Evolution of magnetic field and anomalous thermal losses in a dense plasma

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Nonlinear magnetothermal processes are considered in a dense magnetized high-pressure plasma, when $\beta \equiv 8\pi nT/H^2 \gg 1$. The different regimes of cooling of such a plasma are classified. It is shown that the effective thermal conductivity of a plasma with large β can be much larger than the classical (of the order of the Bohm thermal conductivity) even in the absence of turbulence in the plasma. The radiative cooling wave produced by contact of a hot and cold plasma in a magnetic field is investigated. The structure of the transition region and the rate of condensation of the hot plasma are obtained.

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INTRODUCTION

The interest in research into the properties of high-pressure plasma, i.e., a plasma whose gaskinetic pressure is much higher than the pressure of the magnetic field (the quantity $\beta \equiv 8\pi nT/H^2 \gg 1$) is due to a considerable degree to a number of suggestions made to use such a plasma for controlled thermonuclear fusion.[1–4] In this case the role of the magnetic field reduces only to suppression of the thermal conductivity, while the mechanical plasma confinement is ensured by the rigid walls of the chamber. The main question here is the rate of cooling of the hot plasma via heat flow to the walls and bremsstrahlung that leaves freely the volume of the plasma. The reason is that the cooling of a plasma with $\beta \gg 1$ has a number of features that distinguish it from cooling of a low-pressure ($\beta \ll 1$) plasma, the behavior of which has already been well investigated. Principal among these features is the motion of the plasma and the strong distortion of the external magnetic field, both on account of this motion and as a result of thermoelectric effects (the Nernst effect). Since the thermal conductivity of the plasma depends in turn substantially on the magnetic field, a rather complicated and varied picture of interrelated magnetothermal phenomena appears here. Therefore this problem was solved up to now only by numerically integrating the transport equations for some specific values of the parameters,[5–9] or by using certain simplifying assumptions concerning the structure of the magnetic field.[10] In the present paper we consider analytically various regimes of cooling plasma with $\beta \gg 1$. It is shown that the features noted above lead to a noticeable increase of the thermal losses, and accordingly to a decrease of the energy-dependent plasma-confinement time.[11]

The problem of contact between a hot plasma and a cold plasma or a cold gas is frequenty encountered in astrophysics. Such a situation arises, e.g., as a result of the development of thermal instabilities in hot intergalactic plasma. In the absence of external energy sources, the radiation losses from the plasma lead to condensation of the hot phase on clouds of cold gas. This problem was considered in a recent paper,[12] where the rate of condensation of the hot plasma without a magnetic field was obtained. We solve below a similar problem for a strongly magnetized plasma. In this case the picture of the plasma flow changes substantially, and the condensation rate is found to be anomalously large.

§1. BASIC EQUATIONS

We consider a one-dimensional planar problem, assuming the gradients of all the quantities to be directed along the x axis and the magnetic field along the z axis. The thermal processes of interest to us are much slower than the hydrodynamic processes, therefore the plasma flow velocity is less than that of sound, and the pressure in the system can be regarded as uniform:

$$\frac{\partial}{\partial x}\left(2nT+\frac{H^2}{8\pi}\right)=0. \tag{1}$$

Here $n=n_e=n_i$ is the density of the electrons and ions of the hydrogen plasma, and $T=T_e=T_i$ is their temperature. As will be shown below, in certain cases the magnetic field in narrow layers increases so much that its pressure becomes comparable with the gaskinetic pressure of the plasma. Therefore account is taken in (1) also of the magnetic-field pressure, although in the bulk of the plasma it is negligibly small. The equations for the magnetic field and for the heat balance of the plasma are given in Braginskiĭ's review.[13] Using his notation we obtain

$$\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(\frac{c^2}{4\pi\sigma_\perp}\frac{\partial H}{\partial x}+\frac{c}{e}\beta_\wedge\frac{\partial T}{\partial x}-vH\right), \tag{2}$$

$$3\frac{\partial(nT)}{\partial t}=\frac{\partial}{\partial x}\left(\varkappa_\perp\frac{\partial T}{\partial x}+\frac{cT}{4\pi e}\beta_\wedge\frac{\partial H}{\partial x}-5nTv\right)$$
$$+2v\frac{\partial(nT)}{\partial x}-Q_r+\frac{c^2}{16\pi^2\sigma_\perp}\left(\frac{\partial H}{\partial x}\right)^2+\frac{c\beta_\wedge}{4\pi e}\frac{\partial H}{\partial x}\frac{\partial T}{\partial x}, \tag{3}$$

where v is the plasma flow velocity, Q_r is the volume radiation power, $\sigma_\perp$ and $\varkappa_\perp$ are the plasma electric and thermal conductivities transverse to the magnetic field, and $\beta_\wedge$ is the thermoelectric coefficient. The last terms in the right-hand side of (3) describe the dissipation of the current in the plasma because of the finite conductivity and thermoelectric power (the viscous effects are insignificant here). Together with the continuity equation

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv)=0 \tag{4}$$

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Eqs. (1)–(3) form a complete system of equations that describes the cooling of the plasma in a magnetic field. The kinetic coefficients $\sigma_\perp, \varkappa_\perp$, and β_Λ in this system have entirely different forms in the case of strongly magnetized ($\omega_H\tau \gg 1$) and unmagnetized $\omega_H\tau \ll 1$) plasma (ω_H is the cyclotron frequency and τ is the time between the collisions). Even approximate formulas[13] suitable for all values of the parameter $\omega_H\tau$ are quite cumbersome. We use therefore simple model expressions that describe qualitatively correctly the behavior of the plasma transport coefficients in both limiting cases. To this end we consider separately three regions of the plasma parameters: region I—relatively hot plasma with magnetized ions $(\omega_H\tau)_i > 1$; region III—cold unmagnetized plasma, where $(\omega_H\tau)_e < 1$; intermediate region II, where the electrons are magnetized but the ions are not. Recognizing that $(\omega_H\tau)_i = \mu^{1/2}(\omega_H\tau)_e$, where $\mu \equiv m_e/m_i \ll 1$ is the ratio of the electron and ion masses, the limits of region II are expressed as follows: $\mu^{1/2} < (\omega_H\tau)_i < 1$. Now the kinetic coefficients of the plasma can be represented in the following form[13]:

$$\sigma_\perp = \frac{ne^2\tau_e}{m_e}, \quad \varkappa_\perp = \begin{cases} ncT/eH(\omega_H\tau)_i & \text{in region I} \\ ncT/eH & \text{in region II}, \\ ncT(\omega_H\tau)_e/eH & \text{in region III} \end{cases}$$

$$\beta_\Lambda = \begin{cases} \dfrac{3}{2(\omega_H\tau)_e} & \text{(I, II)} \\ 3(\omega_H\tau)_e/2 & \text{(III)} \end{cases}. \tag{5}$$

We have already noted above the important role of the Nernst effect in the evolution of the magnetic field in a plasma with $\beta \gg 1$. It is convenient to interpret Eq. (2) as the continuity equation for the magnetic field, where the magnetic flux is

$$q_H = vH - \frac{c^2}{4\pi\sigma_\perp}\frac{\partial H}{\partial x} - \frac{c}{e}\beta_\Lambda\frac{\partial T}{\partial x}. \tag{6}$$

We see therefore that the Nernst effect leads to an additional drift of the magnetic field, directed opposite to the plasma temperature gradient. We introduce the velocity of this drift:

$$V_H = \frac{c}{eH}\beta_\Lambda\frac{\partial T}{\partial x}.$$

It is connected with the rate of thermal-conductivity diffusion of the heat

$$V_T = \frac{\varkappa_\perp}{nT}\frac{\partial T}{\partial x}$$

by the following relations which follow from (5):

$$V_H = \begin{cases} {}^3/_2\,\mu^{1/2}V_T, & (\omega_H\tau)_i > 1 & \text{(region I)} \\ {}^3/_2\,V_T/(\omega_H\tau)_e, & 1 < (\omega_H\tau)_e < \mu^{-1/2} & \text{(region II).} \\ {}^3/_2\,V_T, & (\omega_H\tau)_e < 1 & \text{(region III)} \end{cases} \tag{7}$$

This connection will be used later to find the magnetic field.

§2. ANOMALOUS THERMAL CONDUCTIVITY OF PLASMA

The relative role of the thermal conductivity and of radiation in the cooling of a plasma depends on the dimensions of the system. Obviously, at sufficiently small dimensions (quantitative criteria will be indicated below) the loss to radiation can be neglected. To determine the cooling rate of the plasma we consider in this case the following problem. Let at the initial instant of time a homogeneous hot plasma with temperature T_0 and density n_0, situated in an external magnetic field H_0, be brought in contact with a cold wall having zero temperature (the plasma occupies the half-space $x > 0$). In this problem there is no characteristic scale of length, so that its solution should be self-similar. Since further evolution of the system is connected with the thermal conductivity of the plasma, the self-similarity is diffusive and the self-similar variable ξ can be expressed in terms of the thermal diffusivity of the hot plasma $\chi_0 = \varkappa_{\perp 0}/n_0$:

$$\xi = \frac{x}{(\chi_0 t)^{1/2}}, \quad T(x,t) = T_0 T(\xi), \quad n = n_0 n(\xi),$$
$$H = H_0 H(\xi), \quad v = -(\chi_0/t)^{1/2} v(\xi) \tag{8}$$

(the plasma flow is directed towards the wall, so that $v(x,t) < 0$). Now $T(\xi)$, $n(\xi)$, $H(\xi)$, and $v(\xi)$ are here dimensionless functions, for which we obtain from (1)–(4) the following equations:

$$nT + H^2/\beta_0 = 1 + \beta_0^{-1}, \tag{9}$$

$$-\frac{\xi}{2}\frac{dH}{d\xi} = \frac{d}{d\xi}\left(\frac{4\mu^{1/2}}{\beta_0\sigma}\frac{dH}{d\xi} + \frac{3}{2}\mu^{1/2}\alpha\frac{dT}{d\xi} + vH\right), \tag{10}$$

$$-\frac{3}{2}\xi\frac{d(nT)}{d\xi} = \frac{d}{d\xi}\left(\varkappa\frac{dT}{d\xi} + \frac{6\mu^{1/2}}{\beta_0}\alpha T\frac{dH}{d\xi} + 5nTv\right)$$
$$-2v\frac{d(nT)}{d\xi} + \frac{16\mu^{1/2}}{\beta_0^2\sigma}\left(\frac{dH}{d\xi}\right)^2 + \frac{6\mu^{1/2}}{\beta_0}\alpha\frac{dH}{d\xi}\frac{dT}{d\xi}, \tag{11}$$

$$-\frac{\xi}{2}\frac{dn}{d\xi} = \frac{d}{d\xi}(nv). \tag{12}$$

Here $\beta_0 \equiv 16\pi n_0 T_0/H_0^2 \gg 1$, but in this case the hot plasma is assumed to be strongly magnetized, so that $(\omega_{Hi}\tau_i)_0 \equiv \delta_0 \gg 1$. In this notation, $\sigma = T^{3/2}$

$$\varkappa = \begin{cases} n^2/H^2T^{1/2}, & \text{I}, & HT^{3/2}/n > \delta_0^{-1} \\ \delta_0 nT/H, & \text{II}, & \mu^{1/2}\delta_0^{-1} < HT^{3/2}/n < \delta_0^{-1}; \\ \mu^{-1/2}\delta_0^2 T^{5/2}, & \text{III}, & HT^{3/2}/n < \mu^{1/2}\delta_0^{-1} \end{cases}$$
$$\alpha = \begin{cases} n/HT^{3/2}, & \text{I, II} \\ \mu^{-1}\delta_0^2 HT^{3/2}/n, & \text{III} \end{cases}. \tag{13}$$

As the first step we shall assume β_0 to be infinitely large (it will be seen from the solution when this quantity must be regarded as finite). It follows then from (9) that $nT = 1$, and from (11) that the heat flux is constant; this flux receives a contribution from the plasma flow and from the thermal conductivity:

$$\varkappa(dT/d\xi) + 5v = \text{const} = 5v_0. \tag{14}$$

Here v_0 is the flow velocity of the hot plasma as $\xi \to +\infty$, and determines in fact the heat flow to the wall and by the same token the plasma cooling rate. The effective thermal conductivity $\varkappa_{\text{eff}}$ of the plasma is in this case obviously

$$\varkappa_{\text{eff}} = v_0^2\varkappa_{\perp 0}. \tag{15}$$

Since v_0 turns out to be large ($v_0 \gg 1$), we call this thermal conductivity anomalous. When the temperature in the layer next to the wall decreases, the plasma flow velocity decreases, therefore the convective heat flux is significant only in the region of hot plasma with $T\sim 1$, while at $T\ll 1$ the principal role is played by thermal conductivity, and it follows from (14) that

$$\varkappa(dT/d\xi)\approx 5v_0. \tag{16}$$

This equation determines in principle the temperature profile $T(\xi)$, which contains the quantity v_0 as an as-yet-unknown parameter. The thickness of the wall layer is $\Delta\xi\propto v_0^{-1}$. To find v_0 we must use the continuity equation (12). The plasma flux

$$nv=-\frac{1}{2}\int_0^{\xi}\xi\frac{dn}{d\xi}d\xi$$

(we take into account the fact that on the whole, i.e., at $\xi=0$, the flux is zero). Therefore

$$v_0=-\frac{1}{2}\int_0^{\infty}\xi\frac{dn}{d\xi}d\xi. \tag{17}$$

The integral in the right-hand side is proportional here to v_0^{-1}, so that knowing the solution of (16) we can determine from it the value of v_0. As will be shown below, the main contribution to the integral (17) is made by the region of relatively cold plasma with temperature $T\sim T_*\ll 1$. This means that the substance brought in by the flow of the hot plasma accumulates in a region with $T\sim T_*$. Therefore at $T>T_*$ the plasma flux remains constant:

$$nv=\mathrm{const}=v_0,\quad v\approx v_0T, \tag{18}$$

and at $T>T_*$ the flux decreases rapidly and vanishes on the wall.

Since the thermal conductivity $\varkappa$ of the plasma depends substantially on the magnetic field, Eq. (16) must be solved jointly with Eq. (10) for the magnetic field, and the solution of the latter is very sensitive to the boundary condition on the wall. We consider here two limiting cases: and ideally conducting and non-conducting wall. We begin with the latter, when the magnetic field at the wall remains unchanged: $H(\xi=0)=1$. The system (9)–(13) contains two dimensionless parameters, β_0 and δ_0, different values of which are conveniently represented as points on the (β_0,δ_0) plane, see Fig. 1. We can separate here four regions (a,b,c,d) in which the structure of the wall layer turns out to be different. The boundaries of these regions are shown schematically in Fig. 1, and the meaning of this subdivision will be made clear by the exposition that follows (we recall that we are interested only in values of β_0 and δ_0 larger than unity). In region b, Fig. 1, where β_0 is large enough, we can neglect the magnetic viscosity and, using the previously introduced magnetic-field drift velocity V_H, due to the thermoelectric effect, write (10) in the form

$$-\frac{\xi}{2}\frac{dH}{d\xi}=\frac{d}{d\xi}\{(v+V_H)H\}=\frac{dq_H}{d\xi}. \tag{19}$$

The quantity V_H is connected with the heat diffusion rate V_T by the relations (7). But Eq. (16) means that V_T is now a constant equal to $5v_0$. Therefore

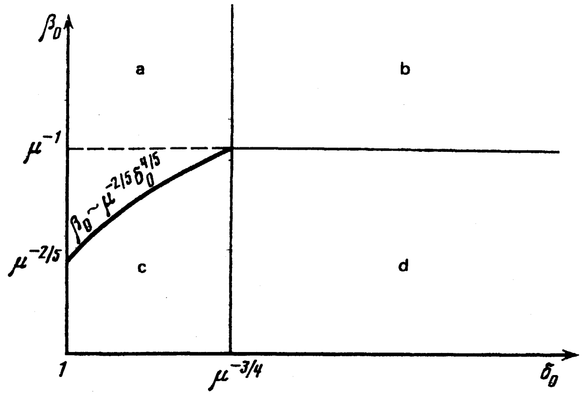

FIG. 1. Breakdown of the plane of the dimensionless parameters (β_0,δ_0) into regions with different structure of the wall layer of the plasma in the case of a nonconducting boundary.

$$V_H=\begin{cases}{}^{15}/_2\mu^{1/2}v_0, & \mathrm{I},\quad T\ll 1\\ {}^{15}/_2\mu^{1/2}v_0/\delta_0HT^{5/2}, & \mathrm{II}\\ {}^{15}/_2v_0, & \mathrm{III}\end{cases} \tag{20}$$

We see therefore that the magnetic-field flux carried to the wall as a result of the Nernst effect is of the same order as the convective magnetic flux from the hot plasma $(q_H\infty)\sim q_H(0)\sim v_0)$. At the same time, as follows from (19), the change of the magnetic flux is

$$\Delta q_H\infty\int\xi\frac{dH}{d\xi}d\xi\infty v_0^{-1}.$$

It can then be assumed that $\Delta q_H\ll q_H$ for sufficiently large v_0, and the magnetic field can be obtained from the condition that the flux q_H be constant:

$$q_H=(v+V_H)H=v_0. \tag{21}$$

From relations (18) and (20) for the velocities v and V_H and from (21) it follows that at $T\gg\mu^{1/2}$ the velocity $v\gg V_H$, therefore the magnetic field is frozen-in there in the plasma: $H\sim n=T^{-1}$. The Nernst effect comes into play at $T\sim\mu^{1/2}$, and then, with further decrease of temperature, it assumes the principal role in the outflow of the magnetic field ($V_H\gg v$ at $T\ll\mu^{1/2}$). As a result, the profile of the magnetic field takes the form shown in Fig. 2. At $T\ll\mu^{1/2}$ the field $H=(2/15)\mu^{-1/2}$ in region I and $H=2/15$ in region III. The lower (temperature) limit of region I coincides here with the upper limit of the region III: $T=T_1(15/2)^{2/5}\mu^{1/5}\delta_0^{-2/5}$, so that at $T=T_1$ a jump of the magnetic field takes place, and region II actually does not exist. A jump of the field takes place also at the wall $(\xi=0)$, where it changes from $H=2/15$ to $H=1$. We have assumed here that the magnetization of the hot plasma δ_0 is large enough,so that $T_1\ll\mu^{1/2}$, i.e., $\delta_0\gg\mu^{-3/4}$. As can be seen from the obtained solution, the magnetic field is strengthened in the wall layer to a value $H_{\max}\sim\mu^{-1/2}$. Thus, the pressure of the magnetic field can be neglected and we can put $nT=1$ if $H_{\max}\ll\beta_0^{1/2}$, i.e., $\beta_0\gg\mu^{-1}$.

Allowance for the magnetic viscosity in Eq. (10) leads, of course, to a smoothing of the magnetic-field profile. Under the condition $\beta_0\gg\mu^{-1}$, however, when the magnetic

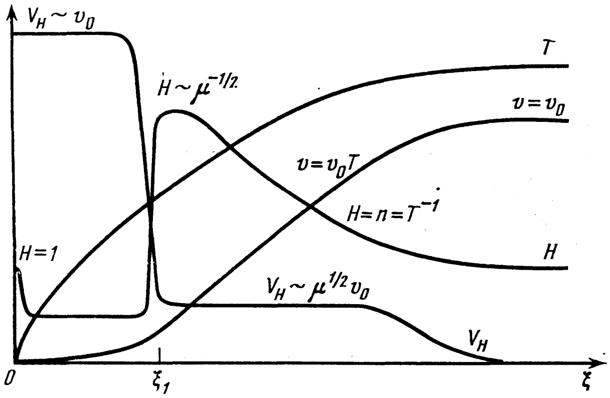

FIG. 2. Profiles of the temperature T, of the magnetic field H, of the plasma velocity v, and of the velocity of the thermal drift of the magnetic field V_H in the case of a nonconducting boundary at $\beta_0 > \mu^{-1}$ and $\delta_0 > \mu^{-3/4}$ ($nT = 1$).

pressure in the wall layer remains less than the plasma pressure, the jumps of the magnetic field become weakly smeared out. To prove this, we write down the analog of Eq. (21) with account taken of the diffusion contribution to the magnetic flux:

$$\frac{4\mu^{1/2}}{\beta_0 T^{3/2}} \frac{dH}{d\xi} + V_H H = v_0 \qquad (22)$$

(at $T \ll \mu^{1/2}$ the drift of the field due to plasma flow is insignificant). Using (13), (16), and (20) we can write this equation in region II in the form:

$$\frac{20\mu^{1/2} H}{\beta_0 \delta_0 T^{3/2}} \frac{dH}{dT} + \frac{{}^{15}/_{2}\mu^{1/2}}{\delta_0 T^{5/2}} = 1,$$

whence

$$H^2 = \left(\frac{2}{15}\right)^2 + \frac{3}{2}\beta_0 \left\{\frac{1}{5}\left[\left(\frac{T}{T_1}\right)^{5/2} - 1\right] - \frac{1}{2}\ln\frac{T}{T_1}\right\}.$$

It can now be seen that the increase of the magnetic field from $H = 2/15$ in region III to $H = (2/15)\mu^{-1/2}$ in I takes place in a narrow temperature interval $\Delta T \sim (\beta_0 \mu)^{-1/2} T_1$. Such an isothermal jump of the magnetic field is analogous to a phenomenon known in hydrodynamics, the isothermal jump of the density in shock waves.[14] For the onset of the latter it is necessary that the viscosity of the gas be small compared with its thermal conductivity. In our case of a magnetized plasma, however, the small quantity is the magnetic viscosity, which at $T \sim T_1$ is smaller by a factor $\beta_0 \mu$ than the thermal diffusivity of the plasma.

The solution of Eq. (22) in region III with the boundary condition $H(T = 0) = 1$ on the wall is of the form

$$H = {}^1/_{15}[2 + 13\exp(-{}^3/_{40}\beta_0 \mu^{-1} \delta_0^2 T^5)],$$

so that at $\beta_0 \gg 1$ the jump of the field on the wall remains narrow, inasmuch as $H \approx 2/15$ already at $T \ll T_1 \sim \mu^{1/5} \delta_0^{-2/5}$.

Having determined the magnetic field and by the same token the thermal conductivity $\varkappa(H,T)$ of the plasma, we now obtain from (16) the temperature profile in the wall layer. Starting from the wall ($\xi = 0$), we obtain in region III, i.e., at $T < T_1$,

$$\xi = {}^2/_{35} \mu^{-1/2} v_0^{-1} \delta_0^2 T^{7/2}.$$

Next, at $T > T_1$, an increase of the magnetic field by a factor $\mu^{-1/2}$ suppresses strongly the thermal conductivity of the plasma. Therefore the layer with temperature $T_1 < T < \mu^{1/2}$ turns out to be relatively narrow:

$$\Delta\xi \ll \xi_1 = {}^2/_{35} \mu^{-1/2} v_0^{-1} \delta_0^2 T_1^{7/2}.$$

With further rise in temperature ($T > \mu^{1/2}$) it follows from (14) and (18) that

$$\xi \approx \xi_1 + {}^2/_5 v_0^{-1} \operatorname{arcth} T^{1/2}$$

(as should be the case, a hot plasma with $T \to 1$ corresponds to $\xi \to +\infty$). Knowing the $\xi(T)$ dependence, we can obtain from the condition (17) the value of v_0—the enhancement of the heat flux to the wall

$$v_0 = -\frac{1}{2}\int_0^\infty \xi \frac{dn}{d\xi} d\xi = -\frac{1}{2}\int_0^1 \xi \frac{dn}{dT} dT = \frac{1}{2}\int_0^1 \xi(T) \frac{dT}{T^2}.$$

It is easy to verify that the main contribution to this integral is made by a region with temperature $T \sim T_1$ (this is in fact the previously introduced plasma "accumulation" temperature T_*), and $v_0 = (3\delta_0/10)^{1/2}$. According to (15) this means that the effective thermal conductivity of the plasma

$$\varkappa_{\text{eff}} = \frac{3\delta_0}{10} \frac{n_0 c T_0}{eH_0(\omega_{Hi}\tau_i)_0} = \frac{3n_0 c T_0}{10 eH_0} \qquad (23)$$

turns out in this case to be of the order of the so-called Bohm thermal conductivity.

The solution obtained is valid if the inequalities $\beta_0 \gg \mu^{-1}$ and $\delta_0 \gg \mu^{-3/4}$ are satisfied. The second of them means that when the hot plasma flows towards the wall, the drift of the magnetic field due to the Nernst effect is "turned-on" earlier than the plasma "accumulation" ($T_* \sim T_1 < \mu^{1/2}$). It was this which made it possible to regard the flux of the magnetic field to be constant ($q_H = \text{const} = v_0$). We consider now the case of Fig. 1a, when $1 \ll \delta_0 \ll \mu^{-3/4}$, but the value of β_0 is sufficiently large as before (so that $H^2_{\max}/\beta_0 \ll 1$). Just as before, in the region III the magnetic field is uniform and equal to 2/15, inasmuch as $V_H \gg v$ in this region. Nor does the boundary of this region change: $T < T_1$. On going into II, the rapid growth of the magnetic field leads already at $T \sim T_1$, according to (20), to a decrease of the value of V_H, so that there $V_H \sim v$. Since accumulation of the plasma takes place here, at $T \sim T_1$ the velocity is $v \sim \xi$, therefore all the terms in (19) become of the same order of magnitude. The simplifying circumstance is that the increase of the magnetic field narrows down the corresponding region and $\xi \approx \text{const} = \xi_1$ at $T \gtrsim T_1$. It follow then from (12) that at $T \gtrsim T_1$

$$nv = -\frac{1}{2}\int_0^T \xi \frac{dn}{dT} dT = \frac{1}{2}\int_0^{T_1} \xi(T) \frac{dT}{T^2}$$

$$+ \frac{\xi_1}{2}\left(\frac{1}{T_1} - \frac{1}{T}\right) = \frac{7}{10T_1}\xi_1 - \frac{\xi_1}{2T}, \qquad v = v_0 T - \frac{\xi_1}{2}.$$

Substituting this expression for the plasma flow velocity in (19) and assuming that $\xi = \xi_1$, we obtain the magnetic field

$$H \approx T^{-1}(1 - T_1^{5/2}/T^{5/2}). \qquad (24)$$

It can be seen that at $T > T_1$ the magnetic field is frozen into the plasma. Although the temperature profile in this case (at $T > T_1$) does in fact differ from the previously obtained one, the value of v_0, obviously, remains unchanged. Thus, at all $\delta_0 \gg 1$ the effective thermal conductivity of the plasma is of the order of the Bohm value.

This result turns out to be correct also when it is necessary to take into account the magnetic-field pressure in (9) [and consequently the magntic viscosity in (10)]. We consider first the solution in the case of strong magnetization of the plasma, when $\delta_0 \gg \mu^{-3/4}$. In this case the magnetic pressure becomes substantial if $\beta_0 \lesssim \mu^{-1}$ (region *d* in Fig. 1). Although now the plasma pressure nT is no longer a constant quantity, it differs from unity, as will be seen later on, only in a narrow layer. Therefore it is possible, as before, to use the condition (16) that the heat flux be constant, and this condition together with the equation $q_H = v_0$ makes it possible to determine the magnetic field and the plasma density. In region III, where $H = 2/15$, the solution remains obviously the same as before. At $T > T_1$, in region II, we obtain from the condition $q_H = v_0$ the following equation:

$$\frac{T_1^{5/2}}{3T^{3/2}(nT)}\frac{d}{dT}\left(\frac{H^2}{\beta_0}\right)=1-\frac{T_1^{5/2}}{T^{5/2}}, \tag{25}$$

the solution of which with allowance for (9) is of the form

$$nT=\left(\frac{T}{T_1}\right)^3\exp\left[\frac{6}{5}\left(1-\frac{T^{5/2}}{T_1^{5/2}}\right)\right]. \tag{26}$$

It follows therefore that at $T > T_1$ the plasma pressure drops off rapidly to values $nT \ll 1$, and the magnetic field becomes equal to $\beta_0^{1/2}$. The solution (26) is valid up to $T = T_2 \sim T_1 \ln(\beta_0\mu)^{-1}$, which is the limit of the region II. In region I ($T > T_2$) we obtain in place of (26)

$$nT\approx\varepsilon/[1-(T_2/T)^{5/2}],\qquad \varepsilon={}^{15}/_2(\beta_0\mu)^{1/2}\ll 1, \tag{27}$$

i.e., at $T \gg T_2$ the plasma pressure is equal to the constant quantity $nT = \varepsilon$. This has the following meaning. From the constancy of the heat flux (16) it follows now that the heat-diffusion rate is $V_T = 5v_0/nT$, therefore the velocity of the thermal drift of the magnetic field is equal according to (7) to $V_H = \frac{15}{2}\mu^{1/2}v_0/nT$ and at $H = \beta_0^{1/2}$ the equality $nT = \varepsilon$ ensures the required magnetic flux: $q_H = V_H H = v_0$. Relation (27) remains valid up to a temperature $T \sim \mu^{1/2}$ at which the plasma flow velocity $v = v_0/n$ becomes comparable with V_H. At $T \gg \mu^{1/2}$ the magnetic field is frozen into the plasma, $H = n$, so that in the interval $\mu^{1/2} \ll T \ll \beta_0^{-1/2}$ the field is $H = n = \beta_0^{1/2}$, and at $T > \beta_0^{-1/2}$ we have $H = n = T^{-1}$. In this case the integral (17), which determines the value of v_0 receives the main contribution as before from the temperature region $T \sim T_1$. Therefore the effective thermal conductivity differs here only by a numerical factor from the value (23).

If the magnetization of the hot plasma is such that the parameter $1 \ll \delta_0 \lesssim \mu^{-3/4}$, the effects connected with the magnetic-field pressure begin to manifest themselves at $\beta_0 \lesssim T_1^{-2} \sim \mu^{-2/5}\delta_0^{4/5}$ (region *c* of Fig. 1). The solution is obtained here as follows. The magnetic field remains frozen in the plasma up to temperatures $T > T_1$, therefore $H = n = T^{-1}$ up to $T \sim \beta_0^{-1/2}$. Subsequently $H = n = \beta_0^{1/2}$ at $T < \beta_0^{-1/2}$, and the plasma pressure is here much lower than the magnetic-field pressure. At $T \sim T_1$ the magnetic field decreases to a value $H = 2/15$ and nT increases to unity. At $T < T_1$ the solution remains the same as before. Accordingly, no change takes place also in the order of magnitude of the effect thermal conductivity of the plasma.

Thus, in the case of a non-conducting wall the effective thermal conductivity of the plasma at $\beta_0 \gg 1$ and $\delta_0 \gg 1$ turns out to be of the order of the Bohm value. The reason for this is the rapid flow of the magnetic field towards the wall as a result of the Nernst effect, which decreases the magnetic field in the non-magnetized wall plasma to a value of the order of unity. The solution of this problem has a substantially different form in the case of an ideally conducting wall which does not pass the magnetic flux through. Now the magnetic field carried away towards the wall by the flow of the hot plasma accumulates in the wall region. As a result the magnetic field is strengthened here and the increase of the heat losses from the plasma are much less pronounced.

Let us examine this solution in greater detail. The boundary condition on the wall

$$q_H(\xi=0)=\left(\frac{4\mu^{1/2}}{\beta_0\sigma}\frac{dH}{d\xi}+V_H H+vH\right)_{\xi=0}=0$$

means that the thermal drift of the magnetic field should be offset by an oppositely directed diffusion flux. Therefore, in contrast to the case of a conducting wall, the magnetic viscosity (and the magnetic-field pressure) must now be taken into account at arbitrarily large values of the parameter β_0. Since the velocity V_H increases in the direction from the hot plasma towards the wall and becomes larger in the wall layer than the plasma flow velocity v, the compensation condition

$$\frac{4\mu^{1/2}}{\beta_0 T^{3/2}}\frac{dH}{d\xi}+V_H H=0 \tag{28}$$

should be satisfied not only on the wall itself, but in the entire region where $v_H > v$. This can be explained in the following manner. The flow of the hot plasma produces fluxes of matter and of magnetic field towards the wall. From the continuity equation (12) it follows that in the region of plasma "accumulation" ($T \sim T_*$) we have $\xi \sim v$, and at $T > T_*$ the velocity is $v > \xi$, and the flux of matter remains constant. The outflow of the magnetic field towards the wall proceeds with a velocity $v + V_H$. We therefore have here two possibilities. The first is that in the region of accumulation of the magnetic field (we designate its temperature by $\tilde{T}_*$) we have $V_H \gg v$. Inasmuch as in this case $V_H \gg \xi$, it can be seen from (10) that at $T < \tilde{T}_*$ the magnetic field should satisfy Eq. (28), and at $T > \tilde{T}_*$ we have a constant magnetic-field flux $V_H H = v_0$. It turns out, however, that no such solutions exist. The reason is that the magnetic field in the wall layer has an upper bound $H_{max} = \beta_0^{1/2}$, which follows simply from the pressure balance (9). Consequently, we are left with the second possibility. In this case the magnetic field is frozen into the plasma in the region where $V_H < v$, and at $V_H > v$ the compensation condition (28) is satisfied. It can be rewritten in the form

$$\frac{4\mu^{1/2}}{\beta_0 T^{3/2}}\frac{dH}{dT}+\frac{3}{2}\mu^{1/2}\alpha=0. \tag{29}$$

Since α has different dependences on n, T, and H in different regions [see (13)], we consider first the solution of this equation for magnetized electrons (regions I and II). Taking (9) into account, we find that

$$nT^{3/4}=\text{const.} \qquad (30)$$

This is the well-known result for a stationary inhomogeneous plasma. It follows therefore that the plasma pressure decreases with decreasing temperature ($nT \propto T^{3/4}$), so that a region of a buffer magnetic field with $H=\beta_0^{1/2}$ and an instantaneous value $\beta \ll 1$ is produced near the wall. The solution (30) does not hold at very low temperatures, where the plasma is already unmagnetized (region III). From (29) and (9), with allowance for (13), we obtain here the result

$$nT=(\gamma^2+{}^3/_{10}\mu^{-1}\beta_0\delta_0^2T^5)^{1/2},$$

where $\gamma \ll 1$ is the plasma pressure on the wall. This solution is valid at $T<\tilde{T}_1 \sim \gamma^{2/5}\mu^{1/5}\beta_0^{-1/5}\delta_0^{-2/5}$, where magnetization of the electrons takes place. Therefore the plasma pressure $nT \sim \gamma$ in the entire region III.

We can now visualize the complete solution of the problem. We start with the case of sufficiently large values of the parameters δ_0 and β_0, when $\delta_0>\mu^{-3/4}$ and $\beta_0>\mu^{-1}$. Then $V_H<v$ at $T>\mu^{1/2}$ and the magnetic field is frozen in the plasma: $H=n=T^{-1}$. At $T\sim\mu^{1/2}$ the velocity V_H of the thermal drift of the magnetic field becomes comparable with the plasma flow velocity, therefore the solution (30) is valid at $T<\mu^{1/2}$ and $nT\sim\mu^{-3/8}T^{3/4}$, while the magnetic field $H=\beta_0^{1/2}$. The decrease of the plasma pressure continues to $T=\tilde{T}_1\sim\mu^{1/14}\delta_0^{-4/7}\beta_0^{-2/7}$, where the plasma becomes demagnetized. At $T\lesssim\tilde{T}_1$ we have

$$nT\sim\gamma\sim(\mu^{9/28}\delta_0^{3/7}\beta_0^{3/14})^{-1}\ll 1.$$

The plasma and the magnetic field accumulate in a region with $T\sim\mu^{1/2}$, and the flow velocity of the hot plasma $v_0\sim\mu^{-1/8}$. Therefore the effective thermal conductivity of the plasma in this case is

$$\varkappa_{\text{eff}}\sim v_0^2\varkappa_{\perp 0}\sim(m_i/m_e)^{1/4}\varkappa_{\perp 0}. \qquad (31)$$

It is interesting to note that in the case of an ideally conducting wall the Nernst effect "works" not to increase the effective thermal conductivity of the plasma (as was the case for a non-conducting wall), but to decrease it, since the effect enhances the magnetic field near the wall. The qualitative picture of the wall layer at an ideally conducting wall is shown in Fig. 3.

Expression (31) for the effective thermal conductivity of the plasma was obtained under the condition that the thermal drift of the magnetic field becomes substantial (on going from a hot plasma to the wall) earlier than the magnetic pressure ($\beta_0>\mu^{-1}$) and the demagnetization of the ions ($\delta_0>\mu^{-3/4}$). If, however, one of these inequalities is not satisfied, the "accumulation" of the plasma and of the field takes place in a temperature region where the Nernst effect and the diffusion of the magnetic field play no role. Therefore the magnetic field is still frozen-in in the plasma and the estimates of $\varkappa_{\text{eff}}$ obtained assuming exact freezing-in of the field[10] become valid.

Let us consider, e.g., the case when β_0 is large but

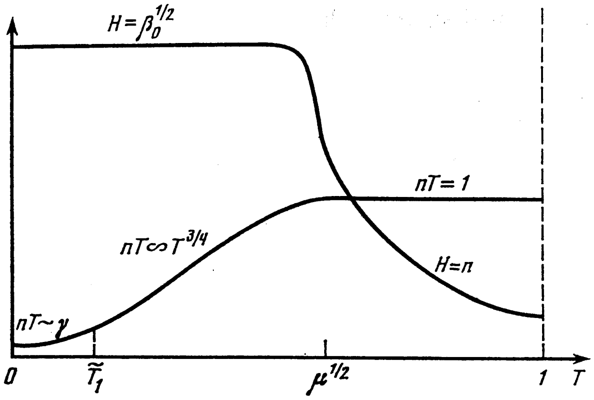

FIG. 3. Profiles of plasma pressure nT and of the magnetic field H in the case of an ideally conducting boundary at $\beta_0>\mu^{-1}$ and $\delta_0>\mu^{-3/4}$.

$\delta_0<\mu^{-3/4}$. The solution is obtained here as follows. In the entire region I, where $T>T_2=\delta_0^{-2/3}$, the field is frozen into the plasma. The "accumulation" of the field and of the plasma takes place at $T\sim T_2$. Therefore at $T<T_2$, the flux of matter and field decreases in region II in accordance with the law

$$nv=Hv\sim\delta_0T/v_0, \quad \text{i.e.,} \quad v\sim\delta_0T^2/v_0,$$

where $v_0\sim\delta_0^{1/6}$. At the same time, the velocity of the thermal drift of the field in region II is $V_H\sim\mu^{1/2}v_0/\delta_0T^{5/2}H$ and becomes comparable with v at $T\sim\mu^{1/7}\delta_0^{-10/21}$ Consequently, above this temperature the field is still frozen-in into the plasma: $H=n=T^{-1}$, and at lower temperatures we have $H=\beta_0^{1/2}$ and the plasma pressure decreases in accordance with Eq. (30). If, however, the plasma is strongly magnetized but $\beta_0<\mu^{-1}$, then "accumulation" of the plasma and of the field takes place at $T\sim\beta_0^{-1/2}$, where the pressure of the frozen-in magnetic field becomes comparable with the plasma pressure. In this case $v_0^2\sim\beta_0^{1/4}$ at $T<\beta_0^{-1/2}$ the field is still frozen-in at first, so that $H=n=\beta_0^{1/2}$, but then the thermal drift of the field comes into play and the condition (30) is satisfied. In the general form, the effective thermal conductivity of the plasma in the case of an ideally conducting wall can be written in the form

$$\varkappa_{\text{eff}}\sim\varkappa_{\perp 0}\min\{\delta_0^{1/3},\beta_0^{1/4},\mu^{-1/4}\}. \qquad (32)$$

§3. RADIATIVE COOLING WAVE

We proceed now to radiative cooling of a hot plasma with large β in contact with a cold wall. For the sake of argument we assume bremsstrahlung from the plasma, so that the plasma volume power is $Q_r=an^2T^{1/2}$. At constant plasma pressure, $Q_r\propto T^{-3/2}$ and increases rapidly with decreasing temperature.[1] Therefore at not too large plasma-column dimensions a situation is possible wherein the total radiation power from the cold wall layer exceeds the total radiation from the hot plasma. Cooling of the hot plasma takes place in this case on account of convective heat flow from the center to the wall, i.e., because of the adiabatic expansion of the hot plasma. The resultant plasma flow can be described in the following manner. The low-temperature wall layer of the plasma cools off rapidly because of radiation and is compressed (since the plasma pressure should remain uniform). The thermal conductivity leads to heat being drawn from the next layer of the plasma, which is cooled, contracts, etc. Thus, a cooling wave propagates from the wall to the hot plasma. Ahead of the wave there is a hot plasma with temperature T_0, density n_0, and magnetic field H_0, while behind the wave there is the "cooled" plasma with a certain low temperature $T_{\text{cold}} \ll T_0$. The value of T_{cold} can be determined, e.g., from the fact that at low temperatures (and accordingly at high densities) the plasma radition becomes blocked. The actual value of T_{cold} is immaterial to us, since all the characteristics of the cooling wave turn out to be independent of this quantity.

It will be convenient hereafter to change to a reference frame fixed in the wave, where all the quantities are independent of time. Introducing the characteristic time $\tau_0 = 3n_0T_0/Q_{r0}$ of the cooling of the hot plasma, we write down the basic equations (1)–(4) in terms of dimensionless variables defined in the following manner: $y = x(\chi_0\tau_0)^{1/2}$, $u = -v(\tau_0/\chi_0)^{1/2}$, χ_0 is the thermal diffusivity of the hot plasma. The density, temperatures, and magnetic field are measured respectively in units of n_0, T_0, and H_0. As a result we obtain from (1)–(4)

$$nT=1, \tag{33}$$

$$nu=u_0, \tag{34}$$

$$\frac{d}{dy}\left(\varkappa\frac{dT}{dy}+5u\right)=f(T), \qquad f(T)=\frac{Q_r}{Q_{r0}}; \tag{35}$$

$$uH+{}^3/_2\mu^{1/2}\alpha(dT/dy)=u_0. \tag{36}$$

These equations are written in the approximation $\beta_0 \to \infty$, which makes it possible to neglect the magnetic-field pressure, the magnetic viscosity, and the Joule heating of the plasma. The dimensionless thermal conductivity $\varkappa$, and the thermoelectric coefficient α are specified by Eqs. (13), and u_0 is here the dimensionless flow velocity of the hot plasma. This formulation of the problem is meaningful if the total radiative losses are determined by radiation from the transition layer, therefore it is possible now to neglect the radiation of the hot and "cooling-off" plasma, i.e., to assume formally that $f(1) = f(0) = 0$. As follows from (35), the next flux q_T in the plasma, which consists of thermal-conductivity and convective terms, can be represented in the form

$$q_T=\varkappa\frac{dT}{dy}+5u_0T=5u_0-\int\limits_y^\infty f\,dy=5u_0-\int\limits_T^1 f\frac{dy}{dT}\,dT, \tag{37}$$

so that the quantity u_0 is connected with the radiation from the transition layer by the equation

$$u_0=\frac{1}{5}\int\limits_0^1 f\frac{dy}{dT}\,dT. \tag{38}$$

The main contribution to this integral is made, as will be shown below, by the cold-plasma region with $T \sim T_* \ll 1$. It follows therefore from (37) that at $T \gg T_*$ the radiation can be disregarded and the heat flux in the plasma can be regarded as constant:

$$q_T=\varkappa(dT/dy)+5u_0T=5u_0,$$

so that at $T_* \ll T \ll 1$, when the convective contribution to the heat flux is small,

$$q_T\approx\varkappa(dT/dy)=5u_0. \tag{39}$$

Relations (36) and (39) make it possible to find the magnetic field. Indeed, from (39), under the condition (33), follows constancy of the heat-diffusion velocity: $V_H = 5u_0$. Therefore, using the introduced rate of thermal drift of the magnetic field V_H and Eq.(37), we get from (36)

$$H=(T+{}^{15}/_2\mu^{1/2})^{-1}, \quad T>T_1; \qquad H={}^2/_{15}, \quad T<T_1; \qquad T_1=(15\mu^{1/2}/2\delta_0)^{2/5}. \tag{40}$$

Just as before, the magnetic field is frozen into the plasma at $T \gg \mu^{1/2}$, and at lower temperatures its profile is determined by the Nernst effect. At $T = T_1$ the magnetic field experiences a jump. Knowing the magnetic field and by the same token the thermal conductivity $\varkappa$ of the plasma, we can determine the structure of the cooling wave. We recall that (39) is valid only at $T > T_*$. At $T < T_*$ the heat flux decreases because of radiation, so that the rate V_H decreases, and, as can be seen from (36), the magnetic field will be stronger than given by (40). It follows from the sequel, however, that the radiation begins to play a role at $T < T_1$, where the plasma is already unmagnetized and consequently the magnetic field does not affect the wave structure. The correct value of H there can be found *post facto*, after solving Eq. (35). In an unmagnetized plasma (region III) this equation is of the form

$$\frac{d}{dy}\left(\mu^{-1/2}\delta_0^2T^{5/2}\frac{dT}{dy}\right)=T^{-3/2}$$

(we have left out here the inessential convective contribution to the heat flux and put $f(T) = T^{-3/2}$ for the bremsstrahlung). Its solution with heat flux that vanishes as $T \to 0$ is of the form

$$\varkappa\frac{dT}{dy}=\mu^{-1/2}\delta_0^2T^{5/2}\frac{dT}{dy}=\mu^{-1/4}\delta_0T. \tag{41}$$

Since $q_T(T = T_1) = (\varkappa dT/dy)_{T_1} = 5u_0$, we obtain for the rate of expansion of the hot plasma

$$u_0=({}^{15}/_2)^{2/5}\mu^{-1/20}\delta_0^{3/5}/5\gg1. \tag{42}$$

The magnetic-field jump at $T = T_1$, from $H = 2/15$ to $H = (2/15)\mu^{-1/2}$, causes the thermal conductivity of the plasma to decrease abruptly at $T > T_1$, and this region no longer contributes to the total radiation. Therefore formulas (39) and (40) are valid here and can be used to find the temperature profile in this part of the transition layer.

The characteristic width of the cooling wave front is $\Delta y \sim \mu^{1/4}$. We obtain now the magnetic field at $T < T_1$. It follows from (41) that the thermal drift velocity is $V_H = (15/2)u_0T/T_1$ and is much larger than the plasma flow velocity $u = u_0T$. Therefore at $T < T_1$ the field is $H = u_0/V_H = 2T_1/15T$. An approximate structure of the front of the radiative cooling wave is shown in Fig. 4.

In the foregoing analysis it was implied that the degree of magnetizatioin of the plasma is high enough, namely $\delta_0 \gg \mu^{-3/4}$. In this case $\mu^{1/2} \gg T_1$ and the thermal effect set is in region I. If, however, $\delta_0 \ll \mu^{-3/4}$, then the profile of the

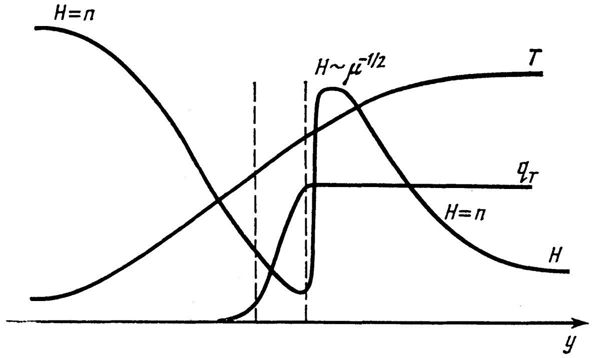

FIG. 4. Structure of the radiative cooling wave: T—plasma temperature, H—magnetic field, q_T—heat flux. The dashed lines mark the region that makes the principal contribution to the energy lost to radiation.

magnetic field in the front changes. In this case $H = T^{-1}(1 - T_1^{5/2}/T^{5/2})$, the magnetic field remains frozen into the plasma up to temperatures $T > T_1$, and then decreases at $T = T_1$ to a value $H = 2/15$. The rate of expansion of the hot plasma remains, as can be easily seen, the same as before.

We discuss now briefly the effects connected with the finite value of the parameter β_0. In the case of strong magnetization, when $\delta_0 \gg \mu^{-3/4}$, they begin to manifest themselves at $\beta_0 \lesssim \mu^{-1}$, and it is necessary here already to take into account the magnetic pressure and the finite conductivity of the plasma. We obtain therefore in place of (33) and (36)

$$nT + H^2/\beta_0 = 1,$$

$$\frac{4\mu^{1/2}}{\beta_0 T^{5/2}} \frac{dH}{dy} + \frac{3}{2} \mu^{1/2} \alpha \frac{dT}{dy} + uH = u_0.$$

In region III at $T < T_1$ the solution remains the same as before. Next, at $T > T_1$ the plasma pressure decreases in accordance with (26) and (27), and the magnetic field increases to a value $H \approx \beta_0^{1/2}$. Starting with temperatures $T > \mu^{1/2}$, the magnetic field becomes frozen into the plasma, so that $H \approx n = \beta_0^{1/2}$ in the interval $\mu^{1/2} \ll T \ll \beta_0^{-1/2}$. At $T > \beta_0^{1/2}$, the magnetic-field pressure is already negligible: $H = n = T^{-1}$.

If, however, $\delta_0 < \mu^{-3/4}$, the finite character of β_0 manifests itself when $\beta_0 < \mu^{-2/3} \delta_0^{4/5}$. In this case the field remains frozen into the plasma up to temperatures $T \gtrsim T_1$, therefore $H = n = T^{-1}$ at $T > \beta_0^{-1/2}$ and $H = n = \beta_0^{1/2}$ at $T < \beta_0^{-1/2}$. In the region $T \sim T_1$ the thermal drift of the magnetic field and its diffusion lead to a rapid decrease of H to a value $H = 2/15$ at $T = T_1$. Inasmuch as at $T < T_1$ the solution remains unchanged, in all the considered cases the rate of expansion of the hot plasma is determined as before by Eq. (42).

In conclusion, we consider the regions of applicability of the described different regimes of plasma cooling. To this end it is convenient to plot the characteristic plasma-cooling time τ_E against its dimensions. At small thicknesses of L plasma column we have $\tau_E \sim L^2/\chi_{\text{eff}}$, and according to (23)

$$\tau_E \sim L^2 e H_0 / c T_0.$$

This holds true so long as the heat flux to the wall by heat conduction exceeds the total loss to radiation. It is easy to verify that at

$$L \sim L_1 \sim \mu^{1/20} \delta_0^{2/5} (\chi_0 \tau_0)^{1/2}$$

the radiation from the wall layer becomes comparable with the heat flux to the wall. Therefore at $L > L_1$ the cooling of the plasma is determined by the radiative wave. In this case

$$\tau_E \sim \frac{L}{u_0} \left(\frac{\tau_0}{\chi_0} \right)^{1/2} \sim \frac{\mu^{1/20}}{\delta_0^{3/5}} \left(\frac{\tau_0}{\chi_0} \right)^{1/2} L$$

and the cooling time turns out to be proportional to the first power of the thickness of the plasma column. At

$$L \sim L_2 \sim \mu^{-1/20} \delta_0^{3/5} (\chi_0 \tau_0)^{1/2} \gg L_1$$

an important role is assumed by the radiation from the entire volume of the hot plasma, so that at $L \gtrsim L_2$ we have $\tau_E \sim \tau_0$.

As noted in the Introduction, the radiative cooling wave can describe not only the energy lost from the hot plasma, which is of interest for thermonulclear applications, but also the condensation of the hot plasma on intergalactic clouds under astrophysical conditions. In the latter case such a wave is meaningful also over times exceeding the radiative time τ_0 of the hot plasma. It follows from the solution obtained that cooling of the hot plasma takes place adiabatically slowly relative to the radiation wave in a plasma with a magnetic field. Therefore Eq. (42) for the rate of onflow of the hot plasma remains valid also in this case, but the quantities δ_0, χ_0, and τ_0 now vary with time.

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[1)]For recombination radiation we have $Q_r \propto n^2 T^{-1/2} \propto T^{-5/2}$ and the growth of the radiation power is even stronger.

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