Nonlinear effects in bismuth under cyclotron resonance conditions

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It is shown that the magnetic-field dependence of the power of a double-frequency (2ω) signal generated when bismuth crystals are irradiated by an intense monochromatic signal of frequency ω, observed by Gantmakher *et al.* [Sov. Phys. JETP **55**, 921 (1982)], can be ascribed to the dependence of the impedance Z_ω on the magnetic field strength.

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Gantmakher *et al.*[1] investigated the influence of a magnetic field on the degree of conversion of a microwave signal of frequency $f = \omega/2\pi = 9.3$ GHz into radiation at double the frequency. They observed a considerable increase of the signal power $P_{2\omega}$ of frequency 2ω both when the cyclotron resonance condition

$$\omega = n\Omega = n(eH/m^*c),\ n=1,\ 2,\dots \tag{1}$$

was satisfied (m^* is the effective mass of the electron) and at

$$\omega = {}^1/_2(n+1)\Omega, \tag{2}$$

i.e., at half-integer frequencies. Arguments to explain the nature of the first of these effects are presented in Ref. 1. However, an attempt to connect the "half-integer" resonance with the fact that condition (2) corresponds to resonance at frequency 2ω did not yield convincing results. The purpose of the present communication is to show that the $P_{2\omega}(H)$ dependence correlates with the relation for the impedance $Z_\omega(H) = R_\omega(H) + iX_\omega(H)$ at the frequency ω.

Since we mean to obtain only a qualitative picture of the phenomenon, and since the singularities of $R_\omega(H)$ are observed at the same value of the field as for $X_\omega(H)$, we refer hereafter for simplicity not to the complex Z_ω but to its real part R_ω. Proceeding in analogy with Ref. 2, we write for the impedance in a magnetic field H_0 in the presence of a weak alternating field $\tilde{H}\exp(i\omega' t)$, the following expansion:

$$R_\omega(H) = R_\omega(H_0 + \tilde{H}e^{i\omega' t}) \approx R_\omega(H_0) + (dR_\omega/dH)\tilde{H}e^{i\omega' t}. \tag{3}$$

Since the field of the reflected wave is determined by the impedance of the sample, it is clear from (3) that the reflected signal will acquire a component with a frequency $(\omega + \omega')$ and with a magnetic-field amplitude proportional to

$$(dR_\omega/dH)\tilde{H}H_\omega,$$

where H_ω is the amplitude of the incident-wave field. If we now let ω' approach ω, Eq. (3), strictly speaking, ceases to be correct. Nonetheless, assuming that the nonlinearity is due to the action of Lorentz forces, i.e., to the same causes that lead to the magnetic-field dependence of the microwave currents, and with them also of the impedance, and remaining within the framework of a definitely qualitative treatment, we shall use Eq. (3) also at $\omega' = \omega$. Then, recognizing that $\tilde{H} \propto H_\omega$, we obtain

$$H_{2\omega} \propto (dR_\omega/dH)H_\omega^2,$$

i.e.,

$$P_{2\omega} \propto (H_{2\omega})^2 \propto (dR_\omega/dH)^2 P_\omega^2. \tag{4}$$

It is known that the impedance of a metal has singularities connected not only with excitation of cyclotron resonance, but also with the end point of the spectrum of the cyclotron and of other waves.[3,4] Their influence on the impedance of bismuth is frequently comparable with (or even larger than) the contribution of cyclotron resonance (see, e.g., Fig. 8 of Ref. 5).

As shown by our measurements, the same situation obtains also in the present case. Figure 1 shows the experimental dependence of the active part of the impedance R_ω of bismuth and of dR_ω/dH on the magnetic field at $\mathbf{H}\|C_2$ (where C_2 is the binary axis of the bismuth crystal) and $f = \omega/2\pi = 10.22$ GHz, i.e., under approximately the same conditions as for the experiments of Ref. 1. This plot (these results were not published before) was obtained in the course of the study reported in Ref. 6, where the experimental procedure is described in detail. Attention is called to the fact that the $R_\omega(H)$ dependence is quite smooth (the same was observed also by the authors of Ref. 1), but dR_ω/dH shows clearly singularities corresponding to the "steep" sections of $R_\omega(H)$.[1)] It can be seen that the plot of $(dR_\omega/dH)^2$ based on the experimental curve correlates well with the function $P_{2\omega}(H)$ obtained in Ref. 1 and shown in Fig. 4 of that refer-

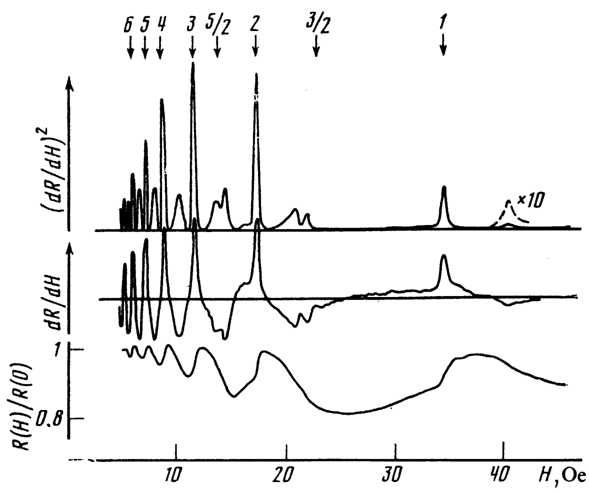

FIG. 1. Active part R_ω of the impedance of bismuth (lower curves), dR_ω/dH (middle curve) and $(dR_\omega/dH)^2$ (upper curve) as functions of the magnetic field. The experimental plots of $R_\omega(H)$ and dR_ω/dH were obtained at $\mathbf{H}\|C_2, f = \omega/2\pi = 10.22$ GHz, and $T \approx 0.4$ K for a sample whose normal $\mathbf{n}$ to the flat surface was parallel to the C_3 trigonal axis.

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ence. The reason why in Ref. 1 the cyclotron-resonance amplitude ratio and the fine structure near the field values corresponding to "half-integer" resonances are different is that for our samples the electron relaxation time was 2–3 times longer than in Ref. 1. The additional singularities are apparently due to the end-points of the cyclotron-wave spectrum, in analogy with what was observed for holes in bismuth.[4] An exact answer calls for a computer calculation.

The proposed approach makes it possible to explain qualitatively also the dependence of $P_{2\omega}$ on the microwave-field amplitude. Indeed, Eq. (3) is valid if H_ω is not too large to make $\widetilde{H}$ smaller than the line width, i.e., for cyclotron resonance $\widetilde{H}/H_0 \lesssim 1/\omega\tau$. To estimate $\widetilde{H}$ we introduce the magnetic-field flux Φ encircled by an electron orbit having an average radius $r(H)$. Its alternating component Φ_ω is obviously of the order $\Phi_\omega \approx H_\omega \delta^{3/2} r^{1/2}$ (where δ is the depth of the skin layer). Then the small-amplitude condition takes the form

$$\Phi_\omega/\Phi \approx (H_\omega/\pi H_0)(\delta/r)^{1/2} \lesssim 1/\omega\tau, \qquad (1)$$

from which it follows that the deviation from the $P_{2\omega} \propto P_\omega^2$ relation sets in at

$$H_\omega \approx \pi H_0 (r/\delta)^{1/2} (\omega\tau)^{-1}.$$

Substituting $\omega\tau \approx 50$ (the value given in Ref. 1), $\delta \approx 10^{-4}$ cm (Ref. 7), and $r \approx 10^{-3}$cm at $\Omega = \omega$, we obtain $H_\omega \approx H_0$, in agreement with the experiment of Ref. 1.

In conclusion we note that in Ref. 1 $P_{2\omega}(H)$ had no singularities connected with resonant transitions between magnetic surface levels.[8] Since the electron orbits lie in this case almost entirely in the skin layer, obviously the singularities should be observed at $H_\omega \lesssim \delta H \approx 0.1$ Oe (where δH is the width of the resonance in terms of the magnetic field), i.e., the effective doubling should take place at power levels smaller by approximately five orders of magnitude than in the case of doubling as a result of cyclotron resonance. No measurements at such low signal levels were performed in Ref. 1.

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[1)] A similar behavior was observed by us earlier in samples of poorer quality.

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