Pulsating "heterophase" regions in nonequilibrium systems

B. S. Kerner and V. V. Osipov

(Submitted 10 May 1982)
Zh. Eksp. Teor. Fiz. **83**, 2201–2214 (December 1982)

Analysis of the stability of inhomogeneous structures (IS) has shown that in initially homogeneous nonequilibrium systems it is possible to excite pulsating inhomogeneous regions whose volume or shape oscillates at a certain characteristic frequency. It is shown that such pulsating "heterophase" regions can arise, for example, when electrons are heated in semiconductors, when gases are ionized, and in the model that describes the Belousov-Zhabotinskiĭ reaction. Heating of electrons in a semiconductor by an electric field produces only the simplest of the pulsating IS, in the form of a pulsating current layer or pinch. In other more complicated systems the simplest pulsating IS are distributions in the form of a breathing stratum wall, a solitary pulsating spot or drop, as well as several pulsating strata, spots, or drops that are separated from one another. From the form of the critical fluctuations with respect to which the IS become unstable it is deduced that when the distance between the drops is small they become unstable with respect to fluctuations that constitute small in-phase changes of the volumes of these drops. Explanations are offered for the experimental data on local oscillations in nonequilibrium systems, including the leading center observed in experimental investigations of a gas discharge and of the Belousov-Zhabotinskiĭ reaction.

PACS numbers: 05.40 + j

1. INTRODUCTION

Stable inhomogeneous structures (IS) of large amplitude can be produced in nonequilibrium systems with a kinetic phase transition.[1–4] Stable IS are produced in phase transition also in equilibrium systems.[5–7] In contrast to equilibrium systems, however, in nonequilibrium systems there can be produced homogeneous oscillations, and pulses that travel without damping as well as other complicated autowave processes can be excited.[8–15] In a number of experiments, local inhomogeneous oscillations are observed,[15] including some in the form of a leading center that constitutes a local source of generation of spherically diverging undamped waves.[8–11]

In this paper we propose a mechanism and determine the conditions for the onset of pulsating "heterophase" regions (including some in the form of a pulsating single drop), whose size or amplitude varies periodically with time. We consider a rather large class of nonequilibrium systems,[1–4,8–18] whose properties are described by the equations

$$\tau_\theta \partial\theta/\partial t = l^2\Delta\theta - q(\theta, \eta, A, \dots, G), \tag{1}$$

$$\tau_\eta \partial\eta/\partial t = L^2\Delta\eta - Q(\theta, \eta, A, \dots, G). \tag{2}$$

These include, in particular, a weakly ionized gas[14,16] and the Belousov-Zhabotinskiĭ autocatalytic reaction,[8,11] which were experimentally investigated in detail. In the model of the latter reaction[11] the activator θ and the inhibitor (damping parameter) η are the concentrations of the intermediate reaction products, while $A,\dots, G$ are the kinetic coefficients and certain constants that characterize the rates of the reactions as well as the concentrations of the initial and final products. For an ionized gas, the system (1) and (2) constitutes, under conditions when the gasdynamic description is applicable, the equations for the electron-density balance and for the effective electron temperature.[14] In this case the role of the activator θ is played by the electron density and that of the damping parameter η by the temperature.[16]

The general requirements imposed on the form of the functions $q(\theta, \eta, A)$ and $Q(\eta, \theta, A)$ for systems with kinetic phase transitions were formulated in Refs. 3 and 4, where, in particular, it was shown that the local $\theta(\eta)$ relation that corresponds to a zero isocline $q(\eta, \theta, A) = 0$ is single-valued in three (or two) sections separated by the points $\theta_0(A_0)$ and $\theta'_0(A'_0)$, at which $q'_\theta = 0$. On the second (unstable) section we have $q'_\theta < 0$. The homogeneous state of the system is unstable only in a certain range $[A_c, A'_c]$ of the bifurcation parameter A, where $q'_\theta < 0$. We shall call the region corresponding to $A < A_0$ cold, and to $A > A'_0$ hot, while the region $A_0 < A < A'_0$ will be called heated. The spontaneous formation of IS in the considered systems is due to spatial decoupling of the activator θ from the damping parameter η (Ref. 19), which is easier the more the characteristic length L of the variation of $\eta(\mathbf{r})$ exceeds the characteristic length l of the variation of $\theta(\mathbf{r})$.

In some systems, L exceeds not only l but also the system dimensions $l_{x,y,z}$. In this case η hardly changes in space, and its value is determined by Eq. (2) averaged over the volume of the system[4]:

$$\tau_\eta \partial\eta/\partial t = -\langle Q(\theta(\mathbf{r}), \eta, A, \dots, G)\rangle, \tag{3}$$

where the symbol $\langle\cdots\rangle$ denotes averaging of the function over the system volume V on the boundaries Σ of which[3]

$$\mathbf{n}\nabla\theta|_\Sigma = \mathbf{n}\nabla\eta|_\Sigma = 0, \tag{4}$$

and $\mathbf{n}$ is a unit vector normal to the surface. The system (1) and (3) describes the properties of a number of concrete physical systems. Reducible to this system are the equations that describe stratification of the current in semiconductors with single-valued current-voltage characteristics (CVC), includ-

 0038-5646/82/121275-08$04.00

ing in the presence of superheat instability of the electrons when the phonons are in disequilibrium[20] and Joule heating of certain semiconducting structures.[18,19]

Also reducible to the system (1) and (3) are the equations that describe the pinching of current in superconductors with S-shaped CVC,[12,17,21] when the role of (3) is assumed by the equation of the external circiut. The conditions for the onset of pulsating current pinches under superheat instability of the electrons in semiconductors will be analyzed in Sec. 3. First, however, we consider in Sec. 2, the general conditions for the appearance of pulsating "heterophase" regions in systems with infinite length of variation of the damping parameter η. In Sec. 4, the results are generalized to the case of a finite value of L. In Sec. 5 we analyze critical fluctuations to which the IS become unstable, and advance some hypotheses concerning the type of the possible pulsations in the considered class of nonequilibrium systems, as well as the effects connected with them.

2. SYSTEMS WITH INFINITE LENGTH OF VARIATION OF THE DAMPING PARAMETER

Stationary inhomogeneous solutions of the system (1) and (3) under the boundary conditions (4) were analyzed in Refs. 4, 12, 17, and 18. It follows from their results that the only stable states in systems with $L=\infty$ and $\tau_\eta=0$ are IS in the form of a single stratum, of a spot (cylindrically symmetrical IS), or of a drop (spherically symmetrical IS). We examine the stability of these IS at finite values of τ_η. To this end it is convenient to analyze the derivative $d\eta/dA$, which in the general case when the bifurcation parameter A has a frequency ω, we shall call the susceptibility of the system.

For the sake of argument we investigate the stability of a spot and of a drop, i.e., of radially symmetrical stationary IS in a system of radius R_b, which satisfy, according to (1) and (3), the equations

$$d^2\theta/d\rho^2+(2^s/\rho)\,d\theta/d\rho+dU_\theta/d\theta=0,\quad U_\theta=-\int q(\theta,\eta,A)\,d\theta, \qquad (5)$$

$$\langle Q(\theta,\eta,A)\rangle=0, \qquad (6)$$

where ρ is measured in units of l, and $s=1$ and 0 respectively for spherically and cylindrically symmetrical states. We linearize Eqs. (1) and (3) near the stationary state $\theta(\rho)$ that satisfies (5) and (6), relative to perturbations of the form

$$\delta\eta(t)=\delta\eta e^{-\gamma t},\qquad \delta A(t)=\delta A e^{-\gamma t}, \qquad (7)$$

$$\delta\theta(\rho,\alpha,\varphi,t)=\delta\theta e^{-\gamma t},\qquad \delta\theta=\delta\theta^{(\beta)}(\rho)P_\beta^{(m)}(\cos\alpha)\,e^{im\varphi}, \qquad (8)$$

where $P_\beta^{(m)}(\cos\alpha)$ are associated Legendre polynomials in the case of spherical symmetry and

$$\delta\theta(\rho,\varphi,t)=\delta\theta e^{-\gamma t},\qquad \delta\theta=\delta\theta^{(\beta)}(\rho)\,e^{i\beta\varphi} \qquad (9)$$

in the case of cylindrical symmetry. As a result we find that the spectrum of the fluctuations of γ is determined from the equations

$$(\hat H_\theta^{(\beta)}-\gamma)\,\delta\theta=-q_\eta'\delta\eta-q_A'\delta A, \qquad (10)$$

$$\delta\eta=-[\langle Q_\theta'\delta\theta^{(0)}\rangle+\langle Q_A'\rangle\delta A]\,(\langle Q_\eta'\rangle-\gamma\alpha^{-1})^{-1}, \qquad (11)$$

where

$$\hat H_\theta^{(\beta)}=-d^2/d\rho^2-(2/\rho)\,d/d\rho+\beta(1+\beta)\rho^{-2}+V_\theta,\ V_\theta=q_\theta'(\theta(\rho)) \qquad (12)$$

or

$$\hat H_\theta^{(\beta)}=-d^2/d\rho^2-(1/\rho)\,d/d\rho+\beta^2\rho^{-2}+V_\theta \qquad (13)$$

respectively for spherical or cylindrical symmetry; $\alpha=\tau_\theta/\tau_\eta$; the time is measured in units of τ_θ.

We expand the functions $\delta\theta^{(\beta)}$ in terms of the orthonormalized eigenfunctions $\delta\theta_n^{(\beta)}$ of the operator $\hat H_\theta^{(\beta)}$. We substitute this series in (10), multiply the resultant expression from the left by $\delta\theta_k^{(\beta)}$, and average over the volume of the sample. As a result, putting $\gamma=i\omega$, we obtain

$$\frac{d\eta}{dA}(\omega)=[D^{(0)}(\omega)]^{-1}\Big[\sum_{n=0}^{\infty}\langle Q_\theta'\delta\theta_n^{(0)}\rangle\langle q_A'\delta\theta_n^{(0)}\rangle(\lambda_n^{(0)}-i\omega)^{-1}-\langle Q_A'\rangle\Big](\mu_0-i\omega\alpha^{-1})^{-1}, \qquad (14)$$

where $\lambda_n^{(\beta)}$ are the eigenvalues of the operator $\hat H_\theta^{(\beta)}$, with $\mu_0>0$ and all $a_n^{(0)}\geqslant 0$ (Refs. 1–4).

$$D(\omega)=1+\sum_{n=0}^{\infty}\frac{a_n^{(0)}[(\lambda_n^{(0)}-\alpha^{-1}\mu_0^{-1}\omega^2)+i\omega(1+\alpha^{-1}\mu_0^{-1}\lambda_n^{(0)})]}{[(\lambda_n^{(0)})^2+\omega^2](1+\alpha^{-2}\mu_0^{-2}\omega^2)}. \qquad (15)$$

$$a_n^{(0)}=-\langle q_\eta'\delta\theta_n^{(0)}\rangle\langle Q_\theta'\delta\theta_n^{(0)}\rangle\mu_0^{-1},\quad \mu_0=\langle Q_\eta'\rangle. \qquad (16)$$

To find the spectrum of the fluctuations of γ at $\delta A=0$ we substitute the perturbation $\delta\eta$ from (11) in (10). Next, expanding the function $\delta\theta^{(\beta)}$ in a series in $\delta\theta_n^{(\beta)}$, we arrive at a system of homogeneous algebraic equation with respect to the coefficient of this expansion. The condition for the solvability of system reduces to

$$\Big[1+\sum_{n=0}^{\infty}a_n^{(0)}\mu_0(\mu_0-\alpha^{-1}\gamma)^{-1}(\lambda_n^{(0)}-\gamma)^{-1}\Big]\prod_{n,\beta}(\lambda_n^{(\beta)}-\gamma)=0. \qquad (17)$$

It can be seen from (17) that $\gamma=\lambda_n^{(\beta)}$ at $\beta\neq 0$. This result follows directly from (10) if it is recognized that, according to (11), $\delta\eta=0$ for the functions $\delta\theta_n^{(\beta)}$ with $\beta\neq 0$. We note that for the same reason only the functions $\delta\theta_n^{(0)}$ are contained in (14)–(16).

Differentiating Eq. (6) with respect to ρ, we obtain

$$(\hat H_\theta^{(0)}+2^s\rho^{-2})\,d\theta/d\rho=\hat H_\theta^{(1)}\,d\theta/d\rho=0, \qquad (18)$$

i.e., $\delta\theta\propto d\theta/d\rho$ at a system radius $R_b\to\infty$ is an eigenfunction of the operator $\hat H_\theta^{(1)}$ and corresponds to $\lambda_n^{(1)}=0$ (see Ref. 22). It follows therefore that in the considered systems with $L=\infty$ (Ref. 4) (just as in semiconductors with S-shaped CVC[17,22]) the only stable IS are those in the form of a single drop or a spot (current pinch), for which $\theta(\rho)$ is monotonic and consequently only $\lambda_0^{(0)}<0$, with $\lambda_0^{(1)}=0$. For a drop or spot with large radius $\rho_0\gg l$, according to Refs. 23 and 3, we have $\lambda_0^{(0)}\sim-(l/\rho_0)^2$. At a finite radius, R_b, the function $\delta\theta\propto d\theta/d\rho$ does not satisfy the boundary condition (4), since it is the function itself and not its derivative which vanishes on the boundary. The true value $\lambda_0^{(1)}$ corresponds to the eigenfunction $\delta\theta_0^{(1)}$, which satisfies (4), of the operator $\hat H_\theta^{(1)}$. From (18), using the fact that the operators $\hat H_\theta^{(\beta)}$ are Hermitian and the condition[24] that $\delta\theta_0^{(1)}(0)=0$, we obtain

$$\lambda_0^{(1)}=2\pi(1+s)R_b^{1+s}V^{-1}\delta\theta_0^{(1)}d^2\theta(\rho)/d\rho^2|_{R_b}\langle\delta\theta_0^{(1)}d\theta/d\rho\rangle^{-1}$$
$$\sim -\exp\{-\min[V_\theta^{1/2}(0)\rho_0, V_\theta^{1/2}(R_b)(R_b-\rho_0)]/l\}. \quad (19)$$

The estimate (19) takes into account the fact that the function $\delta\theta_0^{(1)}$ is localized in the wall of the drop or the spot (of the current filament[23]). Thus, for a drop (spot) $\lambda_0^{(1)}$ is negative at finite $R_b \gg l$, but it is an exponentially small quantity. Therefore, at finite values of L, even a small change of $\eta(\rho)$ leads to suppression of the growth of $\delta\theta_0^{(1)}$ (see also Sec. 4) and to instability of the drop (spot) at the center of the system.[3,4] Formally, however, at $L=\infty$ the drop (spot) at the center of a spherically (cylindrically) symmetric system of radius R_b is unstable. What can be stable is a distribution in the form of 1/8 (1/4) part of the drop (spot) located in a system that has the form of one-eighth of a sphere (one-quarter of a cylinder) of radius R_b. For such distributions of $\theta(\rho)$ the function $\delta\theta_0^{(1)}$ is not included among the eigenfunctions $\delta\theta_n^{(1)}$ of the operator $\hat{H}_\theta^{(1)}$, since it does not satisfy the boundary conditions (4), and therefore only $\lambda_0^{(0)}<0$.

Thus, for a potentially stable single drop (spot) only $\lambda_0^{(0)}<0$, and it follows therefore from (17) that the drop is unstable when $N=0$, where N is the number of zeros in the upper $\omega=-i\gamma$ complex plane of the function $D^{(0)}(\omega)$ (15). According to the argument principle,

$$N=P+(2\pi)^{-1}\arg D^{(0)}(\omega),$$

where P is the number of poles of the function $D^{(0)}(\omega)$ (15) in the upper half-plane. In the case considered, only $\lambda_0^{(0)}<0$, i.e., $P=1$. It can be seen from (15) that $\operatorname{Re} D^{(0)}(\omega)$ is an even function of ω and $\operatorname{Im} D^{(0)}(\omega)$ is odd, with $D^{(0)}(\pm\infty)=1$. It follows therefore, that when $D^{(0)}(0)>0$, we have $N=1$ (curve 1, Fig. 1a), and consequently the drop (spot) is unstable.

At $D^{(0)}(0)<0$ the quantity $\arg D^{(0)}(\omega)$ depends on the sign of the quantity

$$K^{(0)}(\omega)=\sum_{n=0}^{\infty}a_n^{(0)}(\alpha\mu_0+\lambda_n^{(0)})[(\lambda_n^{(0)})^2+\omega^2]^{-1}$$
$$\propto\frac{1}{\omega}\operatorname{Im}D^{(0)}(\omega). \quad (20)$$

In the case when $\alpha=\tau_\theta/\tau_n$ is not small, or more accurately when

$$\alpha\mu_0+\lambda_0^{(0)}>0, \quad (21)$$

the quantity $K^{(0)}(\omega)$ at all finite ω is certainly larger than zero, and $\arg D^{(0)}(\omega)=-2\pi$ (curve 2, Fig. 1a) i.e., $N=0$, and consequently the drop (spot) is stable. Thus, when (21) is satisfied the condition $D^{(0)}(0)=0$ determines the stability limit, i.e., the point $A=A_b$ on the bifurcation characteristic (on the plot of η on A, where $d\eta/dA\ (\omega=0)=\infty$, Fig. 2), is according to (14) the demarcation point.[1,3,4] In this case only drops (spots) of large amplitude are stable, since it is precisely for them that $D^{(0)}(0)<0$. (Refs. 3 and 4).

At $\alpha\ll 1$, since $\mu_0=\langle Q'_\eta\rangle\sim 1$, the condition (21) no longer holds even for drops (spots) of large radius $\rho_0\gg l$, for which $\lambda_0^{(0)}\sim-(l/\rho_0)^2$. Inasmuch as for them we have $|\lambda_0^{(0)}|\ll 1$, it follows that $D^{(0)}(0)$ (15) is certainly less than zero, and it can be seen from (20) that $K^{(0)}(0)<0$, but reverses sign already at a certain $\omega=\omega_1\ll 1$. Analogously, at a certain $\omega=\omega_c$ we have $\operatorname{Re}D^{(0)}(\omega_c)=0$, since $\operatorname{Re}D^{(0)}(0)=D^{(0)}(0)<0$, while $\operatorname{Re}D^{(0)}(\infty)=1$. From an analysis of $D^{(0)}(\omega)$ (15) it follows that at $K^{(0)}(0)<0$ the drop (spot) is stable ($N=P-1=0$) if $\operatorname{Re}D^{(0)}(\omega_1)<0$ (curve 3, Fig. 1b) and unstable ($N=P+1=2$) if $\operatorname{Re}D^{(0)}(\omega_1)>0$ (curve 4, Fig. 1b). Thus, at a certain critical dimension ρ_c of the filament, when $\omega_1=\omega_c$, we have $D^{(0)}(\omega_c)=0$, i.e., the real frequency ω_c is a zero of the function $D^{(0)}(\omega)$ at the stability threshold of the drop (spot). Since $\operatorname{Re}D^{(0)}(\omega)$ (15) and $K^{(0)}(\omega)$ (20) are functions of ω^2, it follows that $D^{(0)}(\omega)$ vanishes at $\omega=\pm\omega_c$, and it is this which explains the simultaneous appearance of two zeros of the function $D^{(0)}(\omega)$ in the upper half plane of $\omega (N=2)$.

We ascertain now the conditions (in particular, we estimate the critical radius of the drop ρ_c) under which $\omega_1=\omega_c$, and estimate also the value of ω_c. Recognizing that among the $\lambda_n^{(\beta)}$ only $\lambda_0^{(0)}<0$, and for a drop (spot) of large radius we have $|\lambda_0^{(0)}|\ll 1$, we can retain in the sum (15) for $\operatorname{Re}D^{(0)}(\omega)$ only the first term and find that

$$\omega_c^2\approx\alpha\mu_0a_0^{(0)}=-\alpha\langle q'_\eta\,\delta\theta_0\rangle\langle Q'_\theta\,\delta\theta_0^{(0)}\rangle\sim\alpha(l\rho_0^{1+s}R_b^{-2-s}). \quad (22)$$

In the estimate of (22) it was taken into account[1)] that the function $\delta\theta_0^{(0)}$ is localized in the wall of the drop (spot), where $q'_\eta Q'_\theta<0$ (Ref. 3), and the thickness of the drop (spot) wall is of the order of l. Taking into account the smallness of the quantities $|\lambda_0^{(0)}|$, α and ω_c, we can obtain from (20) the instability condition corresponding to $K^{(0)}(\omega_c)<0$, i.e., $\omega_1>\omega_c$, when $\operatorname{Re}D^{(0)}(\omega_1)>0$ (curve 4, Fig. 1b):

$$\alpha\mu_0+\lambda_0^{(0)}<-\alpha\mu_0\sum_{n=1}^{\infty}a_n^{(0)}/\lambda_n^{(0)}<0(\gg-\alpha). \quad (23)$$

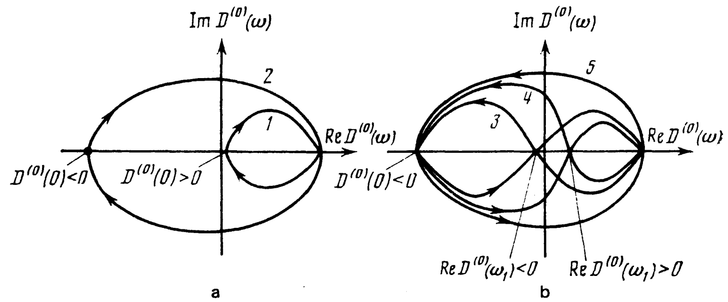

FIG. 1. Qualitative behavior of the complex function $D^{(0)}(\omega)$ (15) on circling around the upper half-plane of ω: a–at $K^{(0)}(\omega)>0$ (20); b–at $K^{(0)}(0)<0$.

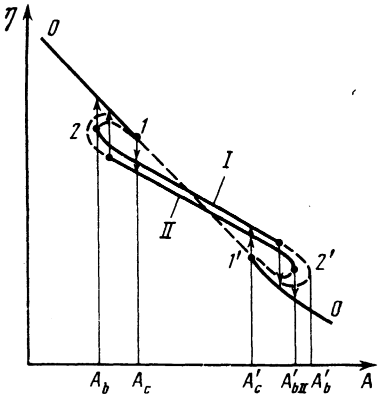

FIG. 2. Form of the bifurcation characteristic of a spherically (cylindrically) symmetrical system of radius R_b with a drop (spot) for the case $L=\infty$, $Q'_\theta>0$, $Q'_A=0$, $q'_A<0$. The curve O corresponds to the homogeneous state, I to a hot drop (spot), and II to a cold drop (spot). (The unstable sections at $\alpha>\alpha_c$ are shown dashed.) The arrows show the possible jumps when the given state loses stability. 1, 1'–points where the homogeneous state loses stability.

For a drop (spot) of large radius $\lambda_0^{(0)}\sim-(l/\rho_0)^2$, therefore the condition (23) at $\alpha\ll1$ is satisfied at $\rho_0<\rho_c\sim l\alpha^{-1/2}$, when $|\lambda_0^{(0)}|\ll1$. In this case, as already noted, $D^{(0)}(0)<0$ (15), i.e., the drop (spot) is stable relative to fluctuations with $\omega=0$. Thus, at $\alpha\ll1$, with decreasing bifurcation parameter A and corresponding decrease of the radius ρ_0 of the drop at $A=A_\omega$, at which $D^{(0)}(\omega_c)=0$, and $d\eta/dA\ (\omega=\omega_c)=\infty$, the drop (spot) loses stability and does not reach the point 2 ($A=A_b$) on the bifurcation characteristic (see Fig. 2), where $d\eta/dA\ (\omega=0)=\infty$.

The result has a simple physical meaning. At $A_\omega>A>A_b$ a uniform slow growth of the activator fluctuation $\delta\theta_0^{(0)}$ with a growth rate $\lambda_0^{(0)}<0$ is damped by the corresponding change of η. At the same time at $\tau_\eta\gg\tau_\theta(\alpha\ll1)$, the change of the lower-inertia activator with frequency $\tau_\theta^{-1}\gg\omega_c\gg\tau_\eta^{-1}$ cannot be followed by the larger-inertia damping parameter η, i.e., it no longer damps the onset of inhomogeneous self-oscillations $\delta\theta\propto\delta\theta_0^{(0)}\cos\omega_c t$. In other words, the physics of the appearance of drop (spot) fluctuations with frequency ω_c is the same as that of the onset of homogeneous oscillations, and is connected with the frequency difference between $\delta\theta_0^{(0)}$ and $\delta\eta$. Indeed, linearizing (1) and (3) [or (2)] relative to the homogeneous fluctuations, we find[3,19] that the homogeneous state loses stability relative to fluctuations of frequency $\omega=\omega_0$, where

$$\omega_0^2=\alpha(Q_\eta'q_\theta'-Q_\theta'q_\eta'),\tag{24}$$

when

$$\alpha Q_\eta'+q_\theta'<0.\tag{25}$$

If it is recognized that $\mu_0=\langle Q'_\eta\rangle$, it can be seen from a comparison of (25) with (21) and of (23), (25) with (22) that the condition for the onset of the pulsations (23) correlates with the condition for the onset of homogeneous oscillations (25). At the same time, according to (24) we have $\omega_0\sim(\tau_\theta\tau_\eta)^{-1/2}$, and for a drop (spot) of large radius we have $\omega_c\sim(\tau_\theta\tau_\eta)^{-1/2}(l\rho_0^{1+s}/R_b^{2+s})^{1/2}$, i.e., the frequency ω_c of the critical pulsations of the drop is noticeably larger than ω_0. Such a difference between ω_c (22) and the frequency ω_0 (24) for the onset of homogeneous oscillations is connected with the fact that the unstable section in the drop (spot) of radius ρ_0, where $q'_\theta<0$, in contrast to the homogeneous state, is concentrated only in a narrow spherical (cylindrical) layer of thickness of the order of l, where $\delta\theta_0^{(0)}$ differs from zero[3] (see, however, footnote 1).

We note[3,4] that the IS in the form of a single drop (spot or stratum) exists also in the cold region ($A<A_0$), where the homogeneous state is stable and the condition (23) for the onset of the pulsations of a drop is easiest to satisfy. The latter is due to the fact that when the system is cooled the drop radius decreases[3,4] and consequently $\lambda_0\sim-(l/\rho_0)^2$ becomes a more negative quantity. When the system is heated (when A increases) the drop expands and at $A\gtrsim A'_c$ it is transformed into a narrow (with dimension of order l) cold spherical layer at the system boundary in the form of a sphere of radius R_b. Such a layer becomes unstable even at $\alpha\gg1$, without reaching the point 2' ($A=A'_b$) where $d\eta/dA\ (\omega=0)=\infty$ (see Fig. 2). The latter is connected with the fact that there are several $\lambda_0^{(\beta)}<0$ in the spectrum of the fluctuations of the narrow spherical layer.[3]

In the hot region, besides the narrow cold spherical layer at the surface of the system, there is realized an IS in the form of a cold drop at the center of the system. The presence of two different states, corresponding to a hot and to a cold drop existing approximately in one and the same range of values of A (Fig. 2), follows directly from the procedure of the construction of the IS (Refs. 1 and 3). Indeed, Eq. (5) can be formally regarded as an equation for one-dimensional motion of a particle with coordinate θ and time ρ, moving in a potential U_θ in the presence of a friction force of constant sign that decreases with increasing ρ (Ref. 3). The distribution of $\theta(\rho)$ in the form of a drop (spot) of large radius corresponds here to a particle trajectory from one saddle point of the potential E_0 to another.[3,17,22] In a certain range of the bifurcation parameter A there exist a certain $\eta=\eta_s$, at which the values of the potential U_θ at the two maxima, i.e., at the saddle points of Eq. (5), coincide.[2–4] Because of the friction, the particle moves on a descending trajectory, therefore $\theta(\rho)$ in the form of a hot drop with $\rho_0\gg l$ is realized only at $\eta>\eta_s$, while in the form of a cold drop it is realized that $\eta<\eta_s$ (Ref. 3) for a system whose bifurcation characteristic is shown in Fig. 2. When the system is heated, the radius of the cold drop decreases, and in the case $\alpha\gg1$ the drop vanishes jumpwise at the point $A=A'_{bII}$, where $d\eta/dA\ (0)=\infty$ (see Fig. 2). When the system is cooled, the cold drop expands and is transformed into a narrow hot spherical layer at the surface of the system, and the layer loses stability at a lower value of A than in the case $d\eta/dA\ (0)=\infty$ (Fig. 2). If $\alpha\ll1$ the cold drop, for the same reason as the hot one, becomes unstable at $A=A'_\omega<A'_{bII}$, which corresponds to the limit of satisfaction of the condition (23), relative to the fluctuation $\delta\theta\sim\delta\theta_0^{(0)}(\rho)\cos\omega_c t$, which is a small oscillation of the volume of the drop with frequency ω_c (22).

The fluctuation $\delta\theta_0^{(0)}(\rho)$, relative to which stability is lost, is centrosymmetric and is localized at the center of the drop. At $\alpha \ll 1$, as already noted, the condition (23) is satisfied at a drop radius $\rho_0 \gg l$, and consequently the instability relative to the fluctuation $\delta\theta_0^{(0)}(\rho)\cos\omega_c t$ denotes instability relative to the onset of pulsations of the drop, i.e., of oscillations of its radius. At not too small a value of α, the condition (23) is satisfied for a drop (spot) of small radius, inasmuch as for it $\lambda_0^{(0)} \sim -1$ (Ref. 3). For such a drop, $\delta\theta_0^{(0)}(\rho)$ is localized over the entire region of the drop, and its extrema are located at $\rho = 0$. Consequently, stability is lost in this case relative to small oscillations of the drop amplitude, with frequency ω_c. With further increase of α, the value of A_ω tends to the value of A_b at which $d\eta/dA\,(\omega=0)=\infty$ (Fig. 2). In this case the instability that leads to pulsations of the drop, becomes aperiodic at $\alpha=\alpha_c$, at which $A_\omega = A_b$. Thus, at $\alpha > \alpha_c$ there will be no drop pulsations, and at $A < A_b$ the hot drop vanishes jumpwise (Fig. 2).

3. PULSATING CURRENT PINCH[2)]

We consider an electronic semiconductor of cylindrical shape with radius R_b and height l_z for which the necessary condition of superheat instability[17,22] is satisfied:

$$d\ln\tau_p\tau_\varepsilon/d\ln T_e > 1,\quad \text{i.e.}\quad \zeta+\xi>1. \tag{26}$$

Here ζ and ξ determine the dependence of the momentum relaxation time $\tau_r \propto T_e^{\xi}$ and of the energy $\tau_\varepsilon \propto T_e^{\xi}$ of the hot electrons on the electron temperature T_e. The condition (26) is satisfied at low lattice temperatures T^0 in certain semiconductors, when the carrier momentum relaxes on ionized impurities, and the energy relaxes on piezoacoustic phonons.[17] In addition, we assume satisfaction of the condition under which the electron gas can be regarded as nondegenerate and incompressible, so that the energy-balance equation of the electron gas can be written in the form[17,22]

$$c_e n\partial T_e/\partial t = \nabla_\perp \varkappa \nabla_\perp T_e + \sigma E^2 - nc_e(T_e - T^0)\tau_\varepsilon^{-1}, \tag{27}$$

where account is taken of the fact that the stratification of the temperature of the electron gas takes place in a direction perpendicular to the electric field $\mathbf{E}$ (the z axis), i.e., $\nabla_\perp = \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y$; the quantities n, σ, $\varkappa$, and c_e are the concentration, specific conductivity, thermal conductivity, and specific heat of the electron gas. Negative differential resistance and instability of the homogeneous state in a semiconductor of sufficiently large cross section ($R_b \gg l = (\varkappa\tau_\varepsilon/nc_e)^{1/2}$, Ref. 22) is realized at[17,22]

$$T_e > T_0 \equiv [(1+\zeta+\xi)/(\zeta+\xi)]T^0. \tag{28}$$

This leads to pinching of the current density at a given current in the external circuit, or more accurately at a sufficiently large load resistance R_l.[22] In other words, when pinching of the current is considered, Eq. (27) must be considered together with the equation of the external circuit

$$l_z C\, dE/dt = \mathscr{E}/R_l - El_z/R_l - E\int\sigma\, dS, \tag{29}$$

where $\mathscr{E}$ is the power-supply voltage and, in contrast to Refs. 12, 17, 22, and 23, account is taken of the influence of the capacitance C. Using the notation

$$\tau_\eta = CR^0,\quad R^0 = l_z(\sigma^0 S)^{-1},\quad \sigma^0=\sigma(T^0);$$
$$S=\pi R_b^2;\quad A=\mathscr{E}(E^0 l_z)^{-1};$$
$$\eta = E/E^0;\quad \tau_\theta = \tau_\varepsilon^0;\quad \tau_\varepsilon^0=\tau_\varepsilon(T^0);\quad T=T_e/T^0;\quad \varkappa^0=\varkappa(T^0);$$
$$\theta = \int^T \frac{\varkappa^0}{\varkappa(T')}\,dT'$$

we easily write Eq. (29) in the form (3), in which

$$\langle Q(\eta,\theta,A)\rangle = -AR^0R_l^{-1} + \eta R^0 R_l^{-1} + \eta l^2 R^0 l_z^{-1}\, 2\pi\int^{R_b}\sigma(\theta)\rho\, d\rho, \tag{30}$$

and Eq. (27) in the form

$$\tau_\theta(\varkappa^0/\varkappa)(\partial\theta/\partial t) = \Delta_\perp\theta - q(\theta,\eta), \tag{31}$$

$$q(\theta,\eta) = [T(\theta)-1]T^{-\xi}(\theta) - T^{\zeta}(\theta)\eta^2 g_0;$$
$$g_0 = \sigma^0\tau_\varepsilon^0(E^0)^2/T^0 nc_e. \tag{32}$$

Equation (31) differs from (1) by an insignificant factor $\varkappa^0/\varkappa$ and by replacement of the operator Δ by $\Delta_\perp$. The only effect of the factor $\varkappa^0/\varkappa$ is that the eigenfunctions $\delta\theta(\rho)$ in the equations derived in Sec. 2 must now be normalized with a weight $\varkappa^0/\varkappa$; the appearance of the operator Δ_T, on the other hand, means that in the concrete case considered it is impossible to have an IS with spherical symmetry in the form of a drop, but an IS of cylindrical symmetry, a current filament, is realized.[23] Thus, all the general relations and results obtained in Sec. 2 are fully applicable to the present model. This makes it possible, using (22) and (23), write down the condition for the onset of radially symmetrical oscillations of the wall of the large-radius current pinch, for which $\lambda_0^{(0)} \sim -\tau_\varepsilon^{-1}(l/\rho_0)^2$, in the form

$$\tau_\varepsilon C^{-1}[R_l^{-1} + Z_{d1}^{-1}(0) + Z_{d3}^{-1}(0)] < (l/\rho_0)^2 \tag{33}$$

and estimate the frequency of such pulsations of the pinch:

$$\omega_c \sim [2\pi\rho_0 l(\sigma_3-\sigma_1)/l_z\tau_\varepsilon C]^{1/2} = (L_w C)^{-1/2}, \tag{34}$$

where $\sigma_3=\sigma(T_3)$, $\sigma_1=\sigma(T_1)$; T_3 and T_1 are the temperatures of the electrons in the filament ($0 \leqslant \rho < \rho_0$) and outside it ($\rho_0 < \rho \leqslant R_b$); L_w is the inductance corresponding to the inertia of the pinch wall[17-23]; $Z_{d3}^{-1}(\omega)$ and $Z_{d1}^{-1}(\omega)$ are the differential admittances of the pinch and of the remaining part of the sample, regarded as homogeneous phases with cross sections $\pi\rho_0^2$ and $\pi(R_b^2-\rho_0^2)$ (Ref. 23).

The relations (33) and (34) obtained for the analyzed simplest model can be obtained also directly from an analysis of the zeros of the function $Z^{-1}(\omega) + R_l^{-1} - i\omega C$, that follows from Kirchoff's law, where $Z^{-1}(\omega)$ is the admittance of a sample with a pinch. According to (23), the equivalent circuit of a sample with a current pinch of large radius consists of parallel-connected impedances Z_{d1} and Z_{d2} and the impedance of the pinch wall, which comprises a series-connected of the negative resistance $R_w = L_w\lambda_0^{(0)}$ and of the inductance L_w. Therefore the addition in the external circuit of the capacitance C satisfying the condition (33) leads to oscillations of the walls of the pinch with frequency ω_c(34). From the viewpoint of the general approach developed in the present paper, the damping parameter η is in this case the electric field E (the increment of E in the regime of a given total current in the absence of capacitor damps out the perturbations of the activator—of the temperature θ). On the other hand, in the presence of a capacitance C the field E does not manage to follow the fluctuation $\delta\theta_0^{(0)}$ with frequency ω_c, since the characteristic time of variation of the field E at sufficiently large R_l and C is

$$\tau_\eta \approx C(Z_{d1}^{-1}+Z_{d3}^{-1})^{-1} \gg \omega_c^{-1}.$$

In other words, the temperature fluctuations $\delta\theta_0^{(0)}$ in the wall of the pinch (activator) and of the damping parameter $\eta \propto E$ differ in frequency. Obviously, a similar situation is realized also in systems with N-shaped CVC. In this case the impedance of a static or of a moving domain contains a resistance $R_c<0$ connected in parallel with a capacitance $C_w = 1/R_c\lambda_0$ (Ref. 17); therefore addition of a certain inductance $\mathscr{L}$ in the external circuit leads to pulsations of the domain wall with frequency $\omega_c \sim (C_w\mathscr{L})^{-1/2}$.

In the model considered above, that of superheat instability of the electron gas in the semiconductor, it was assumed that the phonons are at equilibrium.[17,22] At low temperatures this assumption is not justified, since heating takes place of the system of the long-wave phonons (LP) with which the hot electrons interact. As a result, the section of the negative differential conductivity on the CVC vanishes,[25] since the Joule power obtained by the electrons from the electric field goes to heating not only the electron system but also the LP, whose specific heat increases like $T_e^{3/2}$. Nevertheless, stratification of the current will take place in such a system,[20] and in accordance with the results of the present study the current pinch can pulsate. In other words, stratification of the current takes place at positive differential resistance of the semiconductor and even in the case when its CVC is single-valued. The physical meaning of this result is the following.[20] A homogeneous increase of T_e leads to a homogeneous heating of the LP, and this damps out the superheat instability of the electron gas. A different situation is realized in the case of inhomogeneous change of T_e. Indeed, relaxation of the LP at low temperatures takes place as a rule at the boundaries of the sample,[25] i.e., they can be only uniformly heated. It follows from this that the inhomogeneous fluctuations of T_e, which do not change the total current in the circuit, will not cause a damping heating of the LP. In other words, the stratification of the temperature T_e of the hot electrons is due to the spatial separation of the electron system and of the LP, while the possibility of the appearance of a pulsating pinch is connected with their frequency separation. From (22) we can obtain for this model that the pinch pulsation frequency is

$$\omega_c \sim (\tau_\varepsilon \tau_{ph,e})^{-1/2}(l\rho_0 R_b^{-2})^{1/2},$$

where $\tau_{ph,e}$ is the relaxation time of the LP energy on the electrons.[25] In this case the bifurcation characteristic of the sample with hot and cold current pinches has the form shown in Fig. 2, where η is the average energy of the LP and $A \propto E^2$.

4. SYSTEMS WITH FINITE LENGTH OF VARIATION OF THE DAMPING PARAMETER

Methods of constructing stationary solutions of the system (1) and (2) and for the analysis of their stability were developed in Refs. 1–4 where it was shown, in particular, that the spectrum of the fluctuations of γ near the investigated IS is determined from the equation

$$\det\left[(\lambda_n-\gamma)\delta_{ni}+\sum_{l=0}^{\infty}\mathscr{P}_{lni}(\mu_l-\alpha^{-1}\gamma)^{-1}\right]=0, \tag{35}$$

where

$$\mathscr{P}_{lni} = -\langle\delta\eta_l q_\eta' \delta\dot\theta_i\rangle\langle\delta\dot\eta_l Q_\theta'\delta\theta_n\rangle, \tag{36}$$

and $\langle\cdots\rangle$ denotes averaging of a function over the volume of the system. Owing to the damping action η, all the fluctuations $\delta\eta_l$ correspond to eigenvalues $\mu_l>0$, and the larger the index l the larger μ_l (Ref. 1). In the spectrum λ_n of the fluctuations $\delta\theta_n$ of the activator θ there are negative values. They correspond to a discrete spectrum, and the functions $\delta\theta_m$ corresponding to them are localized in the transitions layers between the regions of different phases, e.g., in the walls of the strata,[1,2] spots, or drops.[3] Taking into account the damping character of the fluctuations $\delta\eta$, it is easy to understand that if account is taken of the influence of all the $\delta\eta_l$ when considering the growth of any particular m-th eigenfunction $\delta\theta_m$ with $\lambda_m<0$, but no account is taken of some others $\delta\theta_n$ with $n\neq m$, the value of γ will be overestimated. This upper bound for γ can be obtained from the determinant (35), by equating its diagonal terms to zero. The result, for each $\lambda_m<0$ we obtain an equation for γ:

$$\Phi^{(m)}(\gamma)=\lambda_m-\gamma+\sum_{l=0}^{\infty}a_m^{(l)}(1-\alpha^{-1}\mu_l^{-1}\gamma)^{-1}=0, \tag{37}$$

where $a_m^{(l)} = \mathscr{P}_{lmm}\mu_l^{-1}$. On the contrary, if account is taken of all other $\delta\theta_n$, and of only $\delta\eta_m$ in the diagonal terms of the matrix in (35), we arrive at a lower bound for γ:

$$D^{(m)}(\gamma)\equiv 1+(1-\alpha^{-1}\mu_m^{-1}\gamma)^{-1}\left(\frac{a_m^{(m)}}{\lambda_m-\gamma}+\sum_{n=0,\,n\neq m}^{\infty}\frac{a_n^{(m)}}{\lambda_n'-\gamma}\right)=0, \tag{38}$$

where $\lambda_n' = \lambda_n + a_n^{(n)}(1-\alpha^{-1}\mu_n^{-1}\gamma)^{-1}$. It follows from (37) that the necessary condition for the stability for the IS is[1,2,4]

$$\lambda_n+\sum_{l=0}^{\infty}a_n^{(l)}>0. \tag{39}$$

The sufficient stability condition, according to (38), reduces to the absence of zeros in the upper ω half-plane for the function $D^{(m)}(\gamma=i\omega)$ for each $\lambda_m<0$. The function $D^{(m)}(\omega)$ at $N=0$ for the functions $\lambda_n'(\omega)$ has only one pole $i\omega=\lambda_m<0$, therefore the problem of finding itz zeros (N) reduces to the problem considered in Sec. 3 in the analysis of the zeros of the function $D^{(0)}(\omega)$ (15). It follows from the results of Sec. 3 that at $\alpha \gg 1$ the IS is stable if for each $\lambda_m<0$ we have $D^{(m)}(0)<0$ (Ref. 4). At $\alpha \ll 1$ and $|\lambda_m| \ll 1$, the sufficient condition for the stability is violated even at $D^{(m)}(0)<0$ relative to one of the fluctuations $\delta\theta_m \cos\omega_c^{(m)}t$ with

$$\omega_c^{(m)} \approx (\alpha\mathscr{P}_{mmm})^{1/2}, \tag{40}$$

when

$$-\alpha \lesssim \lambda_m + \alpha\mu_m < 0. \quad (41)$$

Investigating the zeros of the function $\Phi^{(m)}(\gamma = i\omega)$ (37), we can easily verify that it has two zeros in the upper half-plane with Re $\omega = \pm\omega_c$, when the following condition is satisfied for one of the $\lambda_m < 0$:

$$\lambda_m = -\sum_{l=0}^{\infty} \mathscr{P}_{lmm}\mu_l(\mu_l^2 + \alpha^{-2}\omega_c^2)^{-1};$$
$$\sum_{l=0}^{\infty} \mathscr{P}_{lmm}(\mu_l^2 + \alpha^{-2}\omega_c^2)^{-1} = \alpha. \quad (42)$$

Recognizing that according to Refs. 1 and 3 the coefficients $\mathscr{P}_{lmm} \ll \mathscr{P}_{mmm}$ (the functions $\delta\eta_l$ and $\delta\theta_m$ have different numbers of zeros), then (42) leads in fact to the instability condition (41) connected with the frequency difference between the activator fluctuation, which is close in form to $\delta\theta_m \cos\omega_c^{(m)}t$ with $\lambda_m < 0$, and the damping variation $\delta\eta_m$ of the inhibitor.

5. PROPOSED BASIC TYPES OF PULSATIONS

5.1. In the cold region near the loss of stability of the inhomogeneous state ($A = A_c$), in spherically symmetrical systems or in systems with dimension much larger than L, there exists[3] at $\varepsilon = l/L \ll 1$ an IS in the form of a single large-radius drop with $\rho_0 \gg l$. The spectrum of the fluctuations $\delta\theta_0^{(\beta)}$ and $\delta\eta_0^{(\beta)}$ of such a drop, corresponding to $\lambda_m \equiv \lambda_o^{(\beta)} < 0$, is according to Ref. 3

$$\lambda_0^{(\beta)} \sim -\varepsilon - (l/\rho_0)^2 + \beta(1+\beta)(l/\rho_0)^2,$$
$$\mu_0^{(\beta)} \sim \mu_0 + \beta(1+\beta)\varepsilon^{-2}(l/\rho_0)^2. \quad (43)$$

Recognizing that $\mu_0 \sim 1$ and $\mathscr{P}_{mmm} > |\lambda_m|$, it follows from (43) that the necessary stability condition (39) is satisfied. At the same time, at $\alpha \ll 1$ the condition (41) is satisfied when the system is cooled and the drop dimension is $\rho_0 < \rho_c \sim l_\alpha^{-1/2}$, and according to (43) primarily relative to the spherically symmetrical fluctuation $\delta\theta_0^{(0)}$ corresponding to $\lambda_0^{(0)} < 0$ and localized in the drop wall (at $\rho = \rho_0$). It follows thus that the instability relative to $\delta\theta_0^{(0)}$, which oscillates at a frequency $\omega_c^{(0)}$ (40), leads to pulsations of the drop, i.e., to oscillations of its radius with frequency $\omega_c^{(0)} \approx (\alpha\mathscr{P}_{000})^{1/2}$.

In systems with $\alpha \sim 1$, only a small-radius drop with $\rho_0 \sim l$, for which $\lambda_0 \sim -1$, can pulsate.[3] In this case the function $\delta\theta_0^{(0)}$ is localized in the entire region of the drop,[3] therefore the pulsation of the small-radius drop constitutes an oscillation of its amplitude. Naturally, depending on the parameters α and ε, the onset of pulsations can be both soft and hard, and in the latter case the pulsations can be relaxational with a period of the order of τ_η and with a drop-amplitude growth time of the order of τ_θ.

These results pertain fully to a two-dimensional or one-dimensional IS in the form of a single spot of stratum. Such oscillating layers (current pinches) were observed in experiment,[26] and were also revealed by a numerical investigation of a model of one chemical reaction.[27] The observed stability of such pulsating single drops, spots, or strata is due to the fact that vibrating wall is bounded on both sides by stable regions.[3]

All the foregoing results can also be easily generalized to include the case of a traveling stratum (spot, drop), realized in a system of sufficiently large size.[3] In other words, it is possible to excite in the considered systems, at $\alpha \ll 1$ and $\varepsilon \ll 1$, a pulsating stratum (drop, spot) that travels without damping.

5.2. It is natural to assume that at parameters A that are close to A_c (or A_c') values corresponding to loss of stability of the homogeneous state (see Fig. 2), a drop (spot or stratum) pulsating with sufficiently large amplitude can excite spherically (cylindrically or one-dimensionally) diverging waves and undamped waves (traveling IS). The reason is that according to Refs. 3 and 4, traveling IS of large amplitude are produced in the systems considered at $\alpha \ll 1$ and $\varepsilon \ll 1$. In other words, a single pulsating drop (spot or stratum) can act as the leading center observed experimentally in a high-frequency gas discharge[15] and in the Belousov-Zhabotinskiĭ reaction.[8,11]

The fluctuation spectrum of a single drop contains several $\lambda_0^{(\beta)} < 0$ (Ref. 3) and, according to (43), the condition (41) may be satisfied not only with respect to spherically symmetrical fluctuations with $\beta = 0$. It follows therefore that the drop (spot) can execute also radially asymmetric oscillations. Similarly, from the form of the spectrum λ_n for the stratum[3] it can be concluded that a stratum can become unstable to inhomogeneous oscillations in the planes of the stratum walls. As a result, a stratum with sinuous walls can arise.

5.3. In the cold (hot) region there can exist besides a single stratum (spot or drop) several strata, spots (and particularly current pinches) or drops that lie far from one another.[9,3] When the distance between the strata $L_1 \gg l$, the critical funtions $\delta\theta_m$ with $\lambda_m < 0$, localized in the walls of the neighboring strata, do not overlap in practice.[1–3] Such strata (spots, drops) therefore become unstable to oscillations of their walls practically independently of one another. With decreasing L_1, the phases of the pulsations should become synchronized, because at $L_1 \lesssim L$ the quantity μ_m increases abruptly with increasing m.[1] As a result, the condition (41) can be satisfied only for $m = 0$, to which the fluctuation $\delta\theta_0$, which has no nodes, corresponds. This synchronization effect is due to the fact that diffusion processes cause the inhibitor to follow up more rapidly the shorter-wavelength fluctuations $\delta\theta_m$ with $m \neq 0$ (the time of change of the inhibitor is approximately τ_η/μ_m). These results pertain to a sequence of traveling IS.

5.4. In the systems considered there can exist complicated IS, including some in the form of strata that differ in amplitude and in dimension.[1,3] Since the condition (41) is satisfied for narrow strata also at not too small α, such complicated IS can constitute inhomogeneous states in some regions of which (with size of the order of l) local pulsations take place. Such pulsating regions can be far enough from one another, and can therefore oscillate independently of one another with frequencies that are different albeit close. At $\alpha \ll 1$ these pulsations recall either a breathing wall of a broad stratum, or a narrow wall that oscillates in amplitude.

5.5. According to Ref. 3, at $\varepsilon \ll 1$ stable two- and three-dimensional IS, including those having a complicated form,

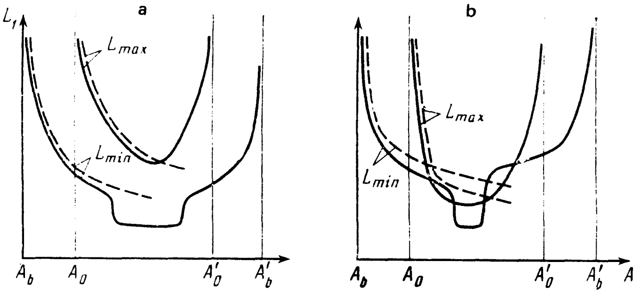

FIG. 3. Qualitative character of the dependences of the minimum $L_1 = L_{\min}$ and maximum $L_1 = L_{\max}$ period of stable strata on the bifurcation parameter A.

constitute stable extended cold and hot regions separated by transition "heterophase" layers with dimension of the order of l, in which is located a warmed "unstable" region of the system. In addition, a stable IS can contain warm regions that are close in form to the hot (cold) drop (spot) of small radius ($\sim l$), separated from one another and from the "heterophase" surface layer by distances much larger than l. Fluctuations $\delta\theta_m$ with $\lambda_m < 0$ are localized in such "unstable" heated regions (or surfaces) (Ref. 3). Therefore at $\alpha \ll 1$ these heated regions will pulsate because of the local frequency difference between the inhibitor η and the activator θ. Such an IS can be regarded as a set of local self-oscillators with different frequencies (which are identical for a periodic structure), located in surface layers separated by stable extended ($\gg l$) regions. At large amplitude of the oscillations, such local self-oscillators can serve as sources for autowave processes in stable regions of the IS, and also to the onset of irregular inhomogeneous oscillations in the system.

We note in conclusion that irregular inhomogeneous oscillations can arise also at $\alpha \gg 1$. In fact, according to Refs. 1 and 2, stable strata in one-dimensional systems with $l_x \gg L$ with a given period L_1 exist in one or two ranges of A, on the boundaries of each of which ($A_{\min}$ and $A_{\max}$) the IS vanishes, and the boundary values $A_{\min}$ and $A_{\max}$ depend on L_1 (Refs. 1 and 2). In other words, in a certain range A ($A_b < A < A_b'$, Fig. 3), at each given value of A, there exist strata from a certain minimum period $L_1 = L_{\min}$ to a certain maximum $L_1 = L_{\max}$. The functions $L_{\min}(A)$ and $L_{\max}(A)$, plotted in accordance with the dependence of the number of strata $N = l_x/L_1$ on A, which follow from Fig. 9 of Ref. 1 and Fig. 3 of Ref. 2, are shown in Fig. 3 by dashed and solid curves, respectively. Since $L_{\min}$ and $L_{\max}$ are generally speaking independent, two cases are possible: a) stable IS exist at all $A_b \leqslant A \leqslant A_b'$ (Fig. 3a); b) in regions of A where $L_{\max} < L_{\min}$, there are no stable IS (Fig. 3b) and turbulence is possible in the entire system. These conditions are apparently satisfied in a gas discharge for which the appearance of turbulence in the plasma is typical.[14]

[1]It was also assumed in the estimate that in the estimate that in the region of abrupt variation of θ (drop wall) we have $|\langle q_\eta' \rangle \mathrm{sh} \langle Q_\theta' \rangle \mathrm{sh}| \sim 1$. However, for a large-amplitude drop it is possible to have $|\langle q_\eta' \rangle \mathrm{sh} \langle Q_\theta' \rangle| \mathrm{sh} \gg 1$. As a result ω_c can exceed substantially the value that follows from the estimate in (22).

[2]The appearance of a pulsating current pinch when condition (21) is satisfied was established earlier[18] in an analysis of the pinching of current in semiconducting structures.

[1]B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz. **74**, 1675 (1978) [Sov. Phys. JETP **47**, 874 (1978)].
[2]B. S. Kerner and V. V. Osipov, Fiz. Tekh. Poluprov. **13**, 721 (1979) [Sov. Phys. Semicond. **13**, 424 (1979)].
[3]B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz. **79**, 2218 (1980) [Sov. Phys. JETP **52**, 1122 (1980)].
[4]B. S. Kerner and V. V. Osipov, Microelektronika **10**, 497 (1981).
[5]I. E. Dzyaloshinskiĭ, Zh. Eksp. Teor. Fiz. **46**, 1420 (1964); **47**, 336 (1964) [Sov. Phys. JETP **19**, 960 (1964), **20**, 223 (1965)].
[6]A. P. Levanyuk and D. G. Sannikov, Fiz. Tverd. Tela (Leningrad) **18**, 423 (1976) [Sov. Phys. Solid State **18**, 245 (1976)].
[7]A. G. Khachaturyan, Teoriya fazovykh prevrashcheniĭ i struktura tverdykh rastvorov (Theory of Phase Transitions and Structure of Solid Solutions), Nauka, 1974.
[8]A. M. Zhabotinskiĭ, Kontsentratsionnye kolebaniya (Density Oscillations), Nauka, 1974.
[9]A. Scott, Active and Nonlinear Wave Propagation in Electronics, Wiley, 1970.
[10]G. Nicolis and I. Prigogine, Self-Organization in Nonequilibrium Systems, Wiley, 1977.
[11]V. A. Vasil'ev, Yu. M. Romanovskiĭ, and V. G. Yakhno, Usp. Fiz. Nauk. **128**, 625 (1979) [Sov. Phys. Usp. **22**, 615 (1979)].
[12]V. L. Bonch-Bruevich, I. P. Zvyagin, and A. G. Mironov, Domain Electrical Instabilities in Semiconductors. Consultants Bureau, 1975.
[13]Yu. M. Romanovskiĭ, N. V. Stepanova, and D. S. Chernavskiĭ, Matematicheskoe modelirovanie v biofizike (Mathematical Modeing in Biophysics), Nauka, 1975.
[14]A. V. Nedospasov and V. D. Khait, Kolebaniya i neustoichivosti nizkotemperaturnoĭ plazmy (Oscillations and Instabilities of Low-Temperature Plasma), Nauka, 1979.
[15]A. A. Zaĭtsev and Kh. A. Dzherpetov, Zh. Eksp. Teor. Fiz. **24**, 516 (1953).
[16]B. S. Kerner and V. V. Osipov, Dokl. Akad. Nauk SSSR **257**, 1352 (1981) [Sov. Phys. Dokl. **26**, 420 (1981)].
[17]A. V. Volkov and Sh. M. Kogan, Usp. Fiz. Nauk **96**, 633 (1968) [Sov. Phys. Usp. **11**, (1969)].
[18]B. S. Kerner and V. V. Osipov, Mikroelektronika **6**, 337 (1977).
[19]B. S. Kerner and V. V. Osipov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 122 (1973) [JETP Lett. **18**, 70 (1973)].
[20]V. V. Osipov, Author's abstract of doctoral dissertation, Leningrad Polytech. Inst. 1975.
[21]V. V. Osipov, and V. A. Kholodnov, Mikroelektronika **2**, 529 (1973).
[22]A. V. Volkov and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **52**, 1647 (1967) [Sov. Phys. JETP **25**, 1095 (1967)].
[23]Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **54**, 1228 (1968) [Sov. Phys. JETP **27**, 656 (1968)].
[24]L. D. Landau and E. M. Lifshitz. Quantum Mechanics. Nonrelativistic Theory, Pergamon, 1978.
[25]L. É. Gurevich and T. M. Gasymov, Fiz. Tverd. Tela (Leningrad) **9**, 106 (1967) [Sov. Phys. Solid State **9**, 78 (1967)]. T. M. Gasymov and L. É. Gurevich, Fiz. Tverd. Tela (Leningrad) **11**, 2946 (1969) [Sov. Phys. Solid State **11**, 2386 (1970)].
[26]H. A. Schafft, Proc. IEEE **55**, 1272 (1967).
[27]S. Koga and Y. Kuramoto, Progr. Theor. Phys. **63**, 106 (1980).

Translated by J. G. Adashko