Resonance effects in the spectra of transformation of electromagnetic waves at the interface lines of surfaces with spatial dispersion

V. M. Agranovich, V. E. Kravtsov, and T. A. Leskova

Spectroscopy Institute, USSR Academy of Sciences
(Submitted 7 May 1982)
Zh. Eksp. Teor. Fiz. **84**, 103–117 (January 1983)

An exact solution is obtained of the problem of diffraction of surface electromagnetic waves by an impedance step in the presence of spatial dispersion and additional surface polaritons (SP). It is shown that allowance for spatial dispersion leads to the formation of a novel type of states (one-dimensional edge modes) localized in the region of the impedance step. The SP energy conversion coefficients are calculated and their frequency dependence in the region of SP resonances with oscillations in a transition layer (a thin semiconducting film on the surface of a metal) is studied. It is shown that the edge modes lead to the appearance of additional resonance singularities in the conversion coefficients.

PACS numbers: 41.10.Hv, 71.36. + c, 73.40. − c

1. INTRODUCTION

Surface electromagnetic waves in the optical band have recently attracted the attention of many researchers, both theoreticians and experimenters. This interest is due principally to the fact that the characteristics of surface polaritons (SP) are quite sensitive to the state of the surface, and therefore SP turn out to be in many cases an effective tool for the investigation of the properties of surfaces and thin films.[1] The considerable progress in experimental methods of surface spectroscopy with the aid of SP makes it timely, in particular, to consider the fundamental problems of linear crystal optics of surfaces. Foremost among such problems if that of calculating the coefficients of conversion of the SP energy on the line that separates surfaces with different properties. This problem is the analog of the known Fresnel problem in bulk crystal optics.

We recall that in the case of bulk crystal optics this problem presupposes the presence of an interface between the media and consists of calculating the amplitudes of the transmitted and reflected bulk waves. When the analogous problem is posed in surface crystal optics, it is necessary to assume that the surface along which a surface wave excited by some external source propagates has an impedance step (i.e., a line that separates surfaces with different impedances). The presence of an impedance step leads to the onset of a reflected surface wave, a transmitted surface wave, and also volume radiation. Volume radiation is the result of diffraction of the surface waves by the surface boundary, and it is this which causes the substantial complications in the mathematical description of the phenomenon.

The passage of surface radio waves through a line that separates surfaces of two metals with different impedances was considered quite long ago (see, e.g., Ref. 2). In that case, however, the impedance was assumed to be independent of the SP wave vector, which is equivalent to neglecting spatial dispersion effects. This approximation is indeed justified at low (e.g., radio) frequencies far from all the natural frequencies of the medium.

The situation may turn out to be qualitatively different, however, in the optical band. To explain the foregoing, we assume that a thin macroscopic film [with dielectric constant $\varepsilon_1(\omega)$] is coated on the surface of a metal having an impedance Z_0 that is independent of the wave vector k. The presence of such a film produces in the impedance of the surface a change that is particularly large in the region of the resonances and zeros of $\varepsilon_1(\omega)$. As shown in Refs. 3–5, under these conditions one can no longer neglect the dependence of the impedance on the wave vector, since this dependence, just as the spatial dispersion in bulk crystal optics, leads to the appearance of an additional (in this case, surface) normal wave. The spatial dispersion in the impedance of the surface is due in this case to the appearance of terms of the order of kd (d is the film thickness) and takes place thus even when the dielectric constant of the film $\varepsilon_1(\omega)$ is independent of k.

The Fresnel problem for this case, i.e., at $\varepsilon_1 = \varepsilon_1(\omega)$, under conditions when the separation line is the edge of a thin film coated on the surface of the metal, was first solved in Refs. 4 and 5. It was shown in them, in particular, that if the impedance boundary conditions are consistently derived, a linear current concentrated in the region of the step is produced. It is known that when solving the Fresnel problem for bulk waves in a region where additional waves exist it becomes necessary to use the so-called supplementary boundary conditions on the interfaces between the media.[6] In crystal optics of surfaces, the correct choice of the linear current plays the same role as the choice of the supplementary boundary conditions for bulk waves.

A next natural generalization of the theory developed in Refs. 4 and 5 is allowance for the spatial dispersion in the film, i.e, allowing for the $\varepsilon_1(k)$ dependence. This is precisely the question dealt with in the present article.

It will be shown that even in the simplest variant the spatial dispersion in a film leads to the possible existence of not two but four surface waves of equal frequency. In the impedance boundary conditions there appear in this case supplementary linear currents that attenuate over the characteristic length of the spatial dispersion with increasing distance from the edge of the film.

We obtain in this paper an expression for the impedance

 0038-5646/83/010060-09$04.00 © 1983 American Institute of Physics

of a thin film with dielectric constant $\varepsilon_1(\omega,k)$, deposited on the surface of a metal, and solve the Fresnel problem by a method similar to that developed in Refs. 4 and 5.

We show that all the coefficients of conversion of the SP energy have resonant peaks of dips at definite frequencies $\tilde{\omega}_{\parallel}$ and $\tilde{\omega}_{\perp}$. These frequencies, however, do not coincide with the frequencies $\omega_{\parallel}$ and $\omega_{\perp}$ that satisfy the equations $\varepsilon_1(\omega_{\parallel},0)=0$ and $\varepsilon_1^{-1}(\omega_{\perp},0)=0$, as when spatial dispersion in the film is not taken into account. Since the film is assumed microscopic in this paper, in the calculation of the impedance of the surface on both surfaces of the films we have used the supplementary boundary conditions of bulk crystal optics. It is important that the form of the supplementary boundary conditions for the case of a thin film ($d\ll\lambda_i$), where λ_i is the SP wavelength) determines only the values of the resonant frequencies and does not influence the character of the obtained frequency dependences.

An important and interesting prediction of the theory developed below is the possible existence of quasilocal oscillations of exciton polarization near the edge of the film. These edge modes exist only when account is taken of the spatial dispersion and have a radiative width connected with the possibility of their decay into short-wave SP.

We solve in this paper also the problem of SP diffraction with allowance for the edge modes and obtain the frequency dependences of the energy-conversion coefficients. We show that near the frequencies corresponding to these modes additional peaks or dips should be observed in the frequency dependence of the SP reflection and transmission coefficients. It was found, in contrast to the results of Refs. 4 and 5, that when account is taken of spatial dispersion in the film, the SP energy conversion coefficients generally speaking depend not only on the film thickness and on its dielectric constant, but also on the shape of the transition region at its edge. This dependence is due to the fact that the frequencies of the edge modes are quite sensitive to the character of the change of the thickness of the film in the transition region.

The equations obtained for the energy conversion coefficients for the SP make it possible to separate the contribution of the edge modes to the frequency dependence. This may be useful in the interpretation of the experimental data.

2. IMPEDANCE OF THIN FILM

Let a thin macroscopic film with dielectric constant $\varepsilon_1(\omega,k)$ border on a vacuum along the surface $z=\zeta(x)$ and with the substrate along the surface $z=0$.

We shall assume that $\zeta(x)=df(x/\rho)$, where d is the film thickness, ρ is the width of the transition region in which $\zeta(x)$ changes from zero to d. We assume also that $\zeta(x)$ is a smoothly varying function, i.e., $\rho\gg d$. The limiting transition to the abrupt boundary of the film will be understood in the sense that $d\ll\rho\ll\lambda_i$, where λ_i are the SP wavelengths. The film thickness d is assumed to be small enough, namely $d\ll\lambda_i$, so that we can confine ourselves to the terms linear in d in the expansion when calculating the change of the fields in the film along z. The condition $\rho\gg d$ means that the unit vector normal to the surface $z=\zeta(x)$ can be chosen to be the vector with coordinates $n=(-\zeta',0,1)$ and only terms linear in the derivative $\zeta'=d\zeta/dx$ need likewise be retained.

Taking the foregoing assumptions into account, it follows from Maxwell's equations and from the conditions for continuity of E_t, H_t and D_n on the film boundaries that the vacuum fields E_x and H_y ($z\geqslant0$) at $z\geqslant0$ are connected by the relation

$$E_x\left[1+\frac{i\omega}{c}Z_0(\varepsilon_\infty-1)\zeta(x)\right]=Z_0H_y+\frac{ic}{\omega}(\varepsilon_\infty^{-1}-1)\frac{\partial}{\partial x}\left[\zeta(x)\frac{\partial H_y}{\partial x}\right]-\frac{4\pi i\omega}{c}Z_0\zeta(x)P_x(x)-\frac{4\pi}{\varepsilon_\infty}\frac{\partial}{\partial x}[\zeta(x)P_z(x)]. \quad (1)$$

In (1) we have introduced the exciton polarization vector $\mathbf{P}$ in the film in accordance with the equation $\mathbf{D}=\varepsilon_\infty\mathbf{E}+4\pi\mathbf{P}$, and also the substrate impedance Z_0. We shall neglect below the spatial dispersion effects in the substrate, assuming the dielectric constant of the substrate $\varepsilon(\omega)$ to be high enough: $|\varepsilon|\gg(\lambda/\lambda_i)^2$, where $\lambda=2\pi c/\omega$. In this case $Z_0\equiv(ic/\omega)\varkappa_0=i|\varepsilon|^{-1/2}$.

In the effective-mass approximation, the equations of motion for the exciton polarization take the form[6]

$$(\omega^2-\omega_{\parallel}^2)P_z+\frac{1}{M}\Delta P_z=-\frac{1}{4\pi}(\omega_{\parallel}^2-\omega_{\perp}^2)D_z^{(1)}, \quad (2)$$

$$(\omega^2-\omega_{\perp}^2)P_x+\frac{1}{M}\Delta P_x=-\frac{1}{4\pi}(\omega_{\parallel}^2-\omega_{\perp}^2)\varepsilon_\infty E_x^{(1)}, \quad (3)$$

corresponding to

$$\varepsilon_1(\omega,k)=\varepsilon_\infty(\omega^2-\omega_{\parallel}^2-k^2/M)(\omega^2-\omega_{\perp}^2-k^2/M)^{-1}.$$

The constant M is connected with the effective mass of the exciton m by the relation $M=m/\hbar\omega_{\perp}$. The index 1 in (2) and (3) labels the field components in the film.

Regarding the film as macroscopic, we assume that its thickness $d\gg r_0$, where r_0 is the radius of the excitonic state. Under these conditions, according to the usual bulk crystal-optics scheme,[6] it is necessary to impose on $\mathbf{P}$ supplementary boundary conditions on the boundaries $z=\zeta(x)$ and $z=0$. We choose for them a supplementary boundary condition of general form

$$\alpha\mathbf{P}+\partial\mathbf{P}/\partial n=0. \quad (4)$$

We must stipulate here, however, that $\alpha d\ll1$, which is the necessary condition that the variation of the polarization along z be slow.

We note that although $\mathbf{P}$ changes little over the film thickness, the terms $\partial^2\mathbf{P}/\partial z^2$ in (2) and (3) cannot be neglected. In fact, upon integration with respect to z from 0 to $\zeta(x)$, these terms produce in the derivatives $\partial\mathbf{P}/\partial z$ jumps, that can be represented with the aid of (4) in the form

$$\partial\mathbf{P}/\partial z|_{z=\zeta}-\partial\mathbf{P}/\partial z|_{z=0}=-(\alpha_1+\alpha_2)\mathbf{P}(x,z=0)+\zeta'\partial\mathbf{P}/\partial x|_{z=0}, \quad (5)$$

where α_1 corresponds to the boundary with the vacuum, and α_2 corresponds to the boundary with the substrate.

Recognizing that in the zeroth order in ζ and ζ' we can put

$$D_z^{(1)}=E_z=(ic/\omega)\partial H_y/\partial x,\quad E_x^{(1)}=E_x.$$

We have after integrating (2) and (3) with respect to z and using (5)

$$\hat{L}_{\|}P_z(x)=(cq_\perp^2/4\pi i\omega)\,\partial H_y/\partial x, \tag{6}$$

$$\hat{L}_\perp P_x(x)=-(\varepsilon_\infty/4\pi)\,q_\perp^2 E_x, \tag{7}$$

where

$$q_\perp^2=M(\omega_\|^2-\omega_\perp^2),$$

$$\varkappa_{\|(\perp)}^2=M(\tilde{\omega}_{\|(\perp)}^2-\omega^2),\quad \tilde{\omega}_{\|(\perp)}^2=\omega_{\|(\perp)}^2+(\alpha_1+\alpha_2)/Md,$$

$$\hat{L}_{\|(\perp)}=d^2/dx^2+(\zeta'/\zeta)\,d/dx-\varkappa_{\|(\perp)}^2+(\alpha_1+\alpha_2)\,(d^{-1}-\zeta^{-1}).$$

The Green's function $G(x,x')$ of an equation of the type (6), (7) can be easily expressed in terms of the Green's function $\mathcal{G}(x,x')$ of a Schrödinger equation with a potential

$$V(x)={}^1/_2\zeta''/\zeta-{}^1/_4(\zeta'/\zeta)^2+(\alpha_1+\alpha_2)\,(\zeta^{-1}-d^{-1})$$

and energy $E=-\varkappa_{\|(\perp)}^2$. The indicated relation is of the form

$$\zeta(x)G(x,x')=[\zeta(x)\zeta(x')]^{1/2}\mathcal{G}.$$

The asymptotic form of the function $\mathcal{G}(x,x')$ at $x,x'\gg\rho$ is well known:

$$\mathcal{G}(x,x')=-\frac{\theta(x)\theta(x')}{2\varkappa_{\|(\perp)}}\times[\exp(-\varkappa_{\|(\perp)}|x-x'|)+r_{\|(\perp)}\exp(-\varkappa_{\|(\perp)}|x+x'|)], \tag{8}$$

where $\operatorname{Re}\varkappa>0$, and r is the reflection coefficient corresponding to the energy E and to the potential $V(x)$. At $E<0$ the asymptotic form of the wave function, which decreases as $x\to-\infty$, takes at large $x\to+\infty$ the form

$$a[e^{\varkappa x}+re^{-\varkappa x}].$$

We call attention to the fact that $\mathcal{G}(x,x')$, meaning also $\mathbf{P}(x)$, contains a resonant denominator $\varkappa$. Thus, as $\omega\to\tilde{\omega}_\|$ the smallness of $\zeta(x)$ in (1) can be offset by a resonant increase of P_z, just as the product ζP_x turns out to be not small when $\omega\to\tilde{\omega}_\perp$.

Retaining in (1) only the resonant terms and the terms that do not contain the film thickness d, we have, taking (6)–(8) into account, as $\omega\to\tilde{\omega}_\|$, the following boundary conditions for the field in a vacuum at $z=0$:

$$E_x=Z_0H_y+\frac{ic}{\omega}\frac{\partial}{\partial x}\left[\int_{-\infty}^{+\infty}F(x,x')\frac{\partial H_y}{\partial x'}dx'\right], \tag{9}$$

where

$$F(x,x')={}^1/_2\mu_0\varkappa_\|\theta(x)\theta(x')\,[\exp(-\varkappa_\||x-x'|)+r_\|\exp(-\varkappa_\||x+x'|)],$$

$$\mu_0=d\varepsilon_\infty^{-1}(\omega_\|^2-\omega_\perp^2)\,(\omega^2-\tilde{\omega}_\|^2)^{-1}. \tag{10}$$

Analogously, near $\omega=\tilde{\omega}_\perp$, the connection between E_x and H_y at $z=0$ is given by the expression

$$E_x+\frac{i\omega}{c}\int_{-\infty}^{+\infty}K(x,x')E_x(x')\,dx'=Z_0H_y, \tag{11}$$

where

$$K(x,x')=Z_0\frac{\nu_0\varkappa_\perp}{2}\times\theta(x)\theta(x')\,[\exp(-\varkappa_\perp|x-x'|)+r_\perp\exp(-\varkappa_\perp|x+x'|)], \tag{12}$$

$$\nu_0=d\varepsilon_\infty(\omega_\|^2-\omega_\perp^2)\,(\tilde{\omega}_\perp^2-\omega^2)^{-1}.$$

In (10) and (12) we have replaced $[\zeta(x)\zeta(x')]^{1/2}$ by $d\theta(x)\theta(x')$, inasmuch as at $\rho\ll\lambda_i$ the transition region makes a small contribution to the integrals in (9) and (11).

We shall consider below in detail the frequency region near $\omega=\tilde{\omega}_\|$. The resonant effects at $\omega\approx\tilde{\omega}_\perp$ can be investigated similarly. We note first of all that the impedance boundary condition (9) contains terms localized near the separation line $x=0$ and have the meaning of linear currents. When account is taken of the spatial dispersion in the film, there appear besides the δ-like linear currents (see Refs. 4 and 5) also linear currents of the form $r_\|\theta(x)\exp(-\varkappa_\|x)$, which determine essentially the character of the solution.

To find the reflection coefficient $r_\|$ in the limit as $\omega\to\tilde{\omega}_\|$, we note that the potential $V(x)$ takes the form of a potential barrier of height $\sim\rho^{-2}$ at $x<0$, and at $x>0$ it is a potential well of width $\sim\rho$ and depth $\sim\rho^{-2}$. Thus, at $\rho\ll|\varkappa_\||^{-1}$ the energy $|E|$ is much less than the characteristic scale of the potential $V(x)$. It is known (see, e.g., Ref. 7) that under the indicated conditions we have $r_\|+1\sim\varkappa_\|\rho E_0^{-1/2}$, where E_0 is the energy of the highest discrete level in the dimensionless potential $\rho^2V(x)$. (In the absence of bound states it is necessary to put $E_0=1$ in the potential $V(x)$.) This estimate shows that if the potential $V(x)$ contains a finite number of bound states, the limiting value of the coefficient is $r_\|(\tilde{\omega}_\|)=-1$. The result has a simple physical meaning: it means that the polarization $P_z(x)$ satisfies the boundary condition $P_z=0$ on the line $x=0$ independently of the form of the additional boundary condition on the film boundaries.

It must be noted, however, that in those cases when the potential $V(x)$ has shallow levels, the coefficient $r_\|$ depends strongly on the frequency ω and can differ substantially from -1. In particular, at frequencies $\omega_n^2=\tilde{\omega}_\|^2-|E_n|/M\rho^2$, which correspond to equality of the "energy" $E=-\varkappa_\|^2$ to the energy of the discrete level $E_n\rho^2$ in the potential $V(x)$, the coefficient $r_\|$ becomes infinite. The frequencies ω_n, as can be easily understood, correspond to local oscillations of the polarization, or to the so-called edge modes. Analysis shows that interaction with an electromagnetic fields leads to a considerable radiative width of these modes, due to possibility of their decay into short-wave SP. The frequencies ω_n are also noticeably shifted in this case.

3. SOLUTION OF THE FRESNEL PROBLEM FOR THE CASE OF NORMAL INCIDENCE

Assume that a surface wave is incident on the interface $x=0$ and its magnetic field in vacuum is given by

$$H=(0,H,0),\quad H=H_0\exp(ikx-\varkappa z).$$

At $k^2=\omega^2/c^2+\varkappa^2$ we have in this case $\operatorname{Im}k>0$ if the SP is incident from the side of the pure surface ($x<0$), and $\operatorname{Im}k<0$ is the incident surface wave propagates along the surface $x>0$ covered by the film. Just as in the Refs. 4 and 5, we shall seek the magnetic field in the vacuum in the form $H = (0,H,0)$, where

$$\frac{H}{H_0}=\exp(ikx-\varkappa z)+\int_{-\infty}^{+\infty}\frac{dw}{2\pi i}\exp[-iwx-v(w)z]\frac{\mathcal{F}(w,k)}{v(w)-\varkappa_0},$$
$$v(w)=\sqrt{w^2-\omega^2/c^2}. \quad (13)$$

Substituting (13) in the impedance boundary condition (9), taking the Fourier transform of the resultant expression, and introducing the function

$$\Phi(w,k)=-(\varkappa_\parallel^2+k^2)(\mu_0\varkappa_\parallel^2 wk)^{-1}\mathcal{F}(w,k),$$

we have the following integral equation for this function:

$$\Phi(w,k)=\int\frac{dw'}{2\pi i}F(w,w')[\psi(w')-1]\Phi(w',k)+F(w,-k), \quad (14)$$

$$2\varkappa_\parallel T(w')=\varkappa_\parallel(1-r_\parallel)+iw'(1+r_\parallel),$$
$$F(w,w')=[(w-w'+i\delta)^{-1}-T(w')/(w+i\varkappa_\parallel)], \quad (15)$$

$$\psi(w)=1+\mu_0\varkappa_\parallel^2 w^2/[v(w)-\varkappa_0](w^2+\varkappa_\parallel^2). \quad (16)$$

Equation (14) can be exactly solved by the factorization method (see, e.g., Ref. 2), and the general solution is of the form

$$\Phi(w,k)=P_n(w)/(w+k)(w+i\varkappa_\parallel)\psi_+(w) \quad (\mathrm{Im}\,k>0), \quad (17)$$

where $\psi_+(w)$ and $\psi_-(w)$ are functions that are analytic, differ from zero in the upper and lower half-planes of the complex variable w, respectively, and satisfy the relation $\psi_+(w)\psi_-(w) = \psi(w)$, while $P_n(w)$ is a polynomial of nth degree, which assumes the following values at the point $w = -k$ and $w = -i\varkappa_\parallel$:

$$P_n(-k)=(i\varkappa_\parallel-k)/\psi_-(-k),$$
$$P_n(-i\varkappa_\parallel)=A(i\varkappa_\parallel-k)/\psi_-(-i\varkappa_\parallel). \quad (18)$$

The coefficient A is obtained from the condition

$$A=T(-k)+\int\frac{dw}{2\pi i}\Phi(w,k)[\psi(w)-1]T(w). \quad (19)$$

Relations (18) determine completely the polynomial of degree $n = 1$. However, even at $n = 1$ the integrals in (14) and (19) diverge logarithmically at large w. The reason for this divergence is that in the derivation of the boundary condition (9) we took the limit as $\rho\to 0$ and neglected the nonresonant terms in (1). It can be shown (see the Appendix) that when account is taken of the nonresonant terms in (1) the integral equation for the function $\Phi(w,k)$ is free of the indicated divergence even in the limit as $\rho\to 0$. As $\omega\to\tilde{\omega}_\parallel$, the solution of this equation, which satisfies the Meixner condition,[8] is unique and takes the form (17) with a polynomial $P_n(w)$ of the first degree. The coefficients of the polynomial $P_n(w)$ coincides with those obtained from relations (18) and (19), provided the integration in (19) is restricted to the values

$$|w|<e\mu^{-1}(\infty)\equiv ed^{-1}\varepsilon_\infty(\varepsilon_\infty-1)^{-1}.$$

Obviously, at a finite size of the transition region $\rho\ll\lambda_i$, the function $\Phi(w,k)$ also takes the form (17) with the polynomial

$$P_n(w)=(w+i\varkappa_\parallel)[\psi_-(-k)]^{-1}-A(w+k)[\psi_-(-i\varkappa_\parallel)]^{-1},$$

but the logarithmically diverging integral (19) must be cut off at $w>Q\equiv\min\{\rho^{-1};e\mu^{-1}(\infty)\}$.

Substituting (17) in (19) we obtain an equation for A. The resultant integrals can in this case be easily calculated by using the analytic properties of the functions $\psi_+(w)$ and $\psi_-(w)$. As a result we have

$$A=\frac{\psi_-(-i\varkappa_\parallel)}{\psi_-(-k)}\frac{S(k)}{S(i\varkappa_\parallel)}, \quad (20)$$

$$S(k)=\oint\frac{dz}{2\pi i}\frac{\psi_-(z)}{z+k}T(z). \quad (21)$$

The integration in (21) is carried out along the circle $z = Qe^{i\varphi}(0\leqslant\varphi\leqslant 2\pi)$ with a large radius $Q\gg|k|$.

Taking (17), (18), (20), and (21) into account, we easily obtain the following expression for the function $\mathcal{F}(w,k)$ at $\mathrm{Im}\,k>0$:

$$\mathcal{F}(w,k)=\frac{k\mu_0\varkappa_\parallel^2}{(k+i\varkappa_\parallel)\psi_-(-k)}\frac{w(1-R(w+k))}{\psi_+(w)(w+k)(w+i\varkappa_\parallel)}, \quad (22)$$

$$R=i(r_\parallel+1)[2\varkappa_\parallel+i(r_\parallel+1)\sigma]^{-1}, \quad (23)$$

$$\sigma=\oint\frac{dz}{2\pi i}[\psi_-(z)-1]. \quad (24)$$

It can be similarly shown that at $\mathrm{Im}\,k<0$ it is necessary to replace $\psi_-(-k)$ in (22) by $[\psi_+(-k)]^{-1}$. We note that in (24) it is assumed that $\psi_-(\infty)=\psi_+(\infty)=1$.

Before we proceed to calculate the SP energy conversion coefficients, we investigate the features of the spectra of the surface waves propagating along the surface covered by the film. The SP spectrum is determined[4,5] by the zeros of the function $\psi(w)$, namely $\psi(k(\omega))=0$.

Typical form of the dispersion curve of the SP in the region of frequencies close to $\tilde{\omega}_\parallel$ is shown in Fig. 1. It can be seen that at $\omega<\tilde{\omega}_\parallel$ there can exist three different surface waves on one and the same frequency. The use of the exact expression (1) for the impedance of the thin film leads to the appearance of one more branch of SP.

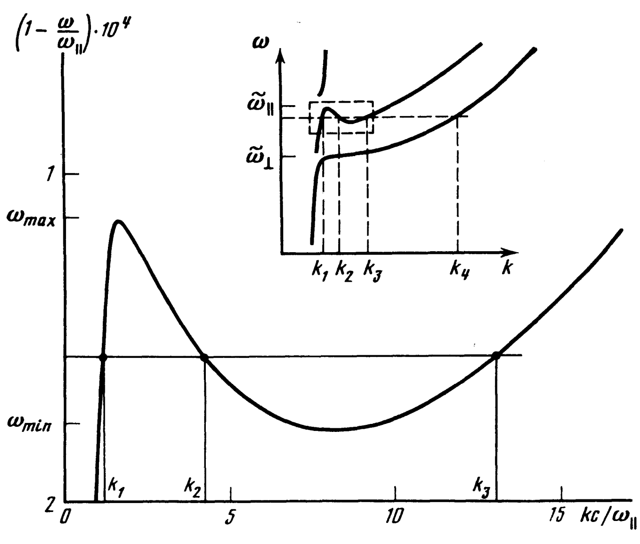

FIG. 1. Dispersion law for SP near $\omega=\tilde{\omega}_\parallel$. The ZnSe film on aluminum has the following characteristics: $d = 50$ Å, $\varepsilon_\infty = 8.7$, $\hbar\tilde{\omega}_\parallel = 2803.75$ meV, $\hbar(\omega_\parallel-\omega_\perp) = 1.45$ meV, $\varkappa_0 c(\omega) = 0.126$, $Mc^2 = 2\cdot 10^5$ (the box in the upper figure is shown enlarged in the lower figure).

Near the resonance $\omega \to \tilde{\omega}_{\parallel}$ the wavelength corresponding to this branch is much shorter than the wavelengths corresponding to the remaining solutions of the dispersion equation. In fact, near the resonance $\omega \approx \tilde{\omega}_{\parallel}$ we have $k_4 \approx q_{\perp}$ and $k_3 \approx d\varepsilon_{\infty}^{-1} q_{\perp}^2$, so that at sufficiently small d we obtain $k_4 \gg k_3$ (see Fig. 1). Also small is the probability of exciting this short-wave SP, and this probability can be neglected as $\omega \to \tilde{\omega}_{\parallel}$.

The amplitudes of the surface waves were determined by the pole contributions to the integral (33). Substituting (22) in (13) we easily obtain expressions for the ratio of the amplitudes of the incident surface waves H_{0i} and the amplitudes of the reflected and transmitted surface waves H_j in the form

$$\frac{H_j}{H_{0i}} = \frac{\mu_0 \varkappa_{\parallel}^2 (i\varkappa_{\parallel} - k_j) \varkappa_j k_i \psi_-(k_j) \psi_+(-k_i) [1 - R(k_i + k_j)]}{(i\varkappa_{\parallel} + k_i)(k_i + k_j)(\varkappa_j - \varkappa_m)(\varkappa_j - \varkappa_n)}. \tag{25}$$

Equation (25) describes the relative amplitudes of the transformed surface waves in the case when both the incident and the transformed surface waves propagate along the surface of the metal covered by the film. In this case the indices i, j, m, n ($j \neq m \neq n$) take on the values 1, 2, and 3. At $i = 0$ (the incident surface wave propagates along the clean metal surface) it is necessary to replace $\psi_+(-k_i)$ in (25) by $[\psi_-(-k_0)]^{-1}$. On the other hand, when the transformed surface wave propagates along a clean metal surface ($j = 0$), it is necessary to make in (25) the substitutions

$$\psi_-(k_j) \to [\psi_+(k_0)]^{-1}, \quad (i\varkappa_{\parallel} - k_j)(\varkappa_j - \varkappa_m)^{-1}(\varkappa_j - \varkappa_n)^{-1} \to (i\varkappa_{\parallel} + k_0)^{-1}.$$

The expression for the SP energy flux, with allowance for the spatial dispersion in the film, can be obtained by a method similar to that used in Ref. 4. Near the resonance $\omega \approx \tilde{\omega}_{\parallel}$ it takes the form

$$W_j = L_y \frac{c^2}{16\pi\omega} |H_y|^2 \left| \frac{k_j}{\varkappa_j} \frac{(\varkappa_j - \varkappa_m)(\varkappa_j - \varkappa_n)}{(\varkappa_j + \varkappa_m)(\varkappa_j + \varkappa_n)} \right|, \tag{26}$$

where L_y is the width of the SP excitation region. H_y is the amplitude of the magnetic field as $z = 0$ and corresponds to the jth surface wave. Obtaining the value of H_y from (25) and substituting it in (26), we can obtain the energy-flux conversion coefficients W_j / W_{0i}. We note first of all that just as in Refs. 4 and 5, the energy-conversion coefficient matrix is symmetric:

$$W_i / W_{0j} = W_j / W_{0i}.$$

Neglecting the damping of the surface waves, the moduli of the functions $\psi_{\pm}(k_i)$ can be relatively simply expressed in terms of the parameters k_i and $\varkappa_i$, using the well-known factorization procedure.[2] The expressions for $|\psi_{\pm}(k_i)|$ have different functional forms in different frequency intervals, depending on the number of solutions of the dispersion equation $\psi(k) = 0$. We present below equations for the energy conversion coefficients of a SP incident from the side of the clean surface, for the frequency region where three different SP can propagate along the surface covered by the film:

$$\frac{W_1}{W_{00}} = 4\varkappa_0 \varkappa_1 \frac{k_1^2}{k_0^2} \times \frac{(|k_2| + |k_0|)^3 (|k_2| - |k_1|)(\varkappa_3 - \varkappa_0)(|k_3| - |k_0|) |1 + R(|k_1| - |k_0|)|^2}{(\varkappa_0 + \varkappa_2)^2 (\varkappa_0 + \varkappa_1)^2 (\varkappa_0 + \varkappa_3)(|k_1| + |k_2|)^2 (|k_3| - |k_1|)} \tag{27}$$

$$\frac{W_2}{W_{00}} = \frac{4k_0^2 \varkappa_0}{\varkappa_2} \frac{(|k_2| - |k_1|)(|k_3| - |k_2|) |1 - R(|k_2| + |k_0|)|^2}{(|k_1| + |k_0|)(|k_2| - |k_0|)^2 (|k_3| + |k_0|)}, \tag{28}$$

$$\frac{W_0}{W_{00}} = \frac{\mu_0^2 \varkappa_{\parallel}^4 k_0^2 \varkappa_0^2 (|k_2| + |k_0|)^2 |1 - 2R|k_0||^2}{(|k_2| - |k_0|)^4 (|k_1| + |k_0|)^2 (|k_3| + |k_0|)^2}. \tag{29}$$

The coefficient W_3 / W_{00} is obtained from W_1 / W_{00} by making the substitution $|k_1| \leftrightarrow |k_3|$. Interest attaches also to the values of W_3 / W_{00} and W_0 / W_{00} in the frequency region that corresponded to the gap in the SP spectrum with allowance for the spatial dispersion in the film.[4,5] Calculation for the case of exact resonance $\omega = \tilde{\omega}_{\parallel}$ leads to the expressions

$$\frac{W_3}{W_{00}} = \frac{4\varkappa_0 \varkappa_3}{(\varkappa_0 + \varkappa_3)^2} |1 + R(|k_3| - |k_0|)|^2, \tag{30}$$

$$\frac{W_0}{W_{00}} = \frac{\varkappa_0^2 (|k_3| - |k_0|)^2}{k_0^2 (\varkappa_3 + \varkappa_0)^2} |1 - 2R|k_0||^2. \tag{31}$$

The angular dependence of the intensity of the volume radiation $W^{(V)}(\theta)$ (see (13) with $r = (x^2 + z^2)^{1/2} \to \infty$), which appears as a result of diffraction, can also be easily determined from (13) and (12). To this end it suffices to calculate the integral (13) by the saddle-point method (see also Ref. 4). It can be easily seen that the saddle point is $w_0 = (\omega / c) \sin\theta \equiv k_x$ ($\tan\theta = -x/z$). As a result we have

$$\frac{W^{(V)}(\theta)}{W_{00}} = C(\omega) \frac{k_x^2 k_z^2 (|k_2| + k_x) |1 - R(|k_0| + k_x)|^2}{(|k_1| + k_x)(|k_3| + k_x)(|k_2| - k_x)^2 (|k_0| - k_x)(|k_0| + k_x)^2}, \tag{32}$$

where

$$C(\omega) = 2\varkappa_0 k_0^2 \mu_0^2 \varkappa_{\parallel}^4 (|k_2| + |k_0|) / (|k_1| + |k_0|)(|k_2| - |k_0|)^2 (|k_3| + |k_0|).$$

We shall now clarify the physical meaning of the coefficient R and determine its value. As already noted, when the frequencies of the local oscillations of the polarization are located far from the investigated region $\omega \approx \tilde{\omega}_{\parallel}$, the coefficient $r_{\parallel} + 1$, and with it also R, is negligibly small. Thus, the value $R = 0$ corresponds to exclusion of effects connected with edge modes from consideration.

Let now $r_{\parallel} \neq -1$. Since σ, as can be seen from (24), is determined by the asymptotic form of the function $\psi_-(w)$ as $|w| \to \infty$, we can express σ, as well as $\psi_-(w)$, in terms of the parameters of the surface waves k_i and $\varkappa_i$. However, the form of the functional dependence of $\sigma(k_i, \varkappa_i)$ depends on the number p surface waves that can propagate on the film-coated surface at the given frequency. We present below results for the frequency region in which $p = 3$, neglecting the damping of the surface waves. In this case we have

$$2\,\mathrm{Re}\,\sigma = \mu_0 \varkappa_{\parallel}^2 + |k_0| + 3|k_2| - |k_1| - |k_3| \equiv -\gamma,$$

$$\mathrm{Im}\,\sigma = \varkappa_{\parallel} - \frac{1}{\pi} \left\{ \mu_0 \varkappa_{\parallel}^2 (\ln 2Q + 1) + \sum_{i=1}^{3} |k_i| \ln \frac{|k_i| + \varkappa_i}{\omega / c} - |k_0| \ln \frac{|k_0| + \varkappa_0}{\omega / c} \right\}. \tag{33}$$

Substituting in (23) $\sigma = -(1/2)\gamma + i\operatorname{Im}\sigma$, where γ and $\operatorname{Im}\sigma$ are given by (33), we obtain the following expression for R:

$$R=-\frac{2i(1+r_{\parallel})}{A(\omega)+i(1+r_{\parallel})\gamma}=\frac{1}{\gamma}\Lambda(\omega),$$
$$A(\omega)=2(1+r_{\parallel})\operatorname{Im}\sigma-4\varkappa_{\parallel}. \tag{34}$$

The dimensionless function $\Lambda(\omega)$ has the property $2\operatorname{Re}\Lambda = -|\Lambda|^2$, so that when account is taken of the edge modes, one more parameter $|\Lambda|^2$ appears in the equations for the energy-conversion coefficients (27)–(31) in addition to k_i and $\varkappa_i$. As seen from (34), $\Lambda(\omega)$ contains a resonant denominator. At frequencies close to the edge-mode frequency ω_0 that satisfies the equation $A(\omega_0) = 0$, the function $\Lambda(\omega)$ takes the Lorentz form

$$\Lambda(\omega)=2i\Gamma/(\omega-\omega_0-i\Gamma). \tag{35}$$

In this case $|\Lambda|^2$ has a characteristic maximum at the frequency ω_0, with a width[1)]

$$\Gamma=-(1+r_{\parallel})\gamma(\omega)\,[dA/d\omega\,|_{\omega=\omega_0}]^{-1}.$$

It follows from (27)–(31) that the conversion-coefficient spectrum should also reveal resonant peaks or dips of width Γ near the frequency ω_0. We note that Γ does not depend explicitly on $r_{\parallel}$. In fact, as already noted, $1 + r_{\parallel} = B\varkappa_{\parallel}$, where B does not depend on the frequency. We therefore have

$$dA/d\omega\,|_{\omega=\omega_0}=2(1+r_{\parallel})\,d\operatorname{Im}\sigma/d\omega\,|_{\omega=\omega_0},$$

from which we obtain for $\Gamma(\omega)$ the following simple expression:

$$\Gamma=\frac{\operatorname{Re}\sigma(\omega)}{d\operatorname{Im}\sigma/d\omega\,|_{\omega=\omega_0}}. \tag{36}$$

Equations (33)–(36) determine completely the quantity $R(\omega,\omega_0)$, in which the frequency of the edge mode ω_0 enters as a parameter.

4. DISCUSSION OF RESULTS

To explain the mechanism whereby the edge modes are formed, we derive an equation of motion for the exciton polarization P with allowance for only the longitudinal field. Since the dimension l of the localized state is of the order of the characteristic length of the spatial dispersion at its frequency, i.e., $l \sim [M(\omega_0 - \omega_{\parallel})]^{-1/2}$, in real cases it is small compared with $\lambda = 2\pi c/\omega$. Under these conditions, analysis of the transverse field leads to relatively small corrections and does not change the qualitative results.

As $\omega \to \tilde{\omega}_{\parallel}$, taking only the resonant part of (1) into account, we have

$$E_x=-\frac{4\pi}{\varepsilon_\infty}\frac{\partial}{\partial x}(\zeta(x)P_z(x)). \tag{37}$$

Expression (6) can be rewritten in the following manner:

$$\left[\frac{d^2}{dx^2}+\frac{\zeta'}{\zeta}\frac{d}{dx}-\varkappa_{\parallel}^2\right]P_z(x)=-\frac{q_\perp^2}{4\pi}E_z(x), \tag{38}$$

where $E_z(x)$ is the z-component of the electric field in vacuum at $z=0$, and we have assumed for simplicity $\alpha_1 = \alpha_2 = 0$. It is easy to verify that the Fourier components of the longitudinal field $E(x)$ satisfy the relation

$$ikE_x(k)=|k|E_z(k). \tag{39}$$

Eliminating the fields E_x and E_z with the aid of (37)–(39) we obtain the following equation for polarization:

$$\left[\frac{d^2}{dx^2}-V(x)-\varkappa_{\parallel}^2\right]\varphi(x)=q_\perp^2\varepsilon_\infty^{-1}\int_{-\infty}^{+\infty}\frac{dx'}{2\pi}[\zeta(x)\zeta(x')]^{1/2}\mathscr{M}(x-x')\varphi(x'), \tag{40}$$

where

$$\mathscr{M}(x-x')=(x-x'+i\delta)^{-2}+(x-x'-i\delta)^{-2},$$
$$\varphi(x)=\zeta^{1/2}(x)P_z(x).$$

At a sufficient film thickness $\zeta(x) = d = \text{const}$ the eigenfunctions (40) constitute surface waves with dispersion

$$-\varkappa_{\parallel}^2=k^2-q_\perp^2d\varepsilon_\infty^{-1}|k|. \tag{41}$$

We note that the interaction of the exciton polarization with the longitudinal electric field has led to the appearance of a term linear in $|k|$ in the SP dispersion law (this question has a long history, see, e.g., Refs. 9 and 10).

Let now $\zeta(x) \neq \text{const}$. At sufficiently small d we can neglect the right-hand side of (40). Then (40) takes the form of a Schrödinger equation with a potential $V(x)$. The region of the continuous spectrum in this equation corresponds to $\varkappa_{\parallel}^2 < 0$. The discrete levels corresponding to local vibrations of the polarization are located in the region $\varkappa_{\parallel}^2 > 0$. With increasing d, as can be seen from (41), the boundary of the continuous spectrum shifts towards lower frequencies, so that the discrete levels land in the region of the continuous spectrum. In this case, as is well known, quasilocal states are produced and lead to the appearance of resonant peaks in the SP scattering amplitude. The width of these peaks is proportional to the probability of the decay of the local mode into surface waves, and their position, generally speaking, does not coincide with the position of the discrete levels.

The dependence of the width Γ of the resonant peak on the frequency ω_0 of the edge mode is given by Eq. (36), in which all the edge-mode decay processes are taken into account. Estimates in accord with (33) and (36) show that at $\omega_0 \lesssim \omega_{\min}$ ($\omega_{\min}$ corresponds to the minimum in the dispersion dependence shown in Fig. 1) the width of the resonance peak is

$$\Gamma_\perp\approx(\tilde{\omega}_{\parallel}-\omega_0)\,\omega_0/c\varkappa_{\parallel},$$

which is much less than the characteristic scale of $\tilde{\omega}_{\parallel} - \omega_0$. Thus, at $\omega_0 < \omega_{\min}$ there should be observed in the spectrum of the conversion coefficients sharp resonant peaks or dips at the frequency ω_0. At $\omega_0 > \omega_{\min}$ a new channel appears for the decay of the edge mode into short-wave SP. This leads to an abrupt increase of $\Gamma(\omega)$ at $\omega > \omega_{\min}$, so that even at a small deviation $\omega_0 - \omega_{\min} \sim \Gamma_\perp$ the resonant singularities in the conversion coefficient, which are connected with the edge mode, become smeared out.

We call attention to a characteristic feature of the radiative width of the edge modes at $\omega_0 < \omega_{\min}$. In this case, as already noted, the decay of the edge mode is due to its inter-

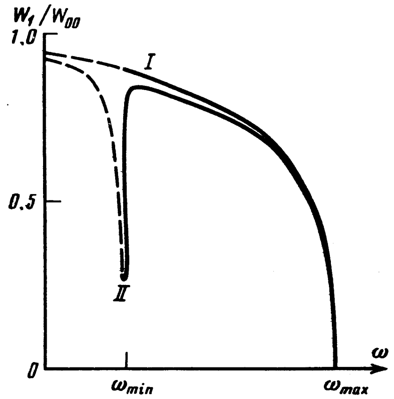

FIG. 2. Transmission coefficient W_1/W_{00}.

action with the transverse electromagnetic field. From the estimate shown above for $\Gamma_\perp$ it follows that $\Gamma_\perp \propto c^{-1}$. Since the radiative width of the atomic terms $\Gamma_0 \propto c^{-3}$, the result obtained can be represented in the form $\Gamma_\perp \approx \Gamma_0(\omega a/c)^{-2}$, where a is a quantity of atomic scale. The parameter $(\omega a/c)^{-2} \sim 10^5$–$10^{-6}$, so that the radiative width of the edge mode turns out, in a certain sense, to be gigantic, as is the case for one-dimensional excitons.[11] We emphasize, however, that in the case considered by us the enhancement effect is much more considerable than for one-dimensional excitons in vacuum, where $\Gamma = \Gamma_0(\omega a/c)^{-1}$. The reason is that the quantity $\Gamma_\perp$ is determined mainly by decay processes into a surface wave, and not into volume radiation as in Ref. 11.

Figures 2–5 show the frequency dependences of the conversion coefficient for the case $R = 0$, when there are no edge modes (curves I) and when account is taken of the edge modes with frequency $\omega_0 \approx \omega_{\min}$ (curves III). We note that at $\omega_0 \gtrsim \omega_{\min}$ the quantity $|\Lambda(\omega)|^2$ does not have a characteristic Lorentz peak at the frequency ω_0, owing to the strong dependence of Γ on the frequency at $\omega_0 \gtrsim \omega_{\min}$. Nonetheless, as seen from Figs. 2–7, the conversion coefficients contain resonant singularities near $\omega_{\min}$, which are due to the abrupt decrease of the quantity $\gamma(\omega)$ [see (33)] as $\omega \to \omega_{\min}$.

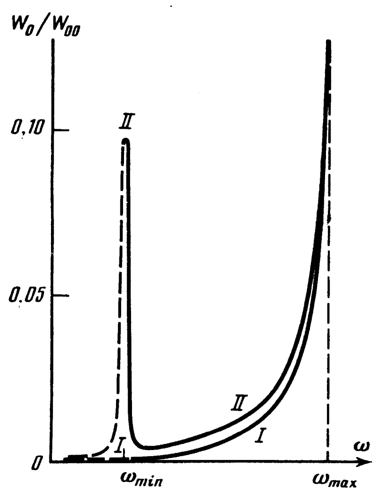

FIG. 3. Reflection coefficient W_0/W_{00}.

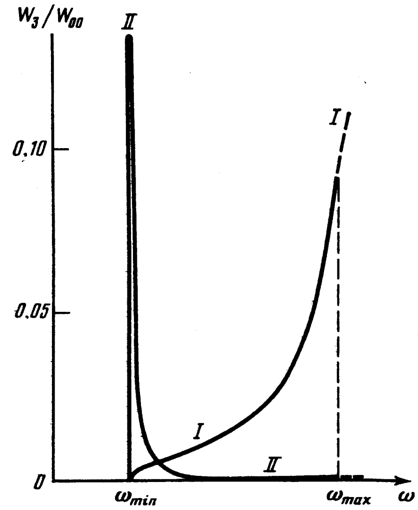

FIG. 4. Coefficient W_3/W_{00} of short-wave surface-polariton excitation.

We wish to note that the formulas obtained above for the conversion coefficients are apparently quite general, since they contain only the characteristics k_i and $\varkappa_i$ of the surface waves and the function $|\Lambda|(\omega)$, which can be approximated within the framework of the phenomenological approach by Eq. (35) of the Lorentz type.

It can be shown that the energy conservation law

$$\sum_i \frac{W_i}{W_{0j}} + \int_{-\pi/2}^{\pi/2} \frac{d\theta}{\pi} \frac{W^{(V)}(\theta)}{W_{0j}} = 1$$

is satisfied identically for any function $\Lambda(\omega)$ that satisfies the condition $|\Lambda|^2 = -2\mathrm{Re}\Lambda$, thus indicating once more that the expressions obtained above for conversion coefficients are quite general.

In conclusion, we emphasize once more the connection between the presence of linear currents in the impedance boundary conditions and the uniqueness of the determination of the power of the polynomial P_n. For an infinite film, the impedance boundary condition analogous to (9) is of the form

$$E_x = Z_0 H_y + \int_{-\infty}^{+\infty} F(x-x') \frac{\partial^2 H_y}{\partial x'^2} dx',$$
$$F(x-x') = {}^1/_2 \mu_0 \varkappa_\parallel \exp[-\varkappa_\parallel |x-x'|]. \qquad (42)$$

The impedance boundary condition without allowance for linear currents, for a bounded film, is obtained with the aid of the simple substitution $f(x-x') \to \theta(x) F(x-x') \theta(x')$.

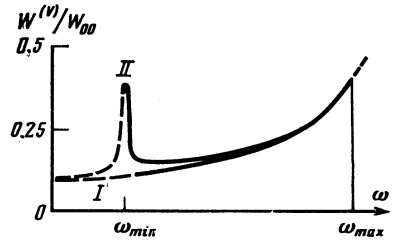

FIG. 5. Integral power $W^{(V)}/W_{00}$ of the volume radiation.

In this case the equation of the type (14) will be satisfied not by the function $\Phi(w,k) \propto w^{-1}\mathscr{F}(w,k)$, but by the function $\mathscr{F}(w,k)$. Then, as $|w|\to\infty$, the function $\mathscr{F}(w,k)$ will be proportional to w^{n-2}, where n is the degree of the polynomial P_n in (17), whereas when the linear currents are taken into account we have $\mathscr{F}(w,k)\propto w^{n-1}$. Near the impedance step $r=(x^2+z^2)^{1/2}\to 0$, the quantity that diverges most strongly is the electric field **E**. It can be easily found from (13) that $|\mathbf{E}|^2\propto r^{-2(m+1)}$, where m is determined by the asymptotic function $\mathscr{F}$ at large $|w|$: $\mathscr{F}(w,k)\propto w^m$.

It follows from the foregoing that the degree of singularity of the energy density of the electromagnetic field as $r\to 0$ depends only on whether or not account is taken of the linear current in the impedance boundary condition. In the former case the field energy is finite at $n\leqslant 1$, and in the latter at $n\leqslant 2$. The condition for finite energy of the electromagnetic field near the geometric singularities (edge, tip, etc.) are widely used in the theory of diffraction and is called the Meixner condition.[8] As already noted, this condition leads to a different number of undetermined constants (coefficients of the polynomial P_n), depending on whether or not the contribution of the linear currents is taken into account. Introduction of a linear current in the impedance boundary conditions decreases by unity the number of undetermined constants in the general solution (17), and this is sufficient to make the solution fully determinate. This is precisely why we can state that a correct choice of the linear current is equivalent to a certain supplementary boundary condition which, generally speaking, does not reduce to any simple supplementary boundary condition such as the vanishing of a linear combination of fields and of their first derivatives at $x=z=0$.

The authors are grateful to L. I. Vaĭnshteĭn for interest in the work and helpful discussions and to N. I. Gapotchenko for the numerical calculations.

APPENDIX

Substituting (13) in (1) we have the following integral equations for the functions $\Phi(w,k) = -\mathscr{F}(w,k)(wk)^{-1}$:

$$\Phi(w,k)=\int_{-\infty}^{+\infty}\frac{dw'}{2\pi i}\Phi(w',k)\frac{w'^2}{v(w')-\varkappa_0}F(w,w')+F(w,-k), \tag{A1}$$

where

$$F(w,w')=\frac{1}{w-w'+i\delta}\left[\mu(w')-\frac{\varkappa_0\bar{v}(w')\nu(w')}{(w+i\delta)w'}\right]$$
$$-\frac{\mu_0\varkappa_\parallel^2}{w+i\varkappa_\parallel}\frac{T(r_\parallel,w')}{\varkappa_\parallel^2+w'^2}+\frac{\varkappa_0\nu_0\varkappa_\perp^2}{(w+i\varkappa_\perp)(w+i\delta)}\frac{v(w')T(r_\perp,w')}{w'(\varkappa_\perp^2+w'^2)},$$
$$\mu(w)=d(\varepsilon_t^{-1}(\omega,w)-1),\qquad \nu(w)=d(\varepsilon_t(\omega,w)-1), \tag{A2}$$
$$\mu_0=\mu(0)-\mu(\infty),\qquad \nu_0=\nu(0)-\nu(\infty).$$

The factorization methods (see Ref. 2) makes it possible to obtain the exact solution of Eq. (A1), which takes the form

$$\Phi(\omega,k)=\tilde{P}_n(w)\left[(w+i\delta)(w+i\varkappa_\parallel)(w+i\varkappa_\perp)(w+k)\tilde{\psi}_+(w)\right]^{-1}, \tag{A3}$$

where $\tilde{\psi}_\pm(w)$ are functions that are analytic, differ from zero in the upper and lower half-planes of the complex variable w, respectively, and satisfy the condition

$$\tilde{\psi}_+(w)\tilde{\psi}_-(w)=\tilde{\psi}(w)\equiv 1+[w^2\mu(w)-\varkappa_0 v(w)\nu(w)]/[v(w)-\varkappa_0]; \tag{A4}$$

$\tilde{P}_n(w)$ is a polynomial of degree n, which takes on given values at the points 0, $-i\varkappa_\parallel$, $-i\varkappa_\perp$, and $-k$. We do not present these values, in order not to clutter up the exposition with excessive formulas. We note only that they are integrals of the function $\tilde{\Phi}$ in analogy with (19).

Thus, there are four conditions for the coefficients of the polynomial $\tilde{P}_n(w)$, therefore its degree is $n\geqslant 3$.

A substantial difference between Eq. (A1) and Eq. (14) is the fact that as $|w|\to\infty$ the function $\tilde{\psi}(w)\sim w$, whereas $\psi(w)\to 1$. It follows from this that $\tilde{\psi}\pm(w)\sim\sqrt{w}$ as $|w|\to\infty$, and this explains the convergence of the integral in the right-hand side of (A1) at $n=3$. It is important, however, that at $n>3$ the indicated integral diverges at large w, so that (A3) is not a solution of (A1) at $n>3$. We note that the Meixner condition also requires that $n\leqslant 3$, since $|E|^2\sim r^{5-2n}$ at small distances r from the impedance step.

It follows from the foregoing that the Meixner condition determines uniquely the degree of the polynomial $\tilde{P}_n(w)$, $n=3$, which is fully specified by its value at the points 0, $-i\varkappa_\parallel$, $-i\varkappa_\perp$, and $-k$.

As a result we obtain from (A3) the following expression for the function $\mathscr{F}(w,k)$:

$$\mathscr{F}(w,k)=\frac{\varkappa_0\varkappa\nu(k)-k^2\mu(k)}{k\tilde{\psi}_-(-k)\tilde{\psi}_+(w)}\left[\frac{w}{w+k}+\frac{\alpha w}{w+i\varkappa_\parallel}+\frac{\beta w}{w+i\varkappa_\perp}+\gamma\right], \tag{A5}$$

where the coefficients α,β, and γ are obtained from the equations

$$\begin{aligned}&\alpha\sigma(i\varkappa_\parallel)+\beta\sigma(i\varkappa_\perp)+\gamma\sigma(0)+\sigma(k)=0,\\&\alpha\chi(i\varkappa_\parallel)+\beta\chi(i\varkappa_\perp)+\gamma\chi(0)+\chi(k)=0,\\&\alpha h(i\varkappa_\parallel)+\beta h(i\varkappa_\perp)+\gamma h(0)+h(k)=0.\end{aligned} \tag{A6}$$

The functions $\sigma(w)$, $\chi(x)$, and $h(w)$ are defined as follows:

$$\sigma(w)=\sum_i\frac{q_i(\varkappa_\perp^2+q_i^2)}{\partial Q/\partial w^2|_{w=q_i}}\left\{\frac{\tilde{\psi}_-(q_i)T(r_\parallel,q_i)}{w+q_i}-\frac{\tilde{\psi}_-(-q_i)T(r_\parallel,-q_i)}{w-q_i}\right\} \tag{A7}$$

$$\chi(w)=\sum_i\frac{\varkappa_0(\varkappa_\parallel^2+q_i^2)}{\partial Q/\partial w^2|_{w=q_i}}\left\{\frac{\tilde{\psi}_-(q_i)T(r_\perp,q_i)}{w+q_i}+\frac{\tilde{\psi}_-(-q_i)T(r_\perp,-q_i)}{w-q_i}\right\}, \tag{A8}$$

$$h(w)=\sum_i\frac{\varkappa_0(\varkappa_\parallel^2+q_i^2)(\varkappa_\perp^2+q_i^2)\nu(q_i)}{q_i\,\partial Q/\partial w^2|_{w=q_i}}\left\{\frac{\tilde{\psi}_-(q_i)}{w+q_i}-\frac{\tilde{\psi}_-(-q_i)}{w-q_i}\right\}, \tag{A9}$$

where

$$Q(w^2)=[w^2\mu(w)-\varkappa_0^2\nu(w)](w^2+\varkappa_\parallel^2)(w^2+\varkappa_\perp^2).$$

The quantities $q_i (i=1,2,3)$ satisfy the equation $Q(q_i)=0$ ($\mathrm{Im}\,q_i<0$), which is equivalent to

$$\varepsilon_l(\omega,q_3)=1;\qquad q_{1,2}^2+\varkappa_0^2\varepsilon_t(\omega,q_{1,2})=0.$$

Using (A6)–(A9), it can be shown that at $\omega\to\tilde{\omega}_\parallel$ the coefficients $\beta\sim|\varkappa_\parallel/\varkappa_\perp|^3$ and $\gamma\sim|\varkappa_\parallel/\varkappa_\perp|^2$ are small, and the coefficient α is given by

$$\alpha=-1+\tilde{R}(k-i\varkappa_\parallel),\quad \tilde{R}=i(1+r_\parallel)[2\varkappa_\parallel+i(1+r_\parallel)\tilde{\sigma}]^{-1}, \tag{A10}$$
$$\tilde{\sigma}={}^1/{}_2q_3[\tilde{\psi}_-(q_3)-\tilde{\psi}_-(-q_3)].$$

Thus, in the limit as $\varkappa_{\parallel}/\varkappa_{\perp}\to 0$ expression (A5) coincides with (22) if the quantity σ defined in (24) is replaced by $\tilde{\sigma}$ from (A10). Calculations show that as $\omega\to\tilde{\omega}_{\parallel}$ the quantity $\tilde{\sigma}$ is described by (33) in which $q = e\mu^{-1}(\infty)$.

In the second limiting case, $\omega\to\tilde{\omega}_{\perp}$, the small coefficient is $\alpha\sim|\varkappa_{\perp}/\varkappa_{\parallel}|$, and the coefficients β and γ are connected by the relation $1+\beta+\gamma=0$. As a result, the function $\mathscr{F}(w,k)$ as $\omega\to\tilde{\omega}_{\perp}$ takes the form

$$\mathscr{F}(w,k_0)=\frac{\nu_0\varkappa_0^2\varkappa_\perp^2}{(k_0+i\varkappa_\perp)\tilde{\Psi}_-(-k_0)}\frac{1-R_\perp(w+k_0)}{\tilde{\Psi}_+(w)(w+i\varkappa_\perp)(w+k_0)},\tag{A11}$$

$$R_\perp=i(1+r_\perp)[2\varkappa_\perp+i(1+r_\perp)\sigma_\perp]^{-1},$$

$$\sigma_\perp=\frac{1}{4}\sum_{i=1,2}q_i[\tilde{\Psi}_-(q_i)-\tilde{\Psi}_-(-q_i)].$$

We call attention to the fact that according to (A11) we have $\mathscr{F}(0,k_0)\neq 0$ as $\omega\to\tilde{\omega}_\perp$, whereas $\mathscr{F}(0,k_0)=0$ near $\omega=\tilde{\omega}_\parallel$ [see (22)]. This leads to a substantial difference in the angular distribution of the intensity of the volume radiation $W^{(V)}(\theta)$, namely, $W^{(V)}(0)=0$ as $\omega\to\tilde{\omega}_\parallel$, whereas as $\omega\to\tilde{\omega}_\perp$ an appreciable fraction of the energy is radiated in a direction close to normal to the surface ($\theta\approx 0$).

[1)] At $\omega<\tilde{\omega}_\parallel$ the coefficient $r_\parallel$ is real, and $|r_\parallel|=1$ at $\omega>\tilde{\omega}_\parallel$ (see the explanation of Eq. (8) and Ref. 7).

[1]Surface Polaritons, V. M. Agranovich and D. L. Mills, eds., North-Holland, 1982.
[2]L. A. Vainshtein, Teoriya difraktsii i metod faktorizatsii (Diffraction Theory and Factorization Method), Sov. radio, 1966.
[3]V. M. Agranovich, Zh. Eksp. Teor. Fiz. **77**, 1124 (1979) [Sov. Phys. JETP **50**, 567 (1979)].
[4]V. M. Agranovich, V. E. Kravtsov, and T. A. Leskova, Zh. Eksp. Teor. Fiz. **81**, 1828 (1981) [Sov. Phys. JETP **54**, 968 (1981)].
[5]V. M. Agranovich, V. E. Kravtsov, and T. A. Leskova, Sol. St. Commun. **40**, 687 (1981).
[6]V. M. Agranovich and V. L. Ginzburg, Spatial Dispersion in Crystal Optics and the Theory of Excitons, Wiley, 1965,
[7]L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Nonrelativistic Theory, Pergamon, 1977.
[8]J. Meixner, Tech. Rept. EM-72, Inst. Math. Sci. New York Univ., 1954.
[9]V. M. Agranovich, Usp. Fiz. Nauk **115**, 199 (1975) [Sov. Phys. Usp. **18**, 99 (1975)].
[10]A. Ya. Blank and V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **75**, 2317 (1978) [Sov. Phys. JETP **48**, 1170 (1978)].
[11]V. M. Agranovich and O. A. Dubovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **3**, 223 (1966) [JETP Lett. **3**, 142 (1966)]. H. Freedhoff, and J. Van Kranendonk, Can. j. Phys. **45**, 1833 (1967). M. R. Philpott, J. Chem. Phys. **62**, 1812; **63**, 485 (1975).

Translated by J. G. Adashko