Anomalous penetration of electric field of a point contact into a metal located in a magnetic field

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(Submitted 9 April 1982)
Zh. Eksp. Teor. Fiz. **83**, 1464–1482 (October 1982)

The distribution of the electric potential produced in a metal by a small point contact on the surface of the sample is investigated theoretically. It is shown that distance from the current-carrying junction smaller than or of the order of the carrier mean free path l, in a strong field **H** (where the characteristic radius of the electron orbit is much smaller than l), along with the major potential spikes (MPS) due to focusing of the electrons injected by the emitter) there is produced a series of auxiliary potential spikes (APS) due to trajectory transfer of the electric field by electrons moving from the MPS. The spike intensity is analyzed for various angles between the vector **H** and the metal surface and for a wide range of the emitter current. The spike intensity is found to be sensitive to the state of the metal surface, and significantly different for specular and diffuse reflection of the carriers from the sample boundary. At certain experimental geometries the MPS and APS can be made to reach the surface at different points. The potential distribution in the interior of the sample can therefore be analyzed by using transverse focusing. At large emitter currents the potential spikes contain information about the electron-phonon interaction.

PACS numbers: 73.40.Cg

I. INTRODUCTION

Microjunction technology is now being extensively used to investigate the electronic properties of normal metals (see the reviews[1–3]). One of the application of point junctions is the conduction-electron focusing method proposed in 1965 by Sharvin,[4] who pointed out the feasibility in principle of studying the electron energy spectrum by this method.[1)] Experiments on the observation of longitudinal[7] and transverse[8] electron focusing in metals and semimetals by a uniform magnetic field were subsequently performed. Transverse electron focusing was found to be an effective method for studying the character of carrier interaction with a conductor surface.[1] From the ratio of the amplitudes of neighboring transverse-electron-focusing lines one can determine the probability specular reflection of conduction electrons from a boundary[8,9] and the probability of intervalleys scattering processes,[10] while the distance between the focusing line depends on the character of the carrier spectrum[9] and on the structure of the metal surface.[10]

Another application of small junctions is connected with the determination of the determination of the electron-phonon interaction function from the nonlinear sections of the current-voltage characteristics of point junctions.[2,3,11–13]

In electron-focusing experiments one usually measures the distribution of the electric potential on the surface of a metal as a function of the magnitude and direction of the magnetic field. The same characteristic was calculated theoretically in Refs. 9, 10, and 14–16. It is also undoubtedly of interest to investigate the electric-field distribution in the interior of a metal, since this distribution describes in detail the focusing of the emitter-injected conduction electrons by the magnetic field.

A method for determining the electric potential in a bounded metallic sample of arbitrary shape and an arbitrary placement of the current-conducting junctions was formulated in a paper by Azbel' and one of us,[17] where the conductor resistance was analyzed, in an approximation linear in the weak electric field, for the case of point and linear junctions at an infinite mean free path and for an arbitrary intensity of the magnetic field **H**, including $H=0$. The distribution of the electric potential near an emitter of the constriction type in the absence of a magnetic field was determined in Ref. 12. The author of that reference have shown that the electric field is localized in the junction region having a characteristic size of the order of the constriction radius b, and decreases monotonically at larger distances. It will be shown below that turning on a strong magnetic field for which $r_H \ll l$ (r_H is the Larmor radius and l is the electron mean free path) leads to the onset two types of potential inhomogeneities at distances smaller than l from the current-carrying junction. One of them is due to focusing, at definite points inside the metal, of the electrons accelerated by the electric field near the emitter and making up the system of major potential spikes [MPS). The second type of inhomogeneity is due to carriers that drag the electric field from the MPS and produce auxiliary potential spikes (APS). The appearance of the APS recalls in many respects the effect of anomalous penetration of an electric field into a metal along a "trajectory chain,"[18] while the MPS are the analog of the skin layer. Our problem, however, has a large number of distinguishing features. In particular, an electron emitted from a MPS gains a nonequilibrium energy increment if it experiences a collision inside the spike. In the opposite case, moving with and against the electric field, the carriers traverse as a result a zero potential difference. Since the maximum length of that part of the orbit which is situated in the MPS is of the order of $(br_H)^{1/2}$, the relative number of electrons scattered in the spike is proportional to $(br_H)^{1/2}/l$. Corresponding to an electron having a specified momentum and located at a given point in the bulk of the metal is a maximum number of initial states ($\sim (b/r_H)^{1/2}$), located in the spike if the electron "starts out" from the MPS at small angle to the central line of the spike. It is precisely these carriers, focused at a distance from the MPS equal to diameter of their orbit, which form the APS. The second potential spike, in turn, leads to the onset of a third, etc. At a definite experimental geometry the major and secondary spikes can be brought out to the sample surface at different points, so that transverse electron focusing makes it possible to analyze the electric field distribution inside the metal.

II. FORMULATION OF PROBLEM AND COMPLETE SYSTEM OF EQUATIONS

We investigate the distribution of the electric field in a semi-infinite metal sample ($x \geqslant 0$), on whose surface is placed an emitter with a characteristic dimension b. We assume that a second current-carrying junction is separated from the emitter by a distance greatly exceeding the carrier mean free path l, and that the constant and uniform magnetic field $\mathbf{H}$ is strong, but the condition $b \ll r_H \ll l$ is always satisfied.

The complete system of equations from which to determine the electric potential $\varphi(\mathbf{r})$ at an arbitrary point $\mathbf{r}$ of the sample, including its surface, was formulated in papers by Azbel' and one of us[17,19] in an approximation linear in the electric field $\mathbf{E}$, and generalized in Ref. 20 to include the case of strong electric field. It consists of the transport equation for the nonequilibrium electron distribution function $n(\mathbf{r}, \mathbf{p})$

$$\mathbf{v}\frac{\partial n}{\partial \mathbf{r}}+\left(e\mathbf{E}+\frac{e}{c}[\mathbf{v}\times\mathbf{H}]\right)\frac{\partial n}{\partial \mathbf{p}}=\hat{W}\{f_0(\varepsilon)-n(\mathbf{r},\mathbf{p})\} \quad (1)$$

and the electroneutrality equation

$$\frac{2e^2}{(2\pi\hbar)^3}\int d^3\mathbf{p}\{n(\mathbf{r},\mathbf{p})-f_0(\varepsilon)\}=0. \quad (2)$$

Here, e, $\mathbf{r}$, and $\mathbf{p}$ are the charge, coordinate, and momentum of the electron; $\varepsilon(\mathbf{p})$ and $\mathbf{v}=\partial\varepsilon/\partial\mathbf{p}$ are its energy and velocity; $f_0(\varepsilon)$ is the Fermi distribution function. The collision integral $\hat{W}$ describes the electron scattering inside the volume, and their intraction with the metal surface is taken into account by the boundary condition for Eq. (1). We confine ourselves below to an approximation linear in the electric field, and defer the generalization to the case of strong fields E to the last section.

Linearizing the Boltzmann equation,[1] it is convenient to seek its solution in the form

$$n(\mathbf{r},\mathbf{p})=f_0(\varepsilon)-e\frac{\partial f_0}{\partial \varepsilon}\psi(\mathbf{r},\mathbf{p}). \quad (3)$$

The integral $\hat{W}$ of the collisions in the volume is a linear operator even when account is taken of the electron-phonon interaction. A weak nonlinearity of $\hat{W}$ can subsequently be taken into account by perturbation theory. A solution of Eq. (1) in the form (3) can be easily obtained by using the method of characteristics in the τ-approximation for the collision integral $\hat{W}\psi=-\psi/\tau$, where τ is the average time between the electron scatterings in the volume and the function $\psi(\mathbf{r},\mathbf{p})$ takes the form

$$\psi(\mathbf{r},\mathbf{p})=f(\mathbf{r}-\mathbf{r}(t))e^{(\lambda-t)/\tau}-\varphi(\mathbf{r})+\frac{1}{\tau}\int_\lambda^t dt' e^{(t'-t)/\tau}\varphi(\mathbf{r}+\mathbf{r}(t')-\mathbf{r}(t)), \quad (4)$$

$$\mathbf{r}(t)=\int^t \mathbf{v}(t')\,dt',$$

where $\lambda(\mathbf{r},\mathbf{p})\leqslant t$ is the instant of the last reflection of the electron from the sample surface; $f(\mathbf{r}-\mathbf{r}(t))$ is an arbitrary function of the characteristics, which should be obtained with the aid of the boundary condition. For the electrons in the volume that do not interact with the surface, we must put $\lambda=-\infty$ in (4). the first term of (4) is then zero. Although the problem can be solved also under an arbitrary condition on the function $\psi(\mathbf{r},\mathbf{p})$ on the boundary, in the form of a linear integral equation,[21] we avoid unwieldy equations by using the specularity parameter q:

$$\psi(\mathbf{r}_s,\mathbf{p}')=q\psi(\mathbf{r}_s,\mathbf{p})+\frac{\langle(1-q)v_x\psi\rangle_-}{\langle v_x\rangle}+\psi^*(\mathbf{r}_s,\mathbf{p}), \quad (5)$$

where $\mathbf{r}_s$ is the coordinate of the surface point from which the reflection took place; $\psi^*(\mathbf{r}_s,\mathbf{p})$ is a specified electron distribution function at the junctions; the angle brackets $\langle\ldots\rangle$ denote integration with a factor $2e^2/(2\pi\hbar)^3$ over that part of the Fermi surface on which the electron-velocity component normal to the boundary is $v_x\geqslant 0$. The momenta $\mathbf{p}$ and $\mathbf{p}'$ of the incident and specularly reflected electrons satisfy the relations

$$\varepsilon(\mathbf{p})=\varepsilon(\mathbf{p}'),\quad \mathbf{p}_t=\mathbf{p}_t' \quad (6)$$

($\mathbf{p}_t$ is the quasimomentum component tangential to the surface). For simplicity we assume hereafter that the distribution function $\psi^*(\mathbf{r}_s,\mathbf{p})$ of the injected carriers, which is determined by the model of the junction, is independent of the momentum $\mathbf{p}$. In this case a simple relation exists between $\psi^*(\mathbf{r}_s)$ and the current distribution $i(\mathbf{r}_s)$ on the surface, namely $i(\mathbf{r}_s)=\psi^*\langle v_x\rangle_+$. In a real situation the number of electrons leaving the emitter at small angles α to the surface is small, and the function $\psi^*(\mathbf{r}_s,\mathbf{p})$ is generally speaking proportional to $|v_x|$. Allowance for this circumstance that alters the potential $\varphi(\mathbf{r})$ is important at the points r at which the presence of an electric field $\mathbf{E}=-\nabla\varphi$ is connected with motion of carriers for which $\alpha\lesssim(b/r_H)^{1/2}\ll 1$, and does not change the results qualitatively.

Taking the Fourier transforms in (4)

$$\begin{Bmatrix}F(\mathbf{k},x)\\ \Phi(\mathbf{k},x)\end{Bmatrix}=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}d^2\mathbf{R}e^{-i\mathbf{k}\mathbf{R}}\begin{Bmatrix}f(x,\mathbf{R})\\ \varphi(x,\mathbf{R})\end{Bmatrix} \quad (7)$$

($\mathbf{R}$ is a two-dimensional vector in a plane parallel to the sample surface) and substituting the result in the electroneutrality condition (2) we obtain an equation for the Fourier transform $\Phi(\mathbf{k},x)$ of the electric potential at an arbitrary depth x:

$$\Phi(\mathbf{k},x)=A(\mathbf{k},x)+\int_0^\infty dx'[B_s(\mathbf{k};x,x')+B_V(\mathbf{k};x,x')]\Phi(\mathbf{k},x'). \quad (8)$$

The integral terms of (8) with kernels B_s and B_V describe respectively the contributions made to the potential by the "surface" and "volume" electrons. At arbitrary carrier reflection from the surface and at arbitrary angle between the surface and the magnetic field, the functions $A(\mathbf{k}, x)$ and $B_s(\mathbf{k}; x,x')$ are extremely unwieldy, and we present below explicit expressions for these functions and an analysis of the distribution of the potential $\varphi(\mathbf{r})$ for the most important particular cases.

III. DISTRIBUTION OF POTENTIAL IN METALS WITH ISOTROPIC CARRIER DISPERSION IN A PARALLEL MAGNETIC FIELD

1. Almost specular electron reflection from surface

If the magnetic field is parallel to the sample surface, the motion of electrons drifting along the boundary as a result of specular reflections is periodic with a period T. In this case the function F of the characteristics takes on only one value on the entire trajectory broken up by specular reflections, and the condition (5) leads to a linear integral equation with a degenerate kernel. At almost specular carrier reflection on the metal boundary $q \gg (b/r_H)^{1/2}$, the term $A(\mathbf{k}, x)$ in (8) can be represented in the form

$$A(\mathbf{k}, x)=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{I(\mathbf{k})}{\langle 1\rangle\langle v_x\rangle_+}\sum_{n=0}^{\infty}q^n\int dp_z\int_0^{T/2}d\lambda v_x(\lambda)$$

$$\times[\mathscr{E}_{\mathbf{k}}(T_\lambda+\lambda,\lambda)]^n\int_{\lambda}^{T_\lambda+\lambda}dt\mathscr{E}_{\mathbf{k}}(t,\lambda)\delta(x-\Delta x(t,\lambda)). \quad (9)$$

The integral term in (8), which is connected with the motion of the electrons near the metal surface, can be written in the form

$$\int_0^\infty B_s(\mathbf{k},x,x')\Phi(\mathbf{k},x')dx'=\frac{2e^3H}{c(2\pi\hbar)^3\langle 1\rangle}\sum_{m=0}^{\infty}q^m\int dp_z'$$

$$\times\int_0^T dt_1\theta(t_1,p_z')[\mathscr{E}_{\mathbf{k}}(T_\mu+\mu,\mu)]^m$$

$$\times\int_\mu^{t_m}\frac{dt'}{\tau}\mathscr{E}_{\mathbf{k}}(t_1,t')\cdot\Phi(\mathbf{k},x-\Delta x(t_1,t')). \quad (10)$$

The last term, due to the "volume" carriers, is given by

$$\int_0^\infty B_V(\mathbf{k},x,x')\Phi(\mathbf{k},x')dx'$$

$$=\frac{2e^3H}{c(2\pi\hbar)^3\langle 1\rangle}\int dp_z'\int_0^T dt_1(1-\theta(t_1,p_z'))$$

$$\times[1-e^{-T/\tau}]^{-1}\int_{t_1-T}^{t_1}\frac{dt'}{\tau}\mathscr{E}_{\mathbf{k}}(t_1,t')\Phi(\mathbf{k},x-\Delta x(t_1,t')). \quad (11)$$

Here $I(\mathbf{k})$ is the Fourier transform of $i(\mathbf{R}_s)$, T is the period of carrier revolution in the field $\mathbf{H}$, p_z is the momentum projection on the magnetic-field direction, and $\theta(t_1, p'_z)$ is a function equal to unity for electrons colliding with the metal surface and to zero for the "volume" carriers:

$$\mathscr{E}_{\mathbf{k}}(t_1,t_2)=\exp\left[-\frac{t_1-t_2}{\tau}-i\mathbf{k}\Delta\mathbf{R}(t_1,t_2)\right],$$

$$\Delta\mathbf{R}(t_1,t_2)=\mathbf{R}(t_1)-\mathbf{R}(t_2),\quad \Delta x(t_1,t_2)=x(t_1)-x(t_2), \quad (12)$$

$$t_m=\begin{cases}t_1, & m=0\\ T_\mu+\mu, & m\neq 0,\quad \mu=\mu(x,t_1).\end{cases}$$

We investigate now the distribution of the potential $\varphi(\mathbf{r})$ at distances $r<l$ from the emitter E, through which a current flows with density $i_E(\mathbf{r}_s)$. We solve Eq. (8) by successive approximation, expressing $\varphi(x, \mathbf{R})$ as a series[2)]

$$\varphi(x,\mathbf{R})=\sum_{\nu=0}^{\infty}\varphi_\nu(x,\mathbf{R}), \quad (13)$$

where

$$\varphi_0(x,\mathbf{R})=\int\int_{-\infty}^{\infty}d^2\mathbf{k}e^{i\mathbf{k}\mathbf{R}}A(\mathbf{k},x), \quad (14)$$

$$\varphi_{\nu+1}(x,\mathbf{R})$$

$$=\int\int_{-\infty}^{\infty}d^2\mathbf{k}e^{i\mathbf{k}\mathbf{R}}\int_0^{\infty}dx'[B_s(\mathbf{k};x,x')+B_V(\mathbf{k};x,x')]\Phi_\nu(\mathbf{k},x'). \quad (15)$$

Let us analyze the contribution of each of the term to the total distribution of the electric potential in the sample. Substituting (9) in (14) we have

$$\varphi_0(x,\mathbf{R})=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{1}{\langle 1\rangle\langle v_x\rangle_+}$$

$$\times\sum_{n=0}^{\infty}q^n\int dp_z\int_0^{T/2}d\lambda v_x(\lambda)\exp\left(-\frac{nT_\lambda}{\lambda}\right)$$

$$\times\int_{\lambda}^{T_\lambda+\lambda}dt\exp\left(\frac{\lambda-t}{\tau}\right)i_E(\mathbf{R}-\Delta\mathbf{R}(t,\lambda)$$

$$-n\Delta\mathbf{R}(T_\lambda+\lambda,\lambda))\delta(x-\Delta x(t,\lambda)). \quad (16)$$

Calculating the integral with respect to t with the aid of a δ-function, we estimate the integrals with respect to λ and p_z by the Laplace method, using the "sharpness" of function $i_E(\mathbf{R}-\Delta\mathbf{R}_n)$, which differs from zero only in a small region with a characteristic dimension b in the vicinity of the point $\mathbf{R}=\Delta\mathbf{R}_n$. The contribution to the integral is made by small intervals $\Delta\lambda$ and Δp_z near those values of λ and p_z for which

$$\mathbf{R}=\Delta\mathbf{R}_n(\lambda,p_z)=\Delta\mathbf{R}(t,\lambda)+n\Delta\mathbf{R}(T_\lambda+\lambda,\lambda). \quad (17)$$

Obviously, the potential $\varphi_0(\mathbf{r})$ (16) is a maximum at the points $\mathbf{r}=(x, \mathbf{R})$ corresponding to the extremum of the function $i_E(\mathbf{R}-\Delta\mathbf{R}_n(\lambda,p_z))$, i.e., at the points where the Jacobian

$$D_n=\frac{\partial(\Delta y_n,\Delta z_n)}{\partial(\lambda,p_z)}=0,\quad \Delta\mathbf{R}_n=(\Delta y_n,\Delta z_n) \quad (18)$$

vanishes. Recognizing that

$$\Delta y_n = \frac{c}{eH}\left[p_{x2} - p_{x1} + n\left(p_{x1}' - p_{x1}\right)\right], \tag{19}$$

$$\Delta z_n = \frac{c}{eH}\left[\int\limits_{p_{y1}(v_{x1}<0)}^{p_{y2}(v_{x1}>0)} \frac{v_z dp_y}{v_x} + n \int\limits_{p_{y1}(v_{x1}<0)}^{p_{y1}(v_{x1}>0)} \frac{v_z dp_y}{v_x}\right],$$

we can rewrite Eq. (16) for D_n in the form

$$D_n = \frac{cv_{x1}}{eH}\begin{vmatrix} \alpha_{yx}^{(n)} & \alpha_{zx}^{(n)} \\ \alpha_{zx}^{(n)} & \beta^{(n)} \end{vmatrix}. \tag{20}$$

Here

$$\alpha_{\mu\nu}^{(n)} = \frac{v_{\mu 2}}{v_{\nu 2}} - \frac{v_{\mu 1}}{v_{\nu 1}} + n\left(\frac{v_{\mu 1}'}{v_{\nu 1}'} - \frac{v_{\mu 1}}{v_{\nu 1}}\right), \tag{21}$$

$$\beta^{(n)} = \frac{\partial^2}{\partial p_z^2}\left[\int\limits_{p_{y1}(v_{x1}<0)}^{p_{y2}(v_{x1}>0)} p_x dp_y + n \int\limits_{p_{y1}(v_{x1}<0)}^{p_{y1}(v_{x1}>0)} p_x dp_y\right], \tag{22}$$

$\mathbf{v}_1 = \mathbf{v}(\lambda, p_z)$, $\mathbf{v}_2 = \mathbf{v}(t, p_z)$, $t = t(\lambda, p_z; x)$, and $\mathbf{v}_1$ and $\mathbf{v}_1'$ are connected by the specularity condition (6) (Fig. 1).

Analyzing (16) with account taken of (21) and (22), it is easy to note that at $r_H \gg b$ the electric field, just as at $\mathbf{H} = 0$ (Ref. 12), is localized mainly in a region of space of the order of the junction radius b, and the potential difference between points in the emitter region ($r \lesssim b$) and a peripheral point of the sample ($r \gg l$) is of the order of $U = U_{\max} \approx J_E R_E$; J_E and R_E are the total current through the emitter and its resistance:

$$J_E = \int d^2\mathbf{R}\, i_E(\mathbf{R}), \quad R_E^{-1} = S_E \langle v_x \rangle_+,$$

S_E is the area of the current-carrying junction.

In the case of an isotropic Fermi surface, $D_0 = 0$ if $P(\lambda, p_z) = |\mathbf{p}_1 - \mathbf{p}_2|$ corresponds to the extremal diameters P^{extr} that belong to the central section $p_z = 0$. Therefore electrons injected by the junction and not interacting with the sample boundary form a potential spike (PS) with a characteristic size b near the geometric locus of the points located at a distance cP^{extr}/eH from the emitter. These carriers, moving along a periodic trajectory over the metal surface, are subsequently focused by the magnetic field in the vicinity $\sim b$ of the points $\mathbf{r} = \Delta\mathbf{r}_n = (\Delta x, \Delta\, \mathbf{R}_n)$, for which $D_n = 0$,

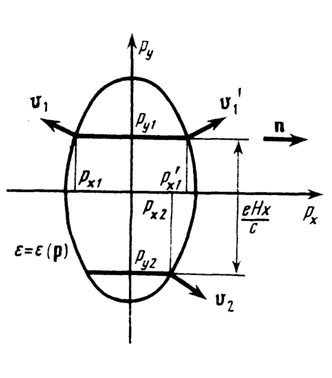

FIG. 1. Electron trajectory in momentum space; $\mathbf{v}_2$-velocity of electron at a depth x, the velocities $\mathbf{v}_1$ and $\mathbf{v}'_1$ are related by the reflection-specularity conditions (6), and $\mathbf{n}$ is the inward normal to the metal surface.

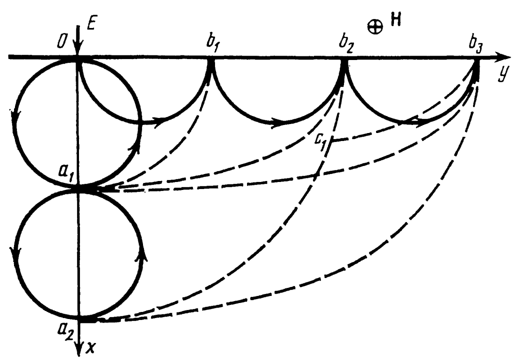

FIG. 2. Schematic arrangement of the potential spikes in the plane $z = 0$ that passes through the junction, for specular reflection of the electrons by the surface ($\mathbf{H} \perp \mathbf{n}$). The amplitudes of the MPS, to which the curves a_1b_1 ($n = 0$), a_2b_2 ($n = 1$) and a_1b_3 ($n = 2$) correspond in the figure, are determined by Eq. (23), and the amplitudes of the APS (the lines a_2b_2, $n = m = 0$; a_2b_3, $n = 1$, $m = 0$; c_1b_3, $n = 0$, $m = 1$) by Eqs. (27) and (28).

forming a series of MPS (the spikes a_1b_1, a_1b_2, and a_1b_3 in Fig. 2). Each of the MPS is due to the motion of electrons specularly reflected n times by the surface. In the situation considered, the spikes lie near the plane $z = 0$($|z| \lesssim b$), and the amplitude of the MPS numbered n is

$$U_n \approx q^n U_{max}(b/r_1)^{3/2}; \quad r_1 = |\Delta\mathbf{r}_n|, \quad n = 0, 1, 2\ldots, \tag{23}$$

whereas the potential far from the spikes (at a distance $\Delta r \gg b$ from the MPS) at a depth $x < 2r_H$ does not exceed the value $U_{\max}(b/r_H)^2$.

We proceed now to an analysis of the term $\varphi_1(\mathbf{r})$ in expression (13) for the potential $\varphi(\mathbf{r})$. Substituting in (15) at $\nu = 0$ expression (19) for $\Phi_0 = A$, we find that $\varphi_1(x, \mathbf{R})$ has a maximum near the points

$$\mathbf{r} = \Delta\mathbf{r}_{nm} = \Delta\mathbf{r}_n + \Delta\tilde{\mathbf{r}}_m, \tag{24}$$

where

$$\Delta\tilde{\mathbf{r}}_m = (\Delta x(t_1, t'), \quad \Delta\tilde{\mathbf{R}}_m = \Delta\mathbf{R}(t_1, t') + m\Delta\mathbf{R}(T_\mu + \mu, \mu)),$$

if the Jacobian

$$\tilde{D}_m = \frac{\partial(\Delta x(t_1, t'), \Delta\tilde{y}_m, \Delta\tilde{z}_m)}{\partial(t_1, t', p_z')}. \tag{25}$$

vanishes at these points simultaneously with D_n. Expressing $\tilde{D}_m$ (25) in a form similar to (20) for D_n, we can easily show that $\varphi(\mathbf{r})$ is a maximum if, besides the conditions that relate the velocities $\mathbf{v}(\lambda, p_z)$ and $\mathbf{v}(t, p_z)$ at which $D_n = 0$, the following relations are satisfied:

$$\frac{v_y(t)}{v_x(t)} = \frac{v_y(t')}{v_x(t')} = \frac{v_y(t_1)}{v_x(t_1)}, \quad v_z(t) = v_z(t') = v_z(t_1) = 0. \tag{26}$$

Consequently, the "volume" and "surface" electrons, which have at the point $\mathbf{r} = \Delta\mathbf{r}_n = (\Delta x(t, \lambda), \Delta\, \mathbf{R}_n(t, \lambda))$ a velocity component $\mathbf{v}_\perp(t')$ perpendicular to $\mathbf{H}$ and antiparallel to the velocity $\mathbf{v}_\perp(t)$ of the carriers that produce the MPS, interact effectively with the electric field of the spike and subsequently "reproduce" it at $\mathbf{r} = \Delta\mathbf{r}_{n0}$ away from the MPS at a distance equal to the extremal diameter of electron trajectory, i.e., they form APS (the spikes a_2b_2 and a_2b_3 in Fig. 2). Since the position of the APS, just as that of the MPS, is determined by the value of p^{extr}, the spikes of both types emerge to the surface at the same points.

Besides the APS considered, series of spikes are produced in the sample by "surface" electrons accelerated by the electric field of one of the MPS and focused by the magnetic field (after m specular reflections) at the points $\mathbf{r} = \Delta \mathbf{r}_{nm}$ for which $D_n = D_m = 0$ (spike c_1b_3 on Fig. 2). The amplitude of the APS with a characteristic size of the order of the dimension b of the junction, on the sections formed by the "volume" electrons (segment a_2c_1 of the APS a_2b_2 in Fig. 2), is proportional to

$$U_{n0}^{(2)} \approx q^n U_{max} \left(\frac{b}{r_1}\right)^{3/2} \left(\frac{b}{r_2}\right)^2 \chi, \quad n=0, 1, 2\ldots, \tag{27}$$

$$\chi = \frac{T}{\tau}\left[1 - \exp\left(-\frac{T}{\tau}\right)\right]^{-1}\Bigg|_{p_z=0}, \quad r_2 = |\Delta \tilde{\mathbf{r}}_m|,$$

and for the surface section of the spike (segment c_1b_2 of the APS a_2b_2 on Fig. 2), produced by the electrons interacting with the boundary,

$$U_{nm}^{(2)} \approx q^{n+m} U_{max} \left(\frac{b}{r_1}\right)^{3/2} \left(\frac{b}{r_2}\right)^2 \frac{T(p_z=0)}{\tau}; \quad n, m=0, 1, 2\ldots \tag{28}$$

In strong magnetic fields $\chi \approx 1$, and the intensity of the APS in the volume of the metal is $l/rH \gg 1$ times larger than its amplitude near the surface. The reason for the latter is that, in contrast to the "surface" electrons the "volume" carriers return l/rH times to the spike within the time τ. This increases effectively the number of nonequilibrium electrons than form at a given instant the auxiliary spike, and consequently increase its amplitude. It follows from (27) and (28) than the amplitude of the APS is smaller by a factor $(b/r_H)^2$ that the amplitude (23) of the MPS. Nonetheless, at depths $2r_H) < x < 4r_H$ far from the spikes the potential is $\varphi(\mathbf{r}) \lesssim U_{\max}(b/r_H)^4 \ll U_{n0}^{(2)}$. In the region $x < 2r_H$, in which all the APS with $m \neq 0$ are located, the amplitudes $U_{nm}^{(2)}$ are generally speaking smaller than the potential-distribution monotonic part produced by electrons injected by the emitter and gliding along the sample surface.

Similarly, using the recurrence relation (15), we can investigate the subsequent drawing of the APS into the interior of the metal and along its surface. We onsider now the potential distribution in the plane $b < |z| < |v_{0z}|T/2$, perpendicular to the magnetic field but not passing through the junction (v_{0z} is the electron velocity at the limiting point). The only electrons that can land on this plane from the emitter are those from noncentral sections of the Fermi surface, with velocity components along the vector $\mathbf{H}$. Since the determinants D_n (20) no longer vanish, an asymptotic estimate of the integrals in the expression (16) for $\varphi_0(\mathbf{r})$ shows that the amplitudes of the MPS are smaller by a factor $(r_H/b)^{1/2}$ than the amplitudes (23) of the spikes in the planes $|z| \lesssim b$, and their position is determined from the condition that $\alpha_{yx}^{(n)}$ vanishes [see Eq. (21)] on the central line of the spike $\mathbf{r} = \Delta \mathbf{r}_n(t, \lambda)$ (spikes a_1b_1, $a_1'b_2$, and $a_1''b_3'$ in Fig. 3). We note that spikes with $n \neq 0$ are formed at each point by electrons

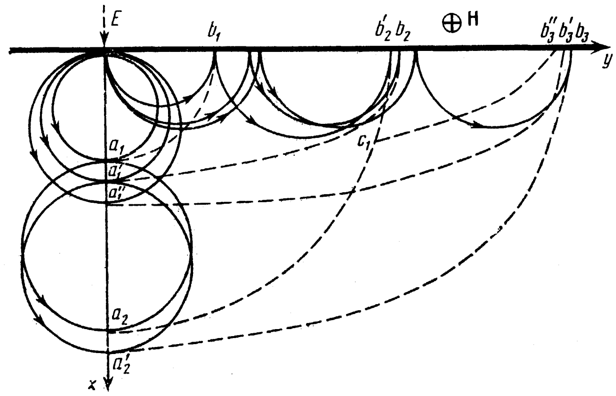

FIG. 3. Schematic distribution of the potential in the z plane, which does not pass through the junction, for specular reflection of the electrons from the surface ($\mathbf{H} \perp \mathbf{n}$, $b \ll |z| < |v_{z0}|T/2$) the lines a_1b_1, $a_1'b_2$ and $a_1''b_3$ correspond to the MPS and a_2b_2', $a_2'b_3'$, c_1b_3'' to the APS.

from different sections $p_z =$ const of the Fermi surface. For this reasons the MPS, and consequently the APS (lines a_2b_2', $a_2'b_3$, and c_1b_3'' in Fig. 3) do not start out from a single point on the x axis, inasmuch as to each n there corresponds a different characteristic radius of electron motion along a helical trajectory. Further penetration of the field into the metal is due, as before, to the motion of carriers belonging to the central section of the Fermi surface, and the ASP emerge to the surface at points that do not coincide with the potential maxima due to the electrons from the emitter.

With increasing distance from the plane passing through the junction, the spike lines decrease in amplitude, approach the z axis, and at $z_n^{(1)} = (n+1)v_{z0}T/2$ the amplitude of the n-th MPS on the surface becomes equal to zero, while at $z_n^{(2)} = (n+1/2)v_{z0}t$ this spikes vanishes also in the interior.

2. Almost diffuse electron reflection from surface

We consider now the case when the electron scattering by the metal surface is close to diffuse, $q \ll (b/r_H)^{1/2}$. The terms A and

$$\int_0^\infty dx' B_s \Phi$$

in the right-hand side of (8) are given by

$$A(\mathbf{k}, x) = \frac{I(\mathbf{k})}{\langle v_x \rangle_+ \langle 1 \rangle} \langle \mathscr{E}_{\mathbf{k}}(t, \lambda(x, t)) \rangle$$

$$\times \sum_{n=0}^{\infty} \left(\frac{1-q}{\langle v_x \rangle_-}\right)^n [\langle v_x \mathscr{E}_{\mathbf{k}}(t, \lambda(0, t)) \rangle_-]^n;$$

$$\int_0^\infty dx' B_s(\mathbf{k}, x, x') \Phi(\mathbf{k}, x') = \frac{1}{\langle 1 \rangle} \langle \mathscr{E}_{\mathbf{k}}(t, \lambda(0, x)) \rangle \tag{29}$$

$$\times\left\langle v_x \int\limits_{\lambda(0,t)}^{t} \frac{dt'}{\tau} \mathscr{E}_{\mathbf{k}}(t,t')\,\Phi(x-\Delta x(t,t'))\right\rangle_{-} \sum_{n=0}^{\infty}\left(\frac{1-q}{\langle v_x\rangle_{-}}\right)^{n+1}$$

$$\times[\langle v_x \mathscr{E}_{\mathbf{k}}(t,\lambda(0,t))\rangle_{-}]^{n}+\frac{1}{\langle 1\rangle}$$

$$\times\left\langle \int\limits_{\lambda(0,t)}^{t} \frac{dt'}{\tau} \mathscr{E}_{\mathbf{k}}(t,t')\,\Phi(\mathbf{k}, x-\Delta x(t,t'))\right\rangle. \qquad (30)$$

Using (14), we obtain an expression for the function $\varphi_0(\mathbf{r})$ that describes the MPS system at a depth $x \lesssim 2r_H$:

$$\varphi_0(x,\mathbf{R})=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{1}{\langle v_x\rangle_{+}\langle 1\rangle}\int dp_z\int\limits_{0}^{T/2} d\lambda\, v_x(\lambda)\int\limits_{\lambda}^{T_\lambda+\lambda} dt\, e^{(\lambda-t)/\tau}. \qquad (31)$$

$$\sum_{n=0}^{\infty}\left(\frac{1-q}{\langle v_x\rangle_{-}}\right)^{n}\langle v_x(t_1)\ldots\langle v_x(t_n)\, i_E(\mathbf{R}-\Delta\mathbf{R}(t,\lambda(x,t))$$

$$-\Delta\mathbf{R}(t_1,\lambda(0,t_1))-\ldots$$

$$-\Delta\mathbf{R}(t_n,\lambda(0,t_n)))\rangle_{-}\ldots\rangle_{-}\delta(x-\Delta x(t,\lambda)).$$

Since the first MPS ($n=0$) is produced by electrons emitted from the junction and not interacting with the surface, its amplitude is of the same order as the amplitude U_0 given by (23), and its position in the volume is given by condition (18) at $n=0$ (spike a_1b_1 in Fig. 4). The remaining terms of the series in (31) describe weaker potential spikes due to focusing of carriers injected by emitters and diffusely scattered $n \geqslant 1$ times by the metal surface (spikes a_2b_2, and a_3b_3 in Fig. 4). The equation

$$\mathbf{r}=\Delta\mathbf{r}_n'=(\Delta x(t,\lambda),$$

$$\Delta\mathbf{R}_n'=\Delta\mathbf{R}(t,\lambda)+\sum_{i=0}^{n}\Delta\mathbf{R}(t_i,\lambda(0,t_i))\,(1-\delta_{i0}))$$

of the central line of the n-th spike is obtained from the conditions

$$D_0=0,\quad D_i'=\frac{\partial(\Delta y(t_i,\lambda(0,t_i)),\,\Delta z(t_i,\lambda(0,t_i)))}{\partial(t_i,p_{zi})}=0,$$

$$i=1,2,3\ldots \qquad (32)$$

(δ_{ij} is the Kronecker delta). since $\Delta\mathbf{R}(t_i,\lambda(0,t_i))$ is the displacement of the electron over the sample surface during the time between to successive collisions with the surface, it follows that $D'_i=0$ if

$$|\Delta\mathbf{R}(t_i,\lambda(0,t_i))|=cP^{extr}/eH,$$

and the picture of the spikes is a repetition of the first MPS displaced along the y axis by distances ncP^{extr}/eH ($n=1, 2, 3\ldots$). The MPS amplitudes decrease in power-law fashion with increasing number n:

$$U_n\approx(1-q)^nU_{max}(b/r_H)^{\frac{1}{2}(n+1)},\quad n=0,1,2\ldots \qquad (33)$$

Using the recurrence relation (15) and Eqs. (30) and (31), it is easy to obtain an expression for the term in the potential $\varphi_1(\mathbf{r})$ which describes the set of MPS at a depth $x\lesssim 4r_H$ (spikes $a_1'b_2$ and $a_2'b_3$ in Fig. 4). Without dwelling on the details of the asymptotic estimate of the integrals, we note that $\varphi_1(\mathbf{r})$ is a maximum at the points $\mathbf{r}=\Delta\mathbf{r}_n'+\Delta\tilde{\mathbf{r}}_0$ for which $D_0=\tilde{D}_0=D_i'=0$ and $\mathbf{v}_\perp(t')=\mathbf{v}_\perp(t_i)$, while the mechanism of the anomalous penetration of the field into the metal differs from the case of almost specular reflection in the absence of sets of APS due to the periodic motion of the spike-carrying electrons over the surface.

In the planes $b<|z|<|v_{z0}|T/2$ the first MPS ($n=0$) is produced by electrons from a noncentral section of the Fermi surface near the curve $\mathbf{r}=\Delta\mathbf{r}_0'$ if the condition $\alpha_{xy}^{(0)}=0$ is satisfied. The MPS at $n\geqslant 1$ are due as before to electron focusing in the vicinities of the points $\mathbf{r}=\Delta\mathbf{r}_n'$ that satisfy the condition $\alpha_{xy}^{(n)}=0$ and relations (32). It is easy to show that the amplitudes of the potential spikes in planes $|z|>b$ that do not pass through the junction are smaller by a factor $(r_H/b)^{1/2}$ than the amplitudes Un given by Eq. (33). The MPS contained in the term $\varphi_1(\mathbf{r})$, regardless of the coordinate z of the plane, are produced by electrons from the central section of the Fermi surface.

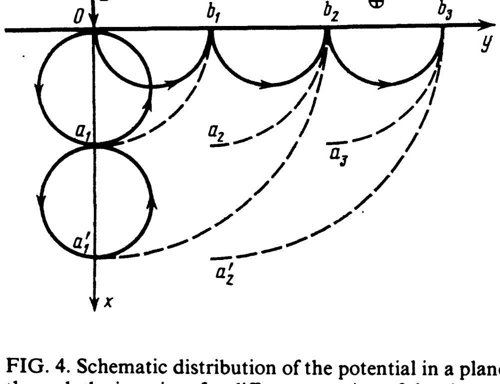

FIG. 4. Schematic distribution of the potential in a plane $z=0$ passing through the junction, for diffuse scattering of the electrons from the boundary ($\mathbf{H}\perp\mathbf{n}$). The lines a_1b_1 ($n=0$), a_2b_2 ($n=1$), and a_3b_3 ($n=2$) designate MPS with amplitudes (33), and $a_1'b_2$ and $a_2'b_3$ designate ABS.

IV. POTENTIAL DISTRIBUTION IN METALS WITH COMPLICATED CARRIER DISPERSION IN A PARALLEL MAGNETIC FIELD

1. Multiply connected Fermi surface

If the Fermi surface is multiply connected, the conservation conditions for the energy and for the tangential component of the quasimomentum (6) do not always ensure uniqueness of the state of the reflected electron.[3)] We consider by way of example a Fermi surface consisting of two isotropic valleys. The boundary condition for the nonequilibrium increment $\psi M(\mathbf{r},\mathbf{p})\partial f_0/\partial\varepsilon_M$ to the Fermi distribution of the electrons of the M-th valley ($M=1$ or 2) is

$$\psi_M(\mathbf{r}_s,\mathbf{p}_M')=P\psi_M(\mathbf{r}_s,\mathbf{p}_M)+Q\psi_N(\mathbf{r}_s,\mathbf{p}_N)+\frac{W_1\langle v_{xM}\psi_M\rangle_{-}}{\langle v_{xM}\rangle_{-}}$$

$$+\frac{W_2\langle v_{xN}\psi_N\rangle_{-}}{\langle v_{xN}\rangle_{-}}+\frac{i_M(\mathbf{r}_s)}{\langle v_{xM}\rangle_{+}};\quad M\neq N. \qquad (34)$$

Here P and Q are respectively the probabilities of the intravalley and intervalley "specular" reflections ($P+Q=q$); W_1 and W_2 are the probabilities of diffuse intravalley and intervalley scattering; $\mathbf{v}_M=\partial\varepsilon_M/\partial\mathbf{p}$ is the electron velocity in the M-th valley; $i_M(\mathbf{r}_s)$ are the partial densities of the current through the junctions; $\mathbf{p}_M$ and $\mathbf{p}_N$ are the solutions of the system of equations (6) for M-th and N-th valleys of the Fermi surface, respectively.

Writing down the solution of the transport equation (1) for each of the valleys in the form (3) and determining the functions f contained in (4) with the aid of the conditions (34), we substitute the sum of the solutions in the electroneutrality equation (2). After taking the Fourier transforms in the plane parallel to the surface, the equation for the function $\Phi(\mathbf{k},x)$ takes the form (8).

If the scattering of the conduction electrons by the metal boundary is only weakly diffuse ($P\ll W_1$, $Q\ll W_2$) and the magnetic field is parallel to the surface, the Fourier component that describes the MPS is

$$\Phi_0(\mathbf{k},x)=A(\mathbf{k},x)=\frac{2e^3H}{c(2\pi\hbar)^3\langle 1\rangle}\sum_{M=1}^{2}\int dp_z\int_0^{T_M}d\lambda_M v_{xM}(\lambda_M)$$
$$\times\int_{\lambda_M}^{T_{\lambda_M}+\lambda_M}dt\,\mathscr{E}_{\mathbf{k}}^{(M)}(t,\lambda_M)\tag{35}$$
$$\times\frac{[I_M(\mathbf{k})(1-P\alpha_N)/\langle v_{xM}\rangle_+ + QI_N(\mathbf{k})\,\alpha_N/\langle v_{xN}\rangle_+]}{(1-P\alpha_1)(1-P\alpha_2)-Q^2\alpha_1\alpha_2}$$
$$\times\delta(x-\Delta x_M(t,\lambda_M));\quad M\neq N,$$

where $\alpha_N=\mathscr{E}_k^{(N)}(T_{\lambda N}+\lambda_N,\lambda_N)$; $N=1,\ 2$. The subscripts M and N in (35) label quantities pertaining to the M-th and N-th valleys of the Fermi surface, $M\neq N$. It follows from (35) that the presence of a second channel for "specular" reflection of electrons leads to a lamination of the $(n+1)$-st MPS ($n=0,1,2,\ldots$) into $2n+1$ spikes that emerge to the surface at $n+2$ points, and the intensities of the spike lines depend on the probabilities P and Q. For example, the amplitudes of the MPS with $n=1$, produced by carriers that have collided once with the boundary, are

$$U_2^{(M,N)}\approx U_{max}\left(\frac{b}{r}\right)^{1/2}\cdot\begin{cases}P, & M=N\\ Q, & M\neq N,\ M,\ N=1,2\end{cases}.\tag{36}$$

If the carrier scattering by the sample surface is close to diffuse ($P\ll W_1$, $Q\ll W_2$), the Fourier component of the function $\varphi_0(\mathbf{r})$ takes the form

$$\Phi_0(\mathbf{k},x)=A(\mathbf{k},x)=\frac{2e^3H}{c(2\pi\hbar)^3\langle 1\rangle}\sum_{M=1}^{2}\int dp_z\int_0^{T_M}d\lambda_M v_{xM}(\lambda_M)$$
$$\times\int_{\lambda_M}^{T_{\lambda_M}+\lambda_M}dt\,\mathscr{E}_{\mathbf{k}}^{(M)}(t,\lambda_M)$$
$$\times\frac{I_M(\mathbf{k})(1-W_1\beta_N)/\langle v_{xM}\rangle_+ + W_2I_N(\mathbf{k})\,\beta_N/\langle v_{xN}\rangle_+}{(1-W_1\beta_1)(1-W_1\beta_2)-W_2{}^2\beta_1\beta_2}$$
$$\times\delta(x-\Delta x_M(t,\lambda_M));\quad M\neq N,\tag{37}$$

where

$$\beta_M=\frac{\langle v_{xM}\mathscr{E}_{\mathbf{k}}^{(M)}(t,\lambda_M(0,t))\rangle_-}{\langle v_{xM}\rangle_-}.$$

Expanding the denominator of (37) in a series and taking the inverse Fourier transform we find that, just as in the preceding case, $2n+1$ MPS are produced by the electrons diffusely scattered n times by the metal surface ($n=0$, 1, 2 ...). The spike amplitude, which depends on W_1 and W_2, decreases with increasing number n likewise in accordance with (33).

The functions $\varphi_\nu(x,\mathbf{R})$ ($\nu\geqslant 1$) describe the penetration of the electric field of the point junction into the metal along a "chain of trajectories," and in the case of a doubly connected Fermi surface it is a sum in which each term contains an APS located at a distance $c(mP_i^{\text{extr}}+sP_2^{\text{extr}})/eH$ from the MPS ($m+s=\nu+1$, m and s are integers, P_M^{extr} is the extremal dimension of the M-th cavity of the Fermi surface).

The foregoing results remain qualitatively valid for a Fermi surface consisting of an arbitrary number of closed cavities, since the geometry of the equal-energy surfaces determines only the amplitude and position of the potential spike. The central lines of the spikes are generally speaking not planar curves.

2. Role of open orbits

If the Fermi surface is not closed, the carriers belong to the open sections can drift, after leaving the emitter, into the interior of the sample even in a magnetic field parallel to the surface, and can produce potential spikes in the interior of the metal.

We shall assume that the Fermi surface is a surface of revolution of the corrugated-cylinder type, with an axis congruent with p_y. The momentum projections on the magnetic field direction $|p_z|<p_1$ and $p_1<|p_z|<p_0$ on the magnetic-field direction correspond respectively to closed and open electron trajectories in momentum space. Inasmuch as in this case the character of the carrier reflection from the metal surface does not play an important role, we assume, to simplify the equations, a pure diffuse carrier scattering by the boundary. In expression (13) for $\varphi(\mathbf{r})$ the term with $\nu=0$, which makes the principal contribution to the amplitudes of the spikes produced in the interior of the metal, is of the form

$$\varphi_0(x,\mathbf{R})=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{1}{\langle 1\rangle\langle v_x\rangle_+}\int dp_z\int_0^{T_0}d\lambda v_x(\lambda)\exp\left(\frac{\lambda-t-NT_0}{\tau}\right)$$
$$\times i_E(\mathbf{R}-\Delta\mathbf{R}(t,\lambda))\,\delta(x-\Delta x(t,\lambda)-NL),\tag{38}$$

where T_0 is the time required for the electron to negotiate the distance $L=c\hbar g_y/eH$ along the z axis, g_y is the reciprocal-lattice period in the open direction, $N=[x/L]$, $[a]$ is the integer part of a, and $\Delta x(t,\lambda)<L$.

For the asymptotic estimate of the integrals in (38) we shall use expression (20) for the Jacobian D_0. If the open trajectories of the carriers have no points at which the electron-velocity component perpendicular to the magnetic field reverses sign, the maximum points

$$\mathbf{r}=\mathbf{r}_N(t,\lambda)=(\Delta x(t,\lambda)+NL,\ \Delta\mathbf{R}(t,\lambda))$$

($D_0=0$) correspond to $\mathbf{v}_\perp(t)=\mathbf{v}_\perp(\lambda)$, and $t=\lambda, \lambda+T_0$. Within a time equal to an integer number of periods NT_0 all the electrons of the open Fermi-surface sections penetrate to the same depth NL in the surface, regardless of the initial velocity $\mathbf{v}_1=\mathbf{v}(\lambda)$, and have at the end point of the orbit a displacement $\Delta y(\lambda+NT_0, \lambda)$ equal to zero. Consequently these carriers produce potential spikes with characteristic size b near lines located in the xy plane at distances $x=NL$ from the surface. Calculation shows that the potential-spike amplitudes at $x=NL$ are proportional to

$$U_N\approx U_{max}e^{-NT_0/\tau}\cdot\begin{cases} b/NL, & b/NL\ll p_1/p_0 \\ p_1/p_0, & b/NL\gg p_1/p_0\end{cases} \tag{39}$$

and exceed by at least $\bar{r}_H/b$ times the potential in the remaining space ($\bar{r}_H=c(p_0-p_1)/eH$, $\bar{r}_H/l\ll1$). The inequality $p_1/p_0\ll b/NL$ means that the layer of open orbits along pz is narrow and after a time NT_0 the electron displacement along the z axis is smaller than b, i.e., the potential spikes exist only in the vicinity of the points $x=NL$ on the x axis. When the inverse inequality $p_1/p_0\gg b/NL$ holds, the potential spike has finite dimensions along the z axis, and since the period T_0 increases with increasing p_z, and goes logarithmically to infinity as $|p_z|\to p_1$, the intensity of the potential spike is a maximum near the plane $z=0$ and decreases with increasing distance from this plane as a result of electron collisions in the volume.

V. EFFECT OF INCLINATION OF THE MAGNETIC FIELD ON THE POTENTIAL DISTRIBUTION IN THE METAL

We consider the distribution of the electric potential in a magnetic field inclined to the surface of the metal, assuming that the Fermi surface is isotropic. In this case the electrons that interact with the sample surface have nonperiodic trajectories. At each specular reflection, the projection p_ζ of the momentum on the magnetic-field direction changes by a certain amount δp_ζ, and the carriers reflected once or several times from the boundary go into the interior of the conductor or else, after a countless number of specular collisions, land on the line $v_x=0$.

At an arbitrary inclination $\vartheta=z\hat{\zeta}$ of the magnetic field and a specularity parameter q it is no longer possible to obtain in closed form an equation for the function f in the solution (4). However, using Eq. (5) as a recurrence relation connecting $f[\mathbf{r}-\mathbf{r}(\lambda_i)]$ with the function $f[\mathbf{r}-\mathbf{r}(\lambda_{i+1})]$ at the an earlier instant of collision, $\lambda_{i+1}>\lambda_i$, we can write down the solution of the transport equation in series form,[19] while the terms A and

$$\int_0^\infty dx' B_s\Phi$$

in Eq. (8) for the potential, which are due to "surface electrons" take the form

$$A(\mathbf{k},x)=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{I(\mathbf{k})}{\langle v_x\rangle_+\langle1\rangle}\int dp_{\zeta1}\int_0^{T/2} d\lambda_1 v_x(\lambda_1) \times\sum_{n=0}^{M(\lambda_1,p_{\zeta1})} q^n\mathcal{F}_{\mathbf{k}}^{(n)}\times\int_{\lambda_1}^{T_{\lambda_1}+\lambda_1} dt\mathcal{E}_{\mathbf{k}}(t,\lambda_1)\,\delta(x-\Delta x(t,\lambda_1)); \tag{40}$$

$$\int_0^\infty dx' B_s(\mathbf{k};x,x')\Phi(\mathbf{k},x') =\frac{2e^3H}{c(2\pi\hbar)^3}\frac{1}{\langle1\rangle}\int dp_{\zeta1}\int_0^T dt_1\theta(p_{\zeta1},t_1)$$

$$\times\Big\{\mathcal{E}_{\mathbf{k}}(t_1,\lambda_1)\sum_{n=0}^{M(\lambda_1,p_{\zeta1})} q^n\mathcal{F}_{\mathbf{k}}^{(n)}\int_{\lambda_{n+1}}^{\lambda_n}\frac{dt'}{\tau}\mathcal{E}_{\mathbf{k}}(\lambda_n,t')\Phi(\mathbf{k},\Delta x(t',\lambda_n))$$

$$+\int_{\lambda_1}^{t}\frac{dt'}{\tau}\mathcal{E}_{\mathbf{k}}(t,t')\Phi(\mathbf{k},x-\Delta x(t,t'))\Big\}, \tag{41}$$

$$\mathcal{F}_{\mathbf{k}}^{(n)}=\begin{cases} 1 & \text{at}\quad n=0; \\ \prod_{i=1}^{n}\mathcal{E}_{\mathbf{k}}(\lambda_{i+1},\lambda_i) & \text{at}\quad n\neq1\end{cases}$$

$M(\lambda_1, p_{\zeta_1})$ is the total number of electron collisions with the surface.

At small inclination angles $\vartheta\ll r_H/l$ the "surface" terms (40) and (41) in Eq. (8) are the major ones. Solving (8), just as in the case of a parallel magnetic field, by iterations, we find that the first MPS ($n=0$) lies in the plane $\zeta=0$ and is produced by electrons from the central section. The remaining MPS ($n\geqslant1$) are due to focusing of electrons that go over, upon collision with the surface, from one Fermi-surface section $p_\zeta=\text{const}$ to another. The geometric locus of the points making up the central line of the spike is determined from the condition that a determinant analogous to D_n (18) vanish on the curve

$$\mathbf{r}=(\Delta x(t,\lambda_1);\quad \Delta\mathbf{R}(t,\lambda_1)+\sum_{i=0}^{n}\Delta\mathbf{R}(\lambda_{i+1},\lambda_i)(1-\delta_{i0})).$$

These lines, which are not planar at $n\neq0$, emerge to the surface at points lying on a straight line that passes through the junction and is perpendicular to the magnetic field.[22] The MPS amplitudes, which depend strongly on the angle ϑ, are proportional to the $(b/r_H)^{3/2}$.

If the carriers are diffusely scattered by the metal boundary ($q\ll1$) the distribution of the electric potential in the interior of the metal is not as sensitive to the inclination $\vartheta<rH/l$ of the magnetic field, since there is no correlation between the momenta of the incident and reflected electrons. In particular, the potential spike in the $\zeta=0$ plane takes a form similar to Fig. 4.

With increasing ϑ ($\tan\vartheta>2r_H/l$) the number of intense potential spikes that emerge to the surface decreases even if the carriers are specularly reflected from the boundary, and is determined no longer by the mean free path but by the inclination of the magnetic field. In this situation, however, peculiar spikes of another type can be produced in the interior of the conductor, and are due to longitudinal focusing of the electrons by the magnetic field. We shall analyze this effect using as an example a very simple geometry, when the vector $\mathbf{H}$ is perpendicular to the diffuse surface of the sample ($\vartheta=\pi/2$, $q=0$). Deflection of the magnetic field from the x axis by a small angle $\Delta\vartheta$ ($\Delta\vartheta\ll1$) does not change the qualitative distribution of the potential in the conductor.

As in the case of an open Fermi surface, a potential spike in a normal magnetic field is described by the term with $\nu = 0$ in (13):

$$\varphi_0(x,\mathbf{R})=\frac{2e^3H}{c(2\pi\hbar)^3}\frac{1}{\langle 1\rangle\langle v_x\rangle_+}\int dp_x\int_0^T d\lambda v_x(\lambda)\int_\lambda^{T+\lambda} dt e^{(\lambda-t)/\tau}$$

$$\times e^{-NT/\tau}i_E(\mathbf{R}-\Delta\mathbf{R}(t,\lambda))\delta(x-\Delta x(t,\lambda)-NL(p_x)). \qquad (42)$$

Here

$$N=[x/L(p_x)], \quad \Delta x(t,\lambda)<L(p_x)=\bar{v}_xT,$$

$$\bar{v}_x=\frac{1}{T}\int_0^T v_x(t)\,dt.$$

Calculations similar to those performed above show that the function $\varphi_0(r)$ is a maximum near the points

$$\mathbf{r}=(\Delta x(t,\lambda)+NL(p_x), \quad \Delta\mathbf{R}(t,\lambda)),$$

at which the Jacobian

$$D=\frac{cv_{x1}}{eH}\begin{vmatrix}\alpha_{yx}^{(0)} & \alpha_{xz}^{(0)}+\dfrac{v_{y2}}{v_{x2}}\gamma\\ \alpha_{zx}^{(0)} & \alpha_{xy}^{(0)}+\dfrac{v_{z2}}{v_{x2}}\gamma\end{vmatrix} \qquad (43)$$

vanishes. In (43), the $\alpha_{\mu\nu}^{(0)}$ are determined by Eq. (21):

$$\gamma=\frac{\partial^2}{\partial p_z^2}\left[\int_{p_{y1}}^{p_{y2}} p_z dp_y+NS(p_x)\right],$$

p_{y_1} and p_{y2} are the y components of the momentum at the instants of time λ and t; $S(p_x)=\int p_z dp_y$ is the area of the intersection of the Fermi surface with the plane $px=$ const.

It is easily noted that $D=0$ at $\mathbf{v}_1=\mathbf{v}_2$, i.e., carriers with definite values of p_x are focused at points such that the time of motion to them from the instant of leaving the emitter is a multiple of the period T of the motion in the magnetic field. In our case this is a straight line passing through the junction and perpendicular to the sample surface. After leaving the junction and executing a number of total revolutions lands on this line irrespectively of the value of λ. In this case the Jacobian (43) vanishes at least like $|NL(p_x)-x|$, and the amplitude of the potential near the points lying on the axis is larger than or of the order of $U\approx U_{\max}b/x(b/x\ll 1)$. The potential spikes are produced by carriers that move along trajectories for which the density of states with given $L(p_x)$ is a maximum. These are electrons located in the vicinity of the limiting points of the Fermi surface, $p_x=p_{x0}$, for which $D\sim|NL(p_{x0})-x|^{3/2}$, as well as electrons with extremal displacement along the magnetic field during one period:

$$\frac{\partial}{\partial p_x}(\bar{v}_xT)|_{p_x=p_{x1}}=0, \quad D\sim|NL(p_{x1})-x|^2.$$

The potential-spike amplitude at the points of focusing of the 'effective" carriers is equal to

$$U_N\approx U_{max}(b/x)^{2/3}, \quad x=NL(p_{x0}), \qquad (44a)$$

$$U_N\approx U_{max}(b/x)^{1/2}, \quad x=NL(p_{x1}). \qquad (44b)$$

Potential spikes of type [44(b)] were observed in longitudinal-focusing experiments[7] to emerge to the surface, opposite that of the point contact, of a thin plate of thickness smaller than the carrier mean free path.

VI. POTENTIAL DISTRIBUTION IN A METAL AT LARGE EMITTER CURRENTS

The foregoing results can be generalized also to include the case of strong electric fields. The collision integral $\hat{W}$ is in this case, generally speaking, a nonlinear integral operator. Since the operator for elastic scattering by impurities is linear at any electron statistics, the nonlinearity of $\hat{W}$ in the absence of interelectron collisions with quasimomentum umklapp is due to electron-phonon interaction and can be determined at low temperatures by successive approximations.[12,20] In the zeroth approximation in the parameter $b/l_\varepsilon\ll 1$, where l_ε is the carrier in elastic-relaxation length, the operator $\hat{W}$ in (1) describes only elastic-impurity interaction. It is easy to verify that in the absence of volume scattering the function

$$n_0(\mathbf{p},\mathbf{r})=f_0(\varepsilon-e[f(\mathbf{r}-\mathbf{r}(t))-\varphi(\mathbf{r})])$$

is a solution of the transport equation (1) (we note that t is here the time of motion in the fields $\mathbf{E}=-\nabla\varphi$ and $\mathbf{H}$). The function f is found with the aid of the boundary condition for the electron distribution function $n_0(\mathbf{p},\mathbf{r})$ and the condition that the electric-current component normal to the surface be conserved. At relatively low emitter currents, when $eU_{\max}\ll\varepsilon_0(\varepsilon_0$ is the Fermi energy), the influence of the inhomogeneous electric field on the electron trajectory can be disregarded, and the potential distribution that must be calculated with the aid of the electroneutrality equation (2) is described, at almost specular carrier reflection from the metal surface ($q\approx 1$), in the principal approximation in the parameter $b/r_H(r_H\ll l_\varepsilon)$, by Eq. (16) in which we must put $\tau=\infty$.

To take into account the influence of the inelastic relaxation of the electrons on the MPS amplitude, we obtain the increment $n_1(\mathbf{p},\mathbf{r})$ to the nonequilibrium electron distribution function $n_0(\mathbf{p},\mathbf{r})$. The function $n_1(\mathbf{p}\,\mathbf{r})$ satisfies the equation

$$\mathbf{v}\frac{\partial n_1}{\partial\mathbf{r}}+\left(e\mathbf{E}_0+\frac{e}{c}[\mathbf{v}\times\mathbf{H}]\right)\frac{\partial n_1}{\partial\mathbf{p}}=e\frac{d\varphi_0^{(1)}}{d\mathbf{r}}\frac{\partial n_0}{\partial\mathbf{p}}+\hat{W}_{ef}(n_0) \qquad (45)$$

with the boundary condition

$$n_1(\mathbf{p},\mathbf{r}_s)=qn_1(\mathbf{p}',\mathbf{r}_s),$$

which is valid for nearly specular carrier reflection from the surface. Here $\hat{W}_{ef}(n_0)$ is the electron-phonon collision integral; $\varphi_0^{(1)}(\mathbf{r})$ is the correction to the potential $\varphi_0(\mathbf{r})$ and is necessitated by the electron phonon interaction; $\mathbf{E}_0=-\nabla\varphi_0$; $\mathbf{p}$ and $\mathbf{p}'$ satisfy Eqs. (6). The phonon distribution function must be found by supplementing Eq. (1) with the transport equation for the phonons; at low temperatures the latter can be easily solved by successive approximations. However, allowance for the disequilibrium of the phonon system leads only to a renormalization of the kernel of the integral operator $W_{ef}(n_0)$ of the electron-phonon collisions.[23]

Assuming as before that $eU_{\max} \ll \varepsilon_0$ and the vector $\mathbf{H}$ is parallel to the surface, we obtain a solution of (45) by the method of characteristics

$$n_1(\mathbf{p},\mathbf{r}) = \sum_{i=1}^{\infty} q^i \int_{\lambda_{i+1}}^{\tilde{\lambda}_i} dt' \widehat{W}_{ef}\{n_0(\mathbf{p},\mathbf{r}+\mathbf{r}(t)-\mathbf{r}(t'))\} + e\frac{\partial f_0}{\partial \varepsilon}\varphi_0^{(1)}(\mathbf{r}),$$

$$\tilde{\lambda}_i = \begin{cases} t, & i=0 \\ \lambda_i & i\neq 0 \end{cases}, \tag{46}$$

where t is the time of motion of the electron in a magnetic field only. Substituting the expression obtained for $n_1(\mathbf{p}\ \mathbf{r})$ in the electroneutrality equation, we find that

$$\varphi_0^{(1)}(\mathbf{r}) = -\frac{2e}{(2\pi\hbar)^3\langle 1\rangle} \times \sum_{i=1}^{\infty} q^i \int d^3\mathbf{p} \int_{\lambda_{i+1}}^{\tilde{\lambda}_i} dt' \widehat{W}_{ef}\{n_0(\mathbf{p},\mathbf{r}+\mathbf{r}(t)-\mathbf{r}(t'))\}. \tag{47}$$

Calculating the correction to the distribution of the potential on the sample surface $\varphi_0^{(1)}(\mathbf{r}_s)$ and differentiating the resultant formula twice with respect to the emitter current, we obtain

$$d^2\varphi_0^{(1)}(\mathbf{r}_s)/dJ_E^2 = eR_E^2TG(\mathbf{r}_s, eU_{max}, \tilde{\Theta}), \tag{48}$$

where $\tilde{\Theta}$ is the temperature. At low temperatures, as $\tilde{\Theta}\to 0$,

$$G(\mathbf{r}_s, eU_{max}, 0) = \sum_k \frac{\nu(\varepsilon_0)}{2\langle\langle 1\rangle\rangle} \left\langle\!\!\left\langle Q(\mathbf{p},\mathbf{p}',\mathbf{r}_s) w_{\mathbf{p}-\mathbf{p}',k} \delta\left(\frac{eU_{max}}{\hbar} - \omega_{\mathbf{p}-\mathbf{p}',k}\right)\right\rangle\!\!\right\rangle,$$

$$Q(\mathbf{p},\mathbf{p}',\mathbf{r}_s) = \sum_{i=1}^{N(\mathbf{p})} q^i \frac{2}{T}\int_{\lambda_{i+1}}^{\lambda_i} dt_1 [\theta(\mathbf{v}'\in\Omega(\mathbf{p},t_1,\mathbf{r}_s)) - \theta(\mathbf{v}\in\Omega(\mathbf{p},t_1,\mathbf{r}_s))]; \tag{49}$$

$\nu(\varepsilon_0)$ is the density of the electronic states on the Fermi surface; the symbol $\theta(\mathbf{v}\in\Omega)$ stands for a function equal to unity when $\mathbf{v}$ belongs to the solid angle Ω containing the velocities of the electrons arriving at the given point $\mathbf{r}_s$ from the emitter; $N(\mathbf{p})$ is th total number of times that the electron collides with the surface when it moves from the current-carrying junction to the surface point $\mathbf{r}_s$; $|M_{\mathbf{p}-\mathbf{p}',k}|^2 = \hbar w_{\mathbf{p}-\mathbf{p}',k}/2\pi$ is the squared modulus of the matrix element of the electron-phonon interaction, with account taken of the corresponding renormalization; the subscript k labels the branches of the phonon spectrum.

Thus, the second derivative of the amplitude of the lines of the transverse electron focusing with respect of the emitter current contains the same information on the phonon spectrum of the metal as the nonlinear current-voltage characteristics of point junctions the only difference lies in the expression for the structural form factor $Q(\mathbf{p},\mathbf{p}',\mathbf{r}_s)$, which is determined by the shape of the emitter and by the distributon of the current through it.

VII. CONCLUSION

Thus, the distribution of the electric potential on the surface and in the bulk of the crystal are substantially altered in a strong magnetic field. The structure of the resultant picture of the major and auxiliary spikes depends on the state of the sample boundary, on the orientation of the magnetic-field vector relative to the surface, and on the carrier dispersion law. It turns out that even in the case of a magentic field parallel to the surface the value of the potential at depths appreciably exceeding the Larmor radius is determined by the probability of the specular reflection of the electrons from the metal boundary.

The results of this paper show that transverse focusing of the electrons by magnetic field affords extensive possibilities for the investigation of the potential distribution in the interior of a conductor. The information provided by the focusing line depends substantially on the location of the collector on the sample surface and on the direction of the vector $\mathbf{H}$. In particular, the possibility of observing secondary potential spikes exists if the magnetic field is directed at an angle to a line joining the current-carrying and the measuring junctions, and when the electrons are reflected from a plane that does not with the plane on which the emitter and collector are placed (e.g., a macroscopic defect is present in the gap between the junctions). In these cases the major and secondary spikes emerge to the surface at different points, and this should manifest itself in a splitting of the focusing peaks into several lines of different intensity.

Investigation of electron focusing at various temperatures and various emitter currents makes it possible to study the spectrum and the relaxation properties of conduction electrons, including electron-phonon interaction, as well as obtain information on the phonon spectrum.

[a)] Pippard[5] has proposed to observe transverse electron focusing by measuring the resistance of a system of linear junctions. Such an experiment was subsequently performed by Clarke and Schwartzkopf.[6]

[2)] It will be shown below that the parameter of the expansion (13) of the potential is the square of the ratio of the emitter dimension b to the characteristic Larmor radius r_H.

[3)] "Specular" reflection can also have many channels when the Fermi surface is anisotropic or the sample surface plane is not a symmetry plane of the crystal. Generalization of the results of the present section to include these cases entails no difficulty.

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Translated by J. G. Adashko