rm thr}$ rectangular pulses (with carrier frequency $\omega(k)$) are transformed in the course of propagation into solitary waves–envelope solitons. It can be seen from (14) that as $\tau \to 0$ the power $P_{\rm thr} \to \infty$. By increasing the pulse duration we can decrease the threshold value to zero. In practice, however, τ is bounded, since the soliton should manage to take form during the time of motion between the input and output converters of the MSW, the distance between which is limited by the damping of the wave. An exact answer to this question can be obtained by solving the Cauchy problem for the nonlinear Schrödinger equation with the parameters (10). For approximate estimates, however, we shall assume here that

$$\tau v_g \ll \left| \frac{\partial^2\omega/\partial k^2}{\delta\omega} \right|^{1/2},$$

where $\delta\omega$ is the MSW line width. We put $d = 10\,\mu$m, $L = 0.5$ cm, $4\pi M = 1750$ G, and $\tau = 20$ nsec. We then obtain from (14) a threshold power $P_{\rm thr} = 0.4$ mW. Formation of a large number of solitons calls for the power to be increased by $(2n-1)^2$ times, where n is the number of solitons.

4. We consider now the case when the magnetic field magnetizes the film parallel to its surface. The MSW spectrum is in this case strongly anisotropic, in contrast to the preceding one. Backward bulk and surface MSW can then propagate in the film. The dispersion equation for the former is of the form[14]

$$1+\eta^2 - 2[1+\eta^2]^{1/2}[1+\eta^2(1+\chi_1)]^{1/2}[-(1+\chi_1)]^{1/2} \times \mathrm{ctg}\, k_\parallel d \left[\frac{1+\eta^2(1+\chi_1)}{-(1+\chi_1)}\right]^{1/2} + (1+\chi_1)(1+\eta^2(1+\chi_1)) - \eta^2\chi_2{}^2 = 0, \tag{15}$$

where $\eta = k_\perp / k_\parallel$, $k_\parallel \parallel \mathbf{M}$ and $k = (k_\perp^2 + k_\parallel^2)^{1/2}$ is the wave number.

The dispersion equation of the surface MSW is

$$1+\eta^2 - 2(1+\eta^2)^{1/2}(1+\eta^2+\chi_1)^{1/2}(1+\chi_1)^{1/2}\,\mathrm{cth}\, k_\parallel d \left(\frac{1+\eta^2+\chi_1}{1+\chi_1}\right)^{1/2} + (1+\chi_1)(1+\eta^2+\chi_1) - \chi_2{}^2 = 0, \tag{16}$$

where $\eta = k_\perp / k_\parallel$ and $k_\parallel \perp \mathbf{M}$. It follows from the equations of motion (1) that as $k \to 0$ the eigenmode of the oscillations takes the form

$$\begin{pmatrix} m_x \\ m_y \end{pmatrix} = M\varphi \begin{pmatrix} i\omega_H/\omega_0 \\ 1 \end{pmatrix},$$

where φ is the dimensionless amplitude of the wave. Since $\omega_H = \gamma H$ in the case considered, and

$$\omega_M = \gamma 4\pi M_z \to \omega_M{}^0 [1 - (1 + \omega_H{}^2/\omega_0{}^2)|\varphi|^2/2],$$

the dispersion relation (15) or (16) can be represented in the form $\omega = \omega(k_\parallel, k_\perp^2, |\varphi|^2)$. For such a dispersion relation the evolution of the envelope is described by the equation

$$\left[i\left(\frac{\partial}{\partial t} + v_g \nabla_\parallel\right) + \frac{1}{2}\frac{\partial^2\omega}{\partial k_\parallel^2}\nabla_\parallel{}^2 + \frac{\partial\omega}{\partial k_\perp^2}\nabla_\perp{}^2 - \frac{\partial\omega}{\partial|\varphi|^2}|\varphi|^2 \right]\varphi = 0, \tag{17}$$

where $\nabla_\parallel$ and $\nabla_\perp$ are the derivatives with respect to k_0 in the longitudinal and transverse directions. Using the dispersion equations (15) or (16) we can find the coefficients of Eq. (17). Thus, for surface MSW

$$v_g = \frac{\omega_M{}^2 d}{4\omega_0}, \quad \left.\frac{\partial^2\omega}{\partial k_\parallel^2}\right|_{k,\varphi=0} = -\frac{\omega_M{}^4 d^2}{16\omega_0{}^3}\left(1 + 8\frac{\omega_0{}^2}{\omega_M{}^2}\right),$$
$$\frac{\partial\omega}{\partial k_\perp{}^2} = \frac{\omega_H\omega_M}{k^2\omega_0}, \quad \frac{\partial\omega}{\partial|\varphi|^2} = -\frac{\omega_H\omega_M}{4\omega_0}\left(1 + \frac{\omega_H{}^2}{\omega_0{}^2}\right). \tag{18}$$

At $\nabla_\perp^2 \varphi = 0$ Eq. (17) with coefficients (18) coincides with the nonlinear Schrödinger equation obtained by Lukomskiĭ.[3] The surface MSW are stable to longitudinal perturbations, for in their case $(\partial\omega/\partial|\varphi|^2)/(\partial^2\omega/\partial k_\parallel^2) > 0$, and are unstable to transverse perturbation, since $(\partial\omega/\partial|\varphi|^2)/(\partial\omega/\partial k_\perp^2) < 0$.

In the case of backward bulk waves, when $\mathbf{k} \perp \mathbf{M}$, we have

$$v_g = -\frac{\omega_H \omega_M}{4\omega_0} d, \quad \left.\frac{\partial^2 \omega}{\partial k_\parallel^2}\right|_{k,\varphi=0} = \frac{5}{48} \frac{\omega_H \omega_M}{\omega_0} \left(1 + \frac{3}{5} \frac{\omega_H^2}{\omega_0^2}\right),$$

$$\left.\frac{\partial \omega}{\partial k_\perp^2}\right|_{k,\varphi=0} = \frac{\omega_M d}{4k} \left(1 - \frac{\omega_H}{2\omega_0}\right), \quad \left.\frac{\partial \omega}{\partial |\varphi|^2}\right|_{k,\varphi=0}$$

$$= -\frac{\omega_H \omega_M}{4\omega_0} \left(1 + \frac{\omega_H^2}{\omega_0^2}\right). \tag{19}$$

This type of wave is unstable to both longitudinal and transverse perturbations. The presence of instability of backward bulk MSW to longitudinal perturbations points to the possibility of soliton formation. Using the criterion for the appearance of the soliton solution (1) for a backward bulk wave and the asymptotic (as $k \to 0$) expression for its power

$$P = \pi d^2 L M^2 \frac{\omega_H^2}{\omega_0} \varphi_0^2, \tag{20}$$

we obtain the following formula for the threshold power of formation of a solitary nonlinear backward bulk MSW:

$$P_{\rm thr} = \frac{5\pi^2}{3} \frac{L d^2 M^2}{\omega_M \tau^2} \frac{\omega_0}{\omega_M} \frac{(1 + 3\omega_H^2/5\omega_0^2)}{(1 + \omega_H^2/\omega_0^2)}. \tag{21}$$

We choose $b_\parallel = 0.1$ cm, $d = 10$ μm, $\omega_H/2\pi = 6 \times 10^9$ Hz ($H = 300$ Oe), $\omega_0/2\pi = 10^{10}$ Hz, and $L = 0.5$ cm. We then obtain from (15) $P_{\rm thr} = 1.5$ mW.

5. The wave instability to transverse perturbation can lead to the appearance of solitons in the transverse direction—to nonlinear self-channeling of the wave.[15] This phenomenon likewise has a threshold. Self-channeling is possible for surface and backward bulk MSW. Assume that the wave has a constant amplitude in the longitudinal direction, i.e., $\nabla_\parallel \varphi = 0$. We then have from (17)

$$i \frac{\partial \varphi}{\partial t} + \frac{\partial \omega}{\partial k_\perp^2} \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial \omega}{\partial |\varphi|^2} |\varphi|^2 \varphi = 0, \tag{22}$$

where x is the coordinate in the transverse direction. This nonlinear Schrödinger equation describes the diffraction of the wave. If we seek the solution of this equation in the form $\varphi = \varphi(x) \exp(i\Omega t + \alpha)$, where $\varphi(x)$ is a real function that decreases to zero as $|x| \to \infty$, we can obtain from (22)

$$\varphi = A \exp\left(-i \frac{A^2}{2} t \frac{\partial \omega}{\partial |\varphi|^2} + \alpha\right) \Big/ \mathrm{ch} \left[A x \left(-\frac{1}{2} \frac{\partial \omega / \partial |\varphi|^2}{\partial \omega / \partial k_\perp^2} \right)^{1/2} \right]. \tag{23}$$

In the soliton solution (23) the amplitude A is connected with the frequency by the relation

$$\Omega = -\frac{A^2}{2} \frac{\partial \omega}{\partial |\varphi|^2}.$$

The width Δ of the self-channeling region depends on the wave amplitude

$$\Delta \sim \frac{1}{A} \left| \frac{\partial \omega / \partial k_\perp^2}{\partial \omega / \partial |\varphi|^2} \right|^{1/2}. \tag{24}$$

If the initial beam was homogeneous with width $b_\perp$, the wave beam threshold amplitude φ_0 at which a stable wave of the form (23) is produced in the course of the diffraction is determined for Eq. (22) by the relation

$$(\varphi_0 b_\perp)^2 = \frac{\pi^2}{2} \left| \frac{\partial \omega / \partial k_\perp^2}{\partial \omega / \partial |\varphi|^2} \right|. \tag{25}$$

This condition is obtained in analogy with condition (12) for longitudinal solitons. From (25) we can obtain the threshold values for the corresponding waves. Thus, for a backward bulk MSW, using (15) and expression (20) for the power, we obtain from (25)

$$P_{\rm thr} = \frac{\pi^3}{2} \frac{d^3}{b_\perp k} M^2 \omega_H \frac{(1 - \omega_H/2\omega_0)}{(1 + \omega_H^2/\omega_0^2)}. \tag{26}$$

For a surface MSW we can obtain similarly

$$P_{\rm thr} = 2\pi^3 \frac{d^2}{b_\perp k^2} M^2 \omega_0 \frac{1}{(1 + \omega_H^2/\omega_0^2)}. \tag{27}$$

From a comparison of (26) and (27) it can be seen that at $kd \ll 1$ the threshold self-channeling power for the surface waves is much higher than for the backward bulk waves. At $b_\perp = 0.5$ cm and $k = 102$ cm^{-1} the threshold self-channeling power (26) is of the same order as the threshold powers, estimated above, for the formation of longitudinal solitons—self modulation.

6. In conclusion, we estimate the threshold power for decay-type three-magnon processes. Recognizing that the asymptotic expression for the surface MSW is

$$P = \pi L d^2 M^2 \omega_0 |\varphi_0|^2,$$

we obtain from (8)

$$P_{\rm thr} = \frac{16\pi}{27} L d^2 M^2 \omega_0 \frac{(\delta\omega)^2}{(\omega_0^2 - 4\omega_H^2)}. \tag{28}$$

If we choose $L = 0.5$ cm, $d = 10$ μm, $\omega_0 = 10$ GHz, and $\delta\omega/\omega_0 \sim 10^{-3}$ we obtain from (28) $P_{\rm thr} = 2 \times 10^{-2}$ mW. Thus, three-magnon processes have much lower thresholds than the considered four-magnon ones. Therefore, as noted in fact at the beginning of the article, the effects considered can be observed only outside the region where the resonance conditions for three-magnon processes are satisfied. Outside the resonance regions, when $|\frac{1}{2}\omega - \omega_k| \gg \delta\omega$, the condition (8) must be replaced by (see Ref. 8)

$$|\varphi|^2 = |\omega^2 (\omega/2 - \omega_k)^2 / 9\omega_H^2 \omega_M^2 \sin^2 \theta_k \cos^2 \theta_k|_{max} \geqslant \omega_H^2 / 9\omega_M^2. \tag{29}$$

Comparing the criterion for the presence of auto modulation (12) in the case of forward bulk MSW:

$$|\varphi|^2_{\rm auto} = \frac{\pi^2 d^2}{64 b^2} \frac{\omega_M}{\omega_H} \left(1 + \frac{8}{3} \frac{\omega_H}{\omega_M}\right)$$

with the criterion (29) we find that the latter effect will be observed if

$$\frac{9\pi^2}{64} \left(1 + \frac{8}{3} \frac{\omega_H}{\omega_M}\right) \left(\frac{d}{b}\right)^2 \left(\frac{\omega_M}{\omega_H}\right)^2 \ll 1. \tag{30}$$

At the parameters indicated above, which are contained in the condition (30), this condition is satisfied. Experimental investigations of yttrium-iron garnet films[1] have shown that the threshold powers, outside the resonance regions for three-magnon processes, increase to 100 mW. Thus, the power of the wave, with allowance for the loss to conversion, can reach values of the order of 10^{-2}–10^{-1} W. This points to a realistic possibility of observing the predicted effects.

We have thus shown that in the absence of three-magnon processes nonlinear effects of higher order can lead to the formation of MSW solitons. The formation of the solitary waves has a threshold, and depends also on the initial size (length or width) of the pulse. The effect of formation of transverse MSW solitons is analogous to the phenomenon of nonlinear self-channeling in optics. From the viewpoint of this analogy, a tangentially magnetized film is a focusing medium, while a normally magnetized volume is a defocusing medium. With increasing wave power, further splitting of the initial beam into two and more MSW beams is possible. The required power increases then by a factor $(2n-1)^2$, where n is the number of self-channeled MSW beams.

APPENDIX

Derivation of nonlinear parabolic equation for the wave envelope with forward bulk MSW as an example

The initial equations (1) can be represented in the form

$$\dot{m}_x/\gamma=m_yH_z-M_zh_y,\quad \dot{m}_y/\gamma=-m_xH_z+M_zh_x, \tag{A.1}$$

$$\frac{\partial}{\partial x}(h_x+4\pi m_x)+\frac{\partial}{\partial z}(h_z+4\pi M_z)=0, \tag{A.2}$$

where

$$M_z=M_0\left(1-\frac{m_x^2+m_y^2}{2M_0^2}\right),\quad \mathbf{h}=\nabla\Psi,\quad \frac{\partial\Psi}{\partial y}=0.$$

The nonlinear terms will be regarded as perturbations, assuming the parameter $\eta\neq(m_x^2+m_y^2)/M_0^2$ to be small. In the zeroth approximation in this parameter, Eqs. (A.1) and (A.2) take the form

$$4\pi\begin{pmatrix} m_x^0\\ m_y^0\end{pmatrix}=\begin{pmatrix}\chi_1\\ -i\chi_2\end{pmatrix}\frac{\partial\Psi_0}{\partial x}, \tag{A.3}$$

$$(1+\chi_1)\frac{\partial^2\Psi_0}{\partial x^2}+\frac{\partial^2\Psi_0}{\partial z^2}=0. \tag{A.4}$$

Taking the boundary conditions $\Psi_0, \partial\Psi_0/\partial z|_{z=\pm d/2}$ = const into account, we get (2). We represent the potential Ψ in the form of an expansion in a small parameter

$$\Psi=\Psi_0+\Psi_1+\Psi_2+\ldots, \tag{A.5}$$

where Ψ_0 is the zeroth approximation (2), Ψ_1 is the first-order correction, etc. In first-order approximation we assume that the amplitude φ of the solution (2) (of the zeroth approximation) depends on the "slow" variables $\tilde{x}$ and $\tilde{t}$. We then obtain from (A.1) and (A.2), in first order in η, retaining the terms with the second derivatives with respect to the slow variables,

$$h_x^1=\frac{\partial\Psi_0}{\partial\tilde{x}}+\frac{\partial\Psi_1}{\partial x},\quad h_z^1=\frac{\partial\Psi_1}{\partial z},$$

$$4\pi m_x^1=\chi_1\left(\frac{\partial\Psi_1}{\partial x}+\frac{\partial\Psi_0}{\partial\tilde{x}}\right)+i\frac{\partial\chi_1}{\partial\omega}\left(\frac{\partial^2\Psi_0}{\partial x\,\partial\tilde{t}}+\frac{\partial^2\Psi_0}{\partial\tilde{x}\,\partial\tilde{t}}\right)$$

$$-\frac{\partial^2\chi_1}{\partial\omega^2}\frac{\partial^3\Psi_0}{\partial x\,\partial\tilde{t}^2}-(\chi_1+\chi_1^2+\chi_2^2)\left[\frac{(m_x^0)^2+(m_y^0)^2}{2M_0^2}\right]\frac{\partial\Psi_0}{\partial x}; \tag{A.6}$$

$$(1+\chi_1)\frac{\partial^2\Psi_1}{\partial x^2}+\frac{\partial^2\Psi_1}{\partial z^2}=-2(1+\chi_1)\frac{\partial^2\Psi_0}{\partial x\,\partial\tilde{x}}-(1+\chi_1)\frac{\partial^2\Psi_0}{\partial\tilde{x}^2}$$
$$-i\frac{\partial\chi_1}{\partial\omega}\frac{\partial^3\Psi_0}{\partial x^2\,\partial\tilde{t}}-2i\frac{\partial\chi_1}{\partial\omega}\frac{\partial^3\Psi_0}{\partial x\,\partial\tilde{x}\,\partial\tilde{t}}+\frac{\partial^2\chi_1}{\partial\omega^2}\frac{\partial^4\Psi_0}{\partial x^2\,\partial\tilde{t}^2}$$
$$+(\chi_1+\chi_1^2+\chi_2^2)\left[\frac{(m_x^0)^2+(m_y^0)^2}{2M_0^2}\right]\frac{\partial^2\Psi_0}{\partial x^2}$$
$$-4\pi M_0\frac{\partial}{\partial z}\left[\frac{(m_x^0)^2+(m_y^0)^2}{2M_0^2}\right],\quad |z|<d/2, \tag{A.7}$$
$$\frac{\partial^2\Psi_1}{\partial x^2}+\frac{\partial^2\Psi_1}{\partial z^2}=-2\frac{\partial^2\Psi_0}{\partial x\,\partial\tilde{x}}-\frac{\partial^2\Psi_0}{\partial\tilde{x}^2},\quad |z|>d/2.$$

The boundary conditions for Ψ_1 are of the same form as for the zeroth approximation. The secular perturbing terms in the right-hand side of (A.7) yield the sought nonlinear equation if the last term is neglected. This can be done when

$$k_1\ll\frac{\chi_1+\chi_1^2+\chi_2^2}{4\pi M_0}k^2|\Psi_0|. \tag{A.8}$$

Recognizing that as $k\to 0$ we have $\omega\to\omega_H+\frac{1}{4}\omega_M kd$, we obtain from (A.8)[4)]

$$kd\ll 8|\varphi|^2. \tag{A.9}$$

From the condition that the system (A.7) with the indicated boundary conditions have a solution follows the sought nonlinear equation

$$i\left(\frac{\partial\Psi_0}{\partial\tilde{t}}+v_g\frac{\partial\Psi_0}{\partial\tilde{x}}\right)+v_g'\frac{\partial^2\Psi_0}{\partial\tilde{x}^2}-\frac{(\chi_1+\chi_1^2+\chi_2^2)}{\partial\chi_1/\partial\omega}\left[\frac{(m_x^0)^2+(m_y^0)^2}{2M_0^2}\right]\Psi_0=0, \tag{A.10}$$

where $v_g=\partial\omega/\partial k$, $v_g'=\frac{1}{2}\partial^2\omega/\partial k^2$, and $\omega(k)$ is the dispersion relation for the MSW. In the derivation of the last equation we made in (A.6) and (A.7) the substitution $\partial/\partial t\to -v_g\partial/\partial x$. When $k\to 0$ we get

$$\frac{m_x^2+m_y^2}{2M_0^2}\to|\varphi|^2,\quad \frac{\chi_1+\chi_1^2+\chi_2^2}{\partial\chi_1/\partial\omega}\to\omega_M,$$

as a result of which (A.10) goes over into Eq. (9) obtained above by another method.

1) As usual, in considering the magnetostatic waves we neglect in the Landau-Lifshitz equations the inhomogeneous exchange interaction. This is valid for sufficiently long waves $k\ll(2\pi M^2/A)^{1/2}$, where A is the inhomogeneous exchange constant (exchange rigidity). In yttrium iron garnet (YIG), for example, $A\sim 4\times 10^{-7}$ erg/cm and $2\pi M^2\sim 1.3\times 10^5$ erg/cm^2, therefore this condition is satisfied at $k\ll 2\times 10^5$ cm^{-1}. There is also a lower bound on k, connected with the employed quasistatic-behavior condition $k\gg\omega/c$, where $\omega c\sim 1$ to 10 cm^{-1}.

2) By way of example we present a symmetric solution corresponding to the zeroth mode of practical interest, inasmuch as in this mode $v_g\neq 0$ as $k\to 0$. The remaining modes attenuate strongly as they propagate, for in them $v_g=\partial\omega/\partial k\to 0$ as $k\to 0$.

3) It can be shown (see the Appendix) that the results of the described method agree with the direct derivation of the envelope-evolution equa-

tion from the initial system of equations (1) with the appropriate boundary conditions, when the condition $kd \ll |\varphi|^2 \ll 1$ is satisified.

[4)] This condition may be weakened because the symmetry of the last term in (A.7) is the inverse of the symmetry of the remaining terms.

[1] J. D. Adam, IEEE Trans. Magn. MAG-16, 1168 (1980).
[2] J. D. Adam, M. R. Daniel, and D. K. Schroder, Electronics **53**, 123 (1980).
[3] V. P. Lukomskiĭ, Ukr. Fiz. Zh. **23**, 134 (1978).
[4] V. G. Bar'yakhtar and M. I. Kaganov, in: Ferromagnitnyĭ rezonans (Ferromagnetic Resonance), S. V. Vonsovskiĭ, ed., Fizmatgiz, 1961, p. 266.
[5] V. B. Kadomtsev and V. I. Karpman, Usp. Fiz. Nauk **103**, 193 (1971) [Sov. Phys. Usp. **14**, 40 (1971)].
[6] V. I. Karpman, Nonlinear Waves in Dispersive Media, Pergamon [Russ. orig., Novosibirsk, 1968, p. 111].
[7] A. M. Mednikov, Fiz. Tverd. Tela (Leningrad) **23**, 242 (1981) [Sov. Phys. Solid State **23**, 136 (1981)].
[8] H. Suhl, J. Phys. Chem. Sol. **1**, 209 (1957).
[9] Ya. A. Monosov, Nelineĭnyĭ ferrmagnitnyĭ rezonans (Nonlinear Ferromagnetic Resonance), Nauka, 1971, p. 88.
[10] A. G. Gurevich, Magnitnyĭ rezonans v ferritakh i antiferrmagnetikakh (Magnetic Resonance in Ferrites and Antiferromagnets), Nauka, 1973, p. 589.
[11] O. M. Phillips, in: Nonlinear Waves, S. Leibovich and A. Seebass, eds., Cornell Univ. Press, 1974 [Russ. transl. Mir, 1977, p. 197].
[12] V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **61**, 118 (1971) [Sov. Phys. JETP **34**, 62 (1972)].
[13] V. E. Zakharov, S. V. Manakov, S. P. Novikov, and L. P. Pitaevskiĭ, Teoriya solitonov: metod obratnoi zadachi (Soliton Theory, The Inverse Problem Method) Nauka, 1980, p. 74.
[14] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Sol. **19**, 308 (1961).
[15] V. I. Bespalov and V. I. Talanov, Pis'ma Zh. Eksp. Teor. Fiz. **3**, 471 (1966) [JETP Lett. **3**, 307 (1966)].

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