Relaxation of hot electrons in a semiconductor in strong magnetic fields

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(Submitted 1 July 1982)
Zh. Eksp. Teor. Fiz. **84**, 230–236 (January 1983)

The relaxation rate τ_H^{-1} of the energy of nonequilibrium electrons scattered by holes in a p-type semiconductor placed in a strong magnetic field is calculated. It is assumed that the electrons are excited to the lower Landau level, and the masses of the electrons and holes are substantially different ($m_e \ll m_h$). The character of the relaxation depends on the ratio of the width ΔE of the energy distribution of the electrons to the distance $\hbar\omega_h$ between the Landau levels for the holes. At $\Delta E \ll \hbar\omega_h$ the energy losses by the electrons become discrete. This leads to the appearance of oscillations of τ_H^{-1} in the electron energy region $E \gtrsim \hbar\omega_h$ and to a conversion of the initial narrow energy distribution of the electrons into a sequence of peaks with distances $\hbar\omega_h$ between them along the energy axis. The energy relaxation of the broad distribution ($\Delta E \gg \hbar\omega_h$) is not substantially influenced by a quantizing magnetic field. This distinguishes the mechanisms considered here from ordinary electron-electron scattering. The possibilities of experimentally studying electron-hole scattering in a magnetic field are discussed.

PACS numbers: 72.20.Dp, 72.20.My

INTRODUCTION

A strong magnetic field H acting on a semiconductor alters the wave functions and the energy spectrum of the carriers. This influences the characteristic frequencies of the relaxation processes in the carrier system. It is well known, e.g., that pair collisions of electrons do not lead in the ultraquantum limit to relaxation of the energy disequilibrium and to establishment of an effective electron temperature T_e (Ref. 1). Energy exchange between the electrons becomes possible only in collisions that occur in the field of an impurity or of a third electron.[2] The magnetic field modifies also the elastic and inelastic scattering of electrons by phonons[1,3] and impurities.[4] The changes of the dynamics of the carriers and of the relaxation frequencies in a magnetic field are reflected in the kinetic properties of semiconductors.[1,5–8] When the majority carriers in a semiconductor are excited, the nonequilibrium distribution function is formed with participation of the relaxation mechanisms described above.

When minority carriers are excited, or in the case of intense bipolar excitation,[9] it is necessary to take into account also one more relaxation mechanism, electron-hole scattering. We calculate in this paper the rate τ_H^{-1} of relaxation of the energy of electrons excited to the lower Landau band when they collide with holes. In semiconductors with substantially different effective carrier masses ($m_e \ll m_h$), the cyclotron frequencies of the electrons and holes also differ greatly: $\omega_e \gg \omega_h$. If the characteristic scale ΔE of the change of the electron distribution function is less than ω_h (we assume here and elsewhere $\hbar \equiv 1$), quantization of the hole spectrum by the magnetic field manifests itself in τ_H^{-1}. If, however, $\Delta E, T \gg \omega_h$ (T is the hole temperature), the hole motion can be regarded as classical. In the latter case the three-dimensional character of the hole motion causes the magnetic field to influence the frequency τ_H^{-1} weakly; the $\tau_H^{-1}(E)$ dependence in the region of energies E of electron motion along the field H and of temperatures T,

$$\omega_e \gtrsim E,\ T \gg \omega_h, \tag{1}$$

differs from $\tau^{-1}(E)$ at $H = 0$ only in that the Coulomb logarithm is changed.

Electron-hole collisions can be the basic channel for energy relaxation of hot electrons even at moderate densities n of the equilibrium holes. An estimate shows that, e.g., in p-InSb with $n \sim 10^{16}$ cm^{-3} the value of $\tau_H^{-1}(E)$ in the entire region (1) is larger by two orders of magnitude than the frequency of the electron energy relaxation on phonons.[10]

At $\Delta E \ll \omega_h$ the variation of the electron energy in the course of electron-hole collisions becomes discrete. This transforms the initially narrow energy distribution of the electrons into a sequence of peaks separated along the energy axis by ω_h; in the energy region $E \gtrsim \omega_h$ the quantity $\tau_H^{-1}(E)$ becomes oscillatory. The simplest form of these oscillations occur in a semiconductor with a nondegenerate valence band: the maxima of $\tau_H^{-1}(E)$ are located at the points $E = l\omega_h$, $l = 1,2...$; at $E < \omega_h$ the relaxation rate $\tau_H^{-1}(E)$ is exponentially small.

We calculate the relaxation frequency assuming the density of the excited electrons to be low enough for the collective effects in the electron system[11] not to manifest themselves.

DIFFUSION APPROXIMATION IN THE KINETIC EQUATION FOR ELECTRONS

The condition (1) makes it possible to neglect the subdivision of the hole spectrum into Landau bands. For electrons, on the contrary, only the lower Landau band can be regarded as filled. The electron-hole collision integral can be obtained by suitable simplification of the general expression given in Ref. 12 for a magnetized plasma. Another calculation method is to construct a collision integral on the basis of a simplified expression for the matrix element of the Coulomb interaction: under the assumptions made, the states of

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the holes can be described by plane waves. Both methods lead to the same result:

$$I\{f\}=(2\pi)^{-3}\int dp_1\,dk_{z1}\,dk_z\delta(p+k_z-p_1-k_{z1})\frac{(4\pi e^2)^2}{\pi\varepsilon_0^2}$$
$$\times\int dk_\perp\,dk_{\perp1}k_{\perp1}\delta\left(\frac{p^2-p_1^2}{2m_e}+\frac{k^2-k_1^2}{2m_h}\right)\int dq_\perp[q_\perp^2+(p-p_1)^2]^{-2}$$
$$\times\exp\left\{-\frac{1}{2}\lambda_H^2q_\perp^2\right\}\left[1-\frac{(k_\perp-k_{\perp1})^2}{q_\perp^2}\right]^{-1/2}\{f_1F_1-fF\}. \tag{2}$$

Here $\lambda_H=(c/eH)^{1/2}$ is the magnetic length, the field $\mathbf{H}$ is directed along the z axis, p is the electron momentum (directed along the z axis), $\mathbf{k}$ is the hole momentum, $k_\perp^2=k_x^2+k_y^2$, the subscript l corresponds to the state of the particles after the scattering, f and F are the electron and hole distribution functions, and ε_0 is the dielectric constant. The distributions f and F are assumed to be spatially homogeneous.

In the case of a smoothly varying function $f(E)$ (at $(T/m_h)^{1/2}\lambda_H^{-1}\partial f/\partial E\ll f$) we can use in (2) a diffusion approximation that yields

$$I\{f\}=\frac{\partial}{\partial p}\int k_\perp\,dk_\perp\,dk_z\left\{J_{zz}\left(\frac{\partial}{\partial p}+\frac{\partial}{\partial k_z}\right)+J_{z\perp}\frac{\partial}{\partial k_\perp}\right\}f(p)F(\mathbf{k}). \tag{3}$$

The coefficients J_{zz} and $J_{z\perp}$ are respectively equal to

$$J_{zz}=\frac{e^4}{\pi^2\varepsilon_0^2u_\perp}\int_{-\infty}^{\infty}dq_z\int_0^{\infty}dq_\perp\frac{q_z^2}{(q_z^2+q_\perp^2)^2}$$
$$\times\exp\left\{-\frac{1}{2}\lambda_H^2q_\perp^2\right\}\left[1-\left(\frac{v-u_z}{u_\perp}\right)^2\left(\frac{q_z}{q_\perp}\right)^2\right]^{-1/2}; \tag{4}$$

$$J_{z\perp}=-[(v-u_z)/u_\perp]J_{zz}; \tag{5}$$

$v=p/m_e$ and $\mathbf{u}=\mathbf{k}/m_h$ are the velocities of the electrons and holes. Calculating (4), we eliminate the logarithmic divergence of the integral with respect to $q_\perp$ in the usual manner,[1] introducing the Debye screening radius r_D:

$$J_{zz}=\frac{e^4L_H}{2\pi^2\varepsilon_0^2}\frac{|v-u_z|^3u_\perp^2}{[(v-u_z)^2+u_\perp^2]^3},\qquad L_H=\ln\frac{2r_D^2}{\gamma\lambda_H^2} \tag{6}$$

(γ is the Euler constant). It can be seen from (4)–(6) that the magnetic field leads to anisotropy of J_{zz} and $J_{z\perp}$, but the absolute values of these coefficients are practically independent of H. Substituting in (3) the Maxwellian distribution for the holes and using (5) and (6), we obtain for the electrons an equation of the Fokker-planck type:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial v}\left\{G(v)\left[\frac{\partial f}{\partial v}+\frac{m_ev}{T}f\right]\right\}. \tag{7}$$

The diffusion coefficient $G(v)$ is equal to

$$G(v)=\frac{2^{3/2}\pi}{\varepsilon_0^2}L_H\frac{e^4nm_h}{T^{1/2}m_e^2}g(\mu),$$
$$g(\mu)=e^{-\mu}\int_0^{\infty}dx\,\mathrm{ch}[2(\mu x)^{1/2}]xe^{-x}\left\{\frac{x+1}{2x}-\frac{x+2}{2}e^x\,\mathrm{li}(e^{-x})\right\}. \tag{8}$$

Here $\mu=m_hv^2/2T$, li (x) is the integral logarithm. In the velocity region $v\gg\bar{u}=(2T/m_h)^{1/2}$ of practical interest we have $\mu\gg1$ and $g(\mu)\approx\mu^{-3/2}$; the relaxation rate $\tau_H^{-1}(E)$ of an electron with energy $E=p^2/2m_e$ is

$$\tau_H^{-1}(E)=L_H\tau_0^{-1}(E),\qquad \tau_0^{-1}(E)=2^{3/2}\pi e^4m_e^{1/2}n/\varepsilon_0^2m_hE^{3/2}. \tag{9}$$

Equation (9) differs from the corresponding expression for the case $H=0$ only in the form of the Coulomb logarithm L_H. Thus, the magnetic field does not suppress the energy transfer from the electrons to the holes.

If multiparticle collisions[2] establish in the electron system an effective temperature T_e, the interaction of the electron with the holes causes T_e to approach the value T. The corresponding relaxation frequency ν_T^H, which enters in the equation

$$\dot{T}_e=-\nu_T^H(T_e-T),$$

can be obtained with the aid of (7) and (8). Calculations carried out with allowance for the inequality $m_h/m_e\gg1$, yield

$$\nu_T^H=8\pi^{-1/2}\ln(0.18m_h/m_e)L_H\tau_0^{-1}(T). \tag{10}$$

Comparing ν_T^H with the corresponding quantity ν_T at $H=0$, we find

$$\nu_T^H/\nu_T=6\ln(0.18m_h/m_e)L_H/L. \tag{11}$$

Here L is the "usual" Coulomb logarithm. It follows from (11) that at a sufficiently high ratio m_h/m_e the value of ν_T^H can exceed ν_T. The physical reason for this result is that the electrons interacting most effectively with the holes are those with velocity $v\approx\bar{u}\ll\bar{v}=(2T/m_e)^{1/2}$. At a fixed effective temperature T_e in a strong magnetic field there are more such electrons than at $H=0$, owing to the square-root singularity in the state density.

Experiments[13] on GaAs offer evidence that in the case $H=0$, under intensive photoexcitation of the semiconductor, the effective electron temperature is close to the effective hole temperature. This makes it posible, when describing the evolution of the electron-hole plasma, to assume the effective temperature to be the same for both carrier groups.[9] Equation (11) shows that a similar approximation holds also at $H\neq0$.

RELAXATION OF A NARROW ENERGY DISTRIBUTION OF THE ELECTRONS

Electron-hole collisions with a magnetic field lead to a discrete change of the electron energy by an amount proportional to ω_h. The discrete character of the energy transfer should manifest itself only in the case of relaxation of a narrow (in energy space) electron distribution. Such a distribution can be produced, e.g., by transferring the electrons into the conduction band by monochromatic light. To obtain relaxation of such a distribution or to determine the form of the stationary nonequilibrium distribution it is necessary to calculate the time of departure of the electron from the initial state, with allowance for the splitting of the hole spectrum into Landau bands. Inasmuch as in one collision act,

under condition (1), the electron energy changes by an amount

$$\delta E \lesssim \lambda_H^{-1}(2T/m_h)^{1/2},$$

we can confine ourselves to quasiclassical allowance for the quantization of the hole motion by the magnetic field. Corresponding to this approximation is a change from integration with respect to the continuous variables $k_\perp$ and $k_{\perp 1}$ in (2) by summation over the levels

$$l=k_\perp^2/2m_h\omega_h, \quad l_1=k_{\perp 1}^2/2m_h\omega_h;$$

$$l, l_1 \gg 1, \quad |\Delta l| = |l_1 - l| \ll l, l_1.$$

Carrying out the calculations on the basis of (2) with the indicated substitutions, we find that the reciprocal time of electron transition with a change $\omega_h\ \Delta l$ in its energy is

$$\tau_{\Delta l}^{-1} = \tau_0^{-1}(E)\frac{4\pi ET}{\omega_e\omega_h}\frac{1}{|\Delta l|^3}K\left(\frac{\Delta l}{\lambda_H(2m_hT)^{1/2}}\right), \quad \Delta l \neq 0. \quad (12)$$

Here

$$K(z) = 2\int_0^\infty dx x^3 e^{-x^2}\left\{\frac{z}{x\sqrt{\pi}}\exp\left(-\frac{z^2}{4x^2}\right) + \left(1-\frac{z^2}{4x^2}\right)\left[1-\Phi\left(\frac{z}{x}\right)\right]\right\}, \quad (13)$$

$\Phi(y)$ is the error integral; $K(z)$ is a monotonic function, $K(0) = 1$, and the asymptotic expression $K(z) \sim \exp(-z^2)$ is valid as $z \to \infty$. When deriving (12) we took into account that $v \gg \bar{u}$; this inequality ensures exponential smallness of the probability of the transition with $\Delta l = 0$. Comparison of (12) with (9) shows that $\tau_{\Delta l} \ll \tau_H$ at $E \sim \omega_e$, $|\Delta l| \lesssim \lambda_H(2m_hT)^{1/2} \ll \bar{l}$. Thus, the narrow electron distribution breaks up within the short time $\tau_{\Delta l}$ into approximately $\lambda_H(2m_hT)^{1/2}$ peaks that are symmetric about the initial energy E_0. Next, after times $\sim \tau_H(E)$, this distribution relaxes in energy. In contrast to τ_H, the characteristic time $\tau_{\Delta l}$ given by Eq. (12) describes a process in which the average electron energy remains unchanged, since $\tau_{\Delta l} = \tau_{-\Delta l}$.

At low electron energies ($E \gtrsim \omega_h$) and low temperatures ($T < \omega_h$) one scattering act can transfer an electron to the bottom of the Landau band. The discrete character of the energy transfer in this case leads to a nonmonotonic dependence of the reciprocal relaxation time $\tau_H^{-1}(E)$. (The distribution is assumed narrow, as before.) Investigating $\tau_H^{-1}(E)$, we confine ourselves to the simplest case of a nondegenerate valence band, when the spectrum of the holes in the magnetic field is equidistant and the dependence of the hole energy on k_z is characterized only by the mass m_h. Using the collision integral from Ref. 12, we represent $\tau_H^{-1}(E)$ in the form

$$\tau_H^{-1}(E) = \sum_{l=1}^{\infty}\tau_l^{-1}(E), \quad (14)$$

$$\tau_l^{-1}(E) = \frac{1}{(2\pi)^3 l!}\int dp_1\, dk_z \delta\left(\frac{p^2-p_1^2}{2m_e} + \frac{k_z^2-k_{z1}^2}{2m_h} - l\omega_h\right)F(k_z)$$

$$\times \frac{eH}{2\pi c\varepsilon_0^2}\int_0^\infty q_\perp\, dq_\perp \left(\frac{cq_\perp^2}{2eH}\right)^l \left[\frac{4\pi e^2}{q_\perp^2 + (p-p_1)^2}\right]^2 \exp(-\lambda_H^2 q_\perp^2). \quad (15)$$

In (15) we took into account the fact that the holes fill only the lower Landau band. The rate of relaxation (14) at $T \ll \omega_e$ has sharp maxima due to the vanishing of the final-state electron momentum p_1. At $m_e \ll m_h$ these maxima occur at the points $E = l\omega_h, l = 1,2,\ldots$. The form of the maxima can be established by retaining in (14) only one term $\tau_l^{-1}(E)$, which is "resonant" at $|E - l\omega_h| \ll \omega_h$:

$$\tau_1^{-1}(E) = [2\pi^2\tau_0(E)]^{-1}\ln\left(\frac{e}{2\gamma}\frac{m_h}{m_e}\right)R_1\left(\frac{E}{\omega_h}\right), \quad E \approx \omega_h, \quad (16a)$$

$$\tau_l^{-1}(E) = \frac{\tau_0^{-1}(E)}{2\pi^2(l-1)2^l}R_l\left(\frac{E}{l\omega_h}\right), \quad E \approx l\omega_h, \quad l \geqslant 2, \quad (16b)$$

$$R_l(x) = \begin{cases} (x-1)^{-1/2}, & x-1 \gg T/\omega_e, \\ (2\pi)^{-1/2}\Gamma(1/4)(l\omega_e/T)^{1/4}, & x=1, \\ 1/2(1-\sqrt{x})^{-1/2}\exp\{-(l\omega_e/T)(1-\sqrt{x})\}, & 1-x \gg T/\omega_e. \end{cases} \quad (16c)$$

Here $\tau_0(E)$ is defined by Eq. (9) and e in (16a) is the base of the natural logarithm. Equations (16) show that the approach to the next maximum from the higher-energy side follows a square-root law, and the decrease from the lower energy side is exponential. The amplitudes of the $\tau_H^{-1}(E)$ spikes are proportional to $2^{-l}(\omega_e/T)^{1/4}$ and decrease rapidly with the number l. At $E \gg \omega_h$ the monotonic part in $\tau_H^{-1}(E)$ predominates:

$$\tau_H^{-1}(E) = \frac{E}{4\omega_e}\tau_0^{-1}(E)\ln\frac{2e^2Em_h}{\gamma\omega_h m_e}. \quad (17)$$

The general form of the $\tau_H^{-1}(E)$ dependence is shown schematically in the figure. Formulas (16) can be easily generalized to the case when the strong inequality $m_e \ll m_h$ is not realized, but nevertheless $m_e < m_h$: the $\tau_H^{-1}(E)$ dependence does not change qualitatively at $E < \omega_e$; τ_H^{-1} has maxima at the points

$$E = (1 + m_e/m_h)l\omega_h.$$

Our assumption that the valence band is not degenerate at the point $\mathbf{p} = 0$ is realized in semiconductors with wurtzite crystal structure,[14] e.g., in CdSe and in CdHgS alloys.[15] The degeneracy of the valence band is lifted by uniaxial deformation.[16] The obtained equations (16) and (17) are therefore valid for deformed crystals of III-V semiconductor compounds. The absolute value of $\tau_H^{-1}(\omega_h)$ for InSb (in the absence of degeneracy) at $H \approx 20$ kOe, $T \approx \omega_h, n \sim 0.5 \cdot 10^{16}$ cm^{-3} amounts to $\sim 10^{12}$ sec^{-1}. This exceeds the frequency of electron relaxation on acoustic phonons

$$\nu_{ep}^H(\omega_h) = (m_h/m_e)^2\nu_{ep}^{H=0}(\omega_h).$$

Electron-electron collisions at nonequilibrium-electron densities $n_e < 0.06\, n$ likewise do not mask the nonmonotonic character of $\tau_H^{-1}(E)$. In the case of a degenerate band the Landau levels are not equidistant,[17] and the dependence of the hole energy on k_z becomes nonquadratic; the shape of the Landau band depends on the direction of the field $\mathbf{H}$ relative to the crystallographic axes.[18,19] This leads to a more complicated $\tau_H^{-1}(E)$ dependence than that considered above.

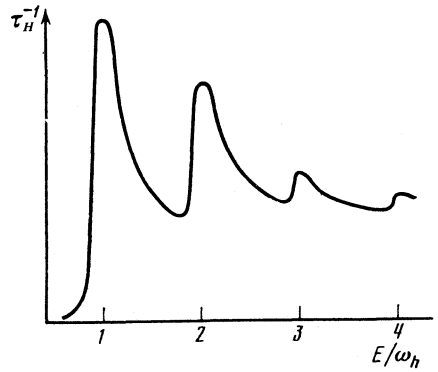

FIG. 1. Schematic form of the dependence of the rate of relaxation of the narrow distribution on the energy, see Eqs. (16).

The foregoing calculations were made under the assumption of a Maxwellian distribution of the holes. Fermi degeneracy under the condition $\omega_e \gg \mu_h$ (μ_h is the Fermi energy of the holes) has practically no influence on the times $\tau_H^{-1}(E)$ determined by Eqs. (9) and (17). The dependence of $\tau_{\Delta l}$ on Δl at $E \gg \omega_h$ [see (12)] becomes asymmetrical: $\tau_{\Delta l} = 0$ at $\Delta l < 0$ and $T = 0$. The positions of the maxima of $\tau_H^{-1}(E)$ remain the same as before; with decreasing temperature the maxima become sharper and at $T = 0$ kinks appear at the extremum points of $\tau_H^{-1}(E)$.

Electron relaxation in a magnetic field can be studied by investigating the luminescence spectrum of the photoexcited electrons.[5] In this case, by varying the spectral composition of the excited light, it is possible to produce either a broad or a narrow distribution of the electrons. The time of the energy relaxation of the broad distribution yields information on the relaxation rate (9). In the case of a monochromatic excitation, on the other hand, the photoluminescence spectrum should contain lines corresponding to discrete changes of the electron energy in collisions with holes. The intensity of these lines relative to the background due to the electron-electron and electron-phonon scattering is proportional to $\tau_{\Delta l}^{-1}$.

Electron-hole scattering should manifest itself also in the photoconductivity produced by the excited electrons. The nonmonotonicity should, e.g., lead to oscillations of the photocurrent.[6] However, to observe these oscillations the degree of doping of the sample and the excitation intensity must apparently satisfy stringent conditions: on the one hand, the electron component of the current should be sufficiently large to be experimentally observable against the background of the hole component, while on the other hand at high electron densities the oscillations will be masked by the electron-electron scattering.

The author thanks V. F. Gantmakher and I. B. Levinson for helpful discussions and useful remarks.

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Translated by J. G. Adashko