Resonant tunneling in superconductor-semiconductor-superconductor junctions

L. G. Aslamazov and M. V. Fistul'

Moscow Institute of Steel and Alloys

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It is shown that in the case of a nondegenerate semiconductor the critical current for superconductor-semiconductor-superconductor junctions can decrease with the thickness of the semiconductor layer more slowly than in the case of ordinary tunneling. The effect is attributed to resonant passage of coherent electrons along trajectories made up of periodically arranged impurity atoms that are produced in the semiconductor with low probability. The corresponding temperature dependence of the critical current and the region in which the effect exists are determined.

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1. INTRODUCTION

The critical current of superconductor-semiconductor-superconductor (S-Sm-S) junction depends essentially on the density of the doping impurities in the semiconductor. At high impurity density such system are similar in their properties to superconductor-normal metal-superconductor (S-N-S) junctions, and at low density they are close to ordinary tunnel junctions.[1–4] In the case of weak doping, the presence of impurity levels in the semiconductor can facilitate the passage of the electrons between the superconductors.

Lifshitz and Kirpichenkov[5] have shown that if the electron energy is near impurity levels, resonant passage of the electrons becomes possible in the junction along special impurity configurations (resonance-percolation trajectories). We investigate here in this connection the flow of a superconducting current through a semiconductor layer under these conditions. It is found that the critical current of the junction can increase appreciably.

When current passes through a semiconductor layer containing impurity levels, the electrons must surmount the barriers between the impurities. The superconducting current is transported by the coherent electrons, and in successive tunneling through several barriers the tunnel resistances are not additive (as in cases of normal current) but are multiplicative. Whereas in the case of a semiconductor-metal junction hopping conduction sets in starting already with very low temperatures, in an S-Sm-S junction resonance passage turns out to preferred in rather wide temperature range.

The analysis is based on the microscopic approach developed earlier[7] for impurities in a degenerate semiconductor. It is assumed that in a nondegenerate semiconductor Schottky barriers are produced near the interfaces with the superconductors, and the potential in the remainder of the semiconductor is equal to V (the bottom of the conduction band). The chemical potential μ is much lower than the bottom of the conduction band, and the impurity levels E_D (donor, for the sake of argument) are scattered about the chemical potential μ. The scatter of the impurity levels can be large compared with the temperature T, but small compared with the barrier height $V-\mu$ (see Fig. 1).

2. RESONANT TUNNELING ALONG IDEAL TRAJECTORIES

We start with the formula obtained in Ref. 7 for the superconducting current through an S-Sm-S junction:

$$J_s=-ieT\sum_{\omega}\int dz\,dz'\int d^2\mathbf{p}_1\,d^2\mathbf{p}_2\,d^2\mathbf{p}_1'\,d^2\mathbf{p}_2'\;G_\omega{}^n(\mathbf{p}_1,\mathbf{p}_2;z,z')\,G_{-\omega}$$
$$\times(\mathbf{p}_2'\mathbf{p}_1';z',z)\,\Delta(\mathbf{p}_1'-\mathbf{p}_1;z)\,\Delta^*(\mathbf{p}_2-\mathbf{p}_2';z')\,[\operatorname{sign} z-\operatorname{sign} z'],\qquad(1)$$

where $G_\omega(\mathbf{p}_1,\mathbf{p}_2;z,z')$ is the Green's function of the junction with the impurities, and depends both on the coordinates z and z' along the direction perpendicular to the plane of the junction, and on the transverse momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ (it is convenient to transform to the momentum representation with respect to the transverse coordinates $\boldsymbol{\rho}$); $\omega=(2n+1)\pi T$ is the Matsubara frequency; the superscript n labels the normal state of the junction. It is assumed here that the order parameter Δ in the semiconductor is zero (in view of the weak electron-phonon interaction) and the mutual influence of the superconductors is small.

Upon averaging of (1) over the impurity positions (the impurity coordinates enter as parameters in the expressions for the Green's functions) it turns out that the main contribution to the current is made by trajectories with periodic arrangement of the impurities. The probability of formation of such a trajectory is low, but at the same time the damping of the superconducting current on passing through the semiconductor region is substantially decreased; we therefore

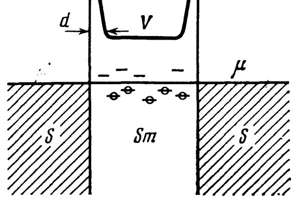

FIG. 1. Band structure of S-Sm-S junction in the case of a nondegenerate semiconductor with impurity levels.

find first the contribution made by such ideal trajectories to the superconducting current.

We assume that the impurities are spaced equal distances $2y$ apart, and that the distance from the first and last impurities to the nearest interface with the superconductor is y. In this case the equation for the Green's function

$$\left[i\omega+\frac{1}{2m}\nabla_r^2+\mu-V-\beta\sum_{j=1}^{N}\delta(\mathbf{r}-\mathbf{r}_j)\right] G_\omega^n(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

where $\mathbf{r}_j=(z_j\ \boldsymbol{\rho}_j)$ is the radius vector of the j-th impurity, can be solved by assuming the barrier to be high enough: $m^{1/2}(V-\mu)^{1/2}y\gg 1$. The solution of (2) can be written in the form

$$G_\omega^n(\mathbf{p},\mathbf{p}';z,z')$$
$$=G_\omega^n(\mathbf{p};z,z')\delta(\mathbf{p}-\mathbf{p}')+\sum_{j=1}^{N} G_\omega^n(\mathbf{p};z,z_j)e^{i\mathbf{p}\boldsymbol{\rho}_j}\varphi_j(z',\mathbf{p}'), \tag{3}$$

where $G_\omega^n(\mathbf{p};z,z')$ is the Green's function of the junction without impurities (the solution of (2) at $\beta=0$), and the values of φ_j are determined by the expressions

$$\varphi_j(z',\mathbf{p}')=\beta\int\frac{d^2\mathbf{p}}{(2\pi)^2}G_\omega^n(\mathbf{p},\mathbf{p}';z_j,z')e^{-i\mathbf{p}\boldsymbol{\rho}_j}. \tag{4}$$

We then obtain from Eq. (3)

$$\varphi_j(z',\mathbf{p}')=\frac{\beta}{(2\pi)^2}\left[1-\beta\int\frac{d^2\mathbf{p}}{(2\pi)^2}G_\omega^n(\mathbf{p};z_j,z_j)\right]^{-1}$$
$$\times\left[G_\omega^n(\mathbf{p}';z_j,z')e^{-i\mathbf{p}'\boldsymbol{\rho}_j}+\sum_{k\neq j}\varphi_k(z',\mathbf{p}')\int d^2\mathbf{p}G_\omega^n(\mathbf{p};z_k,z_j)e^{i\mathbf{p}(\boldsymbol{\rho}_k-\boldsymbol{\rho}_j)}\right]. \tag{5}$$

The integral in the first factor of (5) diverges, owing to the δ-function form of the impurity potential, so that the constant β must be renormalized in the usual manner by introducing a final amplitude g of the scattering by the impurity having the energy level E_D (Ref. 8):

$$g=2^{3/2}\pi m^{-1/2}[(V-\mu)^{1/2}-(V-E_D)^{1/2}]^{-1}. \tag{6}$$

To find the superconducting current in accord with Eq. (1), it suffices to calculate the values of φ_j at z' corresponding to the superconducting regions ($z'<-a; z'>a$, where $2a$ is the thickness of the semiconductor layer). To be specific, we obtain φ_j at $z'>a$ and assume that the first impurity is close to a point with coordinate $-a$, and the last (numbered N) is close to the point with coordinate a. Recognizing that the subbarrier damping is large, we obtain from (5) for φ_j the system of algebraic equations

$$\varphi_1=\frac{gh}{1+gh}\varphi_2, \tag{7}$$

$$\varphi_{j-1}-\frac{1}{gh}\varphi_j+\varphi_{j+1}=0,\quad 2\leqslant j\leqslant N-1, \tag{8}$$

$$\varphi_N=\frac{gh}{1+gh}\left[\varphi_{N-1}-\frac{1}{8\pi^2mh}e^{-i\mathbf{p}'\boldsymbol{\rho}_N}G_\omega^n(\mathbf{p}';z_N,z')\right]. \tag{9}$$

We have introduced here the quantity

$$h=\frac{1}{8\pi y}e^{-2\alpha y},\quad \alpha=(2m)^{1/2}(V-\mu)^{1/2}, \tag{10}$$

which determines the damping of the Green's function between two neighboring impurities. (We neglect the quantity $\omega\sim T$ compared with $V-\mu$.)

Equation (8) has a solution

$$\varphi_j=c_+\lambda_+^j+c_-\lambda_-^j,\quad \lambda_\pm=(2gh)^{-1}[1\pm(1-4g^2h^2)^{1/2}], \tag{11}$$

where the constants c_+ and c_- are obtained by substituting expressions (11) in Eqs. (7) and (8), which serve as boundary conditions.

Using Eq. (11) for φ_j, we can obtain from (3) the Green's function of the system

$$G_\omega^n(\mathbf{p}_1,\mathbf{p}_2;z,z')=\frac{(1-\lambda_+)}{8\pi^2mh(1+\lambda_+)(\lambda_+^N-\lambda_-^N)}\exp[i(\mathbf{p}_1\boldsymbol{\rho}_0-\mathbf{p}_2\boldsymbol{\rho}_N)]$$
$$\times G_\omega^n(\mathbf{p}_2;z_N,z')G_\omega^n(\mathbf{p}_1;z,z_1). \tag{12}$$

As seen from (11), when the number N of the impurities is large, the Green's function of the system is exponentially small. Indeed, using (6), (10), and (11), we have

$$|\lambda_\pm|=1\pm\frac{\omega}{2B}\left[1-\frac{(\mu-E_D)^2}{4B^2}\right]^{-1/2},\quad B=\frac{V-\mu}{\alpha y}e^{-2\alpha y}, \tag{13}$$

where B characterizes the width of the impurity band produced when the impurities are periodically arranged spaced $2y$ apart. Therefore the expression $(\lambda_+^N-\lambda_-^N)$ in the denominator of (2) is exponentially large:

$$|\lambda_+^N-\lambda_-^N|=\exp\left\{\frac{N\omega}{2B}\left[1-\frac{(\mu-E_D)^2}{4B^2}\right]^{-1/2}\right\}. \tag{14}$$

Equations (13) and (14) were derived under the assumption that the Matsubara frequency is low compared with B (as is the case at appreciable values of y), and the parameter $N\omega/B$ is large compared with unity.

When determining the superconductivity of the current through the junction from Eq. (1), we shall assume that the order parameter is constant in the superconducting regions:

$$\Delta(z)=\Delta_1\exp(i\chi_1)\quad \text{at}\quad z<-a,$$
$$\Delta(z)=\Delta_2\exp(i\chi_2)\quad \text{at}\quad z>a$$

[the corrections to Δ_1 and Δ_2 are given by the terms of next order, in view of the smallness of the integrand in (1)].

In addition, to determine the Josephson current for the superconducting Green's functions (without the index n) we can likewise use Eq. (12), where the normal Green's functions of the system without impurities must be replaced by the superconducting Green's functions. In this case, using (12), (13), and (14), we obtain

$$J_s=\frac{e}{m^2}\sin(\chi_1-\chi_2)\Delta_1\Delta_2 T$$

$$\times\sum_\omega\left\langle \exp\left\{-\frac{N\omega}{B}\left[1-\frac{(\mu-E_D)^2}{4B^2}\right]^{-1/2}\right\}\right.$$

$$\times h^{-2}\int\frac{d^2\mathbf{p}_1}{(2\pi)^2}\int_{-\infty}^{-a}dz\,G_\omega^n(\mathbf{p}_1;z,z_1)G_{-\omega}(\mathbf{p}_1;z_1,z)\int\frac{d^2\mathbf{p}_2}{(2\pi)^2}$$

$$\left.\times\int_a^\infty dz'\,G_\omega^n(\mathbf{p}_2;z_N,z')G_\omega(\mathbf{p}_2;z',z_N)\right\rangle_{y,E_D},\qquad(15)$$

where the angle brackets $\langle...\rangle_{y,E_D}$ denote averaging over the impurity positions and over time impurity-level energies

3. CONTRIBUTION OF THE RESONANCE-PERCOLATION TRAJECTORIES

We first average in (15) over the impurity-level energies. The scatter of the impurity levels E_D in the semiconductor is due mainly to fluctuations of the potential, and the correlation radius of the fluctuations in a nondegenerate semiconductor exceeds appreciably the average distance between the impurities (large-scale fluctuations).[6] Therefore in S-Sm-S junctions having not too thick a semiconductor layer it can be assumed that the position of the bottom of the conduction band does not vary along the z axis, and the fluctuations take place in different places of the junction. We can thus average independently over the impurity trajectories that join the superconductors and over the values of the impurity levels E_D in (15).

To average over the values of E_D it is necessary, generally speaking, to know the distribution function of the random potential $F(E)$; this function was obtained in a number of papers.[6] However, as seen from (15), in the averaging over E_D the significant values $E_D-\mu$ are of the order of the width B of the impurity band, which is small compared with the impurity level scatter due to the potential fluctuations. Using this circumstance, we obtain after averaging

$$\left\langle \exp\left\{-\frac{N\omega}{B}\left[1-\frac{(\mu-E_D)^2}{4B^2}\right]^{-1/2}\right\}\right\rangle_{E_D}$$

$$=\sqrt{2\pi}F(\mu)B\left(\frac{N\omega}{B}\right)^{-1/2}\exp\left(-\frac{N\omega}{B}\right).\qquad(16)$$

The exponential factor (16) in the equation for the current has a simple physical explanation. It stems from the loss of electron coherence in the semiconductor layer, where the electron-phonon interaction is assumed weak. Indeed, the coherence length in the semiconductor is $\xi\sim v/T$, where the characteristic velocity v is determined by the probability of the transitions between the impurity levels: $v\sim yB$. The argument $N\omega/B$ of the exponential is therefore of the order of a/ξ (where $a\sim Ny$ is the thickness of the semiconductor layer). The exponential damping of the superconducting current occurs thus when the semiconductor-layer thickness exceeds the coherence length.

Proceeding to the averaging of (15) over the impurity positions, we note that the probability of formation of a trajectory with a strictly periodic arrangement of the impurities is zero, so that we must allow the impurities to deviate from the ideal positions. This does not alter substantially the damping of the current if the impurities are shifted by distances $\sim 1/\alpha$ along the z axis and by distances $\sim y\theta$ in the transverse direction, where θ is the angle characterizing the sinuous character of the trajectory (and is assumed to be small).[5] Such a resonance-percolation trajectory consists of N impurities ($N\gg 1$) located in small volumes $y^2\theta^2/\alpha$ at an average distance $2y$ from one another; in this case the trajectory should be solitary (i.e., there should be no other scattering centers in regions of size $\sim y$ around the impurities). The probability W of formation of such a trajectory was obtained with exponential accuracy in Ref. 5 assuming a Poisson distribution of the impurities:

$$W=\exp\left[N\ln(y^2\theta^2 n/\alpha)-\pi nNy^3\right],\qquad(17)$$

where n is the density of the donor impurities, and for substantial values of y the second term in the argument of the exponential is relatively small).

Using (16) and (17), we obtain from (15) for the density j_c of the critical current

$$j_c=2^{11/2}\pi^{-5/2}ea^{-1}F(\mu)D(\mu)\Delta_1\Delta_2 T\sum_\omega|\omega|^{-1/2}$$

$$\times(\omega^2+\Delta_1^2)^{-1/2}(\omega^2+\Delta_2^2)^{-1/2}$$

$$\times\int N^{-1/2}B^{3/2}\exp\left(-\frac{N|\omega|}{B}+N\ln\frac{y^2\theta^2 n}{\alpha}\right)dy\,d\theta,$$

$$D(\mu)=\frac{V_0-\mu}{\mu d^2}\exp\left\{-2^{3/2}m^{1/2}d(V_0-\mu)^{1/2}\right.$$

$$\left.\times\left[1-\frac{V-\mu}{dA^{1/2}(V_0-\mu)^{1/2}}\ln\frac{dA^{1/2}+(V_0-\mu)^{1/2}}{(V-\mu)^{1/2}}\right]\right\}.\qquad(18)$$

In (18) are used the known expressions for the Green's functions of the system without impurities, and the vlaue of $D(\mu)$ describes the damping of the current on account of the tunneling through the Schottky barriers, which are assumed to be parabolic (d is the width of the barrier, V_0 is the value of the potential on the semiconductor-superconductor interface, and A is the coefficient in the quadratic dependence of the potential on the coordinate).[1,7] The number of impurities N on the trajectory is determined by the expression[5]

$$N=(a/y)(1+\theta^2/2).\qquad(19)$$

The integrals in (18) can be calculated by the saddle-point method. The saddle point is determined from the system of equations

$$-\beta e^{x_0}\theta_0+\frac{\theta_0}{x_0}\ln(x_0^2\theta_0^2c)+\frac{2}{x_0\theta_0}=0,$$

$$-\beta e^{x_0}-\frac{1}{x_0^2}\ln(x_0^2\theta_0^2c)+\frac{2}{x_0^2}-\frac{1}{L}=0,\qquad(20)$$

where we have introduced the dimensionless variables $x=2\alpha y$, $c=n/\alpha^3$, $L=2a\alpha$, $\beta=\pi T/2(V-\mu)$. The asymptotic solution of (20) at $\beta\ll 1$ and $c\ll\ln^{-2}\beta$ is of the form

$$x_0 = \ln(1/\beta) - \ln(\ln^2 1/\beta) + \ln|\ln(c \ln^2 1/\beta)|,$$
$$2\theta_0^{-2} = |\ln(c \ln^2 1/\beta)|. \quad (21)$$

It can be seen that $x_0 \gg 1$ and $\theta_0 \ll 1$, thus justifying the assumptions made.

Calculating in this manner the integrals in (18), we can retain the first term in the sum over ω. We then obtain with exponential accuracy

$$j_c = \frac{eD(\mu)F(\mu)\ln^{7/2}(1/\beta)\Delta_1\Delta_2 T^2}{(\pi^2T^2+\Delta_1^2)^{1/2}(\pi^2T^2+\Delta_2^2)^{1/2}L^{5/2}\ln^5(1/c)} \exp\left(-L\frac{\ln c}{\ln \beta}\right). \quad (22)$$

The exponential superconducting-current damping described by (22) is mainly the result of the low probability of formation of a resonance-percolation trajectory. With increasing impurity density, however, this probablity increases, and even before the onset of degeneracy, if the condition

$$|\ln c| < 2|\ln \beta| \quad (23)$$

is satisfied, the resonance passage turns out to be easier than direct tunneling through the semiconductor layer [in the latter case the critical current is $j_c \sim \exp(-2L)$].

We note also that the inequality (23) ensures satisfaction of the condition, used in the intermediate derivations, that the temperature be low compared with the width of the impurity band. As for the scatter of the impurity levels, it is found to have relatively little effect on the critical current.

4. DISCUSSION OF RESULTS

The results show that in an S-Sm-S junction with a nondegenerate semiconductor there can be realized a resonant mechanism of electron tunneling through the semiconductor layer. Owing to the overlap of the wave functions of the electrons of the impurities equidistantly spaced $2y$ apart, a band of width $B \sim \exp(-2y/a_B)$, is produced, where $a_B = (2m)^{-1/2}(V-\mu)^{-1/2}$ is the Bohr radius of the impurity. Superconducting current can flow through this band at a damping $\sim \exp(-2a/\xi)$ due to the electron coherence loss in the semiconductor layer.

The coherence length $\xi \sim v/T \sim 2yB/T$ decreases with increasing distance between the impurities. However, the probability of formation of a trajectory with a periodic arrangement of impurities (which is also exponentially small) is higher the larger the distance between impurities (the smaller the number of impurities that should be periodically located on the trajectory). The optimal distances are $2y_0 \sim a_B \ln[(V-\mu)/T]$. The width of the band is then

$$B = \pi T \ln[(V-\mu)/T]/|\ln(na_B^3)|.$$

Trajectories on which the impurities are periodically arranged and spaced $2y_0$ apart make the main contribution to the superconducting current. The degree to which the optimal trajecotry is sinuous turns out to be small, so that the problem is close to one-dimensional. In this case the critical current is determined by Eq. (22) and attenuates less with increasing thickness of the semiconductor than the usual tunnel exponential.

Large-scale fluctuations of the potential lead to an energy scatter of the impurity levels on the different trajectories. This scatter is usually large compared with the band width B. The main contribution to the superconducting current is made by electrons with energies $\sim B$ near the chemical-potential level, inasmuch as for electrons with other energies the damping of the current is already determined by the tunnel exponential. The level scatter, however, has less influence on the critical current than the damping due to loss of electron coherence in the semiconductor layer.

Allowance for the electron interaction leads, as is well known, to formation of a "Coulomb gap." The size of the gap is determined by the electron interaction energy at distances on the order of the correlation radius. Therefore in the case of weakly and strongly compensated semiconductors, as well as amorphous semiconductors (when the correlation radius is large), thegap turns out to be less than the level scatter.[6] It can become also less than the width B of the impurity band, as is indeed assumed in the present paper. Otherwise the region of existence of the resonance effect decreases.

Equation (22) for the critical current of the junction corresponds to the resonance mechanism of current flow through the semiconductor layer and is valid at temperature low enough that the width B of the band exceeds T. At these temperatures the transparency of the semiconductor barrier increases in comparison with the usual tunnel transparency. A change in transparency begins with a temperature $T_1 = (V-\mu)(na_B^3)^{1/2}$. The temperature T_1 can be higher as well as lower than the critical temperature T_c of the superconductors. At $T_1 > T_c$ the temperature dependence of the critical current differs from the tunnel dependence already at $T < T_c$, while at $T_1 < T_c$ in the region $T_1 < T < T_c$ the Ambegaokar-Baratov formula is valid.[9]

Equation (22) no longer holds at very low temperatures, when the coherence length ξ becomes larger than the semiconductor thickness $2a$. The limiting temperature T_2 at low impurity densities is given by

$$T_2 = (V-\mu)\exp[-(a/a_B)^{1/2}|\ln(na_B^3)|^{1/2}]$$

and at temperatures lower than T_2 the dependence of the critical current on the layer thickness is given by an equation similar to that obtained in Ref. 5:

$$j_c \sim \exp[-4(a/a_B)^{1/2}|\ln(na_B^3)|^{1/2}].$$

We can thus expect resonant tunneling to be revealed by the characteristic temperature dependence of the critical current of the S-Sm-S junction.

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[1]V. N. Alfeev, Sverhprovodniki, poluprovodniki, paraélektriki v krioélektronike (Superconductors, Semiconductors, and Paraelectrics in Cryoelectronics), Sov. radio, 1979.
[2]F. Andreozzi, A. Barone, G. Paterno, M. Russo, and R. Vaglio, Phys. Rev. **B18**, 6035 (1978).
[3]C. Huang and T. Van Duzer, IEEE Trans. Magn. **11**, 766 (1975).
[4]L. G. Aslamazov and M. V. Fistul', Pis'ma Zh. Eksp. Teor. Fiz. **30**, 233 (1979) [JETP Lett. **30**, 213 (1979)].

[5]I. M. Lifshitz and V. Ya. Kirpichenkov, Zh. Eksp. Teor. Fiz. **77**, 989 (1979) [Sov. Phys. JETP **50**, 499 (1979)].
[6]B. I. Shklovskiĭ and A. L. Efros, Elektronnye svoĭstva legirovannykh poluprovodnikov (Electronic Properties of Doped Semiconductors), Nauka, 1979.
[7]L. G. Aslamazov and M. V. Fistul', Zh. Eksp. Teor. Fiz. **81**, 382 (1981) [Sov. Phys. JETP **54**, 206 (1981)].
[8]L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Nonrelativistic Theory, Pergamon, 1965.
[9]V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **11**, 104 (1963).

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