Scattering of electrons by atoms and photodetachment of negative ions in a homogeneous electric field

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Scattering of electrons by a force center in a homogeneous electric field is considered. The oscillatory energy dependence of the cross section, due to interference effects accompanying scattering, is investigated. The feasibility of experimental observation of the effects is analyzed. A multichannel theory of electron scattering by atoms and a multichannel theory of photodetachment of negative ions in an electric field are developed. It is shown that the phase shifts of electron scattering by atoms can be determined with high resolution by measuring the cross sections for the photodetachment of the corresponding negative ions in an electric field.

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1. INTRODUCTION

Interference effects that occur in photodecay of atoms and negative ions in a uniform electric field were considered earlier in Ref. 1. The same effect should appear in scattering of electrons by atoms. As will be seen below, the organization of an experiment on scattering in an electric field, aimed at observing interference effects, is a rather complicated problem. A theoretical analysis of the scattering problem, however, is of methodological interest. In addition, it is related to the problem of photodetachment of negative ions, and experiments on photodetachment in an electric field are already being performed.[2] The present paper considers therefore also the multichannel theory of photodetachment with allowance for the interaction of the electron with the atoms in the final state.

2. SCATTERING BY A FORCE CENTER

We consider an electron in a field $U(\mathbf{r})$-Fz (the force $\mathbf{F}$ exerted on an electron by a uniform field and directed along the z axis). The potential $U(\mathbf{r})$ is assumed to be short-range in the sense that its radius r_c satisfies the condition

$$Fr_c \ll E, \tag{1}$$

where $E = k^2/2$ is the electron energy (here the below we use the atomic system of units).

The wave function of the electron satisfies the integral equation

$$\chi_{\mathbf{k}}^{+}(\mathbf{r}) = \chi_{\mathbf{k}}^{(0)}(\mathbf{r}) - \int G^{+}(\mathbf{r},\mathbf{r}')U(\mathbf{r}')\chi_{\mathbf{k}}^{+}(\mathbf{r}')d\mathbf{r}', \tag{2}$$

where G^+ is the Green's function of the electron in the electric field.

Let the projection of the current density of the initial beam on the z axis be negative (directed opposite to $\mathbf{F}$). The wave $\chi_{\mathbf{k}}^{(0)}$ consists then of two components, incident and reflected from the barrier produced by the electric field. We express $\chi_{\mathbf{k}}^{(0)}$ in the form

$$\chi_{\mathbf{k}}^{(0)} = 2^{5/6}|k_z|^{1/2}F^{-1/6}\exp[i(k_x x + k_y y)]\Phi[-(z + k_z^2/2F)(2F)^{1/3}], \tag{3}$$

where Φ is the Airy function.

The densities of the incident and reflected beams are equal to k_z as $z \to \infty$. If, however, $z \ll k_z^2/F$, the current density $j = k$. The determination of the effective scattering cross section depends therefore on the chosen value of j. We shall assume hereafter $j = k$. The expressions obtained for the scattering cross sections in the field go over then into the usual cross sections as $F \to 0$.

The Green's function in the quasiclassical approximation was written out in Ref. 1, where a factor $(2s)^{-1/2}l^{-1}$ was left out in the corresponding formula (19). The Green's function is of the form

$$G^{+}(\mathbf{r},\mathbf{r}') = \frac{F}{2^{1/2}\pi k^2[s(\mathbf{r},\mathbf{r}')]^{1/2}}\sum_{n=0}^{1}\left[(-1)^n\left(1 + \frac{F}{k^2}(z + z')\right) - s(\mathbf{r},\mathbf{r}')\right]^{-1/2}\exp[iS_{\tau_n}(\mathbf{r},\mathbf{r}')], \tag{4}$$

where

$$S_\tau(\mathbf{r},\mathbf{r}') = (\mathbf{r}-\mathbf{r}')^2/2\tau + {}^1/_2F\tau(z+z') - {}^1/_{24}F^2\tau^3 + {}^1/_2k^2\tau,$$

$$\tau_n^2 = (2k^2/F^2)[1 + (F/k^2)(z+z') - (-1)^n s(\mathbf{r},\mathbf{r}')],$$

$$[s(\mathbf{r},\mathbf{r}')]^2 = (1 + 2zF/k^2)(1 + 2z'F/k^2) - (F^2/k^4)[(x-x')^2 + (y-y')^2].$$

Equation (2) with the function (4) can be solved for all $\mathbf{r}$ only when U is a zero-radius potential. However, if we are not interested in the angular distribution of the electrons and need determine the total cross section, it suffices to consider the solution at distances $r \gtrsim r_c$. Then Eq. (2) is solved under the condition

$$\alpha_{\mathbf{k}} = (\beta/2)\cos^3\theta_{\mathbf{k}} + \pi/4 \gg 1, \tag{5}$$

$$\theta_{\mathbf{k}} = (\widehat{\mathbf{k},\mathbf{F}}), \quad \beta = {}^2/_3k^3/F. \tag{6}$$

The function $\chi_{\mathbf{k}}^{(0)}$ takes in this case the form

$$\chi_{\mathbf{k}}^{(0)} = 2\exp[i(k_x x + k_y y)]\sin(k_z z + \alpha_{\mathbf{k}}). \tag{7}$$

We consider next the asymptotic form of the Green's function (4) under the condition (1). The two terms in (4) correspond to the contributions of two trajectories in the motion from the point $\mathbf{r}'$ to $\mathbf{r}$. At large z, interference of all the trajectories that start out from the point $\mathbf{r}'$ take place in the xy plane (Fig. 1a). For $r \ll k^2/F$, on a sphere of radius r, the only trajectories that interfere are those which are directed

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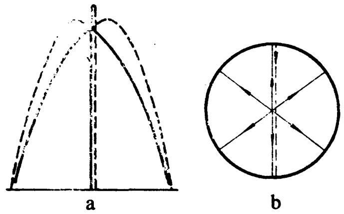

FIG. 1. Trajectories of electrons in a uniform electric field. The trajectories shown by the solid lines correspond to the first term in the Green's function, and those by the dashed lines to the second term. The case a corresponds to the function (4) and the case b to (8).

with and against the field (Fig. 1b). In this case

$$G^+(\mathbf{r},\mathbf{r}')=\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{2\pi|\mathbf{r}-\mathbf{r}'|}+\frac{F}{4\pi ik^2}\exp\{i[\beta+k(z+z')]\}. \quad (8)$$

The second term in (8) is a correction that yields the interference in the scattered wave. A correction of the same order, but having no effect on the interference, was left out.

The solution of Eq. (2) with allowance for (7) and (8) is of the form

$$\chi_{\mathbf{k}}^+=-i[e^{i\alpha_{\mathbf{k}}}\psi_{\mathbf{k}}^+-e^{-i\alpha_{\mathbf{k}}}\psi_{\mathbf{k}'}^+-a_{\mathbf{k}}\psi_{\mathbf{k}^0}^+], \quad (9)$$

where $\psi_{\mathbf{k}}^+$ is the solution of Eq. (2) at $F=0$, $\mathbf{k}'$ is a vector with components (k_x, k_y-k_z), and $\mathbf{k}^0$ is a vector with components $(0, 0, k)$.

Using the integral representation of the scattering amplitude, we obtain

$$a_{\mathbf{k}}=[e^{i\alpha_{\mathbf{k}}}f_{-\mathbf{k}\mathbf{k}^0}-e^{-i\alpha_{\mathbf{k}}}f_{-\mathbf{k}'\mathbf{k}^0}][(2k^2/iF)e^{-i\beta}+f_{-\mathbf{k}^0\mathbf{k}^0}]^{-1}, \quad (10)$$

where $f_{\mathbf{k}_2\mathbf{k}_1}$ is the amplitude of the scattering by the potential $U(\mathbf{r})$ for an initial momentum $\mathbf{k}_1$ and a final momentum $\mathbf{k}_2$.

At $r\gtrsim r_c$ and $Fr\ll k_z^2$ the function $\chi_{\mathbf{k}}^+$ takes the form

$$\chi_{\mathbf{k}}^+(\mathbf{r})\sim-i[e^{i\alpha_{\mathbf{k}}}(e^{i\mathbf{k}\mathbf{r}}+f_{\mathbf{k}_f\mathbf{k}}e^{ikr}/r)-e^{-i\alpha_{\mathbf{k}}}(e^{i\mathbf{k}'\mathbf{r}}+f_{\mathbf{k}_f\mathbf{k}'}e^{ikr}/r)-a_{\mathbf{k}}(e^{i\mathbf{k}^0\mathbf{r}}+f_{\mathbf{k}_f\mathbf{k}^0}e^{ikr}/r)], \quad (11)$$

where $\mathbf{k}_f=k\mathbf{r}/r$.

We consider now interference effects of order not higher than the first in F. We separate first in (11) the coefficient of a spherically diverging wave:

$$A_{\mathbf{k}_f\mathbf{k}}=e^{i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}}-e^{-i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}'}-a_{\mathbf{k}}f_{\mathbf{k}_f\mathbf{k}^0}. \quad (12)$$

The total scattering cross section is

$$\sigma(\mathbf{k})=\sigma_0(\mathbf{k})+\sigma_0(\mathbf{k}')+\sigma_1(\mathbf{k})+\sigma_2(\mathbf{k}), \quad (13)$$

where $\sigma_0(\mathbf{k})$ is the total scattering cross section in the absence of an electric field,

$$\sigma_1=-2\operatorname{Re}\left[e^{2i\alpha_{\mathbf{k}}}\int f_{\mathbf{k}_f\mathbf{k}}f^*_{\mathbf{k}_f\mathbf{k}'}\,d\Omega_{\mathbf{k}_f}\right], \quad (14)$$

$$\sigma_2=-2\operatorname{Re}\int(e^{i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}_i}-e^{-i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}'})\,a_{\mathbf{k}}^*f^*_{\mathbf{k}_f\mathbf{k}}d\Omega_{\mathbf{k}_f}. \quad (15)$$

At any arbitrarily small F, the particles are knocked out of the incident stream twice, before and after reflection from the barrier produced by the electric field. This is manifest by the presence of two terms in (13), which do not depend on F. If the potential U is central, their sum reduces to $2\sigma_0(\mathbf{k})$. The principal contribution to the interference is made by the term σ_1. Applying the unitarity condition, we obtain from (14)

$$\sigma_1=-(4\pi/k)\operatorname{Im}[e^{2i\alpha_{\mathbf{k}}}(f_{\mathbf{k}'\mathbf{k}}-f_{\mathbf{k}\mathbf{k}'}^*)], \quad (16)$$

and in the case of a central potential

$$\sigma_1=(8\pi/k)\sin(\beta\cos^3\theta_{\mathbf{k}})\operatorname{Im}f(2\theta_{\mathbf{k}}-\pi), \quad \pi/2<\theta_{\mathbf{k}}\leqslant\pi, \quad (17)$$

where $f(\varphi)$ is scattering amplitude as a function of the angle φ.

We consider next the interference between the scattered wave χ_1 and the wave χ_2 reflected after scattering. According to (11)

$$\chi_1=(e^{i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}}-e^{-i\alpha_{\mathbf{k}}}f_{\mathbf{k}_f\mathbf{k}'})e^{ikr}/r, \quad \chi_2=-a_{\mathbf{k}}e^{i\mathbf{k}^0\mathbf{r}}.$$

The corresponding current consists of two parts:

$$I_1=r^2\operatorname{Im}\int\chi_1^*\frac{\partial\chi_2}{\partial r}d\Omega_{\mathbf{k}_f}, \quad I_2=r^2\operatorname{Im}\int\chi_2^*\frac{\partial\chi_1}{\partial r}d\Omega_{\mathbf{k}_f}.$$

Calculating the integrals under the assumption $kr\gg1$, we obtain the following correction for the cross section:

$$\sigma_3=\frac{1}{k}(I_1+I_2)$$
$$=-\frac{2\pi F}{k^3}\operatorname{Re}\{e^{i\beta}(f^*_{\mathbf{k}^0\mathbf{k}'}-f^*_{\mathbf{k}^0\mathbf{k}}e^{-2i\alpha_{\mathbf{k}}})(f_{-\mathbf{k}'\mathbf{k}^0}-f_{-\mathbf{k}\mathbf{k}^0}e^{2i\alpha_{\mathbf{k}}})\}. \quad (18)$$

Using (15) and the unitarity condition, we have for the first-order correction in F

$$\sigma_2+\sigma_3=-(2\pi F/k^3)\operatorname{Re}\{(e^{i\beta-2i\alpha_{\mathbf{k}}}f_{\mathbf{k}\mathbf{k}^0}-f_{\mathbf{k}'\mathbf{k}^0})(e^{2i\alpha_{\mathbf{k}}}f_{-\mathbf{k}\mathbf{k}^0}-f_{-\mathbf{k}'\mathbf{k}^0})\}.$$

The use of nonstationary perturbation-theory formula

$$\sigma(\mathbf{k})=\frac{1}{4\pi^2}\int_0^{2\pi}d\varphi_{\mathbf{k}_f}\int_0^{\pi/2}d\theta_{\mathbf{k}_f}\sin\theta_{\mathbf{k}_f}\left|\int\chi_{\mathbf{k}_f}^{(0)*}(\mathbf{r})U(\mathbf{r})\chi_{\mathbf{k}_f}^{(0)}(\mathbf{r})d\mathbf{r}\right|^2,$$

makes it possible to obtain the correction σ_1 in the form (14) and the correction σ_3 (18), in which $f_{\mathbf{k}_f\mathbf{k}}$ is replaced by the corresponding amplitudes in the Born approximation.

3. SCATTERING BY A ZERO-RADIUS POTENTIAL

If $U(\mathbf{r})$ is a zero-radius potential, the solution of (2) is of the form (see Ref. 3)

$$\chi_{\mathbf{k}}^+(\mathbf{r})=\chi_{\mathbf{k}}^{(0)}(\mathbf{r})+\chi_{\mathbf{k}}^{sc}(\mathbf{r}),$$
$$\chi_{\mathbf{k}}^{sc}(\mathbf{r})=-2\pi aG^+(\mathbf{r},0)\left[\frac{\partial(r'\chi_{\mathbf{k}}^{(0)}(\mathbf{r}'))/\partial r'}{1+2\pi a\partial(r'G^+(\mathbf{r}',0))/\partial r'}\right]_{r'=0}, \quad (19)$$

where a is the scattering.

The total scattering cross section is calculated in the same manner as above, and is equal to

$$\sigma=16\pi\frac{\sin^2(^1/_2\beta\cos^3\theta_{\mathbf{k}}+\pi/4)}{|a^{-1}+ik-ike^{i\beta}/3\beta|^2}\left(1-\frac{\cos\beta}{3\beta}\right). \quad (20)$$

On the other hand, it is necessary to find the angular distribution at macroscopic distances from the force center, it is necessary to introduce a plane that is perpendicular to the z axis and on which the electron distribution is given by the differential cross section

$$d^2\sigma(\mathbf{r})=k_z^{-1}\operatorname{Im}\left[\chi_{\mathbf{k}}^{sc}(\mathbf{r})\frac{\partial}{\partial z}\chi_{\mathbf{k}}^{sc}(\mathbf{r})\right]\rho\,d\rho\,d\varphi. \quad (21)$$

Substituting here (4) and (19) under the conditions $z\gg k_z^2/2F$ and $z\gg(2F)^{-1/3}$, we have

$$\frac{1}{\rho}\frac{d^2\sigma(\mathbf{r})}{d\rho\,d\varphi}=\frac{\sin^2(^1/_2\beta\cos^3\theta_k+\pi/4)}{|a^{-1}+ik-ike^{i\beta}/3\beta|^2}\frac{4\pi}{k_zk^3s(\mathbf{r},0)}$$
$$\times\left\{\sum_{n=0}^{1}\frac{1}{\tau_n^2(\mathbf{r},0)}\frac{\partial}{\partial z}S_{\tau_n}(\mathbf{r},0)+\frac{1}{\tau_0(\mathbf{r},0)\tau_1(\mathbf{r},0)}\left[\frac{\partial}{\partial z}S_{\tau_0}(\mathbf{r},0)+\frac{\partial}{\partial z}S_{\tau_1}(\mathbf{r},0)\right]\times\sin[S_{\tau_1}(\mathbf{r},0)-S_{\tau_0}(\mathbf{r},0)]\right\}.$$

The angular distribution takes the same form as in the case of photodetachment.[1,4] The first two terms in the curly brackets give the classical angular distribution, and the next term gives the interference part.

In the case of a zero-radius potential it is also possible to calculate the cross section for scattering in the classically energy-forbidden region. The conditions (1) and (5) are then replaced by

$$Fr_c\ll|k^2|,\quad \beta_z=2\varkappa_z^3/3F\gg1,\quad \varkappa_z^2=-k_z^2.$$

After determining the asymptotic form of the Airy function at $k_z^{(2)}<0$ we obtain in the case $k^2>0$ and $k_z^2<0$

$$\sigma=\frac{4\pi k}{\varkappa_z}\frac{e^{-\beta_z}}{|a^{-1}+ik-ike^{i\beta}/3\beta|^2}\left(1-\frac{\cos\beta}{\beta}\right).$$

If $k^2=-\varkappa^2<0$, the Green's function can be calculated by contour integration, as proposed by Dalidchik and Slonim.[5] Under the condition $Fr\ll\varkappa^2$ we have

$$G(\mathbf{r},0)=\frac{e^{-\varkappa r}}{2\pi r}+\frac{iF}{8\pi\varkappa^2}\exp\left\{-\beta'+\varkappa z\left[1+O\left(\frac{Fr}{\varkappa^2}\right)\right]\right\},\quad(22)$$

where $\beta'=2\varkappa^3/3F$. It follows from (22) that

$$\frac{\partial}{\partial r}[rG(\mathbf{r},0)]_{r=0}=\frac{1}{(2\pi)}\left(-\varkappa+\frac{iF}{4\varkappa^2}e^{-\beta'}\right).$$

Taking into account the next term of the expansion $\partial[r\,\mathrm{Re}\,G(\mathbf{r},0)]/\partial r|_{r=0}$ in F^2, we have

$$\sigma=\frac{\pi F}{\varkappa_z\varkappa^2}\frac{\exp[-(\beta_z+\beta')]}{(a^{-1}-\varkappa-F^2/8\varkappa^5)^2+F^2e^{-2\beta'}/16\varkappa^4}.\quad(23)$$

At $\varkappa$ close to a^{-1}, Eq. (23) describes scattering by a quasidiscrete level; this scattering was investigated by Demkov and Drukarev.[6] At $\varkappa=\varkappa_z$ the resonant value of the cross section is

$$\sigma_r=16\pi\varkappa/F.$$

4. POSSIBILITY OF OBSERVING INTERFERENCE EFFECTS IN SCATTERING

To be able to observe interference effects in experiments with beams, the following condition must be satisfied

$$\beta\Delta E/E\ll1,\quad(24)$$

where ΔE is the energy spread of the particles in the beam. In the opposite case, averaging over the energy distribution "smears out" the effect. The condition (24) imposes the stringent requirement that the electron beam be monokinetic and that the collision region be strongly localized. The situation is aggravated by the fact that when the initial electron is directed opposite to the vector **F**, a kinetic energy of the order X must be imparted to the beam, where X is the distance, which is macroscopic from the beam-formation region to the collision region. Therefore both the initial energy and the monokineticity must be very high; this is not attainable with present-day experimental facilities. In the case of photodetachment by a laser beam, these difficulties disappear—the electric field does not act on the photons and they are monochromatized to a high degree.

There exists, however, another possibility of observing interference, namely, via elastic collision of the electrons with the atoms. In this case the electrons can be launched along the vector **F** and the parameters of the setup can be chosen such that the bulk of the energy accumulated during the motion of the electrons from the source to the collision region is consumed in excitation of the atom. Inelastic scattering will be investigated below.

We shall examine also whether interference effects can appear in scattering indirectly, for example, in investigations of the parameters of a weakly ionized gas. Since the interference length if equal to E/F, in the presence of only elastic collisions it is necessary to have

$$\bar{E}/F<l,\quad(25)$$

where l is the mean free path and $\bar{E}$ the average energy of the electron in the gas.

Using for the electrons in the gas the distribution function obtained in the diffusion approximation,[7] we can easily show that the condition (25) is not satisfied at any value of F. Indeed, if the field is weak compared with T/l (T is the temperature of the gas atoms), it cannot "turn around" the electron within the mean free path, and in the case of a strong field the average energy of the electron is too high to be able to turn it around when it moves against F.

Just as in the analysis of experiments with beams, the situation turns out to be more favorable when account is taken of the excitation of the atoms. Consider a weakly ionized gas in which the predominant role is played by excitation of a certain level, and let the average electron energy be close to the excitation potential. The energy of the bulk of the electrons after the excitation is then low enough for the interference length to be shorter than the mean free path. Since the cross section for elastic scattering by atoms usually exceeds the excitation cross section, such a situation is difficult to realize in a monatomic gas. It can occur, however, if in place of atoms one chooses polar molecules that have large cross section for rotational excitation with change of the rotational quantum number by unity. In addition, it is also possible to consider the distribution function of the electrons in a solid, when the most probable scattering process is phonon excitation. This problem was investigated by Dmitriev and Tsendin.[8] A quantitative analysis of interference effects in kinetics problems, however, is made difficult by the need for solving a transport equation with a complicated dependence [similar to (17)] of the cross section on the energy. Furthermore, a region exists in which there is no known analytic form for the cross section, owing to the condition (1) and (5).

5. MULTICHANNEL SCATTERING THEORY

We write down in place of Eq. (2) a system of strong-coupling equations that take into account the coupling of N scattering channels

$$\chi^{+}_{\mathbf{k}_i i'i}(\mathbf{r})=\delta_{i'i}\chi^{(0)+}_{\mathbf{k}_i i}(\mathbf{r})-\sum_j\int G^{+}_{i'}(\mathbf{r},\mathbf{r}')U_{i'j}(\mathbf{r}')\chi^{+}_{\mathbf{k}_i ji}(\mathbf{r}')d\mathbf{r}', \quad (26)$$

where $\chi^{+}_{\mathbf{k}_i} i'i$ is the function of the channel i' when an incident wave with momentum $\mathbf{k}_i$ is present in the i-th channel.

We separate next the group A of degenerate (or almost degenerate) channels with low energy, in which interference effects are significant. It follows from Sec. 2 that is it necessary to include in group A channels with energies lower than or slightly higher than $F^{2/3}$. In the case of really attainable fields, these may be channels whose energies differ only on account of the Stark splitting of the atomic levels, or else on account of the spin-orbit interaction.

Using the Green's-function representation (8), we obtain the solution of the system (26) in the form

$$\chi^{+}_{\mathbf{k}_i i'i}(\mathbf{r})=\psi^{+}_{\mathbf{k}_i i'i}(\mathbf{r})+i\sum_j\psi^{+}_{\mathbf{k}^0_j i'j}(\mathbf{r})a_{ji}(\mathbf{k}_i). \quad (27)$$

The summation in (27) is only over the group A, and we assume that the initial level i does not belong to A. Otherwise, we would have in place of $\psi^{\pm}_{\mathbf{k}_i i'i}$ just as in (9),

$$-i[\exp(i\alpha_{\mathbf{k}_i})\psi^{+}_{\mathbf{k}_i i'i}-\exp(-i\alpha_{\mathbf{k}_i})\psi^{+}_{\mathbf{k}^{*}_i i'i}].$$

Assuming that the change of the atomic wave function under the influence of the field can be neglected, and using the integral rule representation of the scattering amplitude in the absence of a field, we obtain for the coefficient $a_{ji}(\mathbf{k}_i)$ a system of linear equations

$$a_{i'i}(\mathbf{k}_i)=-\frac{F}{2k^2_{i'}}e^{i\beta_{i'}}\Big[f_{i'i}(-\mathbf{k}^0_{i'},\mathbf{k}_i)+i\sum_j a_{ji}(\mathbf{k}_i)f_{i'j}(-\mathbf{k}^0_{i'},\mathbf{k}^0_j)\Big],$$

where $f_{i'i}(\mathbf{k}_{i'},\mathbf{k}_i)$ is the amplitude of the transition from the state i into i'.

In the first-order approximation in F we have

$$a_{i'i}(\mathbf{k}_i)=-\frac{F}{2k^2_{i'}}e^{i\beta_{i'}}f_{i'i}(-\mathbf{k}^0_{i'},\mathbf{k}_i). \quad (28)$$

To calculate the total cross section it is necessary to determine the current corresponding to the solution (27) at distances $r>r_c$, but such that $Fr\ll k^2_i$. Then, if the final state i' does not belong to the group A, we can introduce a new scattering amplitude

$$A_{i'i}(\mathbf{k}_{i'},\mathbf{k}_i)=f_{i'i}(\mathbf{k}_{i'},\mathbf{k}_i)-\frac{iF}{2}\sum_{j\in A}\frac{e^{i\beta_j}}{k^2_j}f_{i'j}(\mathbf{k}_{i'},\mathbf{k}^0_j)f_{ji}(-\mathbf{k}^0_j,\mathbf{k}_i), \quad (29)$$

so that the cross section is

$$\sigma_{i'i}(\mathbf{k}_i)=\frac{k_{i'}}{k_i}\int|A_{i'i}(\mathbf{k}_{i'},\mathbf{k}_i)|^2 d\Omega_{\mathbf{k}_{i'}}.$$

The sum in (29) has a simple physical meaning. It describes a transition of the atom from a state i into $j\in A$ and scattering of the electron in the direction opposite to $\mathbf{F}$ followed by reflection of the electron from the potential barrier and the transition of the atom into the state i', the summation being taken over all $j\in A$.

If $i'\in A$, it is necessary also to take into account in the asymptotic form of the solution the term $i\exp(ik_{i'}\mathbf{r})a_{i'i}(\mathbf{k}_i)$ that interfers with $A_{i'i}(\mathbf{k}_{i'},\mathbf{k}_i)\exp(ik_{i'}r)/r$. Allowance for this interference in first order in F leads to the contribution

$$\sigma'_{i'i}(\mathbf{k}_i)=-\frac{2\pi F}{k_i k^2_{i'}}\mathrm{Re}\{\exp(i\beta_{i'})f_{i'i}(-\mathbf{k}_{i'}{}^0,\mathbf{k}_i)[f_{i'i}(\mathbf{k}_{i'}{}^0,\mathbf{k}_i)]^*\}. \quad (30)$$

Thus, near each threshold with number j all the cross sections are oscillating and have a phase β_j that is modified by the presence of a factor containing amplitudes of different transitions.

It is interesting that the amplitude of a certain transition in an electric field contains information on amplitudes of other transitions in the absence of a field. This circumstance obtains also in photodetachment of ions.

6. MULTICHANNEL THEORY OF PHOTODETACHMENT

The photoionization of a hydrogenlike atom in a uniform electric field is the subject of a large number of theoretical papers (see, e.g., Refs. 9–11). We consider here a somewhat different problem, namely the photodetachment of a negative ion. In contrast to our earlier paper,[1] we take into account the interaction of the electron with the atom in the final state, and the multichannel character of the problem.

The problem reduces to solution of the system (26), in which $G_{i'}{}^{+}$ is replaced by $G_{i'}{}^{-}=(G_{i'}{}^{+})^*$, followed by determining the matrix elements of the dipole moment with the functions $\chi^{-}_{\mathbf{k}_i i'i}$, where the index i labels the final state.

Solving the system in the same manner as in Sec. 4 and using the relation

$$f_{ji}(-\mathbf{k}_j,-\mathbf{k}_i)=f_{ij}(\mathbf{k}_i,\mathbf{k}_j),$$

we obtain

$$\chi^{-}_{\mathbf{k}_i i'i}=\varphi_{\mathbf{k}_i i'i}-i\sum_j\psi_{-\mathbf{k}^0_j i'j}b_{ji}(\mathbf{k}_i), \quad (31)$$

where

$$\varphi_{\mathbf{k}_i i'i}=\psi^{-}_{\mathbf{k}_i i'i},\qquad b_{ji}(\mathbf{k}_i)=-(F/2k_j^2)e^{-i\beta_j}[f_{ij}(\mathbf{k}_i,\mathbf{k}_j{}^0)]^*, \quad (32)$$

if $i\notin A$ and

$$\varphi_{\mathbf{k}_i i'i}=-i[\exp(i\alpha_{\mathbf{k}_i})\psi^{-}_{\mathbf{k}_i i'i}-\exp(-i\alpha_{\mathbf{k}_i})\psi^{-}_{\mathbf{k}^{*}_i i'i}], \quad (33)$$

$$b_{ji}(\mathbf{k}_i)=(iF/2k_j^2)e^{-i\beta_j}\{\exp(i\alpha_{\mathbf{k}_i})[f_{ij}(\mathbf{k}_i,\mathbf{k}_j{}^0)]^* -\exp(-i\alpha_{\mathbf{k}_i})[f_{ij}(\mathbf{k}_i,-\mathbf{k}_j{}^0)]^*\}, \quad (34)$$

if $i\in A$.

We confine ourselves for simplicity hereafter to photodetachment of an ion consisting of a core with charge $+1$ and two valence electrons in the singlet state (for example, H^- or alkali-element ions). By virtue of the symmetry of the coordinate parts of the functions of the initial and final states, it suffices to consider the matrix element

$$X_{\mathbf{k}_i i}=\int[\Psi^{-}_{\mathbf{k}_i i}(\mathbf{r},\mathbf{r}_a)]^*\mathbf{e}_{ph}(\mathbf{r}+\mathbf{r}_a)\Psi_0(\mathbf{r},\mathbf{r}_a)d\mathbf{r}\,d\mathbf{r}_a, \quad (35)$$

where e_{ph} is the polarization vector of a linearly polarized photon, Ψ_0 is the wave function of the initial negative ion, and $\Psi^{-}_{\mathbf{k}_i i}$ is a nonsymmetrized final-state function:

$$\Psi^{-}_{\mathbf{k}_i\, i}(\mathbf{r},\mathbf{r}_a)=\sum_{i'}\chi^{-}_{i' i}(\mathbf{r})\,\Phi_{i'}(\mathbf{r}_a),$$

where $\Phi_{i'}$ are the atomic wave functions.

We assume that the function Ψ_0 and $\Phi_{i'}$ are insignificantly altered by the electric field. Then, if $i\notin A$ (i.e., we are considering the cross section for photodetachment with formation of an atom in the state i near the threshold of the formation of an atom in a higher state), we obtain from (31), (32), and (35)

$$X_{\mathbf{k}_i\, i}=M_{\mathbf{k}_i\, i}+\frac{iF}{2}\sum_{j\in A}\frac{e^{-i\beta_j}}{k_j^2}[f_{ij}(\mathbf{k}_i,\mathbf{k}_j^0)]^{*}M_{-\mathbf{k}_j^0 j}, \qquad (36)$$

where $M_{\mathbf{k}_i i}$ is the photodetachment matrix element in the absence of a field.

The second term in (36) describes photodetachment with transition of the atom in the state $i\in A$ and of the electron in the state of motion against the vector **F**, followed by reflection of the electron from the potential barrier and transition of the atom into the state i.

Thus, the photodetachment cross section oscillates near each threshold of atom formation in a certain excited state.

If $i\in A$ (i.e., we consider the cross section for photodetachment with formation of an atom in the state i near the threshold of this very same process), then

$$X_{\mathbf{k}_i\, i}=-i[\exp(i\alpha_{\mathbf{k}_i})M_{\mathbf{k}_i\, i}-\exp(-i\alpha_{\mathbf{k}_i})M_{\mathbf{k}'_i\, i}]$$
$$+\frac{F}{2}\sum_{j\in A}\frac{e^{-i\beta_j}}{k_j^2}\{\exp(i\alpha_{\mathbf{k}_j})[f_{ij}(\mathbf{k}_i,\mathbf{k}_j^0)]^{*}$$
$$-\exp(-i\alpha_{\mathbf{k}_j})[f_{ij}(\mathbf{k}_{i'},\mathbf{k}_j^0)]^{*}\}M_{-\mathbf{k}_j^0 j}. \qquad (37)$$

We consider now in greater detail the photodetachment of an electron from the s^2 subshell of an alkali-atom ion. For each state i of the atom we separate the principal quantum number n, the orbital angular momentum l, and its projection m_1. Writing out the strong-coupling expansion in the total-angular-momentum representation[12], we obtain

$$M_{\mathbf{k}_n n l_1 m_1}=\sum_{l_2}C^{1M_L}_{l_1m_1l_2m_2}K_{nl_1l_2}Y^{*}_{l_2m_2}(\Omega_{\mathbf{k}_n}), \qquad (38)$$

where K is a certain combination of the radial matrix elements, the explicit form of which is of no importance to us, C are Clebsch-Gordan coefficients, and M_L is the projection of the total angular momentum on the vector **F** and is equal to zero in the case of π polarization of the photon (parallel to **F**) and to ± 1 for σ polarization (perpendicular to **F**).

We express the scattering amplitude in the form

$$f_{n'l_1'm_1'nl_1m_1}(\mathbf{k}_{n'},\mathbf{k}_n^0)$$
$$=-\frac{1}{2ik_n}\sum_{l_2l_2'}[4\pi(2l_2+1)]^{1/2}i^{l-l'}T_{l_1'm_1'l_2'm_2'l_1m_1l_20}Y_{l_2'm_2'}(\Omega_{\mathbf{k}_{n'}}), \qquad (39)$$

where $T=1-S$ is the transition matrix and S is the scattering matrix. The summation in (39) is over the angular momenta of the scattered electron in the initial and final states.

We consider the photodetachment with formation of an atom in a certain state nl_1m_1 near the threshold of the very same process. Accordingly, we substitute (38) and (39) in (37). Calculating next the cross section of the process with a singled-out oscillating contribution of the order of β_n^{-1}, in exactly the same manner as used in Ref. 1, and summing over m_1, we obtain

$$\frac{\sigma_{nl_1}}{\lambda_n}=\sum_{l_2}|K_{nl_1l_2}|^2-\frac{\cos\beta_n}{3\beta_n}\sum_{l_2l_2'}(-1)^{l_2'}[(2l_2+1)(2l_2'+1)]^{1/2}$$
$$\times C^{1M_L}_{l_1M_Ll_20}C^{1M_L}_{l_1M_Ll_2'0}K_{nl_1l_2}K^{*}_{nl_1l_2'}+\frac{1}{3\beta_n}\mathrm{Re}\Big\{e^{i\beta_n}\sum_{l_2l_2'l_2''}(-1)^{l_2''}$$
$$\times[(2l_2+1)(2l_2''+1)]^{1/2}i^{l_2-l_2'}C^{1M_L}_{l_1M_Ll_2''0}(C^{1M_L}_{l_1M_Ll_20})^{-1}$$
$$\times K^{*}_{nl_1l_2''}K_{nl_1l_2'}T^{1}_{nl_1l_2'nl_1l_2}\Big\}, \qquad (40)$$

where $T^{1}_{nl_1'l_2nl_1l_2}$ are the elements of the T matrix in the representation in which the total angular momentum is equal to unity. Under the conditions that the final-state function is normalized to $\delta(\mathbf{k}_n-\mathbf{k}_n'')$, we have

$$\lambda_n=2\pi^2k_n\omega/c,$$

where ω is the photon frequency and c is its velocity.

At $l_1=0$ we obtain

$$\sigma_{n0}=\sigma_{n0}^{(0)}[1+\beta_n^{-1}\cos(\beta_n+2\eta_{n1})\delta_{M_L0}], \qquad (41)$$

where $\sigma^{(0)}_{n0}$ is the cross section for photodetachment in the absence of a field, η_{n1} is the singlet phase of the p scattering by the n-th s state. The Kronecker delta in the second term means that the oscillating correction is negligibly small in the case $|M_L|=1$ (i.e., in the case of σ polarization of the photon). Indeed, according to Ref. 1, the interference effects are strongest if the emitted electron has an angular momentum projection $m_2=0$ (i.e., is emitted predominantly along and against the field). From the angular-momentum conservation law it follows that at $l_1=0$ this can take place only in the case of π polarization ($|M_L|=0$), and at $l_1\geqslant 1$ this can happen for both π and σ polarization. We note that the results pertaining to the case of σ polarization are equally applicable to the case of unpolarized light whose wave vector is directed along **F**.

We note that it follows from (41) that the phase of the singlet p scattering of an electron by an alkali atom can be determined by measuring the phase of the oscillation of the photodetachment cross section of the corresponding negative ion in an electric field. It is interesting that in experiment one can measure directly the phase, and furthermore with a high accuracy with respect to energy, since the degree of monochromatization of the laser beam greatly exceeds the degree of monokineticity of the electrons when experiments with beams are performed.

If the final state of the atom has a nonzero orbital angular momentum, the interference part of the cross section contain elements $K_{nl_1l_2}$ with different l_2 and there is no such simple relation between σ_{nl_1} and $\sigma^{(0)}_{nl_1}$, as would follow from (41). In this case, however, we can use the fact that near the threshold we have $K_{nl_1l_2}\sim(k_n)^{l_2}$, so that from among all the $K_{nl_1l_2}$ it suffices to retain $K_{nl_1l_1-1}$. We then obtain

$$\sigma_{nl_1}=\sigma_{nl_1}^{(0)}\Bigg[1-\frac{\cos\beta_n}{3\beta_n}(-1)^{l_1-1}(2l_1-1)\left(C^{1M_L}_{l_1M_L,l_1-1,0}\right)^2$$

$$+\frac{1}{3\beta_n}\mathrm{Re}\left\{e^{i\beta_n}(2l_1-1)^{1/2}\cdot i^{l_1-1}C^{1M_L}_{l_1M_Ll_1-10}\sum_{l_2}\left(C^{1M_L}_{l_1M_Ll_20}\right)^{-1}i^{l_2}\right.$$

$$\left.\times(2l_2+1)T^1_{nl_1,l_1-1,nl_1l_2}\right\}\Bigg].$$

By virtue of the conservation of the total angular momentum and of the parity, this formula contains two elements of the T matrix, $T^1_{nl_1l_1-inl_1l_1-1}$ and $T^1_{nl_1l_1nl_1l_1+1}$. Near the threshold they are proportional to k_n because of the quadrupole interaction in the final state (see, e.g., Ref. 13), so that they are of the same order of magnitude. These elements are determined by three real parameters, two of which can be determined by measuring σ_{nl_1} for two polarizations.

When considering the formation of an atom in a certain state nl_1 near the threshold of a higher state $n'l'_1$ [the process described by the matrix element (36)], the expression for the interference part of the cross section contains the product of M_{nkn} and $M_{n'-\mathbf{k}^0_{n'}}$, so that the connection between σ_{nl_1} and the elements of the T matrix is much more complicated for the $n\leftarrow n'$ transition.

To my knowledge, only one experiment has been performed to date on photodetachment in an electric field.[2] It consisted of measuring the cross section for photodetachment of H^- by unpolarized light with formation of a hydrogen atom in the ground state. As follows from (41), no noticeable cross-section oscillations should be observed in this case. It would therefore be of interest to perform the experiment either with polarized light or with measurement of the cross section for photodetachment with formation of an atom in the state with $l_1\geqslant 1$. It must be noted, however, that since the hydrogen atom is subject to the linear Stark effect, the theory of photodetachment of H^- with formation of H in the excited state should be modified somewhat. This remark does not apply to alkali atoms.

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