Multiple acceleration of electrons in plasma resonance

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Electron acceleration under the action of an electromagnetic wave in the region of a resonance localized in an inhomogeneous plasma is collisionless. The effect of collisions on the acceleration process is considered. It is shown that collisions enable the electrons to traverse the resonance region many times. This results in a significant enhancement of the acceleration of the fast electrons. The theory is compared with the results of experiments on the effect of intense radio waves on the ionosphere.

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When electromagnetic waves propagate in an inhomogeneous layer of a tenuous plasma, the region of plasma resonance, where the wave frequency ω coincides with the natural frequency $\omega_0(z)$ of the plasma, becomes sharply pronounced.[1] In this region, plasma oscillations are intensively excited, modulation instability develops, and the electrons are effectively accelerated. The wave incident on the layer is dissipated, and an appreciable part of its energy goes to electron acceleration. Anomalous dissipation of electromagnetic waves and generation of accelerated electrons were observed many times in experiment both in laboratory plasma[2] and in the ionosphere.[3]

The processes that determine the electron acceleration were investigated in detail in Refs. 4–6. The acceleration is attributed to formation in the plasma, during the linear stage of development, of modulation instability of striction density wells—cavitons—filled with local blobs of electric field. When the caviton is crossed the energy of the fast electrons increases. The acceleration has a local character—it takes place in the vicinity of the plasma-resonance point $\omega = \omega_0(z_r)$ in a relatively thin plasma layer.

As will be shown below, however, the acceleration does not end here. Although the thickness of the accelerating plasma layer is usually much less than the electron mean free path, an electron accelerated in a layer as a result of scattering can return to the same layer because of the collisions, and its energy increases again after passing through this layer. The process then repeats. The electron energy will continue to grow until the energy lost by the electrons to collisions becomes comparable with that acquired in the accelerating layer. As a result of multiple passage of the electrons through the layer, their acceleration in the region of plasma resonance becomes considerably enhanced.

The present paper is devoted in fact to this phenomenon. In §1 we present a general theory of multiple scattering of electrons in plasma resonance. In §2 we consider specifically the acceleration of electrons acted upon by high-power radio waves in the upper layers of the ionosphere.

§1. FORMULATION OF PROBLEM. SIMPLIFIED KINETIC EQUATIONS

We consider a plasma that is weakly inhomogeneous along the z axis. Let the plasma-resonance point $z_r = 0$. The region in the vicinity of the plasma resonance, in which modulation instability develops intensively and the particles are accelerated, will be called the accelerating layer. We assume that its thickness is much less than the electron mean free path l. The collisions inside the accelerating layer can then be neglected. Outside the layer, i.e., in the upper ($z > 0$) and lower ($z < 0$) parts of the plasma, which are separated by the accelerating layer, the collisions play an important role. In this case the following kinetic equation is valid:

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} = -S, \tag{1}$$

where $f = f(t, \varepsilon, \mu, z)$ is the electron distribution function. It depends on the time t, on the coordinate z, on the electron energy ε, and on the angle θ between the direction of the velocity and the z axis; furthermore, $\mu = \cos\theta$, $v = (2\varepsilon/m)^{1/2}$, and S is the collision integral. In Eq. (1), generally speaking, it would be necessary to take into account also the electric field, but in the case of a weakly inhomogeneous plasma, when $l dN/dz \ll N$, its influence can be neglected.

The perturbation of the distribution function of the electrons proceeds qualitatively in the following manner. The fast electron crossing the accelerating layer increases in energy. Next, colliding with other plasma particles, it changes its velocity direction as well as its energy. It is important that the electron velocity direction usually changes in the collisions much more rapidly than the energy, i.e.,

$$\delta = 1/\nu_1 \Delta t_\varepsilon \ll 1. \tag{2}$$

Here ν_1 is the electron collision frequency and Δt_ε is the characteristic time of variation of its energy. Therefore within the time Δt_ε the electron manages to change its direction repeatedly and cross the accelerated layer many times ($\propto \delta^{-1/2}$). It is this which leads to enhancement of the acceleration of the electrons via the collisions, and also to the increase of the dimensions of the perturbed zone.

It is known[1,7] that under conditions (2) the electron velocity distribution becomes symmetrical, i.e., dependent mainly on the electron energy. It is therefore natural to expand its angular part in Legendre polynomials $P_n(\mu)$:

$$f(t, z, \varepsilon, \mu) = \sum_{n=0}^{\infty} f_n(t, z, \varepsilon) P_n(\mu).$$

The kinetic equation (1) is then rewritten in the form of a chain:

$$\frac{\partial F_0}{\partial t}+\frac{v}{3}\frac{\partial f_1}{\partial z}=-S_0,\qquad S_0=\frac{1}{2}\int_{-1}^{1} S d\mu, \tag{3}$$

$$\frac{\partial f_1}{\partial t}+v\left(\frac{\partial F_0}{\partial z}+\frac{2}{5}\frac{\partial f_2}{\partial z}\right)=-\nu_1 f_1,\ \ldots\ . \tag{4}$$

Account is taken here of the fact that the collision integral S_1 for the function f_1 is proportional to f_1:

$$S_1=\frac{3}{2}\int_{-1}^{1}\mu s\, d\mu=\nu_1 f_1, \tag{5}$$

where $\nu_1(\varepsilon)$ is the electron collision frequency. The collision integrals S_n are similar in form and have approximately the same values for the succeeding harmonics f_2, f_3, etc. (see Ref. 7). On the other hand, the integral $S_0(F_0)$ turns out to be smaller by a factor $1/\delta$ under the conditions (2). Next, F_0 is the change of the symmetrical part of the electron distribution function as the result of the action of the accelerating layer:

$$F_0=f_0-f^{(0)}, \tag{6}$$

where $f^{(0)}(\varepsilon)$ is the unperturbed value of the distribution function. We emphasize that in the region of the high-energy electrons the function $f^{(0)}(\varepsilon)$ can differ substantially from Maxwellian, being determined by the balance between the source and the losses of the fast particles. The principal role in Eqs. (3) and (4) under condition (2) is played by the spherically symmetrical function $F_0(t, \varepsilon, z)$, i.e., the one that depends only on the electron energy, while the higher harmonics of the expansion with $n = 1, 2,\ldots$ are small in terms of the parameter $\delta^{1/2} \ll 1$.

Equations (3) and (4) should be solved in each of the regions $z > 0$ and $z < 0$ and matched together at the accelerating-layer boundary $z \approx 0$. To find the corresponding boundary conditions we consider in greater detail the acceleration of electrons in a small vicinity of the plasma resonance, i.e., at $|z| \ll l = v/\nu_1$. Following Refs. 4–6, we assume that the pump-wave field excites in the plasma-resonance region intense natural longitudinal oscillations of the plasma. The plasma is then unstable. As a result of the development of a modulated striction instability, the distribution of the amplitude of the longitudinal oscillations in the plasma becomes strongly inhomogeneous—local blobs of a trapped field are produced, namely cavitons with scale $a \gtrsim D_e$, where $D_e = (T_e/m)^{1/2}\omega_0^{-1}$ is the Debye radius and T_e is the temperature of the bulk of the low-energy electrons.

The collisions of the electrons in the narrow accelerating layer can be neglected. Their motion is therefore described here by the simple collisionless equation

$$\frac{\partial v_\parallel}{\partial t}+\frac{1}{2}\frac{\partial}{\partial z}v_\parallel^2=-\frac{e}{m}E(z,t),$$

$$E(z,t)=\frac{1}{2}[E(z)e^{-i\omega t}+\text{c.c.}], \tag{7}$$

where $E = -\partial\varphi/\partial z$ is the longitudinal electric field that oscillates with the frequency of the pump wave $\omega \approx \omega_0$ ($z = 0$); $v_\parallel$ is the velocity of the electron along the z axis.

According to (7), the electron-energy increment $\Delta\varepsilon(z, t)$ (at a given point of space), averaged over the rapid oscillations, vanishes:

$$\langle\Delta\varepsilon(t)\rangle=0. \tag{8}$$

Nonetheless the acceleration prevails, since there are fewer fast eletrons than slow ones. Within the framework of perturbation theory, the energy $\Delta\varepsilon(z, t)$ acquired by the electron after passing through one caviton is[4]

$$\Delta\varepsilon(z,\ t)=-\Delta\varepsilon\cos(\omega t-\omega z/v_\parallel+\psi). \tag{9}$$

The amplitude $\Delta\varepsilon$ is here a function of the "longitudinal" energy of the electron, $\varepsilon_\parallel = mv_\parallel^2/2 = \varepsilon\mu^2$, and is determined by the expression

$$\Delta\varepsilon(\varepsilon_\parallel)=e|E_k|,\quad E_k=\int_{-\infty}^{\infty}E(z)e^{-ikz}dz, \tag{10}$$

$$k=\omega/v_\parallel=\omega(m/2\varepsilon_\parallel)^{1/2},$$

ψ is the phase of the electric field E_k. For example, for caviton of Gaussian and solitonlike form we obtain respectively

$$\Delta\varepsilon=\pi^{1/2}eE_0ae^{-(ka/2)^2}\quad \text{if}\quad E(z)=E_0e^{-(z/a)^2},$$

$$\Delta\varepsilon=\pi eE_0a/\text{ch}(\pi ka/2)\quad \text{if}\quad E(z)=E_0/\text{ch}(z/a).$$

It is important, however, that the main features of the dependence of $\Delta\varepsilon$ on $\varepsilon_\parallel$ have a universal character, i.e., they do not depend on the specific shape of the caviton. Indeed, electrons with sufficiently high energy

$$\varepsilon_\parallel>\varepsilon_1=m(\omega a)^2=T_e(a/D_e)^2,\quad (a/D_e)^2\gg 1 \tag{11}$$

cross the caviton within a time $a/v_\parallel$ which is short compared with the period $1/\omega$ of the high-frequency field ($v_\parallel > \omega a$). The amplitude $\Delta\varepsilon$ is in this case a maximum and equal to

$$\Delta\varepsilon(\varepsilon_\parallel>\varepsilon_1)=\Delta\varepsilon_m=e\int E\,dz\approx 2eE_0a, \tag{12}$$

where E_0 is the electric field at the center of the caviton and a is its half-width. In the case opposite to (11), that of relatively low electron energies, $\varepsilon_\parallel < \varepsilon_1$, the amplitude $\Delta\varepsilon$ decreases exponentially with increasing $\varepsilon_\parallel$ because of the fast oscillations of $E(t)$. The expressions obtained show that only sufficiently fast electrons with energy $\varepsilon_\parallel \gg T_e$ are accelerated by the cavitons [see (11)]. The region of applicability of expressions (9) and (10) is quite large—it is restricted by a condition, weak compared with (11), that the oscillatory velocity of the electron at the center of the caviton be small: $eE_0/m\omega < v_\parallel$.

After determining the electron-energy increment following the passage of the caviton in the accelerating layer $z \approx 0$, it is easy to find the corresponding change of the average distribution function $f(z)$. According to the Liouville theorem we have at $|z| \ll l$

$$f(\varepsilon_\parallel,\ z)=\langle f(\varepsilon_\parallel-\Delta\varepsilon(t),\ -z)\rangle\quad \text{at}\quad \mu z>0, \tag{13}$$

where the symbol $\langle f\rangle$ denotes, as before, averaging over the fast oscillations in time, while $\Delta\varepsilon(t)$ is itself a function of $\varepsilon_\parallel - \Delta\varepsilon(t)$. In the case of repeated passage of the electrons through the accelerating layer, the right-hand side of (13) can be expanded in terms of the small energy increment $\Delta\varepsilon$, retaining only the first nonvanishing term of the expansion. As a result we obtain, taking (8) into account,

$$\Delta f=\frac{1}{2}\frac{\partial}{\partial\varepsilon_{\parallel}}\left\{\langle\Delta\varepsilon^{2}\rangle\frac{\partial f}{\partial\varepsilon_{\parallel}}\right\} \qquad (14)$$

where Δf is the change of the distribution function on going through the caviton.

It is easy to see that by virtue of the small change of f this expression remains valid also when not one but several cavitons that are not correlated in phase are present in the accelerating layer. It must only be taken into account that the mean squared energy increment $\langle\Delta\varepsilon^2\rangle$ in (14) is equal to the sum of the contributions from all the cavitons:

$$\langle\Delta\varepsilon^{2}(\varepsilon_{\parallel})\rangle=\sum_{i}\langle\Delta\varepsilon_{i}^{2}(\varepsilon_{\parallel})\rangle=\frac{1}{2}\sum_{i=1}^{n}\Delta\varepsilon_{i}^{2}(\varepsilon_{\parallel}). \qquad (15)$$

Here $\Delta\varepsilon_i(\varepsilon_{\parallel})$ is the amplitude of the change of energy in the ith caviton (10) and n is the total number of cavitons.

Equations (14) and (15) determine the change of the distribution function on going through the accelerating layer. At the same time, in a weakly inhomogeneous collision-dominated plasma, on the boundaries of a symmetrically accelerating layer at $|z|\ll l$ the distribution function f should satisfy the symmetry condition $f(\varepsilon,\mu,z)=f(\varepsilon,-\mu,-z)$ and can therefore be represented in the form

$$f(\varepsilon,\mu,z)=f^{(+)}(\varepsilon,\mu)+(\operatorname{sign} z)f^{(-)}(\varepsilon,\mu) \quad \text{at} \quad |z|<l, \qquad (16)$$

where

$$f^{(\pm)}(\varepsilon,\mu)=\pm f^{(\pm)}(\varepsilon,-\mu), \quad \operatorname{sign} z=z/|z|.$$

Therefore that part of the distribution function $f^{(+)}$ which is even in $\mu=\cos\theta$ is continued through the accelerating layer without change, while the jump of the odd part is equal to $2f^{(-)}$. It is this jump which is determined by expression (14), in the right-hand side of which it suffices to take into account only the first principal term of the expansion of f in Legendre polynomials, which yields

$$2\Delta f^{(-)}|_{\mu>0}=\frac{1}{2}\frac{\partial}{\partial\varepsilon_{\parallel}}\left\{\langle\Delta\varepsilon^{2}(\varepsilon_{\parallel})\rangle\frac{df_0}{d\varepsilon}\right\}. \qquad (17)$$

Equation (17) stipulates the sought boundary conditions for Eqs. (3), (4), and (6). These equations can be simplified because the principal spherically symmetrical part F_0 of the perturbation of the distribution function (6) is most closely connected only with the first odd harmonic f_1 (and vice versa). With the aid of (3)–(5) it is easy to verify that under conditions (2) the influence of the higher terms of the expansion of the function f in Legendre polynomials (with $n\geqslant 2$) leads only to negligibly small corrections $\propto\delta^{1/2}\ll 1$ in F_0 and f_1. This confirms also the exact solution of the stationary equation (1) without expansion of the distribution function in Legendre polynomials, given in the Appendix for the case of a model collision integral. Recognizing that in the same approximation we can neglect also the term $\partial f_1/\partial t$ in (4), we obtain its solution in the form

$$f_1=-\frac{v}{\nu_1}\frac{\partial F_0}{\partial z}. \qquad (18)$$

The system (3) and (4) reduces thus to a single equation for the perturbation of the symmetrical part of the distribution function:

$$\frac{\partial F_0}{\partial t}-\frac{2\varepsilon}{3m}\frac{\partial}{\partial z}\left(\frac{1}{\nu_1}\frac{\partial F_0}{\partial z}\right)=-S_0(F_0) \qquad (19)$$

with the boundary condition at $z=0$,

$$\frac{\partial F_0}{\partial z}=-\frac{\nu_1}{v}\operatorname{sign} z\frac{3}{2}\int_0^1 f^{(-)}\,d\mu^2$$

$$=-\operatorname{sign} z\frac{3\nu_1}{8v\varepsilon}\frac{\partial}{\partial\varepsilon}\left\{\frac{\partial f_0}{\partial\varepsilon}\int_0^{\varepsilon}\langle\Delta\varepsilon^2(\varepsilon_{\parallel})\rangle d\varepsilon_{\parallel}\right\} \qquad (20)$$

$$f_0=F_0+f^{(0)}(\varepsilon).$$

We note that Eqs. (18)–(20) have simple conservation integrals. The first of them describes the change of the total number of particles as a result of inelastic ionization and recombination processes:

$$\frac{d}{dt}\left\{\int F_0 d^3v\,dz\right\}=-\int S_0(F_0)\,d^3v\,dz,$$

and the second the energy conservation:

$$\frac{d}{dt}W=-\int S_0(F_0)\,\varepsilon d^3v\,dz+q_E, \qquad W=\int F_0\varepsilon d^3v\,dz.$$

Here W is the total energy of the accelerated electrons, the integral term takes into account the energy loss in the collisions, and the source q_E takes into account the acquisition of energy in the accelerating layer:

$$q_E=\frac{2\pi}{m^2}\int_0^{\infty}\langle\Delta\varepsilon^2(\varepsilon)\rangle f_0(\varepsilon)|_{z=0}\,d\varepsilon.$$

Further simplifications of Eqs. (19) and (20), as well as their concrete solutions, depend on the form of the collision integral S_0. They will be considered in the next section for the conditions in ionospheric plasma.

§2. ACCELERATED ELECTRONS IN THE UPPER IONOSPHERE WHEN IT IS PERTURBED BY RADIO WAVES

In experiments on the action of high-power radio waves on the upper ionosphere (heights $h\sim 200$–400 km) one observes almost total absorption of radio waves with polarization, due to their transformations into the natural longitudinal oscillations of the ionospheric plasma.[3] An essential role in this process is played by the earth's weak magnetic field. Owing to the presence of the magnetic field, the excitation of the longitudinal waves by an ordinary radio wave has a more varied and complicated character than the case of an isotropic plasma.[4–6] The longitudinal waves are generated in the ionosphere by various mechanisms: by excitation of striction parametric instability in the region of reflection of the high-power radio wave, as a result of direct transformation of the ordinary radio wave by the smooth gradient of the ionospheric plasma, or via resonant scattering of the radio wave by the small-scale inhomogeneities that stretch out along the magnetic field.[8] It is important, however, that regardless of the actual mechanism that excites the plasma waves, the weakness of the collisional dissipation causes the greater part of their energy to accumulate in the vicinity of the plasma-resonance point, where their group velocity decreases sharply.[1)] The amplitude E_0 of the plasma-wave field increases correspondingly in this region. Therefore, under the ionosphere conditions, just as in an isotropic plasma, modulation instability develops here and an accelerating layer is produced. Its size does not exceed the scale of the collisional damping of the plasma waves (~ 100 m), i.e., it turns out to be much less than the mean free path $l = 1$–10 km of the fast electrons. The width of the individual field blobs is $a \approx 1$–10 cm. Consequently the conditions for applicability of the theory of multiple acceleration of the electrons are well satisfied here.[2)]

We consider next Eq. (19). In the ionosphere, the main contribution to the collision integral S_0 for electrons of energy $\varepsilon \gtrsim 4$–5 eV is made by inelastic collisions with excitation of optical levels and ionization of atomic and molecular oxygen (O, O_2) and molecular nitrogen (N_2). In this case the electron loses a large energy after one inelastic impact, so that the collision integral S_0 in the high-energy region can be represented in the form[9]

$$S_0(F_0)=\nu_0(\varepsilon)F_0, \quad \nu_0(\varepsilon)=v\sigma_0 N_m. \tag{21}$$

Here σ_0 is the total effective cross section of the inelastic collisions, N_m is the total density of the neutral particles:

$$\sigma_0=\sum_k \sigma_{0k}(\varepsilon)N_{mk}/N_m, \qquad N_m=\sum_k N_{mk},$$

σ_{0k} and N_{mk} are the total inelastic-collision cross section and the density of each neutral component O, O_2, and N_2, respectively. The transport cross section σ for the scattering of electrons by neutral particles in the collision integral S_1 (5) is written in similar form:

$$S_1=+\nu_1(\varepsilon)f_1, \quad \nu_1=v\sigma_t N_m, \quad \sigma_t=\sum_k \sigma_{tk}(\varepsilon)N_{mk}/N_m.$$

The expression for the mean fraction δ [Eq. (12)] of the energy lost by the electron in one collision takes the following simple form:

$$\delta=\nu_0/\nu_1=\sigma_0/\sigma_t. \tag{22}$$

The dependence of the parameter δ on the electron energy, for different heights in the ionosphere, is shown in Fig. 1. It can be seen that $\delta \sim 0.1$. Consequently, the principal condition $\delta \ll 1$ for multiple acceleration of electrons in the ionosphere is well enough satisfied. We note also that the relative density N_{mk}/N_m of the neutral components varies in the ionosphere much more slowly than the total density $N_m(h)$. Therefore the parameter δ [Eq. (22)] depends relatively little on the height h and on the coordinate z.

Under the conditions (21), the equation (19) for the distribution function of the acceleration electrons depends on the energy ε as a parameter. Let us obtain the stationary solution of this equation. To this end we rewrite it in the form

$$\frac{\partial^2 F_0}{\partial \xi^2}=3\delta F_0, \quad \xi=\int_0^z \frac{\nu_1}{v}\,dz=\int_0^z \sigma_t N_m(z)\,dz. \tag{23}$$

Recognizing that the parameter δ is a slow function of ξ, we

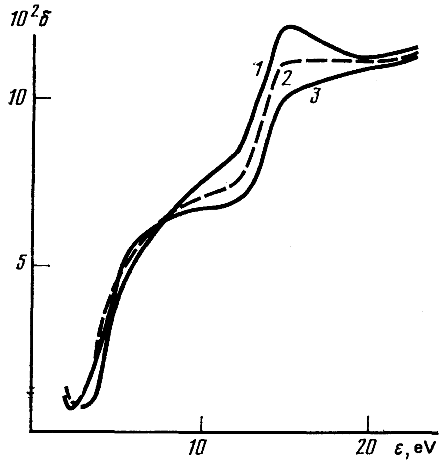

FIG. 1. Dependence of the average fraction δ of the energy loss in the ionosphere on the electron energy ε: 1) height $h = 250$ km; 2) $h = 300$ km; 3) $h > 300$ km.

obtain hence in the WKB approximation

$$F_0(\varepsilon,z)=F_0(\varepsilon)\left[\frac{\delta(z=0)}{\delta(z)}\right]^{1/4}\exp\left\{-\left|\int_0^z \frac{dz}{L_\varepsilon(z)}\right|\right\},$$
$$f_1(\varepsilon,z)=(\operatorname{sign} z)(3\delta)^{1/2}F_0(\varepsilon,z), \tag{24}$$

where L_ε is the characteristic scale of the relaxation of electrons with a given energy ε:

$$L_\varepsilon=v/(3\nu_1\nu_0)^{1/2}=1/N_m(3\sigma_t\sigma_0)^{1/2}. \tag{25}$$

It is shown in Fig. 2. It can be seen that the scale L_ε increases rapidly with the height h because of the exponential decrease of $N_m(h)$. This can lead to a considerable asymmetry in the height distribution of the fast electrons. Moreover, the quantity $L_\varepsilon(z)$ increases so abruptly at large heights that the integral in the exponential of (24) always converges to a finite limit R_∞ as $z \to \infty$. This means that a flux P of fast electrons that go off into the magnetosphere is produced. The value of

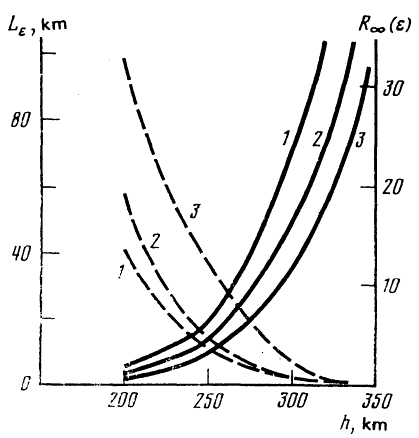

FIG. 2. Change of the relaxation scale L_ε (solid lines) and of the parameter R_∞ (dashed) with height h for different energies: 1) $\varepsilon = 4.5$ eV; 2) $\varepsilon = 10$ eV; 3) $\varepsilon = 20$ eV.

the flux depends exponentially on R_∞:

$$\frac{dP}{d\varepsilon}=\frac{8\pi}{3^{1/2}m^2}[\delta(z)\delta(z=0)]^{1/2}F_0(\varepsilon)\varepsilon e^{-R_\infty},\quad R_\infty=\int_0^\infty\frac{dz}{L_\varepsilon(z)}. \tag{26}$$

The integral R_∞ is also shown in Fig. 2 as a function of the height h of the accelerating layer. It can be seen that R_∞ decreases rapidly with increasing h, all the way to small values $R_\infty < 1$ at heights $h \gtrsim 300$ km, where the characteristic scale of the decrease of $N_m(h)$ becomes smaller than L_ε. Consequently the greater part of the accelerated electrons goes off at these heights to the magnetosphere. We note that the solution (24) is valid only provided that the flux (26) is small, i.e., at $\exp\{-R_\infty\} \ll 1$. In the case $R_\infty < 1$ the effect of multiple acceleration becomes weaker.

Next, the function $F_0(\varepsilon)$ in expressions (24) and (26) is equal to $F_0(\varepsilon) = F_0(\varepsilon, z)|_{z=0}$. According to the boundary condition (20), it satisfies a differential equation of the diffusion type:

$$\frac{1}{\delta^{1/2}\varepsilon}\frac{d}{d\varepsilon}\left\{\delta^{1/2}\varepsilon T_{eff}^2\frac{df_0}{d\varepsilon}\right\}=f_0-f^{(0)}(\varepsilon),\qquad F_0=f_0-f^{(0)}(\varepsilon). \tag{27}$$

We have introduced here the notation

$$T_{eff}(\varepsilon)=\varkappa\overline{\Delta\varepsilon},\quad \overline{\Delta\varepsilon}=\left\{\frac{1}{\varepsilon}\int_0^\varepsilon\langle\Delta\varepsilon^2(\varepsilon_\parallel)\rangle d\varepsilon_\parallel\right\}^{1/2},\quad \varkappa=\frac{3^{1/4}}{2\sqrt{2}}\delta^{-1/4}, \tag{28}$$

where $\overline{\Delta\varepsilon}(\varepsilon)$ is the average increment of the electron energy in the accelerating layer. It will be shown below that under the conditions of strong multiple acceleration [(33), (34b)] the parameter T_{eff} determines a new effective electron temperature in the high-velocity region:

$$T_{eff}=-f_0(\varepsilon)\Big/\frac{df_0(\varepsilon)}{d\varepsilon}. \tag{29}$$

The temperature T_{eff} [(28), (29)] is a function of the electron energy ε. This dependence is due mainly to the change of $\overline{\Delta\varepsilon}(\varepsilon)$, since under the conditions of the ionosphere the coefficient $\varkappa$ depends little on the energy ε and on the height h, namely $\varkappa(\varepsilon, z) \approx 1.0$. The increase of electron energy in the accelerating layer was considered in detail in §1 [see Eqs. (9)–(12) and (15)]. Our analysis enables us to state that the effective temperature $T_{eff}(\varepsilon)$ increases monotonically with increasing ε and reaches at the high energies (11) and (12) a maximum value

$$T_{effm}=\varkappa(n/2)^{1/2}\Delta\varepsilon_m,\quad \Delta\varepsilon_m\lessdot\varepsilon_1, \tag{30}$$

whereas at $\varepsilon < \varepsilon_1$ it decreases rapidly exponentially. Here n is the average number of cavitons in the accelerating layer.

Knowing the behavior of the function $T_{eff}(\varepsilon)$, it is easy to analyze the peculiarities of the electron acceleration. The intensity of the acceleration is determined by the parameter

$$\gamma=\frac{T_{effm}}{T},\qquad T=-f^{(0)}(\varepsilon)\Big/\frac{df^{(0)}(\varepsilon)}{d\varepsilon}, \tag{31}$$

where T is the effective temperature, analogous to (29), for the distribution function $f=f^{(0)}$ of the unperturbed (background) electrons. In the case of weak acceleration $\gamma < 1$ the distribution function $f^{(0)}$ of the background electrons is only insignificantly distorted. Neglecting in this case the small perturbation $F_0 \ll f^{(0)}$ in the left-hand side of (27), we obtain

$$F_0(\varepsilon)=T_{effm}^2\frac{d^2f^{(0)}(\varepsilon)}{d\varepsilon^2}=\left(\frac{T_{effm}}{T}\right)^2 f^{(0)}(\varepsilon)\qquad(\varepsilon>T>T_{effm}). \tag{32}$$

In the opposite case of strong acceleration

$$\gamma=T_{effm}/T\gg 1 \tag{33}$$

Eq. (27) has the following asymptotic solution (in the WKB approximation):

$$f_0(\varepsilon)=f^{(0)}(\varepsilon)\qquad(\varepsilon<\varepsilon_{sad}), \tag{34a}$$

$$f_0(\varepsilon)=f^{(0)}(\varepsilon_{sad})\left[\frac{\pi}{2}\frac{\varepsilon_{sad}T_{eff}(\varepsilon_{sad})\delta^{1/2}(\varepsilon_{sad})}{\varepsilon T_{eff}(\varepsilon)\delta^{1/2}(\varepsilon)}\Big/\frac{dT_{eff}(\varepsilon_{sad})}{d\varepsilon}\right]^{1/2}$$

$$\times\exp\left\{-\int_{\varepsilon_{sad}}^{\varepsilon}\frac{d\varepsilon}{T_{eff}(\varepsilon)}\right\}\qquad(\varepsilon>\varepsilon_{sad}). \tag{34b}$$

The new distribution function (34b) is a continuation of the background function (34a) at the saddle point $\varepsilon = \varepsilon_{sad}$, which is determined by the temperature-equality condition

$$T_{eff}(\varepsilon_{sad})=T(\varepsilon_{sad}). \tag{35}$$

We note that the quasiclassical-approximation conditions

$$dT_{eff}(\varepsilon)/d\varepsilon\ll 1,\qquad T_{eff}(\varepsilon)\ll\varepsilon \tag{36}$$

are always satisfied at $\varepsilon < \varepsilon_1$, and in the case of a small number of cavitons (i.e., at $T_{eff\,m} < \varepsilon_1$) it is satisfied in the entire energy interval.

We present also the function $f_0(\varepsilon)$ in the case of the strongest acceleration of electrons by a large number of cavitons at $T_{eff\,m} \gg \varepsilon_1$, when the second condition of (36) is violated in the region $\varepsilon_1 < \varepsilon < T_{eff\,m}$. Recognizing that in the case

$$T_{eff}(\varepsilon)\approx T_{effm}=\text{const},\qquad \delta^{1/2}(\varepsilon)\approx\text{const}$$

Eq. (27) reduces to a Bessel equation, we obtain at $\varepsilon > \varepsilon_1$

$$f_0(\varepsilon)=c\frac{N_0'}{2\pi}\left(\frac{m}{2T_{effm}}\right)^{3/2}K_0\left(\frac{\varepsilon}{T_{effm}}\right),\qquad c=2^{1/2}\Gamma^{-2}(3/4)\approx 0.944. \tag{37}$$

Here $K_0(z)$ is a Bessel function of imaginary argument, $\Gamma(3/4) = 1.225$ is the Euler gamma function, N_0' is the density of the accelerated electrons, and the normalization constant c is chosen in accord with the condition $\int f_0(\varepsilon)d^3v = N'_0$. The asymptotic forms of (34) and (37) are naturally identical.

The form of the distribution function $f_0(\varepsilon)$ of the fast electrons under the conditions $T_{eff\,m} \gg T$ (33) of strong acceleration is shown schematically in Fig. 3. It can be seen that the number of fast electrons in the region $\varepsilon > \varepsilon_{sad}$ increases abruptly: $f_0 \gg f^{(0)}$. In this case, the rate of decrease of $f_0(\varepsilon)$ with increasing ε slows down substantially already in the transition region $\varepsilon_{sad} < \varepsilon < 1$. The maximum temperature $T_{eff\,m}$ (30) is established in the region of high energies $\varepsilon > \varepsilon_1$ (11) which greatly exceed the temperature T_e of the thermal electrons.

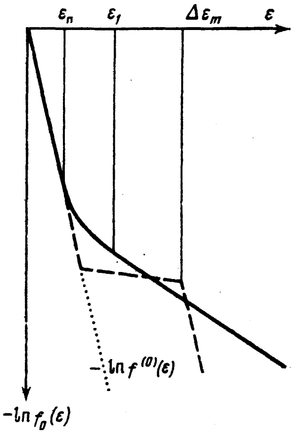

FIG. 3. Distortion of fast-electron distribution function as a result of their multiple acceleration in the vicinity of plasma resonance. $f^{(0)}(\varepsilon)$ is the distribution function of the background electrons in the unperturbed plasma. The deformation of $f^{(0)}(\varepsilon_\parallel)$ in the collisionless case is shown dashed.

Special notice should be taken of the fact that such an energy distribution of the accelerated electrons differs qualitatively from the distribution in a collisionless plasma, where the initial function $f^{(0)}(\varepsilon_\parallel)$ is smeared out over an energy interval $\sim\Delta\varepsilon_m$ on account of acceleration by the caviton. Therefore, in a collisionless plasma the ratio

$$f_0(\varepsilon_\parallel)/f^{(0)}(\varepsilon_\parallel)\approx\exp(\Delta\varepsilon_m/T_e)$$

is independent of $\varepsilon_\parallel$ at high energies $\varepsilon_\parallel>\Delta\varepsilon_m\approx T_{\text{eff}\,m}$ (Ref. 4), whereas in our case it increases exponentially [see Fig. 3, in which the collisionless function $f_0(\varepsilon_\parallel)$ is shown dashed]. The cause of the abrupt enhancement of the effect in the collision-dominated plasma is perfectly understandable: As a result of the collisions the electrons become capable of crossing the accelerating layer many times. Therefore their distribution function $f_0(\varepsilon)$ in the high-energy region is determined by the statistics (by the probability of multiple acceleration in the layer) and not by the form of the rapidly decreasing initial distribution function $f^{(0)}(\varepsilon)$. At the same time, the total number of accelerated electrons turns out to be proportional to the density $f^{(0)}(\varepsilon)$ of the background particles in the region of low energies $\varepsilon\approx\varepsilon_{\text{sad}}$ (35). Therefore the change of the distribution function of the background particles at $\varepsilon\approx\varepsilon_{\text{sad}}$ always influences substantially the number of accelerated electrons. We emphasize that the foregoing quantitative analysis of the mechanism of multiple acceleration of electrons via collisions with neutral molecules (at $\delta\ll1$) remains qualitatively valid also in a fully ionized plasma at $\delta\approx1/2$.

Let us estimate the energy lost by a radio wave to acceleration of fast electrons in the vicinity of the plasma resonance. According to (21) and (24) we have

$$P=\int\nu_0\varepsilon F_0(\varepsilon,z)\,d^3v\,dz$$
$$=\frac{8\pi}{3^{1/2}m^2}\int\delta^{1/4}(z=0)\,\varepsilon^2F_0(\varepsilon)\,d\varepsilon\int\delta^{1/4}(z)$$
$$\times\exp\left\{-\left|\int_0^z\frac{dz}{L_\varepsilon}\right|\right\}\frac{dz}{L_\varepsilon}.$$

From this, under conditions of relatively weak change $\delta=\delta(\varepsilon,z)$, we obtain approximately

$$P\approx\frac{16\pi}{m^2}\left(\frac{\delta}{3}\right)^{1/2}\int\varepsilon^2F_0(\varepsilon)\,d\varepsilon.$$

The generation of the accelerated electrons in the field of high-power radio waves leads to a number of experimentally observable effects. Thus, an increase in the number of fast electrons with energies $\varepsilon>2$ eV enhances the emission of the red ($\lambda_1=6300$ Å) and green ($\lambda_2=5577$ Å) lines of oxygen, with respective excitation potentials $I_1=1.98$ and $I_2=4.17$ eV (Ref. 10). An estimate of the relative luminosities of these lines allows us to conclude that the effective electron temperature (29) in the energy region $\varepsilon\approx(2\text{–}4)$ eV increases under the influence of the high-power radiation at least to (2–3) eV at an initial thermal-electron temperature $T_e\sim0.1$ eV (in the dark time of the day). This increase of T_{eff} agrees well with the results of the theory of multiple acceleration of electrons; see Eqs. (30) and (11). Calculations show in this case[11] that it would be possible to observe in experiment also enhancement of the emission of other lines, $\lambda_3=8446$ Å, $\lambda_4=7774$ Å, and $\lambda_5=3914$ Å with higher excitation potentials: $I_3=10.74$ eV, $I_4=10.99$ eV, and $I_5=18.74$ eV.

Fast electrons with energies $\varepsilon>(12\text{–}15)$ eV lead also to an increase of the ionization of the ionosphere, i.e., an increase of the electron density in the region of reflection of high-power radio waves. This effect was observed in experiments[12] during daytime hours, while at night the effect is much weaker. The enhancement of the action of the accelerating layer with increasing number of background electrons comes into play here. These are the fast photoelectrons that appear only in the sunlit ionosphere and have in the ionization-energy region a high "temperature" T [Eq. (31)] on the order of 3 eV. The magnitude of the observed effect is well described by expression (32).

The increase in the number of fast electrons and in their effective temperature T_{eff} can lead also to enhancement of the plasma noise with frequency ω close to the local plasma frequency ω_0. According to Ref. 13, the intensity of the noise excited by electrons of energy ε is proportional to $T_{\text{eff}}(\varepsilon)$. This phenomenon was observed in Arecibo with the aid of the facility for incoherent backscattering.[14] When the high-power transmitter was turned on at nighttime, a considerable (larger by 1–2 orders of magnitude) enhancement of the intensity of plasma noise excited by the fast electrons with $\varepsilon\gtrsim22$ eV was observed. The perturbing radio wave was reflected at a height 285 km, and amplification of the plasma waves took place in a wide range of heights from 255 to 450 km. The magnitude of the observable effect and its height distribution (the width and asymmetry of the perturbed zone) are sufficiently well described by expressions (25), (28), and (34b).

The authors are grateful to G. N. Milikh for supplying the data and for performing the calculations connected with the ionospheric applications.

APPENDIX

We obtain now the exact solution of the stationary solution (1) for the perturbation of the distribution function $F(\varepsilon,$

$\mu, z) = f - f^{(0)}$ in the case of the model collision integral[3)] S:

$$v\mu(\partial F/\partial z) = -S, \quad \mu = \cos\theta, \tag{A.1}$$

$$S = \nu_0 F_0 + \nu_1 (F - F_0), \quad F_0 = {}^1/_2 \int_{-1}^{1} F(\mu)\, d\mu.$$

We consider for the sake of argument the relaxation of $F(z)$ in the region of positive $z > 0$. The boundary conditions for $F(z)$ will be assumed specified at the point $z = 0$. Introducing a variable analogous to (23)

$$\xi = \int_0^z \nu_1(z') v^{-1}\, dz'$$

and carrying out a unilateral Fourier transformation

$$F_q = \int_0^\infty F(\xi) e^{-iq\xi}\, d\xi,$$

we obtain in the case $\delta \equiv \nu_0/\nu_1 \neq \delta(z)$ [see (20)]

$$F_q = \frac{1 - iq\mu}{1 + q^2\mu}\{(1-\delta)F_{0q} + \mu F(0)\}, \text{ where } F(0) = F(z=0). \tag{A.2}$$

We resolve furthermore $F(\mu)$ into even and odd (relative to μ) components:

$$F(\mu) = F^{(+)}(\mu) + F^{(-)}(\mu), \quad \text{where} \quad F^{(\pm)}(\mu) = \pm F^{(\pm)}(-\mu),$$

and change over to the convenient combinations $\widetilde{F}_q^{(\pm)} = F_q^{(\pm)} \pm F_{-q}^{(\pm)}$. This corresponds to a continuation,[4)] odd in $F^{(-)}$ and even in $F^{(+)}$ of $F(z)$ into the region of negative z, viz., $F^{(\pm)}(-z) = \pm F^{(\pm)}(z)$. According to (2) we have

$$\widetilde{F}_q^{(+)} = (1+q^2\mu^2)^{-1}\{(1-\delta)\widetilde{F}_{0q} + 2\mu F^{(-)}(0)\}, \quad \widetilde{F}_q^{(-)} = -iq\mu \widetilde{F}_q^{(+)}, \tag{A.3}$$

where the symmetrical function $\widetilde{F}_{0q}$ is equal, by virtue of the definition (A.1), to

$$\widetilde{F}_{0q} = \left\{1 - (1-\delta)\frac{\operatorname{arctg} q}{q}\right\}^{-1} \int_0^1 \frac{F^{(-)}(0)\, d\mu^2}{1 + q^2\mu^2}. \tag{A.4}$$

The expressions (A.3) and (A.4) solve our problem—they determine the Fourier transform of the perturbation $F(z)$ in terms of the value of its odd (in μ) component $F^{(-)}(0)$ at the boundary $z = 0$. In particular, at the point $z = 0$ we obtain

$$F_0(0) = \frac{1}{\pi}\int_0^\infty \left\{1 - (1-\delta)\frac{\operatorname{arctg} q}{q}\right\}^{-1} dq \int_0^1 \frac{F_0 - d\mu^2}{1 + q^2\mu^2},$$

$$F^{(+)}(0) = \frac{1-\delta}{\pi}\int_0^\infty \widetilde{F}_{0q} \frac{dq}{1 + q^2\mu^2} + \frac{\mu}{|\mu|} F^{(-)}(0). \tag{A.5}$$

The inversion of $\widetilde{F}_q^{(-)}$ as $z \to +0$ leads to an identity. It is easy to see that in the case of small $\delta = \nu_0/\nu_1 \ll 1$ the first formula of (A.5) goes over into the solution (24):

$$F_0(0) = \frac{1}{(3\delta)^{1/2}} F_1(0), \text{ where } F_1(0) = \frac{3}{2}\int_0^1 F^{(-)}(0)\, d\mu^2$$

independently of the values of the higher harmonics of the expansion of $F^{(-)}(0)$ in odd Legendre polynomials.

1)The plasma-resonance point z_r in the plasma layer $N(z)$ is determined in the presence of a magnetic field by the condition $\omega_0^2(z_r) = \omega^2[1 - \omega_H^2 \sin^2\alpha/(\omega^2 - \omega_H^2\cos^2\alpha)]$. Here $\omega_H = eH/cm$ is the gyromagnetic frequency of the electrons and α is the angle between the magnetic field and the z axis.

2)We note that the accelerated electrons in the ionosphere have a small Larmor radius ($\sim$ 10–20 cm) because they move mainly along the magnetic field. Therefore the restriction on the transverse dimensions of the accelerating layer to a scale of the order of 10–100 km (due to the finite width of the beam of the radio waves that excite the ionosphere) does not really influence the multiple acceleration process considered in the preceding section, provided that the magnetic field crosses the accelerated grating layer at an appreciable angle.
Calculation of the multiple-acceleration effect in a magnetized plasma shows that under the influence of the magnetic field the average number of crossings of the accelerating layer by the fast electrons increases by $1/\cos\widetilde{\gamma} = (\cos^2\alpha + \nu_1^2\sin^2\alpha/\omega_H^2)^{-1/2}$ times, as a result of which the effective temperature (28) in the case $\omega_H \ll \omega$ increases by $1/(\cos\widetilde{\gamma})^{1/2}$ times. Simultaneously, the characteristic scale (25) of the spatial relaxation of the accelerated electrons increases in proportion to $\cos\widetilde{\gamma}$.

3)The chosen integral S describes well the collisions of the high-energy electrons with neutral molecules [Eqs. (5), (21)] under conditions when the differential cross section $\sigma(\varepsilon, \theta)$ for elastic scattering does not depend on the scattering angle θ (the elastic-spheres approximation).

4)We note that precisely such a symmetry of $F(z)$ is realized in a collision-dominated plasma in the presence of an accelerating layer, see (16).

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