Excitation of hydrogen-atom states with large n and l by electron impact

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The distribution in l, following excitation of a hydrogen atom into a state with large n near the threshold, is calculated. The most probable value of l is approximately $(n/2)^{1/2}$.

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Fano[1] called attention to the fact that when states with large n are excited near threshold, a transient complex is temporarily produced, in which both electron (incident and atomic) move at low velocities. Their motion is in this case strongly correlated because of the mutual repulsion, which is similar to that produced by ionization near the threshold. This correlation leads to a specific distributions of the orbital momenta. Namely, it turns out that the excited atom has at large n on the average also large l (although certainly smaller than the maximum possible value $n-1$). Fano estimated the order of magnitude of l by using the picture of the ionization near the threshold and extending it to the case of excitation of discrete-spectrum states with large n.

In this article we calculate the distribution in l for a given n, starting from the assumption that a wave function of definite form is established as a result of the evolution of the transient complex.

We introduce a parabolic coordinate frame for the description of the atomic electron, with the z axis in the direction of the scattered electron moving away from the atom. It is natural to assume that the wave function of the atomic electron is characterized by a maximum asymmetry relative to the plane passing through the nucleus perpendicular to the z axis. In this state, one of the parabolic quantum numbers, n_1 or n_2, vanishes. In addition, we assume that the projection of the orbital angular momentum on the z axis is zero. In the language of classical mechanics this corresponds to the assumption that the trajectories of the atomic and scattered electrons lie in the same plane.

To determine the distribution in l we must expand the assumed wave function, given in parabolic coordinates, in terms of wave functions expressed in spherical coordinates and characterized by the quantum numbers n and l. This expansion is well known.[2] As applied to our case, it takes the form

$$\psi=\sum_{l=0}^{n-1} C_{jjj-j}^{l0}\psi_{nl}, \tag{1}$$

where $j=(n-1)/2$ and C is a Clebsch-Gordan coefficient. The probability of finding a given value of l is

$$W_l=|C_{jjj-j}^{l0}|^2. \tag{2}$$

Substituting the value of C_{jjj-j}^{l0} (Ref. 3, §8.5, Eq. (36), we obtain

$$W_l=(2l+1)\frac{[(n-1)!]^2}{(n+l)!(n-l-1)!}. \tag{3}$$

Using the asymptotic formula for $\ln N!$ at large N, we obtain at $n\gg 1$ and $l\ll n$

$$W_l=\frac{2l+1}{n}\exp\left[-\frac{l(l+1)}{n}\right]. \tag{4}$$

According to (4), the probability W_l has a maximum at

$$l_0\approx(n/2)^{1/2}. \tag{5}$$

Equation (4) does not hold at $l\sim n$. But at such large values of l the probability W_l is very small.

The value l_0 at which W_l is a maximum lies according to (5) in the region $l\ll n$, in which Eq. (4) is fully applicable.

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