Some features of the self-focusing and energy absorption of intense wave beams in an inhomogeneous plasma

N. S. Erokhin and R. Z. Sadgeev

Institute of Space Research, USSR Academy of Sciences
(Submitted 1 February 1982)
Zh. Eksp. Teor. Fiz. **83**, 128–138 (July 1982)

Possible self-focusing regimes of wave beams in a weakly inhomogeneous plasma and the conditions of their realization are investigated for case of static local nonlinearity. Collisional absorption of wave-beam energy is considered and the possibility of its enhancement in an inhomogeneous plasma is studied. Simple equations are derived for the beam width in qualitatively different cases, and the main properties of the solutions of the equations are established.

PACS numbers: 52.35.Mw, 52.25.Ps

Much progress was made recently in the production of high intensity radiation beams. It is clear that the passage of such beams through a plasma is accompanied by strong nonlinear interaction with the medium. One of the central problems is that of the role of various mechanisms whereby the energy of the intense wave beam is dissipated. There are three competing processes responsible for the absorption of the wave-beam energy in the plasma: Coulomb collisions, linear transformation of an obliquely incident transverse wave into a longitudinal wave, and parametric mechanisms (see, e.g., Refs. 1 and 2). At present, the bulk of the experimental data cannot always be uniquely interpreted, inasmuch as at the attained parameters (power, collision frequency, characteristic plasma density gradient, and others) these data lie in a region that borders on the indicated three processes. An investigation of energy absorption in the transition region is therefore of special interest.

It was shown earlier[3,4] that allowance for the transverse dimensions of the wave beam and of the ensuing self-focusing alters quantitatively the picture of energy absorption in an inhomogeneous plasma, and also the ratio between the competing processes. In particular, it was indicated that at not too high wave-beam powers an increasing role is assumed by the collision mechanism of absorption. In the cited references, however, the spatial structure of the field of the self-focusing wave beam was investigated within the framework of a certain parabolic-type equation. This restriction is burdensome in the region where nonlinear perturbation of the dielectric constant of the plasma is far from small, since an estimate of the contributions of the different absorption mechanisms to the wave-beam energy dissipation depends substantially on the self-focusing regime. In the present paper, therefore, the features of the self-focusing and energy-absorption regimes of intense wave beams in an inhomogeneous plasma are investigated on the basis of a more rigorous approach, that includes the description of the spatial dynamics of the wave field with the aid of equations of the elliptic type. It is shown that three qualitatively different regimes of propagation of self-focusing wave beams exist in an inhomogeneous plasma, and the conditions for their realization are investigated. It is proved that the maximum efficiency of the absorption by collisions is reached in the case of broad beams with moderate energy density, while in the greater part of their path these beams propagate quasi-one-dimensionally. At not too high energy densities of the wave beam, the depth of its penetration into a transcritical plasma with increasing density is restricted to a scale of the order of the beam width. The main conclusion of Refs. 3 and 4, that the collisional absorption by self-focusing wave beams can be increased, remains in force. Thus, a complete analysis is presented of the self-focusing regimes of a wave beam in an inhomogeneous plasma in the case of static local nonlinearity.

1. SELF-FOCUSING OF WAVE BEAMS IN AN INHOMOGENEOUS PLASMA

We consider within the framework of the scalar problem an equation for the complex amplitude of the electric field of a wave beam

$$\Delta E+k_0^2\varepsilon(|E|^2)E=0, \qquad (1.1)$$

where $\varepsilon(|E|)\equiv\varepsilon_r+i\varepsilon_i$ is the dielectric constant of the plasma with allowance for nonlinear terms, $k_0=\omega/c$ for transverse or $k=(\omega/3^{1/2}v_{Te})$ for longitudinal waves. The applicability of the scalar model will be justified later.

Let b be the characteristic transverse dimension of the wave beam and $k_z=k_0\varepsilon^{1/2}$ the longitudinal wave number. Under the natural condition $|k_zb|\gg1$ the solution of (1.1) has a large phase and can be investigated by the methods of geometric optics, similar to those presented in Maslov's monograph.[5]

Separating in $E(z,r_\perp)$ the amplitude and the phase, $E=A\exp(-i\varphi)$, we obtain from (1.1) for an axisymmetric wave beam, discarding terms of the order of $1/k_z^2b^2$, equations for the eikonal and for the energy flux

$$(\nabla\varphi)^2=k_0^2\varepsilon_r(A^2),\quad \frac{\partial}{\partial z}A^2\frac{\partial\varphi}{\partial z}+\frac{1}{r_\perp}\frac{\partial}{\partial r_\perp}A^2r_\perp\frac{\partial\varphi}{\partial r_\perp}=k_0^2A^2\varepsilon_i(A^2). \qquad (1.2)$$

Taking into account the self-similar character of the contraction of the wave beam in the axial region, we put

$$A^2(z,r_\perp)=A_c^2(z)\exp(-r_\perp^2/2b^2(z)), \qquad (1.3)$$
$$\varphi(z,r_\perp)=\int_{z_0}^{z}k_z(z)\,dz+r_\perp^2\varphi_1(z)+r_\perp^4\varphi_2(z)+\dots,$$

where $A_c(z)$ is the field on the beam axis. We introduce the effective refractive index $N_z(z)=k_z(z)/k_0$ and the ef-

 0038-5646/82/070069-06$04.00

fective width of the wave beam $l(z)$:

$$l^2(z)=b^2(z)(1+U), \quad U=\frac{l^4}{2N_z}\frac{d}{dz}\left(\frac{N_z}{l}\frac{dl}{dz}\right).$$

From the condition that Eqs. (1.2) and (1.3) be compatible follow equations for the beam width $l(z)$, for the refractive index N_z, and for the field $A_c(z)$ on the beam axis:

$$(2+\Phi)N_z\frac{d}{dz}N_z\frac{d}{dz}l+\frac{\Phi}{l}N_z^2[2-(dl/dz)^2]=\Psi_D,$$
$$N_z^2-\varepsilon_r(A_c^2)=\varepsilon_D, \quad \frac{d}{dz}\ln(A_c^2l^2N_z)=\varkappa_z. \qquad (1.4)$$

Here Ψ_D and ε_D are diffraction corrections that determine the minimum transverse dimension of the beam; $\varkappa_z=(k_0/N_z)\varepsilon_i(A_c^2)$; $\Phi=(A_c/N_z)^2\partial\varepsilon_r/\partial(A_c^2)$ characterizes the nonlinear contribution to the dielectric constant. Equations (1.4) are the starting point for the study of self-focusing of wave beams.

We consider the self-focusing regimes.

1) If the nonlinear part of the dielectric constant ε_n is substantially smaller than the linear $\varepsilon_0(z)$ and the damping is small, the system (1.4) takes the simpler form

$$\left(N_z\frac{d}{dz}\right)^2 l+\frac{A_c}{2l}\frac{\partial\varepsilon_r}{\partial A_c}=\frac{3\lambda^2}{l^3}, \quad N_z^2+\frac{2\lambda^2}{l^2}=\varepsilon_r, \quad A_c^2=\frac{E_m^2l_0^2N_0}{l^2|N_z|}, \quad \lambda=\frac{1}{k_0}, \qquad (1.5)$$

which was investigated in Ref. 6 for cubic nonlinearity $\varepsilon_r=\varepsilon_0(z)+(A_c/E_p)^2$, where E_p is the characteristic nonlinearity field. Equations (1.5) correspond to a transition in (1.1) to the parabolic approximation[6]

$$E=E_mF(N_0/N_z(z))^{1/2}\exp\left(-i\int^z k_z(z)\,dz\right),$$

when, for example for a cubic nonlinearity the complex function F satisfies the equation

$$2i\frac{\partial F}{\partial\xi}=\nabla_\rho^2F+\beta(\xi)|F|^2F, \qquad (1.6)$$

here

$$d\xi=\frac{k_m^2}{k_z}dz, \quad k_m=\frac{k_0E_m}{E_p}, \quad \rho=k_m\mathbf{r}_\perp, \quad \beta(\xi)=\frac{1-\varepsilon_0}{\varepsilon_0^{1/2}}.$$

We can make a number of statements concerning those solutions of (1.6) which describe wave beams of finite diameter. First, we introduce the positive functions

$$I_1=\int d\rho|F|^2, \quad I_3(\xi)=\int d\rho|\nabla_\rho F|^2, \quad I_4=\frac{1}{2}\int d\rho|F|^4, \quad a^2(\xi)=\int d\rho\rho^2|F|^2$$

and, in analogy with the case of a homogeneous plasma, we construct the function $I_2(\xi)=I_3(\xi)-\beta(\xi)I_4(\xi)$. It is easily seen that I_1 is an integral, but in an inhomogeneous plasma I_2 is no longer conserved but obeys the evolution equation $dI_2/d\xi=-I_4d\beta/d\xi$. Just as in a homogeneous plasma, the characteristic dimension of the region of localization of the field upon development of self-focusing instability is determined by the sign of I_2, i.e., $d^2(a^2)/d\xi^2=2I_2$. It follows therefore that wave beams with negative value of I_2 become self-focused, and the characteristic self-focusing length corresponds to $\Delta\xi\sim a_0/(|I_2|)^{1/2}$. Moreover, in an inhomogeneous plasma, upon propagation in the direction of increasing plasma density, the wave beams that spread but on

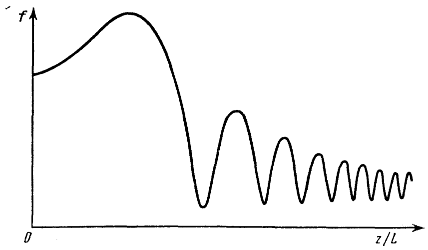

FIG. 1.

account of diffraction can (owing to the decrease of I_2 and to its becoming negative) start to become self-focused at a certain distance from the injection plane (see Fig. 1).

We note in addition that the Talanov transformation[7]

$$\tilde{\xi}=\xi R/(\xi+R), \quad \tilde{\rho}=\rho R/(\xi+R), \quad \tilde{F}=(1+\xi/R)F\exp[i\rho^2/(\xi+R)]$$

establishes a connection between the solutions of equations of the type (1.6) with different coefficients $\beta(\xi)$ and $\tilde{\beta}\equiv\beta(\xi(\tilde{\xi}))$.

The main condition imposed on the parameters of the self-focusing wave beams can be obtained more easily by considering the stability of the plane wave

$$F_0(\xi)=F_m\exp\left(-\frac{i}{2}F_m^2\int^\xi d\xi\beta(\xi)\right), \quad F_m=\text{const}$$

in Eq. (1.6). Linearizing (1.6), we find the spatial increment $\varkappa_\xi(\xi)$ of the self-focusing instability:

$$\varkappa_\xi=(\varkappa_\rho/2)(2\beta F_m^2-\varkappa_\rho^2)^{1/2},$$

where $\varkappa_\rho=\varkappa_\perp/k_m$, and $\varkappa_\perp$ is the transverse wave number of the perturbation. Let L be the characteristic scale of the inhomogeneity of the plasma, for example $\varepsilon_0(z)=1-(z/L)$. Then the perturbations can become accelerated in the beam over a distance L if the following condition is satisfied for the amplitude of the beam field:

$$\left(\frac{E_m}{E_p}\right)^2>\frac{\lambda}{L}\left(\frac{b}{b_*}+\frac{b_*}{b}\right)^2\geqslant\frac{4\lambda}{L}.$$

Here $b_*=(\lambda L)^{1/2}$ is the beam thickness at which the self-focusing length is a minimum. Since usually $E_m<E_p$, it follows therefore that the beam diameter should be less than the characteristic inhomogeneity scale L. In particular, for striction nonlinearity ($E_p^2=16\pi n_cT$, $n_c=m_e\omega^2/4\pi e^2$) and $b>b_*$ we have for the vacuum energy density in a beam of transverse waves the estimate $W_\infty>n_cT(b/L)^2$, which agrees with Ref. 6. At sufficiently small wave damping, the wave-beam energy loss over the self-focusing length is small. Therefore, after going to the first focus, the beam broadens to a certain maximum size, and then starts to be focused again, i.e., a pulsating waveguide is produced in the plasma. To connect the parameters of such a waveguide with the slowly varying plasma parameters, we note the following. As seen from (1.5), the equation for the dimensionless-beam thickness $f\equiv l(z)/l_0$ describes the motion of the nonlinear oscillator with a frequency that varies slowly with time

$$\frac{d^2f}{dt^2}+\Omega^2(f,t)f=0, \quad \Omega^2=\frac{A_c^2}{f^2}\frac{\partial\varepsilon_r}{\partial(A_c^2)}-3\left(\frac{\lambda}{l_0f^2}\right)^2,$$

where $dt \equiv dz/l_0 N_z$. We introduce the potential $\overline{U}(f, t)$ defined by the equation $\partial \overline{U}/\partial f = 2\Omega^2 f$. The condition for the conservation of the adiabatic invariant of the oscillator

$$I = \oint p\, df, \quad p^2 + \overline{U} = \mathscr{E}$$

connects the change of the energy of the oscillator $\mathscr{E}$ with the slow changes of the plasma parameters and determines by the same token the waveguide-parameter dependences of interest to us, particularly of the maximum and minimum radii on the plasma parameters. We demonstrate the foregoing using a simple example that simulates the allowance for diffraction and nonlinearity saturation. We choose the frequency Ω of the nonlinear oscillator in the form

$$\Omega^2 = \Omega_0^2(t)\,[1 - (\delta/f)^2]/f^4,$$

where $\delta \ll 1$ characterizes the size of the focal spot. We note first of all that for self-focusing wave beams the energy $\mathscr{E}$ of the nonlinear oscillator is negative. It can be shown next that the inhomogeneity of the frequency $\Omega_0(t)$ in time is equivalent to introducing into the system friction that is positive when Ω_0 increases with time and negative in the opposite case. Indeed, with increasing Ω_0 the oscillator energy decreases, i.e., the level drops to the potential-well bottom corresponding to the waveguide. As a result, the amplitude of the oscillations of the wave-beam width and the self-focusing length decrease. With increasing Ω_0 the process is reversed, and after $\mathscr{E}$ becomes positive the self-focusing stops and the beam spreads out. It is important to note that at appreciable oscillations of the beam width, when $l_{max} \gg l_{min}$, the time dependence of $\mathscr{E}$ takes the form of jumps in time intervals corresponding to formation of the focal spot. Analogous jumps are made simultaneously also by the function $I_2(\xi)$, as can incidentally be readily seen from the evolution equation for I_2.

In the example presented above, the condition for the conservation of the adiabatic invariant takes the form

$$\Omega_0[(2-k^2)^{1/2}K(k) - 2E(k)/(2-k^2)^{1/2}] = \text{const}, \quad k^2 \equiv 2(1-\sigma)^{1/2}/(1+(1-\sigma)^{1/2}).$$

here $\sigma = -2\mathscr{E}(\delta/\Omega_0)^2 \leqslant 1$, while $E(k)$ and $K(k)$ are elliptic functions. It follows therefore that in the case $\sigma \ll 1$ the energy $\mathscr{E}$ of the oscillator varies in proportion to $\Omega_0^2 \exp(-I/\Omega_0)$. Near the bottom of the potential well, where $(1-\sigma) \ll 1$, we have

$$\Omega_0[1 + 2\mathscr{E}(\delta/\Omega_0)^2] \approx \text{const}.$$

On the whole, the behavior of the dimensionless width of the beam $f(z)$ in the case of increasing Ω_0, which corresponds to propagation of the wave beam towards the increasing plasma density, is shown schematically in Fig. 1.

2) We consider now the self-focusing of a wave-beam for a strong nonlinear perturbation of the plasma dielectric constant, when the linear term $\varepsilon_0(z)$ in the expression for $\varepsilon_r(A_c^2)$ can be neglected. Such conditions arise automatically when the beam approaches the surface of the critical plasma density $n_c(\omega)$, but can be realized in the transparency region also far from the critical surface. It is reasonable to assume that outside the narrow region directly adjacent to the focus the nonlinearity saturation is negligible. Putting $\varepsilon_r = (A_c/E_p)^{2\alpha}$ (where $\alpha > 0$, and for striction nonlinearity $\alpha = 1$), and neglecting damping and diffraction, we obtain from (1.4)

$$\ln N_* = \text{const} - (2\mu/3)\ln l(z), \quad \Phi = \alpha,$$
$$(dl/dz)^2 + \mathscr{E}(l/l_0)^{2\mu} = 2/3, \qquad (1.7)$$

where $\mu = 3\alpha/(2+\alpha)$; $\mathscr{E} = (\frac{2}{3}) - \tan^2\theta_0$ is the integration constant, and $\tan\theta_0 = dl/dz$ at $l = l_0$. It is seen from (1.7) that diverging beams become self-focused if the initial divergence θ_0 does not exceed $\tan^{-1}(\frac{2}{3})^{1/2}$. In the case of zero initial divergence, the self-focusing length is

$$z_f = \left(\frac{3\pi}{2}\right)^{1/2} l_0 \Gamma\left(\frac{7}{6} + \frac{1}{3\alpha}\right) \Big/ \Gamma\left(\frac{2}{3} + \frac{1}{3\alpha}\right).$$

For $\alpha = 1$, in particular, we have $z_f \approx 1.9 l_0$. The described self-focusing regime is realized in the inhomogeneous-plasma region, where $\varepsilon_p(A_c^2) > |\varepsilon_0(z)|$, i.e., at a sufficiently high nonlinearity level. Putting $A_c^2 = N_0 E_m^2 l_0^2/l^2 |N_z|$ and stipulating also $k_z z_f > 1$, we obtain the condition for the nonlinearity, which takes at $\alpha = 1$ the form

$$N_0(E_m/E_p)^2 > (2l_0/L)^{2/3} + (\lambda\!\!\!^{-}/l_0)^3.$$

It follows therefore that the nonlinearity level is a minimum at a beam thickness $l_0 \approx (\lambda\!\!\!^{-2} L)^{1/3}$ and is equal to

$$E_m^2/E_p^2 \approx 4\lambda\!\!\!^{-}/LN_0.$$

We present explicit formulas for the main functions in the case $\alpha = 1$ and at zero initial beam divergence:

$$A_c = \frac{(N_0 E_p E_m^2)^{1/3}}{(\sin\psi)^{2/3}}, \quad N_z = \frac{(N_0 E_m^2/E_p^2)^{1/3}}{(\sin\psi)^{2/3}}, \quad b(z) = \frac{l(z)}{(4 + 2\sin^2\psi)^{1/2}}$$
$$l(z) = l_0 \sin\psi, \qquad (1.8)$$

here $\psi = (\frac{1}{2})\pi(1 - z/z_f)$. We note the qualitative distinguishing features of the self-focusing regime. First, owing to the substantial increase of the group velocity with decreasing beam width, the self-focusing length is independent of the level of the "initial" nonlinearity. This circumstance in the case of cubic nonlinearity was pointed out by Moiseev.[3] Second, the limiting divergence of self-focusing wave beams is large, $\theta_m \approx 39.2°$, and is likewise independent of the level of the initial nonlinearity, this being the consequence of the increase of the parameter $\varepsilon_n/\varepsilon_r$. We note also another important aspect. It is known[8] that in the linear Schrödinger equation diffraction cannot stop the contraction of the beam even for cubic nonlinearity, when $\alpha = 1$. In our case, however, because of the nonlinear increase of the group velocity, the limiting degree of nonlinearity that suppresses the diffraction increases and corresponds to $\alpha = 2$. Indeed, allowance for diffraction leads to the appearance in the left-hand side of Eq. (1.7) for the beam width $l(z)$, of a "centrifugal" potential $u(l/l_0)^{2\gamma}$, where

$$\gamma = \frac{\alpha - 2}{\alpha + 2}, \quad u = \frac{(\lambda\!\!\!^{-}/l_0)^2}{(1+\alpha)N_m^2} \ll 1, \quad N_m(\alpha) = (N_0 E_m^2/E_p^2)^{\mu/3}.$$

It is seen therefore that the limiting degree of nonlinearity corresponding to the condition $\gamma = 0$ equals 5. Thus, for $\alpha > 2$ the size of the focal region is determined by the amplitude E_n of the nonlinearity saturation field and is of the order of $l \sim l_0(N_0E_m^2/N_mE_n^2)^{(2+\alpha)/4}$. In the opposite case $\alpha < 2$, the minimum beam dimension can be reached below the nonlinearity saturation threshold, at the level $l \sim l_0u^{1/2}$.

We now write down the geometric-optics parameter

$$k_z(z)l(z)=l_0N_m/\bar{\lambda}f^{\gamma}.$$

Consequently, at $\gamma > 0$ ($\alpha > 2$) the conditions for the applicability of the nonlinear WKB approximation improve with increased contraction of the beam. For $\alpha < 2$, when $\gamma < 0$, the quasiclassical parameter decreases to the minimum permissible value only in the focal spot.

At a linear plasma-density profile in the region of interest to us, when $\varepsilon_0(z) = -z/L$, the investigated self-focusing regime is realized in the region $|z| < 2LN_m^2$. If this dimension $2LN_m^2$ is large compared with the characteristic self-focusing length $z_f \sim l_0$, the self-focusing beam produces a pulsating waveguide against the background of slow variation of $\varepsilon_0(z)$. Let us indicate now the result of taking $\varepsilon_0(z)$ into account, all the more since $|\varepsilon_0(z)|$ increases with increasing distance from the critical surface and becomes comparable with the term $\varepsilon_n(A_c^2)$ in the dielectric constant ε_r of the plasma.

Confining ourselves for simplicity to the case $\alpha = 1$, we write the equation for the width of the beam:

$$\left(\frac{dl}{dz}\right)^2+\mathscr{E}\left(\frac{l}{l_0}\right)^2+\frac{\bar{\lambda}^2}{2l_0^2N_m^2}\left(\frac{l_0}{l}\right)^{2/3}=\frac{2}{3}. \tag{1.9}$$

The effect of interest to us is due to inclusion in the function Φ of a small correction ε_0/N_z^2. This introduces in the right-hand side of (1.9) small nonconservative terms that produce small changes in $\mathscr{E}$, which should be positive for a self-focusing beam. When $\mathscr{E}$ becomes negative, the beam becomes defocused and is reflected.

Averaging the nonconservative increments in Eq. (1.9) and using Eqs. (1.8), we arrive at the following equation for $\mathscr{E}(z)$:

$$\frac{d}{dz}\mathscr{E}^{2/3}=-\frac{1}{2LN_m^2}.$$

It follows therefore that with increasing penetration of the wave beam into the plasma, $\mathscr{E}$ decreases like

$$\mathscr{E}(z)=[C-(z/2LN_m^2)]^{3/2},$$

where C is a number of the order of unity. The decrease of $\mathscr{E}$ leads to an increase of the amplitude of the oscillations of the beam width $l_{\max}$ and of the self-focusing length z_f:

$$l_{\max}\approx l_0(2/3\mathscr{E})^{1/2},\quad z_f=\pi l_0/2\mathscr{E}^{1/2},$$

which reaches values $z_f \sim l_{\max} \sim LN_m^2$ at the boundary of the region $z \sim 2LN_m^2$. In that region, however, a correct description of the nonlinear evolution of the beam calls for consideration of the complete system of equations (1.4). We note only that the wave beam can penetrate deeper into the plasma only to a distance of the order of LN_m^2.

3) An interesting propagation regime sets in for a sufficiently broad beam of moderate intensity as it approaches the region of reflection of small-amplitude waves. Let $\varepsilon_0(z) = -z/L$ near the cutoff surface. The characteristic values of the dielectric constant and of the size of the region where the reflected wave is formed are then, for small-amplitude waves,

$$\varepsilon_m=1.8(\bar{\lambda}/L)^{2/3},\quad \Delta z_r=1.8(L\bar{\lambda}^2)^{1/3}.$$

If the energy density in the beam ensures that the nonlinear part $\varepsilon_n(A_c^2)$ of the dielectric constant exceeds (not strongly) ε_m, a broad ($l_0 \gg \Delta z_r$) wave beam penetrates farther, into the denser plasma, in the quasi-one-dimensional propagation regime, when the plasma transparency is due in the case of slow beam narrowing to the increase of the beam field as a result of the strong group deceleration of the wave packet. The criterion for the field amplitude follows from the condition

$$(A_c/E_p)^{2\alpha}\gtrsim\varepsilon_m\sim N_z^2.$$

From this, taking into account the relation $A_c^2 = N_0l_0^2E_m^2/l^2|N_z|$ we obtain

$$N_0E_m^2/E_p^2\gtrsim 3(\bar{\lambda}/L)^{1/\mu}.$$

In particular, for a transverse wave incident from vacuum we have a lower bound on the energy flux, $S \gtrsim (2c\bar{\lambda}/8\pi L)E_p^2$. The diffraction spreading of the collimated beam in the region where it is transported to the cutoff surface is then negligible if $l_0 > (L\lambda)^{1/2}$.

Analytic formulas that describe the propagation of the beam in the plasma under the indicated conditions are simplest for a linear profile of the unperturbed plasma density in the region of interest to us, when $\varepsilon_0 = -z/L$. In this case the refractive index is obtained from the equation

$$N_z^2=-z/L+(N_0l_0^2E_m^2/|N_z|l^2E_p^2)^{\alpha}.$$

Now, in contrast to the regime 2), the beam in the region where $|\varepsilon_0| < \varepsilon_n$ is only barely narrower, and the refractive index remains at the level $N_z \approx N_m$. The passage of the wave beam into a denser plasma $z > z_m = LN_m^2$ is due to balancing of the linear and nonlinear parts of dielectric constant. In this case $\Phi \gg 1$ and the refractive index is determined by the formula

$$N_z\approx(N_ml_0^2/l^2)(z_m/z)^{1/\alpha},$$

while the beam width $l(z)$ satisfies the equation

$$l\frac{d^2l}{dz^2}-3\left(\frac{dl}{dz}\right)^2-\frac{l}{\alpha z}\frac{dl}{dz}+2=\frac{2L\bar{\lambda}^2}{\alpha zl^2}. \tag{1.10}$$

With the aid of the substitution

$$X(t)=[z/l(z)]^2,\quad t=2\ln(z/l_0)$$

Equation (1.10) is transformed into the equation of a nonlinear oscillator with negative friction

$$\frac{d^2X}{dt^2}-\frac{1}{2}\left(5+\frac{1}{\alpha}\right)\frac{dX}{dt}+\frac{1}{2}\left(3+\frac{1}{\alpha}\right)X-X^2+\frac{L\bar{\lambda}^2}{\alpha l_0^3}X^3\exp\left(-\frac{3t}{2}\right)=0 \tag{1.11}$$

with initial data

$$X_0=\left(\frac{z_m}{l_0}\right)^2,\quad \frac{dX_0}{dt_0}=X_0\left[1-X_0^{1/2}\frac{dl(z_m)}{dz_m}\right],\quad t_0=2\ln\left(\frac{z_m}{l_0}\right).$$

Leaving out the diffraction term, which is important only near the focal spot, we investigate the phase plane of the solutions of Eq. (1.11). It can be seen first of all that the oscillator trajectories have two singular points,

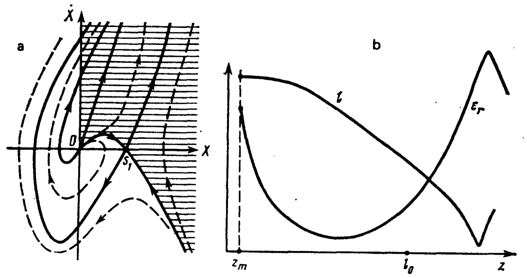

FIG. 2.

a node (0, 0) and a saddle (0, s_1), where $s_1 = (1+\alpha)/2\alpha$. For the trajectories out of the node we obtain at small X

$$(dX/dt) \approx X[1 + C_1 X + (X^{s_1} - X)/(s_1 - 1)],$$

where C_1 is the integration constant. The separatrix $dX/dt = f_s(X)$ that joins the singular point determines the limiting initial divergence of the wave beam

$$dl(z_m)/dz_m = [1 - (f_s(X_0)/X_0)] X_0^{-1/2}.$$

We note in addition that the trajectory goes off to infinity, and the representative point reaches infinity within a finite time. On the whole, the phase portrait of the solutions of Eq. (1.11) is shown in Fig. 2(a). The region of initial data at which the beam becomes self-focused is shaded.

In the regime considered, the self-focused length z_f is of the order of the initial beam width l_0 and depends little on the small parameter z_m/l_0.

The refractive index N_z has a distinctive dependence on the distance. At first, so long as the beam narrowing is negligible, the index decreases with increasing distance in proportion to $(1/z)^{1/\alpha}$. Then, past the point at which the condition $dl/dz = -l/2\alpha z$ is satisfied, N_z increases rapidly as the focus is approached, owing to the faster rate of the decrease of the beam width $l(z)$.

We call attention to the following important peculiarities of this self-focusing regime. First, as can be easily seen from Eq. (1.11), at any $\alpha > 0$ the diffraction stops the contraction of the beam at a certain l_{min}. Second, an analysis of the solutions of Eq. (1.11) shows that in this self-focusing regime the number of the foci is finite. After one or several contractions, the beam becomes defocused and its reflection must be considered. Finally, the parameter

$$k_z(z) l(z) = (l_0^2 N_m / \lambda l)(z_m/z)^{1/\alpha}$$

of the quasiclassical theory increases when the beam is self-focused and decreases when it is defocused; it also decreases monotonically in the direction of beam motion because of the increase of $|\varepsilon_0|$.

The behavior of the dimensionless wave-beam width $f = l(z)/l_0$ and of the plasma dielectric constant $\varepsilon_r(A_c^2)$ are shown schematically in Fig. 2(b).

2. ABSORPTION OF THE ENERGY OF A SELF-FOCUSING WAVE BEAM

We consider the absorption of the energy of a self-focusing wave beam in an inhomogeneous plasma. Since the collisional absorption is most effective at small group velocities of the waves, it suffices to investigate the singularities of the energy dissipation in the second and third self-focusing regimes. To simplify the formulas, we confine ourselves here to cubic nonlinearity.

We write first, using (1.4), an expression for the power $P(z)$ carried by the wave beam along the propagation path

$$P(z) = P_0 e^{-q}, \qquad q(z) = \int_{z_0}^{z} |\varkappa_z| dz. \tag{2.1}$$

The energy-absorption efficiency will be characterized by an absorption coefficient Q, and it is natural to write it in the form $Q = 1 - \exp(-q)$. The absorption increases thus with increasing q and is appreciable at $q \gtrsim 1$; for example, putting $q = 1.5$ we obtain $Q \approx 0.777$.

We consider the second self-focusing regime of the wave beam. With the aid of (1.8) and (2.1) we calculate the increment of the function q over the self-focusing length z_f:

$$\delta q = \frac{l_0 |\varepsilon_i|}{2\lambda N_m} \left(\frac{3\pi}{2}\right)^{1/2} \left(\frac{2}{3\mathscr{E}}\right)^{5/6} \frac{\Gamma(5/6)}{\Gamma(4/3)}. \tag{2.2}$$

Next, integrating (2.2) over the entire region where the second regime is realized, with account taken of the $\mathscr{E}(z)$ dependence, we obtain the total value of q in the self-focusing regime

$$q = \frac{2L}{\lambda} |\varepsilon_i| \left(\frac{N_0 E_m^2}{E_p^2}\right)^{1/3}. \tag{2.3}$$

Comparing (2.3) with the result $q_0 = 2L|\varepsilon_i|/\lambda$ of the linear theory, we see that the contribution of the plasma region $|\varepsilon_0| < N_m^2$ in which the second regime of self-focusing of the wave beam is realized is small in terms of the parameter N_m. It should be noted, however, that for transverse waves the joint contribution of the beam-propagation region $\varepsilon_0(z) > -N_m^2$ in which the first and second self-focusing regimes are possible is barely smaller than the linear contribution, i.e., $q \approx q_0$. This curious fact is due to competition between two tendencies. On the one hand, the nonlinearity, by increasing the group velocity of the waves, decreases the spatial damping decrement, and on the other it increases the depth of penetration of the beam into the plasma that is opaque to low-amplitude waves. On the whole, however, the result is found to be independent of the nonlinearity. Of course, at very high energy densities in the wave beam, corresponding to $E_m^2 \gtrsim E_p^2$, the collisional absorption is much lower than in the linear case.

A much larger increase of the collisional absorption of the wave-beam energy is obtained in the third self-focusing regime for broad beams. It was established above that this regime is realized at a beam width $l_0 \gg \Delta z_r$ and at energy densities $N_0 E_m^2/E_p^2 \gtrsim \varepsilon_m^{(3/2)\mu}$. In this beam-propagation regime, the total collisional absorption contains two contributions. The first takes place in the region $\varepsilon_0(z) < 0$. It is determined from the equations of the linear theory and corresponds to $\Delta q_1 \approx q_0$. The additional contribution is made by the region $\varepsilon_0(z) < 0$, and can substantially exceed the linear contribution, i.e., $\Delta q_2 > q_0$. Let us calculate it over the self-focusing length z_f. Substituting in (2.1) the refractive index in the form corresponding to the third self-focusing regime, we write the result in the form $\Delta q_2 = q_0\chi J$, where

$$\xi = z/l_0, \quad f(\xi) = l(z)/l_0, \quad J = \int_0^{\xi_f} d\xi f^2(\xi)\xi^{1/\alpha}.$$

Since $\xi_f \sim 1$, the factor J does not contain large parameters and is of the order of unity. For example, for $l_0 \approx 20z_m$ and cubic nonlinearity we have $\xi_f \approx 1.42$ and $J \approx 0.3$. Consequently, an appreciable excess of Δq_2 over q_0 can be reached only on account of the factor χ, which is equal to

$$\chi \approx 0.67(\lambda/L)^{1/3}(l_0/\Delta z_r)^{(1+\alpha)/\alpha}(\varepsilon_m/N_m^2)^{3/2\mu},$$

here $N_m^2 \gtrsim \varepsilon_m$. Stipulating $\Delta q_2 > \Lambda q_0$, where Λ is a certain number that determines the desired value of the collisional absorption

$$Q = 1 - \exp[-q_0(1+\Lambda)],$$

we obtain the following condition for the width of the wave beam:

$$\frac{l_0}{\Delta z_r} > \left[\frac{3\Lambda}{2J}\left(\frac{L}{\lambda}\right)^{1/3}\left(\frac{N_m^2}{\varepsilon_m}\right)^{3/2\mu}\right]^{\alpha/(1+\alpha)} . \qquad (2.4)$$

In the case of cubic nonlinearity, taking $\Lambda = 9$ from (2.4), we have

$$(l_0/\Delta z_r) > 3(3/2J)^{1/2}(L/\lambda)^{1/6}(N_m^2/\varepsilon_m)^{3/4},$$

which can be satisfied at $l_0 < L$.

3. CONCLUSION

We have thus investigated above, in the case of static local nonlinearity, the features of stationary self-focusing of intense wave beams and the collisional absorption of their energy in an inhomogeneous plasma. Of course, the analysis presented does not cover all the aspects of nonlinear dynamics of wave beams in an inhomogeneous plasma. In particular, a self-consistent allowance for the parametric and modulational instabilities in the nonstationary problem calls for a special treatment.

We discuss now the conditions for the realization of a scalar model for the investigation of self-focusing of wave beams. We consider first Langmuir oscillations. We write down the known equation for the complex amplitude of the high-frequency potential of a wave packet

$$(2i/\omega)\cdot\frac{\partial}{\partial t}\Delta\Psi + \operatorname{div}\left[\left(\frac{\delta n}{n_c} - \varepsilon_0\right)\nabla\Psi\right] - 3\lambda_{De}^2\Delta\Delta\Psi = 0,$$

where δn is the perturbation of the plasma density. It is seen thus that in the stationary case the component E_z of the electric field of the wave beam satisfies Eq. (1.1).

For transverse waves, the beam field satisfies the vector equation

$$\frac{2i}{\omega}\frac{\partial}{\partial t}\mathbf{E} = \left(\varepsilon_0 - \frac{\delta n}{n_c}\right)\mathbf{E} + \frac{c^2}{\omega^2}(\Delta\mathbf{E} - \nabla \operatorname{div}\mathbf{E}).$$

The problem becomes scalar if the interaction of the polarizations can be neglected; this can be done if $\varepsilon_n \ll \varepsilon_0$. If the inverse condition $\varepsilon_n \gtrsim \varepsilon_0$ is satisfied, the term with div $\mathbf{E}$ is generally speaking not small and the vector problem must be solved. In the case of thick beams of width $b \gg 1/k_z$, the coupling of the polarizations is weak as before. In addition, an analysis of the self-focusing instability at $|\varepsilon_0| \ll \varepsilon_n$ shows that allowance for div $\mathbf{E}$ leads only to small corrections. This allows us to state that the scalar problem describes correctly the character of the self-focusing of beams of transverse waves in the region $\varepsilon_n \gtrsim \varepsilon_0$.

It should be noted that for intense wave beams with electric-field amplitudes $E \gtrsim m_e c\omega/e$ an important role in the penetration beyond the cutoff surface $n = n_c$ is played by the static nonlinearity due to the relativistic dependence of the mass m_c of the carriers (in this case, electrons) on the beam field.

The authors thank S. S. Moiseev for helpful discussions.

[1]Vzaimodeĭstvie sil'nykh élektromagnitnykh voln s besstolknovitel'noĭ plazmoĭ (Interaction of Strong Electromagnetic Waves with a Collisionless Plasma), Collected articles, Gor'kiĭ, Inst. Appl. Phys., 1980.
[2]R. Z. Sagdeev, G. I. Solov'ev, V. D. Shapiro, V. I. Shevchenko, and I. U. Yusupov, Zh. Eksp. Teor. Fiz. **82**, 125 (1982) [Sov. Phys. JETP **55**, 74 (1982)].
[3]N. S. Erokhin, S. S. Moiseev, V. V. Mukhin, V. E. Novikov, and R. Z. Sagdeev, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 451 (1981) [JETP Lett. **33**, 434 (1981)].
[4]N. S. Erokhin, S. S. Moiseev, V. V. Mukhin, V. E. Novikov, and R. Z. Sagdeev, Tenth Europ. Internat. Conf. on Contr. Nucl. Fusion, Moscow, 1981, Report F-1.
[5]V. P. Maslov, Kompleksnyĭ method VKB v nelineĭnykh uravneniyakh (Complex WKB Method in Nonlinear Equations), Nauka, 1977, Chap. 8.
[6]N. S. Erokhin, S. S. Moiseev, and V. E. Novikov, Zh. Tekh. Fiz. **48**, 1769 (1978) [Sov. Phys. Tech. Phys. **23**, 1007 (1978)].
[7]V. I. Talanov, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 303 (1970) [JETP Lett. **11**, 199 (1970)].
[8]V. E. Zakharov, V. V. Sobolev, and V. S. Synakh, Zh. Eksp. Teor. Fiz. **60**, 136 (1971) [Sov. Phys. JETP **33**, 77 (1981)].

Translated by J. G. Adashko