Ion-acoustic plasma turbulence

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A theory is developed of the nonlinear state that is established in a plasma as a result of development of ion-acoustic instability. Account is taken simultaneously of the linear induced scattering of the waves by the ions and of the quasilinear relaxation of the electrons by the ion-acoustic pulsations. The distribution of the ion-acoustic turbulence in frequency and in angle is obtained. An Ohm's law is established and expressions are obtained for the electronic heat flux and for the relaxation time of the electron temperature in a turbulent plasma. Anomalously large absorption and scattering of the electromagnetic waves by the ion-acoustic pulsations is predicted.

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1. INTRODUCTION

The theory of ion-acoustic turbulence (IAT) of a plasma has been developing for many years. Its present state of development, however, is still conceded to be unsatisfactory (see, e.g., Ref. 1). In 1963–1964 Kadomtsev and Petviashvili formulated for a plasma in the absence of a magnetic field an IAT model wherein the mechanism of the nonlinear saturation was assumed to be induced scattering of the waves by ions.[2,3] They have formulated a distribution law for the IAT fluctuations as a function of the absolute value of the wave vector. The qualitative experimental confirmation of this distribution notwithstanding, a large quantitative disparity was obtained, for the Kadomtsev-Petviashvili (K-P) spectrum yielded an excessively high intensity of the IAT pulsations. This suggested the need for decreasing the absolute value of such pulsations by two orders of magnitude.[4] It must be emphasized, however, that it remained uncertain for a long time whether the K-P spectrum actually conforms to the model in Petviashvili's and Kadomtsev's papers,[2,3] since it remained doubtful whether the integral kinetic equation for the waves, which is basic for this model, has a solution. At the same time, the fact that the K-P model had not been sufficiently investigated gave grounds for hoping that the absolute values of the turbulent pulsations can be decreased. These hopes were dashed after the solution of the kinetic equation for the waves in the K-P model was found (see Ref. 5) and confirmed that the pulsation level in the K-P model is excessively high.

In a search for an IAT model capable improving substantially on the K-P model, Sizonenko and Stepanov[6] have advanced an important hypothesis, which is confirmed in the present paper, that the IAT pulsation level should be determined by the scattering of the electrons from these pulsations. It can be said that this hypothesis corresponds to some degree to a quasilinear theory (see Refs. 7–10). To be sure, it must be immediately emphasized that, in a quasilinear theory that takes into account only the interaction of the waves with the particles and neglects the nonlinear wave interaction, the IAT spectrum is found to be concentrated near a definite value of the wave vector, a fact at variance with the experiment. Our paper shows, however, that the results of the quasilinear theory are still methodologically useful when it comes to determining the angular distribution.

Our approach to the IAT theory consists of simultaneously taking into account both the nonlinear induced scattering of the waves by the ions, on which the K-P model is based, and the quasilinear relaxation of the electrons on the ion-acoustic pulsations.[7] We consider here a stationary state of the IAT of a plasma in the absence of a magnetic field, neglecting, as our predecessors, the change of the ion distribution function and the possible onset of runaway electrons. We show within the framework of these assumptions that the Kadomtsev-Petviashvili spectrum is realized in our IAT model. The absolute value of the pulsations, however, turns out to be considerably lower, so that our use of the premises of the weak-turbulence theory can be regarded as justified. Next, in contrast to the frequency spectrum, the IAT angular spectrum is found to be close to that obtained from the quasilinear theory of the IAT, and above all to Rudakov's basic result of this theory,[7] that a sharply directional angular distribution sets in. The singularity of the spectrum of Ref. 7 was eliminated by Kovryzhnykh,[8] who took into account the Čerenkov absorption of the waves by the ions; this is realized in our theory either in the case of a weakly nonisothermal plasma, or at a small excess above threshold. It must be stated that from among the works of our predecessors the paper richest in ideas and closest to our views is that of Kingsep,[9] who undertook the task of eliminating the spectrum singularities of Rudakov's paper[7] by taking into account the induced scattering of the waves by the ions. He, however, adhered to the conclusion that the IAT has a δ-function spectrum in k space, and did not deduce the existence of the K-P spectrum. At the same time, as can be stated on the basis of our article, his regularization of the angular dependence of the turbulence spectrum turns out to be accurate. The reason was an approximation not corresponding to the known and recognized ideas of Refs. 1–4 and 11 concerning the description of the scattering of ion-acoustic waves by ions in a nonisothermal plasma.

Besides the IAT spectrum we derive below an Ohm's law, an expression for the electronic heat flux, as well

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as an expression for the electron-temperature relaxation time. These results eliminate the long-standing contradiction between the theory and the experimental results, a splendid description of the latter being given in the book by Volkov *et al.*[12] Namely, as confirmed by experiment, first, the turbulence spectrum and the K-P spectrum have similar dependences on the absolute value of the wave vector; second, the absolute value of the IAT also agrees with our theory. Third and last, the saturation current coincides with that obtained by us and exceeds the corresponding prediction of the quasilinear theory.[8] Our theory establishes the value of the limiting electronic heat flux, the existence of which was suggested in Ref. 13 and agrees with many experiments (see, e.g., Ref. 14). Furthermore, this limiting value is larger than that obtained in the quasilinear theory.[10] Finally, our theory makes it possible, under the IAT conditions considered by us, to predict an anomalously larger radiation absorption in a plasma, due to transformation of the electromagnetic waves into longitudinal perturbations by the ion-acoustic pulsations. It predicts also an anomalously large Brillouin scattering.

2. INITIAL GENERAL EQUATIONS

For a theory of turbulence with relatively high noise level that causes the mean free path of the electron to be small compared with the characteristic spatial dimension of the particle-distribution inhomogeneity, we can use the usual Chapman-Enskog approach, which allows us to describe the kinetic equation for electrons in the form

$$f_0\left(\frac{\nabla n_e \varkappa T_e}{n_e \varkappa T_e}-\frac{e\mathbf{E}}{\varkappa T_e}+\frac{\nabla T_e}{T_e}\left[\frac{m_e v^2}{2\varkappa T_e}-\frac{5}{2}\right],\mathbf{v}\right)=\left[\frac{\partial f}{\partial t}\right]_{st}. \tag{2.1}$$

Here f_0 is the equilibrium Maxwellian electron distribution functions, from which the nonequilibrium distribution f deviates little; n_e, T_e, e, and m_e are the density of the number of electrons, their temperature, charge, and mass, and $\mathbf{E}$ is the intensity of the constant electric field.

In the turbulent state, the main effect that characterizes the electron scattering is scattering by ion-acoustic fluctuations. We therefore represent the right-hand side of the equation in (2.1) in the form

$$\left[\frac{\partial f}{\partial t}\right]_{st}=\frac{\partial}{\partial v_i}D_{ij}\frac{\partial f}{\partial v_j}+\mathcal{J}_{st}(f). \tag{2.2}$$

Here $\mathcal{J}_{st}$ is the usual Landau collision integral; D_{ij} is the coefficient of quasilinear diffusion and is given by

$$D_{ij}=\frac{e^2}{2\pi m_e^2}\int d\mathbf{k}\frac{k_i k_j}{k^2}\frac{\omega_s^3}{\omega_{Li}^2}N(\mathbf{k})\delta(\omega_s-\mathbf{kv}), \tag{2.3}$$

where ω_{Li} is the ion Langmuir frequency, $\omega_s(k)$ is the frequency of the ion sound, and $N(\mathbf{k})$ is the number of ion-acoustic quanta multiplied by Planck's constant $\hbar$. To obtain approximate results, we confine ourselves here only to ordinary collisions. We allow therefore only for electron-ion collisions, which will enable us also to assess the quantitative results in the case of a high degree of ionization $z=|e_i/e|\gg 1$.

We seek the distribution of the ion-acoustic fluctuations in an approximation that neglects their dependence on the time and on the spatial coordinates. This means that our analysis will be suitable under conditions when the characteristic time of change of the number of quanta is long compared with their free-path time, and the region of their spatial change is large compared with the mean free path of the ion-acoustic quanta. The kinetic equation for $N(\mathbf{k})$ can then be written in the form

$$N(\mathbf{k})=-W_0(k)/\Gamma_{NL}(\mathbf{k}), \tag{2.4}$$

where W_0 characterizes the level of the spontaneous thermal fluctuations, and the nonlinear growth rate

$$\Gamma_{NL}(\mathbf{k})=\gamma(\mathbf{k})+\gamma_i(k)+\gamma_{NL}(\mathbf{k}) \tag{2.5}$$

is made up, first, of the electronic growth rate

$$\gamma(\mathbf{k})=\frac{2\pi^2 e^2}{m_e k^2}\frac{\omega_s^3}{\omega_{Li}^2}\int d\mathbf{v}\delta(\omega_s-\mathbf{kv})\mathbf{k}\frac{\partial f}{\partial \mathbf{v}}, \tag{2.6}$$

which is determined by the nonequilibrium distribution function of the electrons, and second, of the damping decrement of the ion sound

$$\gamma_i=-\frac{4}{5}\frac{k^2 v_{Ti}^2\nu_{ii}}{\omega_s^2}-\left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_s^4}{k^3 v_{Ti}^3}\exp\left(-\frac{\omega_s^2}{2k^2 v_{Ti}^2}\right), \tag{2.7}$$

which is due to the ion-ion collisions and the Čerenkov interaction of the ions with the waves, in an approximation in which the deviation of the ion distribution from Maxwellian is small; third and final, of the nonlinear ion-acoustic wave decrement due to the induced scattering of the waves by the ions:

$$\gamma_{NL}(\mathbf{k})=\frac{\pi v_{Ti}^4 k^2}{(\partial\omega_s/\partial k)}\frac{\partial}{\partial k}\left\{\frac{k^2}{(\partial\omega_s/\partial k)}\int\frac{d\mathbf{k}'}{(2\pi)^3}\delta(k-k')[\boldsymbol{\varkappa}\times\boldsymbol{\varkappa}']^2(\boldsymbol{\varkappa}\boldsymbol{\varkappa}')^2\frac{N(\mathbf{k}')}{n_i\varkappa T_i}\right\}. \tag{2.8}$$

Here $v_{Ti}=(\varkappa T_i/m_i)^{1/2}$ is the thermal velocity of the ions, T_i and n_i are the temperature and the ion-number density, ν_{ii} is the frequency of the ion-ion collisions, $\boldsymbol{\varkappa}=\mathbf{k}/k$, and $\boldsymbol{\varkappa}'=\mathbf{k}'/k'$.

Within the limit of the specified electron distribution function and when account is taken of the induced scattering, Eq. (2.4) is the basis of the Kadomtsev-Petviashvili theory of ion-acoustic turbulence, which has made it possible to establish in this case the distributions of the fluctuations in frequency[2,3] and in angle.[5] On the contrary, if the nonlinear decrement (2.8) is neglected, Eq. (2.4) jointly with the kinetic equation (2.1)–(2.3) is the basis of the quasilinear theory of the ion-acoustic turbulences,[7-10] which leads to qualitatively different results compared with the predictions of the nonlinear theory.[2,3,5] In our analysis we shall make known simultaneously the roles of both the quasilinear and nonlinear effects.

We shall be interested below in a plasma state such that the level of the ion-acoustic fluctuations greatly exceeds the thermal level. This allows us to separate in the wave-vector space a nonturbulent region in which $N(\mathbf{k})$ differs little from the thermal value and in which we can assume approximately $N(\mathbf{k})=0$. The condition

$$\Gamma_{NL}(\mathbf{k})<0 \tag{2.9}$$

should be satisfied in this region. In the turbulent region, on the contrary, $N(\mathbf{k})$ is much larger than the thermal value, therefore Eq. (2.4) can be written in the form

$$\Gamma_{NL}(\mathbf{k})=0. \qquad (2.10)$$

3. SOLUTION OF KINETIC EQUATION OF THE ELECTRONS. INTEGRAL EQUATION FOR ION SOUND AND THE KADOMTSEV-PETVIASHVILI SPECTRUM

We proceed in this section to obtain the consequences of Eqs. (2.1)–(2.10) as applied to the case of axially-symmetric distributions when, for example, the spatial gradients and the electric field in the right-hand side of (2.1) are oriented along one direction, which we assume to be that of the z axis. We can then put

$$N(\mathbf{k})=N(k,\cos\theta_k) \text{ and } f(\mathbf{v})=f_0(v)\left[1+\psi(v,\cos\theta_v)\right],$$

where θ_k and θ_v are respectively the angles between the z axis and the vectors $\mathbf{k}$ and $\mathbf{v}$. In this case

$$\mathcal{I}_{st}(f)=\nu_{st}f_0(v)\frac{v_{Te}^3}{v^3}\frac{\partial}{\partial\cos\theta}\sin^2\theta\frac{\partial\psi}{\partial\cos\theta}, \quad \nu_{st}=\frac{2\pi ze^4n_e\Lambda}{m_e^2v_{Te}^3}, \qquad (3.1)$$

where $v_{Te}=(\varkappa_eT/m_e)^{1/2}$ and Λ is the Coulomb logarithm.

The quasilinear diffusion operator takes in velocity space the form

$$\frac{\partial}{\partial v_i}D_{ij}\frac{\partial}{\partial v_j}=\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\left(D_{vv}\frac{\partial}{\partial v}+\frac{1}{v}D_{v\theta}\frac{\partial}{\partial\theta}\right)\right]$$
$$+\frac{1}{v\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(D_{v\theta}\frac{\partial}{\partial v}+\frac{1}{v}D_{\theta\theta}\frac{\partial}{\partial\theta}\right)\right], \qquad (3.2)$$

where

$$D_{\alpha\beta}(v,\cos\theta_v)=\frac{v_{Te}^4}{v}\int\sin\theta_kd\theta_k\int\frac{k^3dk}{(2\pi)^2}$$
$$\times\frac{\omega_sN(k,\cos\theta_k)d_{\alpha\beta}(\omega_s/kv)}{n_e\varkappa T_e[\sin^2\theta_v\sin^2\theta_k-(\omega_s/kv-\cos\theta_v\cos\theta_k)^2]^{1/2}}, \qquad (3.3)$$

$$d_{vv}(x)=x^2,\quad d_{v\theta}(x)=\frac{x(x\cos\theta_v-\cos\theta_k)}{\sin\theta_v},\quad d_{\theta\theta}(x)=\frac{(x\cos\theta_v-\cos\theta_k)^2}{\sin^2\theta_v},$$

and the integration in wave-spectrum space is over the region

$$\sin^2\theta_v\sin^2\theta_k\geqslant[\omega_s/kv-\cos\theta_v\cos\theta_k]^2.$$

Taking into account the smallness of the phase velocity of the ion sound compared with the thermal velocity of the electrons, we have $D_{\theta\theta}\sim(m_i/m_e)^{1/2}D_{v\theta}\sim(m_i/m_e)D_{vv}$. This enables us now to write the kinetic equation for the electrons in the form

$$\cos\theta_v\left\{\frac{eE_z}{\varkappa T_e}-\frac{\nabla_zn_eT_e}{n_eT_e}-\left[\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right]\nabla_z\ln T_e\right\}$$
$$=\frac{v_{Te}^3}{v^4}\frac{\partial}{\partial\cos\theta_v}\left\{\frac{v_sv}{v_{Te}^2}\sin\theta_v\nu_1(\theta_v)-\sin^2\theta_v[\nu_{st}+\nu_2(\theta_v)]\frac{\partial\psi}{\partial\cos\theta_v}\right\}, \qquad (3.4)$$

where we have used the assumption that the main contribution to the quasilinear diffusion operator is made by the ion-sound waves with $kr_{De}<1$, when $\omega_s=kv_s$ (r_{De} is the electron Debye radius and v_s is the ion-sound velocity), and used also the following notation for the angle-dependent turbulent collision frequencies ($n=1,2$)

$$\nu_n(\theta)=v_{Te}\int\frac{k^3dk}{(2\pi)^2}\int_{-\sin\theta}^{\sin\theta}d(\cos\theta_k)\frac{\omega_sN(k,\cos\theta_k)}{n_e\varkappa T_e}\left(\frac{\cos\theta_k}{\sin\theta}\right)^n\frac{1}{(\sin^2\theta-\cos^2\theta_k)^{1/2}}. \qquad (3.5)$$

Integration of Eq. (3.4) (cf. Ref. 7), which satisfies the condition of regularity at $\theta_v=0$, yields

$$\frac{\partial\psi}{\partial\cos\theta_v}=\frac{v^4}{2v_{Te}^3}\frac{1}{\nu_{st}+\nu_2(\theta_v)}\left\{\frac{eE_z}{\varkappa T_e}-\frac{\nabla_zn_eT_e}{n_eT_e}\right.$$
$$\left.-\left[\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right]\nabla_z\ln T_e\right\}+\frac{v_sv}{v_{Te}^2\sin\theta_v}\frac{\nu_1(\theta_v)}{\nu_{st}+\nu_2(\theta_v)}. \qquad (3.6)$$

Bearing it in mind that the electronic growth rate is given by

$$\gamma(k,\theta_k)=-\gamma_s(k)+\frac{\pi\omega_sv_{Te}^2}{n_e}\cos\theta_k\int_0^\infty f_0\,dv$$
$$\times\int_{-\sin\theta_k}^{\sin\theta_k}\frac{d(\cos\theta_v)}{(\sin^2\theta_k-\cos^2\theta_v)^{1/2}}\frac{\partial\psi}{\partial\cos\theta_v}, \qquad (3.7)$$

it is clear that Eq. (3.6) suffices to describe the contribution made by the electrons to Γ_{NL}. Here

$$\gamma_s=\left(\frac{\pi}{8}\right)^{1/2}\omega_{Li}\omega_s/\omega_{Le}. \qquad (3.8)$$

Equation (2.10) now takes the form

$$\Gamma_{NL}=-\gamma_s(k)\left\{1+\delta+\delta_1-\frac{2}{\pi}\cos\theta_k\int_0^{\sin\theta_k}\frac{d(\cos\theta_v)}{(\sin^2\theta_k-\cos^2\theta_v)^{1/2}}\right.$$
$$\times\frac{\nu_0+\sin^{-1}\theta_v\nu_1(\theta_v)}{\nu_{st}+\nu_2(\theta_v)}-\frac{k^2v_{Ti}^2}{\gamma_s}\frac{\partial}{\partial k}\left[k^4\int_{-1}^{1}d(\cos\theta_k')\right.$$
$$\left.\left.\times Q(\sin\theta_k,\cos\theta_k')\frac{N(k,\cos\theta_k')}{4\pi n_e\varkappa T_e}\right]\right\}=0. \qquad (3.9)$$

The kernel of the nonlinear interaction is defined here by the formula[5]

$$Q(s,t)=(t^2-t^4)+{}^1/_2s^2(1-9t^2+10t^4)+{}^1/_8s^4(-3+30t^2-35t^4). \qquad (3.10)$$

and we use here the notation

$$\delta=\frac{\omega_{Le}}{\omega_{Li}}\left(\frac{r_{De}}{r_{Di}}\right)^3\exp\left(-\frac{r_{De}^2}{2r_{Di}^2}-\frac{3}{2}\right),\delta_1=\frac{8}{5}\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{r_{Di}}{r_{De}}\right)^2\frac{\nu_{ii}\omega_L}{\omega_{Li}^2}\frac{1}{kr_{De}}, \qquad (3.11)$$

$$\nu_0=\left(\frac{9\pi}{8}\right)^{1/2}\frac{v_{Te}^2}{v_s}\left[\frac{eE_z}{\varkappa T_e}-\frac{\nabla_zn_eT_e}{n_eT_e}\right]. \qquad (3.12)$$

Bearing Eq. (3.5) in mind, we see that Eq. (3.9) is a nonlinear integral equation that defines the function $N(k,\cos\theta_k)$. We proceed to solve this equation after first confining ourselves, as did Petviashvili,[2] to not too long waves, when δ_1 can be neglected. It is then possible to separate the variables in (3.9):

$$N(k,\cos\theta_k)=N(k)\Phi(\cos\theta_k). \qquad (3.13)$$

In this case

$$N(k)=\frac{4\pi n_e\varkappa T_e}{v_{Ti}^2}\frac{\gamma_s(k)}{k^5}\ln\frac{1}{kr_{De}} \qquad (3.14)$$

corresponds to the Kadomtsev-Petviashvili spectrum. We see thus that a consistent allowance for the induced scattering by the ions leads to a qualitative change of the spectrum of the ion-acoustic fluctuations, compared with the δ-function distribution in frequency, used in the quasilinear theory.

4. EQUATIONS FOR THE ANGULAR DISTRIBUTION OF THE ION-ACOUSTIC TURBULENCE, AND THE CASE OF WEAK INFLUENCE OF THE NONLINEARITY ON THE ANGULAR DISTRIBUTION

By substituting (3.13) and (3.14) in (3.5) and in (3.9), under conditions when δ_1 can be neglected, we obtain the following integral equation for the function $\Phi(\cos\theta_k)$

$$1+\delta+\int_{-1}^{1} d(\cos\theta_k')Q(\sin\theta_k,\cos\theta_k')\Phi(\cos\theta_k')$$

$$=\frac{2}{\pi}\cos\theta_k\int_{0}^{\sin\theta_k}\frac{d(\cos\theta_\nu)}{(\sin^2\theta_k-\cos^2\theta_\nu)^{1/2}}\frac{\nu_0+\sin^{-1}\theta_\nu\nu_N\chi_1(\sin\theta_\nu)}{\nu_{st}+\nu_N\chi_2(\sin\theta_\nu)}. \tag{4.1}$$

Here

$$\nu_N=v_{Te}\int\frac{k^2dk}{(2\pi)^2}\frac{\omega_s N(k)}{n_e\varkappa T_e}=\frac{\omega_{Li}}{\sqrt{8\pi}}\frac{r_{De}^2}{r_{Di}^2}, \tag{4.2}$$

$$\chi_n(x)=\int_{-x}^{x}\frac{dt}{(x^2-t^2)^{1/2}}\left(\frac{t}{x}\right)^n\Phi(t). \tag{4.3}$$

With the integral operator in the left-hand side of (4.1) neglected, this equation was investigated in Ref. 8, devoted to the quasilinear theory of ion-acoustic turbulence.

We direct the z axis such that ν_0 is positive. It turns out then, since Q, Φ, and χ_n are nonnegative, that the inequality (2.9) holds at $\cos\theta_k<0$. Therefore the region of ion-sound turbulence is possible only if $\cos\theta_k>0$. Following Ref. 8, we assume that the turbulence region is located only inside a cone with apex angle θ_0. In other words, $\Phi(x)\neq 0$ at $1\geq\cos\theta_k=x\geq\cos\theta_0=x_0$. This assumption does not contradict relations (2.9) and (2.10) if at $\cos\theta>\cos\theta_0$ the function

$$w(t)=\left(1+\frac{\chi_1(\sqrt{1-t^2})}{\sqrt{1-t^2}K_N}\right)\Big/\left(\frac{1}{K_{st}}+\frac{1}{K_N}\chi_2(\sqrt{1-t^2})\right) \tag{4.4}$$

satisfies the following Abel integral equation:

$$\int_0^{\sin\theta}\frac{dt\,w(t)}{(\sin^2\theta-t^2)^{1/2}}=\frac{\pi}{2\cos\theta}\left\{1+\delta+\int_{\cos\theta_0}^{1}d\xi Q(\sin\theta,\xi)\Phi(\xi)\right\}, \tag{4.5}$$

and if the following inequality

$$\int_0^{\sin\theta}\frac{dt\,w(t)}{(\sin^2\theta-t^2)^{1/2}}\leq\frac{\pi}{2\cos\theta}\left\{1+\delta+\int_{\cos\theta_0}^{1}d\xi Q(\sin\theta,\xi)\Phi(\xi)\right\} \tag{4.6}$$

is satisfied at $\cos\theta\leq\cos\theta_0$. The quantity $K_{st}=\nu_0/\nu_{st}$ in (4.4) is determined by the Knudsen number, which is connected with the ordinary collisions, while $K_N=\nu_0/\nu_N$ is characterized by the turbulent Knudsen number.

To continue the study of the angular distribution of the noise in the interval $0\leq t\leq 1$ it is convenient to introduce the function

$$w_0(t)=\frac{1+\delta}{1-t^2}+\frac{d}{dt}\int_0^t\frac{s\,ds}{[(t^2-s^2)(1-s^2)]^{1/2}}\int_{\cos\theta_0}^{1}d\xi Q(s,\xi)\Phi(\xi)$$
$$=\frac{1+\delta+A_1}{1-t^2}+A_2+A_3(1-t^2)+\frac{t}{2}\ln\frac{1+t}{1-t}[A_4-A_3(1-t^2)], \tag{4.7}$$

where the quantities $A_n(n=1,2,3,4)$:

$$A_1={}^1/_8(M_0+2M_1-3M_2),\quad A_2={}^1/_{16}(7M_0-78M_1+95M_2),$$
$$A_3={}^1/_{16}(-9M_0+90M_1-105M_2),\quad A_4=(-M_0+12M_1-15M_2) \tag{4.8}$$

are expressed in terms of $(n=0,1,2)$

$$M_n=\int_{\cos\theta_0}^{1}d\xi\xi^{2n}\Phi(\xi) \tag{4.9}$$

which are the moments of the sought angular distribution function. It is easy to verify that by assuming $w(t)=w_0(t)$ at $\cos\theta\geq\cos\theta_0$ we satisfy Eq. (4.5). Thus, $w_0(t)$ is a solution of Eq. (4.5) at $0\leq t\leq\sin\theta_0$.

It must be noted that $\chi_{1,2}(x)=0$ at $x\ll\cos\theta_0$. Therefore, according to (4.4), $w(t)=K_{st}$ at $t\geq\sin\theta_0$. We obtain thus the following expression for the function $w(t)$:

$$w(t)=w_0(t)\quad\text{at}\quad 0\leq t\leq\sin\theta_0,$$
$$w(t)=K_{st}\quad\text{at}\quad 1\geq t\geq\sin\theta_0. \tag{4.10}$$

Since $w(t)$ must be continuous at $t=\sin\theta_0$, we have the following equation for the determination of θ_0:

$$K_{st}=w_0(\sin\theta_0), \tag{4.11}$$

where $w_0(t)$ is determined by Eq. (4.7). Bearing in mind the relation (4.10), we can rewrite the integral inequality (4.6) in the form $(\cos\theta_0\geq\cos\theta)$

$$\int_{\sin\theta_0}^{\sin\theta}\frac{dt}{(\sin^2\theta-t^2)^{1/2}}[w_0(t)-K_{st}]\geq 0. \tag{4.12}$$

Taking into account the statements made above, the problem of finding the angular distribution function $\Phi(x)$ has been reduce to solution of Eqs. (4.4), (4.10) and (4.11) and of inequality (4.12).

According to (4.3), (4.4), and (4.10) the function $\Phi(t)$ satisfies the following integral equation:

$$\int_{x_0}\frac{t\Phi(t)\,dt}{(x^2-t^2)^{1/2}}(tw_0(\sqrt{1-x^2})-1)=K_Nx^2\left(1-\frac{1}{K_{st}}w_0(\sqrt{1-x^2})\right), \tag{4.13}$$

from which it is seen that the function $\Phi(t)$, generally speaking, is small if K_N is small. The smallness $K_N\ll 1$ is easily realized by virtue of the high turbulent collision frequency ν_N. The smallness of the function $\Phi(t)$, and hence of its moments (4.9), permits us under certain conditions (details follow below) to speak of the possibility of approximately neglecting the nonlinear terms in (4.13). This corresponds to the approximation corresponding to the equations

$$w_0(x)=(1+\delta)/(1-x^2), \tag{4.14}$$
$$x_0^2=(1+\delta)/K_{st}, \tag{4.15}$$

from which it follows directly, in particular, that the condition (4.12) is valid.

If we use relations (4.14) and (4.15), we obtain from (4.13) Eq. (26) of Ref. 8:

$$\frac{1+\delta}{K_N}\int_{x_0}^{x}\frac{t\Phi(t)\,dt}{(x^2-t^2)^{1/2}}\left(\frac{t}{x^2}-\frac{1}{1+\delta}\right)=x^2-x_0^2. \tag{4.16}$$

We shall show now that Eq. (4.16) can be easily solved without resorting to the cumbersome series expansion used in Ref. 8. In fact, we multiply both halves of (4.16) by $x/(s^2-x^2)^{1/2}$ and integrate with respect to x in the range from x_0 to s. As a result we obtain

$$\frac{\pi}{2K_N}\left(\frac{1+\delta}{s}-1\right)\int_{x_0}^{s}t\Phi(t)\,dt={}^2/_3(s^2-x_0^2)^{3/2}. \tag{4.17}$$

Solving Eq. (4.17), we have

$$\Phi(x)=\frac{4K_N}{3\pi x}\frac{d}{dx}\frac{x(x^2-x_0^2)^{3/2}}{1-x+\delta}. \tag{4.18}$$

At $\delta=0$, the resultant solution (4.18), which is singular at $x=1$, corresponds to the result of Ref. 7.

The solution obtained by us differs from that given in Ref. 8 [see Eq. (29) there]. It is easy to verify by direct substitution that the solution (29) of Ref. 8 does not satisfy Eq. (4.16).

Equations (3.14) and (4.18) determine completely the spectrum of the ion-acoustic turbulence in the case of a negligibly small influence of the nonlinearity on the angular distribution of the noise. We emphasize at the same time that the distribution in k is determined essentially by the nonlinear interaction of the waves. We note that as $x_0 \to 1$ the turbulence region in the space of the wave vectors becomes vanishingly small. Therefore the equality $x_0 = 1$ corresponds to the threshold of the onset of the ion-acoustic instability.

To conclude this section, we calculate the total energy of the ion-acoustic noise:

$$\frac{E^2}{4\pi} = \int \frac{d\mathbf{k}}{(2\pi)^3} \omega_s(k) N(\mathbf{k}) = \int_{k_{min}}^{r_{De}^{-1}} \frac{k^2 dk}{(2\pi)^2} \omega_s N(k) \int_{x_0}^{1} dx \Phi(x). \qquad (4.19)$$

Substituting (3.14) and (4.18) in (4.19) we obtain

$$\frac{E^2}{4\pi n_e \varkappa T_e} = \frac{1}{3\pi\sqrt{2\pi}} \left\{ (1-x_0^2)^{1/2} \left[\frac{1-x_0^2}{\delta} - \frac{3}{2} - \delta \right] + \frac{x_0^3}{1+\delta} \arccos x_0 + \left[\frac{3}{2} x_0^2 - (1+\delta)^2 \right] \ln \frac{1+(1-x_0^2)^{1/2}}{x_0} + \frac{[(1+\delta)^2 - x_0^2]^{3/2}}{1+\delta} \ln \frac{\delta x_0}{1+\delta - x_0^2 - (1-x_0^2)^{1/2}[(1+\delta)^2 - x_0^2]^{1/2}} \right\} \times K_N \frac{\omega_{Li}}{\omega_{Le}} \frac{r_{De}^2}{r_{Di}^2} \ln^2 \frac{1}{k_{min} r_{De}}. \qquad (4.20)$$

In the derivation of (4.20) we have, just as in Refs. 2–4, set in the integration with respect to k in (4.19) the long-wave limit $k \geqslant k_{min}$. The value of k_{min} can be connected with the value of the wave number at which collisional dissipation (viscosity) of the sound waves becomes appreciable. This occurs when δ_1 [see (3.11)] becomes comparable with $1+\delta$. From the condition $\delta_1 \approx 1+\delta$ we have for k_{min}

$$k_{min} r_{De} \approx \nu_{ii} \omega_{Le} r_{Di}^2 / \omega_{Li}^2 r_{De}^2 (1+\delta).$$

If the threshold $x_0^2 \ll 1$ is considerably exceeded, corresponding to almost the total apex angle of the turbulence-region cone, we obtain for the energy of the ion-acoustic pulsations

$$\frac{E^2}{4\pi n_e \varkappa T_e} = \frac{1}{3\pi\sqrt{2\pi}} \left[\frac{1}{\delta} + (1+\delta)^2 \ln\left(1+\frac{1}{\delta}\right) - \delta - \frac{3}{2} \right] \times K_N \frac{\omega_{Li} r_{De}^2}{\omega_{Le} r_{Di}^2} \ln^2 \frac{1}{k_{min} r_{De}}. \qquad (4.21)$$

The decrease of the damping of the ion-acoustic waves (of δ) leads, as it should, to an increase of the total energy of the turbulent noise.

Despite the fact that in the limit $x_0^2 \ll 1$ the difference between the angular distribution (4.18) and that obtained in Ref. 8 becomes negligible, the difference between the ion-sound intensity distributions in frequency leads to a different total noise energy [cf. (4.21) and Eq. (30) of Ref. 8]. Thus, in the case $\delta < 1$ the expression (4.21) for the total turbulent-noise energy differs from the corresponding result of Ref. 8 by a factor

$$\sim \delta^{-1} (r_{Di}/r_{De})^2 \ln^2 (1/k_{min} r_{De}),$$

which brings about a different dependence of the intensity of the ion-acoustic pulsations on the plasma parameters compared with Ref. 8.

5. ANGULAR DISTRIBUTION OF ION-ACOUSTIC TURBULENCE IN THE CASE OF LARGE NONLINEARITY

As already mentioned, the intensity of the turbulent pulsation increases in the limit of small ion dissipation ($\delta \ll 1$). It must then be borne in mind that at small δ we can no longer assume that the nonlinear effects can be neglected, as was done in the preceding section.

In this section we obtain the angular distribution of the ion-sound noise under conditions when allowance for the nonlinear interaction of the acoustic waves is more important than allowance for the ionic Čerenkov damping of the oscillations, and derive also a criterion for the realization of such conditions.

We note that the influence of nonlinear effects (induced scattering of sound waves) on the ion-acoustic turbulence was the subject of Ref. 9. In that reference γ_{NL} was calculated by using an expression different from (2.8) and valid only if it is assumed that in induced scattering of sound waves by ions the frequencies $\omega_s(k)$ and $\omega_s(k')$ of the interacting oscillations satisfy the condition $\omega_s(k) - \omega_s(k') \ll |\mathbf{k} - \mathbf{k}'| v_{Ti}$. This assumption does not lead to the spectrum (3.14) of Kadomtsev and Petviashvili. The turbulence distribution in the value of the wave vector, given in Kingsep's paper,[9] hardly differed from that obtained in the quasilinear theory. In addition, Kingsep considered the angular distribution of the noise using only qualitative estimates brought about by the need to eliminate the angular-spectrum singularity obtained in Ref. 7.

If the contribution of the Čerenkov effect on the ions to the damping of the sound waves is negligibly small ($\delta = 0$), the angular distribution of the turbulence is given by Eq. (4.13), where

$$w_0(\sqrt{1-x^2}) = \frac{1}{x^2} \left\{ 1 + A_1 + A_2 x^2 + A_3 x^4 + x^2 \sqrt{1-x^2} (A_4 - A_5 x^2) \ln \frac{1+\sqrt{1-x^2}}{x} \right\}. \qquad (5.1)$$

As already mentioned, the values of A_n that lead to the nonlinearity of Eq. (4.13) are small: $A_n \ll 1$, since $K_N \ll 1$. If they are neglected, the expression $w_0(\sqrt{1-x^2}) = 1/x^2$ leads to a singularity of the angular distribution of the noise at $x = 1$. Inclusion of corrections proportional to A_N eliminates this singularity, and by virtue of the smallness of A_n the role of the nonlinear effects can be revealed by merely expanding the expression in the curly brackets of (5.1) near $x = 1$. Writing accordingly

$$w_0(\sqrt{1-x^2}) = (1+\varepsilon)/x^2, \qquad (5.2)$$

where $\varepsilon = A_1 + A_2 + A_3 = M_1 - M_2 \ll 1$, and substituting (5.2) in (4.13) we arrive at Eq. (4.16), in which the role of δ is assumed by the parameter ε. For the function $\Phi(x)$ we have then expression (4.18) with δ replaced by ε, and for x_0 we obtain

$$x_0^2 = (1+\varepsilon)/K_{st} \approx K_{st}^{-1}.$$

It is clear that the function (5.2) satisfies the inequality (4.12).

For a final determination of the angular distribution we must calculate ε. Substituting in explicit form the angular distribution function in expression (4.9) for the moments M_1 and M_2, and calculating their difference, we obtain the following equation for ε:

$$\varepsilon=\frac{4K_N}{3\pi}\int_{x_0}^{1}\frac{x(3x^2-1)(x^2-x_0^2)^{3/2}}{1-x+\varepsilon}dx. \tag{5.3}$$

An approximate solution of (5.3) yields

$$\varepsilon\approx\frac{8K_N}{3\pi}(1-x_0^2)^{3/2}\ln\left[\frac{1-x_0}{K_N(1-x_0^2)^{3/2}}\right]\ll 1. \tag{5.4}$$

From (4.19) and (5.4) we obtain for the total energy of the ion-acoustic fluctuations

$$\frac{E^2}{4\pi n_e\varkappa T_e}=\frac{1}{8\sqrt{2\pi}}\frac{\omega_{Li}r_{De}^2}{\omega_{Le}r_{Di}^2}\left(\ln^2\frac{1}{k_{min}r_{De}}\right)\Big/\ln\left[\frac{1-x_0}{K_N(1-x_0^2)^{3/2}}\right]. \tag{5.5}$$

At the instability threshold $x_0=1$ the intensity of the turbulent noise (5.5) vanishes. At $x_0\lesssim 1$ the noise energy depends weakly (logarithmically) on the excess above threshold, which determines the value of $(1-x_0)$. In the case when the threshold is greatly exceeded, $x_0^2\ll 1$, which is of practical interest, the energy of the sound pulsation does not depend at all on the excess above threshold:

$$\frac{E^2}{4\pi n_e\varkappa T_e}=\frac{1}{8\sqrt{2\pi}}\frac{\omega_{Li}r_{De}^2}{\omega_{Le}r_{Di}^2}\frac{\ln^2(1/k_{min}r_{De})}{\ln K_N^{-1}}. \tag{5.6}$$

In this case the angular distribution of the noise energy is given by

$$\Phi(x)=\frac{4K_N}{3\pi x}\frac{d}{dx}\frac{x^4}{1-x+\varepsilon},\quad \varepsilon\approx\frac{8K_N}{3\pi}\ln\frac{1}{K_N}. \tag{5.7}$$

Equation (5.7) is similar in form to the expression given in Ref. 9 for the angular part of the ion-acoustic spectrum. However, as noted at the beginning of this section, there is a fundamental difference between our approach in the description of the linear interaction and that of Ref. 9, and this leads to different values of ε.

We assess now the conditions under which the results obtained in Secs. 4 and 5 are valid. The corresponding conditions follow directly from a comparison of δ with ε. Thus, in the case

$$\varepsilon\gg\delta, \tag{5.8}$$

when the Čerenkov dissipation of the ion-acoustic waves is small, the angular distribution of the ion-sound noise is determined by the nonlinear interaction of the waves. Equations (5.5) and (5.6) are then valid. If an inequality inverse to (5.8) holds, the Čerenkov ion dissipation effects are decisive and we have Eqs. (4.20) and (4.21) for the ion-sound noise energy. We emphasize the condition (5.8) corresponds to a rather large nonisothermy of the plasma, when

$$3+\frac{r_{De}^2}{r_{Di}^2}>\ln\left[\frac{\omega_{Le}^2r_{De}^6}{\omega_{Li}^2r_{Di}^6}\left(K_N\ln\frac{1}{K_N}\right)^{-2}\right]. \tag{5.9}$$

Comparison of Eqs. (4.21) and (5.6) shows that the intensity of the turbulent noise under conditions (5.9) turns out to be higher.

6. OHM'S LAW, ELECTRONIC HEAT TRANSPORT AND RELAXATION OF THE ELECTRON TEMPERATURE IN A TURBULENT PLASMA

In this section we determine, on the basis of the result of the theory developed above, the electronic kinetic coefficients in a turbulent plasma. We obtain first expressions for the density of the electric current j and of the electronic heat flux $\mathbf{q}_e$:

$$\mathbf{j}=e\int d\mathbf{v}\mathbf{v}f,\quad \mathbf{q}_e={}^1/_2m_e\int d\mathbf{v}\mathbf{v}v^2f. \tag{6.1}$$

Recognizing that j and $\mathbf{q}_e$ are oriented along the z axis, we can rewrite (6.1) in the form

$$\begin{aligned} j_z&=2\pi e\int_0^\infty v^3dvf_0(v)\int_0^\pi d\theta\sin\theta\cos\theta\psi(v,\cos\theta)\\ &=\pi e\int_0^\infty v^3dvf_0(v)\int_0^\pi d\theta\sin^3\theta\frac{\partial\psi}{\partial\cos\theta}, \end{aligned} \tag{6.2}$$

$$q_{ez}=\frac{1}{2}\pi m_e\int_0^\infty v^5dvf_0(v)\int_0^\pi\sin^3\theta d\theta\frac{\partial\psi}{\partial\cos\theta}. \tag{6.3}$$

The sought fluxes are thus determined by the function $\partial\psi/\partial\cos\theta$, which takes according to (3.6) the form

$$\begin{aligned}\frac{\partial\psi}{\partial\cos\theta}=&\frac{1}{2}\frac{v^4}{v_{Te}^3}\frac{1}{\nu_{ei}+\nu_N\chi_2(\sin\theta)}\left\{\frac{eE_z}{\varkappa T_e}-\frac{\nabla_zn_eT_e}{n_eT_e}\right.\\ &\left.-\left(\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right)\nabla_z\ln T_e\right\}+\frac{v_sv}{v_{Te}^2\sin\theta}\frac{\nu_N\chi_1(\sin\theta)}{\nu_{ei}+\nu_N\chi_2(\sin\theta)},\end{aligned} \tag{6.4}$$

where the functions $\chi_{1,2}$ are calculated with the aid of the formulas obtained in Secs. 4 and 5 for the angular distribution of the ion-acoustic turbulence. The general formulas obtained in this case are cumbersome and make a lucid comparison with the result of earlier studies difficult. Therefore, with an aim at demonstrating qualitatively the new results of our theory, we consider the kinetic coefficients under extremal conditions, when the instability threshold is considerably exceeded: $x_0^2\ll 1$, and the influence of the ion Čerenkov damping is weak ($\delta\ll 1$). In this case we obtain for the functions $\chi_{1,2}(x)$

$$\chi_1(x)=\frac{1}{x}\chi_2(x)-xK_N,$$

$$\chi_2(x)=\frac{4K_N}{3\pi}\left\{\frac{x(2x^2-1)}{1-x^2}+\frac{\pi/2+\arcsin x}{(1-x^2)^{3/2}}\right\}-\frac{2K_N}{3}.$$

These equations allow us to rewrite (6.4) in the form

$$\begin{aligned}\frac{\partial\psi}{\partial\cos\theta}=&\frac{v_sv}{v_{Te}^2\sin^2\theta}+\frac{3\pi}{4}\frac{v}{v_{Te}^2}\left(\frac{\pi/2+\theta}{\cos^3\theta}-\frac{\pi}{2}+\frac{\sin^3\theta}{\cos^2\theta}-\sin\theta\right)^{-1}\\ &\times\left\{-v_s+\frac{v^3}{2v_{Te}\nu_0}\left[\frac{eE_z}{\varkappa T_e}-\frac{\nabla_zn_eT_e}{n_eT_e}-\frac{\nabla_zT_e}{T_e}\left(\frac{v^2}{2v_{Te}^2}-\frac{5}{2}\right)\right]\right\}.\end{aligned} \tag{6.5}$$

Now, substituting (6.5) in the right-hand side of (6.2), we obtain the following expression for the electric-current density:

$$j_z=\sigma\left(E_z-\frac{1}{en_e}\nabla_zn_e\varkappa T_e\right)+\alpha\nabla_zT_e, \tag{6.6}$$

where the conductivity σ and the thermoelectric coefficient α are given by

$$\sigma=\frac{e^2n_ev_s}{eE_z-n_e^{-1}\nabla_zn_e\varkappa T_e}\left[\frac{3}{2}(1-\beta)+\frac{16}{\pi}\beta\right], \tag{6.7}$$

$$\alpha=24\beta|e|\varkappa n_ev_s/(eE_z-n_e^{-1}\nabla_zn_e\varkappa T_e). \tag{6.8}$$

The constant β here is the result of the integration

$$\beta=\frac{3\pi}{4}\int_0^1\frac{x^3(1-x^2)\,dx}{{}^1/{}_2\pi(1-x^3)+\arccos x+x(1-x^2)^{1/2}(1-2x^2)}\approx 0.18. \tag{6.9}$$

Expressions (6.7) and (6.8) are similar to a certain degree to those obtained in Refs. 8 and 10. The main reason is that in the limit of a high excess above threshold the nonequilibrium distribution function corresponds both in our analysis and in Refs. 8 and 10 to a nonlinear threshold modified by the turbulence. The qualitative difference between our turbulence spectrum and that obtained in Ref. 8, however, leads in the case of a strongly isothermal plasma[8] to transport coefficients that are

$$l^{1/2}=(zT_e/2T_i)^{1/2}\left\{\ln\frac{z^2m_iT_e^3}{m_eT_i^3}\right\}^{-1/2}\gg 1 \tag{6.10}$$

times larger (cf. Ref. 15). The latter is due to the fact that in our analysis the most significant contribution to the turbulent transport coefficients are made by sound waves whose phase velocity equals the sound velocity v_s, whereas the quasilinear theory predicts the presence of only short-wave turbulence corresponding to relatively small phase velocities of the sound waves.

We proceed now to consider the electronic heat flux. Substituting (6.5) in (6.3) we obtain

$$q_{ez}=n_e\varkappa T_e v_s\left\{\frac{15}{4}(1-\beta)+\frac{64}{\pi}\beta-\frac{160\beta}{\pi}\frac{\varkappa n_e\nabla_z T_e}{en_eE_z-\nabla_z n_e\varkappa T_e}\right\}. \tag{6.11}$$

For comparison with the results of earlier models of the ion-acoustic turbulence, we write down an expression for the electronic heat flux under conditions when there is no electric current in the plasma: $j_z=0$. Then

$$q_{ez}=-n_e\varkappa T_e v_s\left\{\frac{25}{4}(1-\beta)+\frac{128}{3\pi}\beta\right\}\approx -7.6 n_e\varkappa T_e v_s. \tag{6.12}$$

Inasmuch as in this case the condition $j_z=0$ leads to

$$\frac{\nabla\varkappa T_e}{eE_z-n_e^{-1}\nabla_z n_e\varkappa T_e}=\frac{\pi}{16\beta}\left(1-\beta+\frac{32}{3\pi}\beta\right)>0, \tag{6.13}$$

we can assert that the heat flux (6.12) is directed towards the lower temperature. It must be noted in connection with (6.12) that it follows from its comparison with the result of Ref. 13 that the ion-acoustic turbulence that arises in the Kadomtsev-Petviashvili model which neglects the quasilinear effects, leads to a lower limit of the heat flux ($q_e\sim(|\nabla T_e|)^{1/2}$). Next, comparison of (6.12) with the result of Ref. 10 shows that neglect of the nonlinear interaction of the ion-acoustic waves, with account taken of only quasilinear effects, leads to an excessive limitation of the heat flux. This, just as in the case of the electric current, is due to the large phase velocity of the ion-acoustic turbulence waves, which is determined not only by quasilinear but also by nonlinear processes.

In this section it remains for us to consider the relaxation of the electron temperature, when the usual collisions turn out to be insignificant. From the kinetic equation that takes only a quasilinear collision into account we obtain then

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_e\varkappa T_e\right)+\operatorname{div}\mathbf{q}_e=\mathbf{Ej}-\frac{3}{2}\nu_T n_e\varkappa T_e. \tag{6.14}$$

The effective temperature-relaxation frequency is determined here by the formula

$$\nu_T=\frac{4}{3}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\gamma(\mathbf{k})\omega_s(k)N(\mathbf{k})}{n_e\varkappa T_e} \tag{6.15}$$

and characterizes the electronic-plasma-component cooling due to energy transfer to the ion-sound waves. According to (2.10) we have

$$\gamma(k,\theta)=\gamma_s(k)\left[\delta+\int_{x_0}^1 dx\,Q(\sin\theta,x)\,\Phi(x)\right]. \tag{6.16}$$

Therefore

$$\nu_T=A\,\omega_{Li}^3 r_{De}^2/6\omega_{Le}^2 r_{Di}^2, \tag{6.17}$$

$$A=\int_{x_0}^1 dx\,\Phi(x)\left[\delta+\int_{x_0}^1 dx'\,Q(\sqrt{1-x^2},x')\,\Phi(x')\right]. \tag{6.18}$$

The last expression is particularly simple in the limit $x_0^2\ll 1$ and $\delta\ll 1$, when the following interpolation formula can be obtained:

$$A=\frac{4K_N}{3\pi}\frac{1}{\delta+\varepsilon}\left\{\delta+\frac{16}{3\pi}K_N\ln\frac{1}{\delta+\varepsilon}\right\}. \tag{6.19}$$

As a result we obtain for the characteristic temperature relaxation frequency

$$\nu_T=\frac{4}{9}\left(\frac{2}{\pi}\right)^{1/2}\frac{\omega_{Li}^2}{\omega_{Le}^2}B\nu_0, \tag{6.20}$$

where $B=1$ at $\delta\gg\varepsilon$ and $B=2$ at $\delta\ll\varepsilon$. We see thus that the characteristic time of the electron cooling via emissions of sound is $\sim(\omega_{Le}/\omega_{Li})^2\nu_0^{-1}$. The kinetic coefficients obtained in this section characterize the influence of the ion-acoustic turbulence on the electronic component of the plasma.

It is worth noting that since

$$^3/_2\nu_T n_e\varkappa T_e=Bv_s(e\mathbf{E}n_e-\nabla n_e\varkappa T_e)_z,$$

the quantity ν_T can be directly used to describe the rate of transfer of the translational-motion energy to the ion-sound waves. Finally, the relatively large fluxes obtained by us in accord with inequality (6.10) allow us to state that the turbulent state obtained by us corresponds to the conditions of excitation of long-wave IAT.

7. ABSORPTION AND SCATTERING OF ELECTROMAGNETIC RADIATION BY A TURBULENT PLASMA

The distribution obtained above for the turbulent ion-acoustic fluctuations can answer the question of the efficiency of the interaction between electromagnetic radiation and a plasma, effected by Brillouin scattering or when the sound transforms electromagnetic waves into longitudinal electronic Langmuir perturbations. The question of radiation absorption due to transformation into longitudinal perturbation was posed long ago.[17,18] However, the approaches discussed in Refs. 19–21 to the investigation of this absorption, which is due to transformation by ion sound (ion-acoustic absorption) were equivalent in fact to stating that if the level of the ion-acoustic turbulence is high, the absorption can also be large. This situation came about primarily because to determine the ion-acoustic absorption one must know the spectrum of the ion-acoustic turbulence. We have formulated elsewhere[22] a theory of ion-acoustic absorption and scattering, based on a fluctuation distribution obtained in Ref. 5 and corresponding to the Kadomtsev-Petviashvili turbulence model. Since it is clear from the present study that such a model overestimates at $K_N < 1$ the fluctuation level, it is expedient to obtain the necessary estimates based on the result of Secs. 4 and 5 of this article.

Ion-acoustic absorption of electromagnet radiation having a frequency ω_0, a wave vector $\mathbf{k}_0$, and a polarization $\mathbf{e}_0$ on account of transformation into longitudinal perturbations is characterized by the following effective collision frequency[22]

$$\nu_{eff}^{(l)}=\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\omega_{Le}^2\omega_s(k)N(\mathbf{k})(\mathbf{k}\mathbf{e}_0)^2}{\omega_0 n_e\varkappa T_e k^2}\frac{\operatorname{Im}\varepsilon_l(\omega_0,k)}{|\varepsilon_l(\omega_0,k)|^2}, \tag{7.1}$$

where ε_l is the longitudinal dielectric constants and $\omega_0 \gg \nu_{eff}^{(l)}$. Substituting in (7.1) the ion-acoustic fluctuation spectrum obtained by us [see Eqs. (3.13), (3.14), (4.18)] we obtain in the limit of appreciable excess above threshold, $x_0^2 \ll 1$

$$\nu_{eff}^{(l)}=\left(\frac{2}{9\pi^3}\right)^{1/2}\left(\frac{\cos^2\theta_l}{\Delta}+\sin^2\theta_l\ln\frac{1}{\Delta}\right)K_N\omega_{Li}\frac{r_{De}^2\omega_{Le}}{r_{Di}^2\omega_0}$$
$$\times\int_{k_{min}}^{r_{De}^{-1}}\frac{dk}{k}\ln\frac{1}{kr_{De}}\frac{\operatorname{Im}\varepsilon_l}{|\varepsilon_l|^2}. \tag{7.2}$$

Here $\Delta=\max(\delta,\varepsilon)$, θ_l is the angle between $\mathbf{e}_0$ and $en_e\mathbf{E}\,\nabla n_e\varkappa T_e$. In this case $\Delta\ll 1$.

Equation (7.2) does not imply substantial absorption under conditions when the imaginary part of the dielectric constant exceeds the real, as is the case when $\omega_{Le}\lesssim\omega_0/2$, i.e., in a sufficiently rarefied plasma. Accordingly the ion-acoustic absorption in a spatially inhomogeneous expanding laser plasma[22] is possible only if the radiation is incident on the plasma at an angle $\lesssim 60°$. The absorption efficiency in a spatially inhomogeneous plasma is characterized by an optical thickness $\tau^{(l)}\sim\nu_{eff}^{(l)}L/c$, where L is the characteristic scale of the electron-density inhomogeneity. Considerable absorption corresponds to the condition $\tau^{(l)}\gtrsim 1$, which takes according to (7.2), e.g., in the case when there is no current in the plasma ($j=0$), the form

$$\left(\frac{\cos^2\theta_l}{\Delta}+\sin^2\theta_l\ln\frac{1}{\Delta}\right)\frac{v_{Te}}{c}\frac{\omega_{Le}}{\omega_{Li}}\gg 1. \tag{7.3}$$

Although this inequality is more stringent compared with that obtained in Ref. 22 on the basis of the Kadomtsev-Petviashvili turbulence model, in a laser plasma the inequality (7.3) is satisfied as a rule. It can therefore be stated that when interaction of laser radiation with a plasma leads to the onset, e.g., of sufficiently large heat fluxes, anomalously large ion-sound absorption should be observed.

Besides the anomalous absorption, ion-acoustic turbulence causes the onset of anomalously large Brillouin-Raman scattering. The corresponding decrease of intensity because of this scattering is characterized by the following effective collision frequency[22]:

$$\nu_{eff}^{(t)}=\pi\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\omega_{Le}^2\omega_s N(\mathbf{k})}{\omega_0 n_e\varkappa T_e}\frac{[(\mathbf{k}_0-\mathbf{k})\times\mathbf{e}_0]^2}{(\mathbf{k}_0-\mathbf{k})^2}\delta\left(\varepsilon_t(\omega_0,\mathbf{k}_0-\mathbf{k})-\frac{c^2(\mathbf{k}_0-\mathbf{k})^2}{\omega_0^2}\right), \tag{7.4}$$

where ε_t is the transverse dielectric constant. Since, as in the Kadomtsev-Petviashvili turbulence model, long wave fluctuations are possible in our case, in contrast to the turbulence spectrum of Refs. 7–10, it follows that for the onset of anomalous Brillouin scattering it is necessary to satisfy the condition $k_{min}\lesssim\omega_0/c$, which takes the following form:

$$\nu_{ii}\omega_{Le}/\omega_{Li}^2\lesssim(v_{Te}/c)(r_{De}^2/r_{Di}^2). \tag{7.5}$$

The inequality (7.5) is easily realized in contemporary laser-plasma experiments. As for the total scattering intensity, comparison of (7.2) with the equation that follows from (7.4)

$$\nu_{eff}^{(t)}\approx\left(\frac{2}{9\pi}\right)^{1/2}K_N\frac{r_{De}^2}{r_{Di}^2}\frac{\omega_{Le}\omega_{Li}}{\omega_0}\int_{k_{min}}^{r_{De}^{-1}}\frac{dk}{k}\ln\frac{1}{kr_{De}}$$
$$\times\int\frac{d\Omega_\mathbf{k}}{2\pi}\left[1+4\frac{(\mathbf{k}\mathbf{e}_0)^2(\mathbf{k}\mathbf{k}_0)^2}{k^4k_0^2}\right]\frac{(\omega_0/c)^2\delta(k^2-2\mathbf{k}\mathbf{k}_0)}{(1-\cos\theta_\mathbf{k}+\Delta)^2} \tag{7.6}$$

allows us to state that this intensity is close in order of magnitude to the intensity of the absorbed radiation. Equation (7.6) corresponds to a rather broad angular distribution of the radiation reflected from the plasma. Therefore, in accord with Ref. 22, the anomalous Brillouin scattering discussed by us can be compared with the experimentally observed diffuse scattering of laser radiation.[23,24]

8. DISCUSSION

Summarizing the theory expounded above, we must emphasize that it has led to a qualitative decrease of the IAT intensity, while preserving the K-P distribution that agrees with experiment. This, first, agrees with the experimental results and, second, allows us to claim intrinsic consistency for it. Thus, our theory, in contrast to the quasilinear theory, has established a correspondence between the spectral distributions of the IAT and the flux limits in a turbulent plasma.

We discuss now the quantitative relations that follow from our theory and may be applicable to a plasma with current.[12] First, in accord with (4.15) and (5.2), the threshold of the appearance if the IAT with a spectrum of the K-P type and its corresponding manifestation in the anomalous resistance occurs at an electric field intensity (m_p is the proton mass)

$$E_{thr}=\left(\frac{2z}{9\pi A}\frac{m_e}{m_p}\right)^{1/2}(1+\delta+\varepsilon)E_{Dr}\approx 0.006\left(\frac{z}{A}\right)^{1/2}(1+\delta+\varepsilon)E_{Dr}, \tag{8.1}$$

where $A=m_i/m_p$ and the Dreicer field is defined as

$$E_{Dr}=4\pi zn_e|e|^3\Lambda/\varkappa T_e.$$

Equation (8.1) agrees with the experimental data.[12]

It must be noted that if the spatial gradients are neglected we have at the threshold

$$(K_N)_{thr}=390\frac{T_i}{zT_e}\left(\frac{A}{z}\right)^{1/2}\frac{\nu_{ei}}{\omega_{Le}}(1+\delta+\varepsilon). \tag{8.2}$$

This shows that the turbulence of a current-carrying plasma can vary with increasing electric field in two ways, depending on the relation between (8.1) and

$$\delta=43\left[\frac{A}{z}\left(\frac{zT_e}{T_i}\right)^3\exp\left(-\frac{zT_e}{T_i}-3\right)\right]^{1/2}. \tag{8.3}$$

If $(K_N)_{thr}$ exceeds δ substantially, the regime of Sec. 4 is not realized when the electric field intensity is further increased. The energy of the ion-acoustic fluctuations then increases rapidly (at $K_{st} > 1$) in accordance with (5.5) to the value (5.6), after which it changes little. In the opposite case, which apparently corresponds to the experiment of Ref. 12, when (8.2) is much less than (8.3), the IAT energy, after exceeding the threshold value (8.1), increases, as follows from Sec. 4, in proportion to K_N, i.e., in proportion to the electric field intensity, until ε becomes close to the value (8.3). After that, the regime of Sec. 5 is realized.

We emphasize once more that all the foregoing is limited by the condition $K_N \ll 1$. It is therefore useful to point out the regularities that correspond to the inverse limit of so high an excess above threshold, when $K_{st} \gg K_N \gg 1 + \delta$. It follows then from (4.13) that $\Phi(\cos\theta) \sim K_N^{1/2}$. This allows us to state that in this limit the noise level will crease with increasing K_N like (cf. Ref. 1)

$$\frac{E^2}{4\pi n_e \varkappa T_e} \sim \frac{\omega_{Li}}{\omega_{Le}} \frac{r_{De}^2}{r_{Di}^2} K_N^{1/2} \ln^2 \frac{1}{k_{min} r_{De}}. \tag{8.4}$$

The limiting values of the current and of the heat flux will increase simultaneously in proportion to $K_N^{1/2}$, whereas the characteristic electron-cooling time will be determined as before by (6.20) in accord with the result of Ref. 16. At $K_N \gg 1$, for example, in the absence of spatial gradients we have for the current the order-of-magnitude value (cf. Ref. 25)

$$j \sim e n_e v_{Te} \left(\frac{|e\mathbf{E}| r_{Di}^2}{\varkappa T_e r_{De}} \right)^{1/2}, \tag{8.5}$$

and in the absence of current we obtain for the heat flux approximately[16]

$$q_e \sim n_e \varkappa T_e v_{Te} \left(\frac{|\nabla T_e|}{T_e} \frac{r_{Di}^2}{r_{De}} \right)^{1/2}. \tag{8.6}$$

We emphasize in conclusion that the qualitative arguments that supplement our main analysis and lead to Eqs. (8.4)–(8.6) point to the desirability of developing a theory also for the case $K_N > 1$, which corresponds to the turbulent state of a plasma in the presence of strong electric fields and large gradients of the hydrodynamic quantities.

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