Acoustic and magnetosonic waves in the atmosphere

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A mechanism is presented for the vertical propagation of magnetosonic waves in an equilibrium atmosphere with allowance for viscosity and magnetic viscosity. Limitations on the frequencies of acoustic waves that can penetrate into the lower layers of the atmosphere are obtained and the amplitudes of the waves are calculated.

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The theory of the propagation of acoustic and magnetosonic waves in a nonuniform dissipative medium in a gravitational field is of general physical interest.[1,2] In particular, this theory finds practical application in the problem of the action on the lower atmosphere of magneto-hydrodynamic waves excited in the magnetosphere.[3,4]

In this paper we examine the propagation of acoustic modes in a nonuniform dissipative atmosphere in a gravitational field and establish the properties of the atmosphere as a spatial frequency filter.

1. INITIAL EQUATIONS

We shall consider the propagation of magnetohydrodynamic waves with frequencies $\omega \ll \omega_{Bi}$ (ω_{Bi} is the ion cyclotron frequency) in a nonuniform, equilibrium, partially ionized atmosphere under the assumption that the waves come from the edge of the magnetosphere. We shall start from the linearized equations of magnetohydrodynamics (the atmosphere is assumed to be sufficiently dense):

$$\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho_0 \mathbf{v})=0, \qquad (1)$$

$$\rho_0\frac{\partial v}{\partial t}=-\operatorname{grad} p+\frac{1}{4\pi}[\operatorname{rot}\mathbf{b}\times\mathbf{B}_0]+\rho g+\eta\Delta v + \left(\frac{\eta}{3}+\zeta\right)\operatorname{grad}\operatorname{div}\mathbf{v}, \qquad (2)$$

$$\frac{\partial \mathbf{b}}{\partial t}=\operatorname{rot}[\mathbf{v}\times\mathbf{B}_0]+\nu_m\Delta\mathbf{b}, \qquad (3)$$

$$\frac{\partial p}{\partial t}+(\mathbf{v}\operatorname{grad}p_0)=v_S^2\left[\frac{\partial\rho}{\partial t}+(\mathbf{v}\operatorname{grad}\rho_0)\right], \qquad (4)$$

where p, ρ, $\mathbf{v}$, and $\mathbf{b}$ are the deviations of the pressure, density, velocity, and magnetic field strength from their respective equilibrium values p_0, ρ_0, 0, and $\mathbf{B}_0$, while v_S is the adiabatic speed of sound, $\mathbf{g}$ is the strength of the external gravitational field (the acceleration due to gravity), and η, ζ, and ν_m are the first viscosity, second viscosity, and magnetic viscosity coefficients. We shall assume that the characteristic scale $h_0 = |\operatorname{grad}\ln\rho_0|^{-1}$ of the variation of p_0 and ρ_0 is considerably smaller than the characteristic scale for the variations of the magnetic field strength $\mathbf{B}_0$ (this is valid at altitudes above about 1000 km) so that we may treat $\mathbf{B}_0$ as a constant and assume that grad $\ln\rho_0$ is parallel to $\mathbf{g}$.

Now let us go over to Fourier components in Eqs. (1)–(4), writing

$$\{p,\rho,\mathbf{v},\mathbf{b}\}\to\{p,\rho,\mathbf{u},\mathbf{b}\}\exp\{-i\omega t+i\mathbf{k}_\perp\mathbf{r}_\perp+i\Phi\}, \qquad (5)$$

where ω is the wave frequency,

$$\Phi=\int_{z_0}^{z}k_z\,dz$$

is the eikonal (the z axis is parallel to $\mathbf{g}$), and $\mathbf{k}_\perp$ is the components of the wave vector perpendicular to $\mathbf{g}$. After eliminating p, ρ, and $\mathbf{b}$ with the aid of the equilibrium condition

$$\operatorname{grad}p_0=\mathbf{g}\rho_0 \qquad (6)$$

we obtain the following equation for the velocity amplitude:

$$\omega^2\mathbf{u}=v_S^2\hat{\mathbf{k}}(\hat{\mathbf{k}}\mathbf{u})+v_a^2(1+ik^2\nu_m/\omega)^{-1}\{\hat{\mathbf{k}}(\hat{\mathbf{k}}\mathbf{u})+(\mathbf{n}\hat{\mathbf{k}})^2\mathbf{u}-(\mathbf{n}\hat{\mathbf{k}})[\mathbf{n}(\hat{\mathbf{k}}\mathbf{u})+\hat{\mathbf{k}}(\mathbf{n}\mathbf{u})]\}$$
$$-i\omega\eta k^2\mathbf{u}-i\omega(\eta/3+\zeta)\mathbf{k}(\mathbf{k}\mathbf{u})-i\{[(\gamma-1)\mathbf{g}+\operatorname{grad}v_S^2](\hat{\mathbf{k}}\mathbf{u})+(\mathbf{g}\hat{\mathbf{k}})\mathbf{u}+[\mathbf{g}\times[\hat{\mathbf{k}}\times\mathbf{u}]]\}, \qquad (7)$$

where $v_a = B_0/(4\pi\rho_0)^{1/2}$ is the Alfven velocity, $\mathbf{n} = \mathbf{B}_0/B_0$, $\hat{\mathbf{k}}$ denotes the operator $\{\mathbf{k}_\perp, k_z - i\partial/\partial z\}$, and γ is the adiabatic exponent; it is assumed that the waves are so weakly attenuated that we may write $\hat{\mathbf{k}} = \mathbf{k}$ in the dissipative terms.

Assuming that the parameter $(k_z h_0)^{-1}$ is small, we may use the geometric-optics approximation, i.e., we may expand Eq. (7) in $\partial/\partial z \ll k_z$. We obtain the zeroth approximation from Eq. (7) by making the substitution $\hat{\mathbf{k}}\to\mathbf{k}$; it defines waves of four possible types; Alfven waves, fast and slow magnetosonic waves, and intrinsic gravitational waves. To analyze Eq. (7) for the general case of oblique waves is a rather cumbersome task (for example, the Alfven waves split off only when $\mathbf{k}$ lies in the plane defined by $\mathbf{g}$ and $\mathbf{B}_0$) so in what follows we shall consider only two simple cases of vertical propagation ($\mathbf{k}\|\mathbf{g}$) in which the propagation direction is either perpendicular to the magnetic field ($\mathbf{g}\perp\mathbf{B}_0$) or parallel to it ($\mathbf{g}\|\mathbf{B}_0$). In the first case, which is important at low latitudes, when dissipation is neglected the equation for $\mathbf{u}$ equivalent to (7) takes the form

$$\frac{\partial^2u_z}{\partial t^2}=(v_a^2+v_S^2)\frac{\partial^2u_z}{\partial z^2}+\frac{1}{\rho_0}\frac{d}{dz}(\rho_0v_S^2)\frac{\partial u_z}{\partial z} \qquad (8)$$

and describes the only possible longitudinal fast magnetosonic (FMS) wave. In the second case (which is significant at

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polar latitudes) there are waves of two types: transverse Alfven waves, for which

$$\partial^2 \mathbf{u}_\perp/\partial t^2 = v_a^2 \partial^2 \mathbf{u}_\perp/\partial z^2, \tag{9}$$

and longitudinal slow magnetosonic (SMS) waves. When $v_a > v_S$ the equation for the slow waves turns out to be the same as that for ordinary acoustic waves in a gravitational field, which has been discussed in Refs. 5 and 6:

$$\frac{\partial^2 u_z}{\partial t^2} = v_S^2 \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{\rho_0}\frac{d}{dz}(\rho_0 v_S^2)\frac{\partial u_z}{\partial z}. \tag{10}$$

When $v_a < v_S$ there is only one possible longitudinal mode, the equation for which is the same as that for isotropic sound.[1]

2. PROPAGATION OF MAGNETOSONIC WAVES

Let us consider the propagation of FMS waves (Eq. (8)) in the geometric-optic approximation.[1,2] In the zeroth approximation, Eq. (7) or (8) yields the dispersion relation

$$\omega^2 = k^2(v_a^2 + v_S^2) = k^2 v_{ph}^2, \tag{11}$$

which determines the z dependence of k. In the first approximation we obtain

$$u(z) = u(z_0)\left[\frac{v_{ph}(z)}{v_{ph}(z_0)}\right]^{1/2} \exp\left\{-\frac{1}{2}\int_{z_0}^{z} \frac{v_S^2}{v_{ph}^2} |\operatorname{grad} \ln \rho_0 v_S^2| dz'\right\} \tag{12}$$

for the velocity amplitude, where $v_{ph} = (v_a^2 + v_S^2)^{1/2}$ is the phase velocity of the wave and $u(z_0)$ is the velocity amplitude at the initial level $z = z_0$. In view of the fact that when B_0 is constant we have

$$v_S^2 \operatorname{grad} \ln \rho_0 v_S^2 = v_{ph}^2 \operatorname{grad} \ln \rho_0 v_{ph}^2, \tag{13}$$

and we obtain

$$u(z) = u(z_0)[\rho_0(z_0) v_{ph}(z_0)/\rho_0(z) v_{ph}(z)]^{1/2} \tag{14}$$

from (12). It is easy to see that Eq. (14) corresponds to a constant energy flux:

$$S = \rho_0 u^2 v_{ph} = \text{const}. \tag{15}$$

From Eqs. (12)–(15) we find that the velocity amplitude varies with altitude as follows:

$$u \sim \rho_0^{-1/4} \quad \text{at} \quad v_a \gg v_S, \qquad u \sim \rho_0^{-1/2} \quad \text{at} \quad v_a \ll v_S. \tag{16}$$

From the relations

$$\rho/\rho_0 = {}^3/_5\, p/p_0 = b/B_0 = u/v_{ph} \tag{17}$$

we find that the density, pressure, and magnetic field strength amplitudes behave as follows:

$$p, \rho \sim \rho_0^{5/4}, \quad b \sim \rho_0^{1/4} \quad \text{at} \quad v_a \gg v_S, \qquad p, \rho \sim \rho_0^{1/2}, \quad b \sim \text{const} \quad \text{at} \quad v_a \ll v_S. \tag{18}$$

It should be noted that the nonuniformity of the phase velocity will also cause oblique waves to propagate in a more nearly vertical direction as they penetrate more deeply into the atmosphere.

We obtain results analogous to (11)–(18) for SMS waves when $\mathbf{g} \| \mathbf{B}_0$ by setting $v_a = 0$ ($v_{ph} = v_S$). Equation (10) admits an exact solution in the special case of an isothermal atmosphere[5,6]:

$$\omega^2 = \omega_0^2 + k^2 v_S^2, \tag{19}$$

$$u \sim \rho_0^{-1/2}, \quad p, \rho \sim \rho_0^{1/2}, \tag{20}$$

$$\omega_0 = \gamma g / 2 v_S. \tag{21}$$

This solution will also be valid for FMS waves in the region $v_S \gg v_a$ provided v_S is constant.

Thus, the density and pressure amplitudes of magnetosonic waves increase exponentially as the wave penetrates into the atmosphere; the relative amplitudes p/p_0 and ρ/ρ_0 increase for FMS waves in the high-latitude region where $v_a \gg v_S$ and decrease for FMS waves when $v_S \gg v_a$ and for the SMS waves for all values of z.

3. DISSIPATIVE PROCESSES

The frequency range for waves that can reach the Earth's surface will be determined by the dissipation of the waves and the frequency ω_0 (21). Let us first consider the attenuation of the waves associated with the viscosities η and ζ and the magnetic viscosity ν_m. In view of (15), the energy flux of the waves at $z = 0$ will be

$$S(0) = S_0 e^{-\tau}, \tag{22}$$

where the total optical thickness τ is given by the sum

$$\tau = \int_{z_0}^{0} \varkappa_b \, dz' + \int_{z_0}^{0} \varkappa_m \, dz' = \tau_b + \tau_m, \tag{23}$$

while $\varkappa_b$ and $\varkappa_m$ are the attenuation factors due to viscosity and magnetic viscosity, and can be obtained from Eq. (7). The attenuation factor for FMS waves due to viscosity turns out to be

$$\varkappa_b = {}^1/_2({}^4/_3\eta + \zeta)\rho_0^{-1}(v_S^2 + v_a^2)^{-3/2}\omega^2. \tag{24}$$

Assuming that the plasma is weakly ionized and that $\zeta \ll \eta = \lambda v_T \rho_0 / 3$ ($v_T = (3kT/M)^{1/2}$ is the thermal velocity, $\lambda = v_T/\nu_{mol}$ is the mean free path, and ν_{mol} is the collision frequency of the molecules) and taking the exponential behavior of the atmosphere $\rho_0 \sim \exp(-z/h_0)$ into account, we obtain

$$\tau_b = \frac{2}{9} \frac{\omega^2 v_T^2 h_0}{\nu_{mol} v_S^3} \int_0^\infty \frac{d\zeta}{(1+\zeta)^{3/2}}, \tag{25}$$

in which the values of v_T, h_0, v_S, and ν_{mol} are taken at the level $z = z_1$ where $v_S = v_a$. Then we find the upper limit of the pass band from the condition $\tau_b < 1$:

$$\omega_b = ({}^5/_4 \nu_{mol} v_S / h_0)^{1/2}. \tag{26}$$

In accordance with (7), the magnetic viscosity coefficient for FMS waves can be written in the form

$$\varkappa_m = \frac{1}{2} \frac{v_a^2}{(v_a^2 + v_S^2)^{3/2}} \frac{\omega^2}{\omega_m^2}\left(1 + \frac{\omega^2}{\omega_m^2}\right)^{-1}, \qquad \omega_m = \frac{v_a^2 + v_S^2}{\nu_m}, \quad \nu_m = \frac{c^2}{\omega_{pe}^2}\nu_{\text{eff}}^{e}, \tag{27}$$

where $\nu_{\rm eff}^{e}$ if the effective electron collision frequency, ω_{p_e} is the plasma frequency, and c is the velocity of light. When $\omega \ll \omega_m$, the attenuation (27) leads to no substantial change in the structure of the wave. In the opposite case, when $\omega \gg \omega_m$, there is strong diffusion of the magnetic field, as a result of which only acoustic waves can propagate, i.e., propagation can take place only when $v_S \gg v_a$. The attenuation will be greatest at the level where $\omega \approx \omega_m$. Estimates show that for the Earth's atmosphere this level is approximately the same as the level $z = z_1$ where $v_S = v_a$. In analogy with (25), we find

$$\tau_m = \frac{\omega^2 \nu_m h_0}{2 v_S^3} \int_0^\infty \frac{\xi\, d\xi}{(1+\xi)^{1/2}[\xi^2(1+\xi)^2 + \omega^2 \nu_m / v_S^4]}, \qquad (28)$$

where the values of all the parameters are taken at the level $z = z_1$. The corresponding etimate for the upper limit of the frequency band will be

$$\omega_M \approx 2^{7/4} (v_S^3 / \nu_m h_0)^{1/2}, \qquad (29)$$

while the highest possible frequency for FMS waves will be

$$\omega_{max} = \min\{\omega_M,\ \omega_V\}. \qquad (30)$$

We emphasize that it follows from Eq. (25) and (28) that there is a special interval of altitudes $z = z_1 \pm h_0(z_1)$ for the fast waves in which they are mainly attenuated. The dissipation will be small when $z \ll z_1$ because of the high values of the collision frequency $\nu_{\rm mol}$ and $\nu_{\rm eff}^{e}$, and when $z \gg z_1$ because of the high Alfven velocity v_a.

For SMS waves (Eq. (10)), which do not involve magnetic field variations, the attenuation is due to viscosity alone and, as follows from Eq. (24) with $v_a = 0$, the attenuation coefficient increases monotonically with altitude. In this case we have the following estimate for τ:

$$\tau = \tau_V \approx \frac{2}{5} \omega \frac{h_0(z_2)}{v_S(z_2)}, \qquad \omega_{max} = \omega_V = \frac{5}{2} \frac{v_S(z_2)}{h_0(z_2)}, \qquad (31)$$

where z_2 is the altitude at which the hydrodynamic approximation is no longer valid, i.e., where $\nu_{\rm mol}(z_2) = \omega$. On comparing (31) with (26), we conclude that the viscous attenuation of SMS waves exceeds that of FMS waves; for an isothermal atmosphere, $\omega_V^{\rm FMS} / \omega_V^{\rm MMS} \approx (\omega/\omega_0)^{1/2}$.

The lower limit of the frequency range for magnetosonic waves is associated with the fact that waves with $\omega < \omega_0$ (Eq. (21)) are reflected, and as a result their amplitude falls off exponentially as they penetrate more deeply into the atmosphere.[1] The equivalent atmospheric height h_0 is minimal near a certain level $z = z_\perp'$,[7] so at this level ω_0 will be maximal, being smaller when $z < z_\perp'$ and when $z > z_\perp'$. Accordingly, the condition for the field to penetrate to the Earth's surface is $(\omega_0^2 - \omega^2)^{1/2} < v_S / z_\perp'$; hence

$$\Delta\omega = \omega_0 - \omega_{min} \approx {}^1/_2 \omega_0 (h_0 / z_\perp')^2 \qquad (32)$$

and $\Delta\omega \ll \omega_0$ when $z_\perp' \gg h_0$.

Thus, the transmission factor for magnetosonic waves is exponentially small when $\omega < \omega_{\rm min}$ (Eq. (32)), increases sharply when $\omega_{\rm min} < \omega < \omega_0$, is close to unity in the frequency range $\omega_0 < \omega < \omega_{\rm max} = \min\{\omega_V, \omega_M\}$, and decreases with increasing ω as $\exp(-\omega^2/\omega_{\rm max}^2)$ when $\omega > \omega_{\rm max}$. The transparency window is found at the level $z \sim z_1$ where $v_S = v_A$ for FMS waves and at altitudes $z > z_1$ for SMS waves.

4. DISCUSSION

Let us make a few estimates, using known data on the Earth's atmosphere.[7] We shall first consider the region of low latitudes where FMS waves propagating vertically downward across the magnetic field are important. Let us first estimate the boundaries of the frequency pass band. The minimal equivalent atmospheric height h_0 is reached at $z \approx z_\perp' \approx 80$ km corresponding to the frequency $\omega_{\rm min} \approx \omega_0$ which amounts to $\omega_0 \approx 3 \times 10^{-2}$ sec^{-1}. The upper edge of the pass band is associated with magnetic viscosity and amounts to $\omega_{\rm max} = \omega_M \approx 2 \times 10^{-1}$ sec^{-1} ($z_1 \approx 120$ km). The pass band is evidently fairly narrow and corresponds to oscillation periods ranging from 0.5 to 3 min, i.e., to infrasound.

Let us calculate the expected intensity of fluctuations at the Earth's surface. The condition for equilibrium at the shock-wave front is[8]

$$\rho_0^w v_0^2 = B_1^2 / 8\pi, \qquad (33)$$

where ρ_0^w and v_0 are the density and velocity of the solar wind, and B_1 is the magnetic field strength beyond the front. The density variations $\Delta\rho_0^w$ in the solar wind lead to magnetic field-strength variations ΔB_1:

$$\Delta B_1 \approx B_1 \Delta\rho_0^w / \rho_0^w, \qquad (34)$$

which will propagate to the Earth as FMS waves. The contribution from the velocity variations Δv_0 to the magnetic-field variations beyond the front will be considerably smaller than (34) since $v_a \ll v_0$ in the polar wind. The energy density of the waves is

$$S_0 = W v_{a_1} = \frac{\Delta B_1^2}{4\pi} v_{a_1} = 2 \left(\frac{\Delta\rho_0^w}{\rho_0^w} \right)^2 \rho_0 v_0^2 v_{a_1} = W(z=0) v_S, \qquad (35)$$

where v_{a_1} is the Alfven velocity beyond the front. Adopting the values[9] $v_0 \approx 5 \times 10^7$ cm/sec, $\rho_0 \approx 10^{-23}$ g/cm^3, $v_{a_1} \approx 2 \times 10^7$ cm/sec, and $(\Delta\rho_0^w / \rho_0^w) \approx 10^{-1}$ for the frequency range $\omega_0 < \omega < \omega_{\rm max}$, we obtain $S_0 \approx 10^{-1}$ erg/cm^2 sec. If we assume that the flux density in the magnetosphere does not change we obtain the energy density W_0 at the Earth's surface as $W_0 = S_0 / v_S \approx 3 \times 10^{-6}$ erg/cm^3 in accordance with Eqs. (15) and (35); the corresponding amplitude of the pressure variations is

$$p/p_0 = (W_0 / \rho_0 v_S^2)^{1/2} \approx 2 \cdot 10^{-6}, \qquad (36)$$

i.e., the amplitude of the pressure variations will be of the order of 2 μbar. We note that this estimate is evidently too low because of the possible effect of focusing of the wave energy associated with the positive curvature of the boundary of the magnetosphere and the fact that the waves cannot escape into the solar wind. The gain for the energy flux density may be of the order of 10^2, so the amplitude of the pressure variations may reach $p \approx 10$–20 μbar and may be even higher on disturbed days. We also note that the upper limit of the frequency range $\omega_{max} = \omega_M$ (Eq. (29)) depends on the electron concentration in the E layer of the ionosphere and may be several times higher in strongly disturbed periods.

The situation turns out to be more complicated for polar latitudes, where SMS waves are possible. Since the equivalent altitude h_0 at $z > z_1$ increases with increasing z,[7] it turns out that $\omega_{max}(z_2) < \omega_0(z_1)$ when $z_2 > z_1$ [$\omega_{max}(z_2)$ is the upper frequency of SMS waves associated with viscous dissipation—see Eqs. (31)], from which it follows that SMS waves cannot penetrate to the Earth's surface directly from the solar wind. Nevertheless, nonlinear generation of acoustic-type waves by Alfven waves near $z = z_1$ seems possible, as the relative amplitude of Alfven waves may increase to a level of the order of unity during propagation to $z = z_1$ as a result of the nonuniformity of v_a (the Alfven waves cannot penetrate into the region $z < z_1$ because of the strong magnetic viscosity). Linear conversion of FMS waves into sound, associated with the nonuniformity of the medium, may also take place near the level $z = z_1$.

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