Decrease of energy losses in a beam passing through a medium

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It is demonstrated how in principle the energy losses of charged particle beams in a plasma or gas can be decreased as a result of "collective" effects due to collisions. The phenomenon, which is called autorecuperation, may take place in a medium at thermodynamic equilibrium when a suitably modulated beam passes through it. As a result of periodic heat evolution in the medium, oscillations are built up, which in turn may impart energy to the beam particles and decrease in this way the effective losses and hence the energy losses in the reactions involved in the collisions.

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INTRODUCTION

The question of the energy lost when a beam of charged particles passes through matter has by now been investigated in sufficient detail (see, e.g., Refs. 1–4). This loss, disregarding the fine points, can be divided into a part due to collisions of beam particles with the medium particles, and the so-called "collective" loss due to the fact that oscillations can build up in the beam + medium system.[3,4] It is regarded as obvious, since collective processes in a medium in thermodynamic equilibrium decrease in principle the beam energy, that the loss can in no way be less than that due only to collisions.

It will be shown in the present paper that situations are possible in which this statement is incorrect. Namely, when a sufficiently monochromatic beam whose current is modulated in a definite manner passes through a medium in thermodynamic equilibrium, the energy loss due to collective processes may turn out in many cases less than those due to collisions. This can be qualitatively visualized as follows. The wave excited by the modulated beam loses energy to heat the medium. It might seem that if the medium is in thermodynamic equilibrium, it is possible to excite the wave only at the expense of work performed by the beam particles on the wave field. Therefore, at any form of modulation, the energy loss should be larger than in the absence of modulation. This reasoning, however, does not take into account one other possibility: If the modulated beam passes through some section of the medium and releases in it heat periodically, via collisions, then the pressure in this section will also vary periodically; the pressure oscillations, which are phased in a definite manner in all sections through which the beam passes, can also generate waves in the medium.[1)] The wave is built up here at the expense of the energy lost by the beam to collisions in the medium. If the damping is small enough, then a suitable modulation can bring about a situation in which the field of the wave built up in the medium, even when such a generation mechanism is taken into account, performs work on the beam particles and returns thereby part of the heat-loss energy. In this case the field of the wave effectively decreases, as it were, the friction force connected with the collisions. Therefore the energy loss per collision of the beam particles with the medium particles turns out to be less for such a modulated beam than for an unmodulated one. Since the number of collisions determines the number of nuclear reactions generated by the beam in the medium, the energy consumed by one nuclear reaction is therefore decreased in this case.

It is natural to call such a phenomenon autorecuperation. Thus, autorecuperation, if we attempt to define it formally, is a phenomenon in which part of the energy released in the medium in the form of heat by a beam modulated in a special manner goes to build up oscillations which in turn can provide energy to accelerate the beam particles, decreasing thereby the effective loss. Autorecuperation can be of interest in principle for problems in which the heating of the medium is an undesirable side effect, such as in the passage of a tritium beam through a deuterium target (Ref. 8),[2)] in muon generation, etc., decreasing the energy loss per nuclear reaction.

Autorecuperation can be quantitatively characterized by the aid of a coefficient $\eta_{\rm arc}$, which is the ratio of the energy W_r returned by the beam particles to the total energy W_b (without allowance for the return) lost by the beam in the target. For W_b we can write

$$W_b=\Delta\mathscr{E}+Q+W_r,$$

where $\Delta\mathscr{E}$ is the change of the target energy after passage of the beam, and Q is the heat loss (in the general case, energy loss) of the target. Obviously, $\eta_{\rm arc}<1$. Next, if T_1 and T_2 are the maximum and minimum temperatures that are reached in some sections of the target at some instants of time, then $\eta_{\rm arc}\leqslant(T_1-T_2)/T_1$. The same principal limitations on the energy recuperation hold here as for heat engines of any other type, but since the medium (plasma or gas) is in this case not only the working medium, but also serves as the heat engine itself, it is possible to avoid here certain purely technological limitations imposed on T_1, by the thermal endurance of the materials used in the existing heat engines, etc. A modulated beam, since it makes it possible to obtain relatively easily any required law of heat-input to the target, also offers certain advantages in comparison with many other methods of heating the medium. One can hope that because of these advantages the autorecuperation, regardless of the formulation of

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the problem, will be equally or even more effective than the usual energy recuperation wherein the target simply serves as a heating source, the beam is not modulated (since there is no need for it in principle), and any one of the ordinary type heat engines converts part of the released heat into electricity. From among the existing methods, perhaps the most suitable for the recuperation of energy released by a beam in a target might be an MHD generator, but even that is subject to major technical limitations on the maximum and minimum temperatures which are connected, for example, with the interaction of plasma with electrodes.[10]

When considering autorecuperation, we confine ourselves in the present article to situations herein, first, the waves built up in the medium can be investigated in the linear approximation and, second, one can neglect the change of the beam-particle velocity (this condition can be satisfied, for example, in those cases when a beam of sufficiently heavy particle loses a small fraction of its energy to light particles in a medium). Although in the linear approximation, because of the small amplitudes of the oscillations, we have $T=(T_1-T_2)\ll T_1$ and $\eta_{\rm arc}\ll 1$, which is of little interest from the practical point of view, nevertheless even in this case it is possible to demonstrate, first, the feasibility in principle of autorecuperation and, second, those concrete examples of systems in which autorecuperation is possible.

In the first section we consider the simple possible example, autorecuperation in an unbounded plasma with infinitely heavy ions in the absence of a magnetic field. This variant, apparently by virtue of a number of causes, in particular such as the high rate of damping of the oscillation, the need of modulating the beam at a frequency close to the plasma frequency, etc., is of purely methodological interest. Another somewhat more complicated but also more interesting variant is autorecuperation in a plasma located in a constant and homogeneous magnetic field, is considered in the second section. For autorecuperation to occur in this variant it is desirable that the magnetic field be "frozen" in the plasma. The waves built up in the plasma because of the alternating heat release due to themodulated beam cause in this case corresponding oscillations of the intensity of the "frozen-in" magnetic field. These oscillations should in turn generate an alternating electric field which, under suitable conditions indicated in the second section, performs work on the beam particles. In contrast to the plasma without a magnetic field, it is possible here, by increasing the length of the excited wave, to decrease both the contribution from the damping and the frequency of the beam modulation. Finally, in the third section, we consider autorecuperation in a gas target. This last example calls for an additional explanation. On the one hand, in many cases it is much simpler to work with a gas target than with a plasma target, there is no need for magnetic containment, the temperature of a gas is much lower, etc. Next, if we compare, for example, the energy lost to Coulomb collisions when fast charged particles pass through a low-temperature[3] (but almost fully ionized) plasma with the energy loss in a gas of the same density, the loss in the gas is smaller by an amount equal approximately to the Coulomb logarithm Λ, making the gas, naturally, preferable for the problems considered. On the other hand, the oscillations in the gas cannot transfer their energy directly to the beam particles. One can, however, propose a number of schemes in which the oscillations in the gas transfer their energy to excitation of oscillations in some other system, for example (see §3) in the walls of acoustic resonators and in an associated electric circuit, from which this energy can already be used to compensate for the beam losses, or to feed the accelerator that produces the beam. The example given in §3 is not the only one. It would be possible in principle to consider a situation wherein the energy is transferred through a vibrating or rotating conducting piston, metallic or plasma. Such an indirect autorecuperation, while somewhat more complicated then the ordinary one, offers also a number of advantages. For example, by adding external energy to the intermediate system, it is possible in a number of cases to keep the beam-particle energy from decreasing at all, and to cause the energy to be optimal at all time from the point of view of the ratio of the cross sections for the useful and parasitic processes.

In the Conclusion we discuss, on a qualitative level, the situation in the nonlinear region. In most cases, when attempts are made to excite oscillations with high amplitude, shock wave discontinuities are produced and lead to a substantial energy dissipation. Shock waves in an unbounded medium might not be formed in the presence of dispersion. Thus, for those cases when the oscillations can be described with the aid of a Korteweg–de Vries equation, it is known that there exist finite-amplitude solutions that give no shock waves. Oscillations with relatively large amplitudes can be obtained in resonators having finite volumes, provided that the natural frequencies of the oscillations are not multiples of one another. For example, will be stable in a oscillation with small (but finite) amplitudes cylinder, since the natural frequencies determined by the zeros of the Bessel function are not multiples of one another, unlike in an unbounded medium without dispersion. For plasma with a "frozen-in" magnetic field, placed in a cylinder (the autorecuperation coefficient for this case is calculated in the linear approximation in §2), it is possible to estimate approximately the wave amplitude at which discontinuities begin to be formed (the calculations connected with this estimate are not presented in this article). By using such an estimate, we can obtained the following restriction on $\eta_{\rm arc}$; $\eta_{\rm arc}\lesssim 0.2$. No optimization of the system or modulation were considered here at all.

Appreciable ΔT can be obtained without shock-wave formation also if it is possible to change the resonator volume substantially within times much shorter than the characteristic times connected with the thermal conductivity.

§1. AUTORECUPERATION IN A PLASMA WITHOUT A MAGNETIC FIELD

We consider a one-dimensional problem. A beam whose particle density is a periodic function of one coordinate and is independent of the two others is present in an unbounded plasma with infinitely heavy ions. The beam-particle velocity v_0 is constant and it directed along this chosen coordinate. In the hydrodynamic approximation, the equations take the form

$$n_e m\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)=-\frac{\partial}{\partial x}(n_e \varkappa T)-en_e\frac{\partial \varphi}{\partial x}-n_e m v \nu, \quad (1)$$
$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial x}(n_e v)=0, \quad -\frac{\partial^2\varphi}{\partial x^2}=4\pi e(n_e-n_0-N(x;t)),$$
$$\frac{3}{2}n_e\varkappa\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial x}\right)=-n_e\varkappa T\frac{\partial v}{\partial x}+n_e N(x;t)\sigma v_0\mathscr{E}_0+n_e m \nu v^2-Q.$$

Here n_e is the electron density, v is their velocity, m is their mass, T is the temperature, $\varkappa$ is the Boltzmann constant, ν is the frequency of the electron-ion collisions, Q is the heat loss, $N(x;t)$ is the density of the beam particles, σ is the cross section for beam-particle collisions with electrons, $\mathscr{E}_0$ is the average energy transferred to the electron in this collision, and $e=-|e|$ is the electron charge.

We assume that $N(x;t)$ is of the form

$$N(x;\ t)=\bar{N}+\tilde{N}\exp(ik(x-v_0t))+\tilde{N}\exp(-ik(x-v_0t)) \quad (2)$$

[here $\bar{N}$ and $\tilde{N}$ are real quantities; $N(x;t)\geq 0$].

After linearization of (1), the oscillating part of the solution of the obtained equations must be sought in the form

$$v(x;\ t)=v\exp(ik(x-v_0t))+\text{c.c.},\quad \varphi(x;\ t)=\varphi\exp(ik(x-v_0t))+\text{c.c.},$$
$$\delta n(x;\ t)\equiv n_e(x;\ t)-n_0=\delta n\exp(ik(x-v_0t))+\text{c.c.} \quad (3)$$

We have

$$\delta_n=\left(\frac{2}{3}\frac{n_0\sigma\mathscr{E}_0}{km}i-\frac{4\pi n_0e^2}{mk^2}\right)\tilde{N}\left(v_0^2-\frac{5}{3}\frac{\varkappa T_0}{m}-\frac{4\pi n_0 e^2}{mk^2}-\frac{\nu v_0}{k}i\right)^{-1},$$
$$\varphi=\frac{4\pi e}{k^2}\left(\frac{2}{3}\frac{n_0\sigma\mathscr{E}_0}{km}i-v_0^2+\frac{5}{3}\frac{\varkappa T_0}{m}-\frac{\nu v_0}{k}i\right)\tilde{N} \quad (4)$$
$$\times\left(v_0^2-\frac{5}{3}\frac{\varkappa T_0}{m}-\frac{4\pi n_0e^2}{mk^2}+\frac{\nu v_0}{k}i\right)^{-1}.$$

We write down the expression for $\eta_{\rm arc}$:

$$\eta_{\rm arc}=\left(\int eN(x;t)\,v_0\frac{\partial\varphi}{\partial x}dx\,dt\right)\left(\int n_eN(x;t)\,v_0\sigma\mathscr{E}_0dx\,dt\right)^{-1}. \quad (5)$$

Substituting (4) in (5) we obtain

$$\eta_{\rm arc}=\frac{16\pi e^2\tilde{N}^2}{3\bar{N}k^2m}\left[\left(1-\frac{3}{2}\frac{m\nu v_0}{n_0\sigma\mathscr{E}_0}\right)\frac{4\pi n_0e^2}{mk^2}+\frac{5}{3}\frac{\varkappa T_0}{m}\right.$$
$$\left.-v_0^2\right]\left[\left(v_0^2-\frac{5}{3}\frac{\varkappa T_0}{m}-\frac{4\pi n_0e^2}{mk^2}\right)^2+\frac{\nu^2v_0^2}{k^2}\right]^{-1} \quad (6)$$

[the condition for the applicability of (6) is $\eta_{\rm arc}\ll 1$]. If the damping of the oscillations in the plasma is relatively small, i.e., if $m\nu v_0/n_0\sigma\mathscr{E}_0\ll 1$, it is always possible, by proper modulation of the beam, to choose the value of k such that $\eta_{\rm arc}$ is positive. The phase velocity of plasma oscillations having the same k turns out to be in this case larger than v_0. This, generally speaking, agrees with the known intuitive concepts: To give up energy to the particles, the wave must have a velocity somewhat larger than the velocity of these particles.

To describe the autorecuperation process it is meaningful to introduce besides $\eta_{\rm arc}$ one other quantity $\eta'_{\rm arc}$, which is the ratio of the energy lost by the beam (with allowance for the recovery) per "useful" nuclear reaction that takes place when the beam passes through the target, to the energy $\mathscr{E}_1$ lost in this target in one such reaction by an unmodulated beam. In other words, $\eta'_{\rm arc}=(W_b-W_r)/\mathscr{E}_1N_1$, where N_1 is the number of "useful" nuclear reactions that have taken place in the target. In those cases when all the beam particles have the same velocity that remains constant during the flight through the target, and the oscillations produced in the medium do not violate its neutrality (we assume for simplicity that only one species of nuclei is present in the medium), then $W_b=\mathscr{E}_1N_1$ and the identity $1-\eta_{\rm arc}\equiv\eta'_{\rm arc}$ holds. When the neutrality of the medium is violated, if the losses take place on electrons, and the "useful" nuclear reactions take place, naturally, on nuclei, the inequality of n_e and n_i makes $\eta'_{\rm arc}$, generally speaking, not equal to $1-\eta_{\rm arc}$. In our case $\eta'_{\rm arc}$ is given by

$$\eta'_{\rm arc}=1+\frac{8\pi e^2\tilde{N}^2}{3\bar{N}k^2m}\left[\left(\frac{4\pi n_0e^2}{mk^2}+\frac{5}{3}\frac{\varkappa T_0}{m}-v_0^2\right)\right.$$
$$\left.+3\nu v_0\left(\frac{4\pi e^2}{k^2\mathscr{E}_0\sigma}+\frac{2}{3}\frac{\sigma\mathscr{E}_0}{4\pi e^2}\right)\right]\left[\left(v_0^2-\frac{5}{3}\frac{\varkappa T_0}{m}-\frac{4\pi n_0e^2}{mk^2}\right)^2+\frac{\nu v_0^2}{k^2}\right]^{-1}. \quad (7)$$

We consider the situation when $\nu\to 0$. To obtain a gain compared with the unmodulated beam, it is necessary to satisfy the inequality $\eta'_{\rm arc}<1$. If we choose k to satisfy this condition, we find in contrast to the previous situation, that the phase velocity of the free oscillations with this k should be less than v_0. This paradox is explained by the fact that in this case, in addition to autorecuperation, there is one other mechanism whereby the "parasitic" energy losses are decreased. The energy lost to collisions with electrons is decreased if the maxima of the beam particle density occur at the minima of the electron density. But oscillations in which this condition would be satisfied can be excited in this problem only when the beam, in contrast to autorecuperation, performs work against the field of the excited waves. Naturally, in this case v_0 must exceed somewhat the phase velocity of the waves. Since the second mechanism of decreasing the "parasitic" losses is more considerable in our problem, it determines the choice of k.

§2. AUTORECUPERATION IN A PLASMA WITH A HOMOGENEOUS MAGNETIC FIELD

We consider an unbounded plasma placed in a homogeneous magnetic field. We consider an unbounded plasma placed in a homogeneous magnetic field H_0 directed along the z axis. A modulated beam is present in the plasma. The particle density in the beam varies in accord with the law

$$N(x;\ t)=\bar{N}+\tilde{N}\exp(ik(x-v_1t))+\tilde{N}\exp(-ik(x-v_1t)). \quad (8)$$

The velocity v_0 of the beam particles is constant but is not directed along the x axis:

$$\mathbf{v}_0=v_1\mathbf{i}+v_2\mathbf{j} \quad (9)$$

(here $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along the directions of x, y, and z). We shall consider only those cases when the inequality $v_2\gg v_1$ is satisfied. The change of the direction of the beam-particle velocity under the influence of an external magnetic field will be disregarded.

In the hydrodynamic approximation, the equations (for singly charged ions) are of the form

$$
\begin{gathered}
nm_i\frac{d\mathbf{v}}{dt}=-\nabla(2n\varkappa T)+\frac{1}{4\pi}[\mathrm{rot}\,\mathbf{H}\times\mathbf{H}]-\frac{|e|}{c}N(x;t)[\mathbf{v}_0\times\mathbf{H}],\\
\frac{\partial\mathbf{H}}{\partial t}=\mathrm{rot}[\mathbf{v}\times\mathbf{H}]+\frac{c|e|}{\sigma_b}\mathrm{rot}(N(x;t)\mathbf{v}_0)+\frac{c^2}{4\pi\sigma_b}\Delta\mathbf{H},\\
\partial n/\partial t+\mathrm{div}\,n\mathbf{v}=0, \qquad (10)\\
3n\varkappa\frac{dT}{dt}=-2n\varkappa T\,\mathrm{div}\,\mathbf{v}+nN(x;t)\sigma v_0\mathscr{E}_0+\frac{1}{\sigma_b}\left(\frac{c}{4\pi}\mathrm{rot}\,\mathbf{H}-|e|N(x;t)\mathbf{v}_0\right)^2-Q,\\
\mathrm{rot}\,\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t};
\end{gathered}
$$

here n is the plasma density, $\mathbf{v}$ the velocity, $\mathbf{H}$ the magnetic field, σ_b the plasma conductivity, and $\mathbf{E}$ the electric field. For η_{arc} we can write

$$\eta_{arc}=\left(\int|e|N(x;t)\mathbf{E}\mathbf{v}_0dx\,dt\right)\left(\int N(x;t)n\sigma v_0\mathscr{E}_0dx\,dt\right)^{-1}. \qquad (11)$$

Linearizing (10) and solving the resultant equations, we get

$$
\begin{aligned}
\eta_{arc}=&\left\{2|e|\tilde{N}^2H_0\left(\frac{10}{3}\varkappa T_0n_0-n_0m_iv_1^2\right)\right.\\
&\times\left[\frac{|e|v_2}{H_0}\left(n_0m_iv_1^2-\frac{10}{3}\varkappa T_0n_0\right)-\frac{n_0\sigma v_2\mathscr{E}_0c}{6\pi v_1}\right]\Bigg\}\\
&\times\left\{n_0\bar{N}\sigma\mathscr{E}_0\sigma_b\left[\left(n_0m_iv_1^2-\frac{10}{3}\varkappa T_0n_0-H_0^2/4\pi\right)^2\right.\right.\\
&\left.\left.+\frac{c^4k^2}{16\pi^2\sigma_b^2v_1^2}\left(n_0m_iv_1^2-\frac{10}{3}\varkappa T_0n_0\right)^2\right]\right\}^{-1}. \qquad (12)
\end{aligned}
$$

In order for η_{arc} to be larger than zero it is necessary to require, for example, first that $m_iv_1^2$ be larger than $(10/3)\varkappa T_0$ and, second, satisfaction of the inequality

$$\frac{|e|}{H_0}\left(m_iv_1^2-\frac{10}{3}\varkappa T_0\right)<\frac{\sigma\mathscr{E}_0c}{6\pi v_1}. \qquad (13)$$

To increase η_{arc}, it is advisable apparently to choose sufficiently small k and, in addition, choose v_1 to satisfy the relation

$$n_0m_iv_1^2-{}^{10}/_3\varkappa T_0n_0-H_0^2/4\pi\approx 0. \qquad (14)$$

We consider one more problem: We have an infinitely long plasma-filled cylinder of radius R_0 and a magnetic field directed along the cylinder axis; a modulated beam propagates along the axis. Assume that the Larmor radius r_L of the beam particles is equal to $cMv_\perp/|e|H_0$ (here M is the mass of the beam particles and $v_\perp$ is the velocity component perpendicular to the magnetic field) is much less not only than R_0, but also than the characteristic lengths of the waves excited in the cylinder.

The density of the particles in the beam can be represented in the form

$$
\begin{aligned}
N(r;z;t)=&[\bar{N}+\tilde{N}\exp(ik(z-v_\parallel t))\\
&+\tilde{N}\exp(-ik(z-v_\parallel t))]\frac{\delta(r-r_L)}{2\pi r_L}. \qquad (15)
\end{aligned}
$$

Here δ is a delta function and $v_\parallel$ is the velocity component parallel to the axis. We assume that k is small enough ($R_0^{-1}\gg k$) to be neglected everywhere in the equations except in terms of the type $kv_\parallel$, where it is multiplied by a sufficiently large $v_\parallel$. In order not to write out the cumbersome equations, we consider the limiting case $\sigma_b\to\infty$. In the case of modulation of the type (2), many modes can be excited in the cylinder, but at a definite choice of k only one of them can turn out to be close to resonance. Taking into account the contribution of only this one mode, we have

$$\eta_{arc}=\frac{2|e|\tilde{N}^2v_\perp r_LH_0\nu_n}{3\pi\bar{N}cR_0^3J_0^2(\nu_n)}\left[\frac{n_0(kv_\parallel)^2R_0m_i}{\nu_n}-\frac{10}{3}\varkappa T_0\frac{n_0\nu_b}{R_0}-\frac{H_0^2\nu_n}{4\pi R_0}\right]^{-1}, \qquad (16)$$

where J_0 is a Bessel function and ν_n satisfies the equation

$$J_1(\nu_n)=0. \qquad (17)$$

§3. AUTORECUPERATION IN A GAS TARGET

Assume that the gas fills a system of acoustic resonators of the type shown in Fig. 1. The resonators are coupled to each other through inductances L. The walls of the resonators are acoustic membranes that can be displaced by the gas oscillations. Each ith resonator has a capacitance C_i to ground, which can be represented in the form

$$C_i=C_0+\beta\delta R_i. \qquad (18)$$

Here C_0 is a constant independent of the displacement of the membrane, the part δR_i is the change of the radius of the ith resonator (in this case the displacement of the membrane is in fact the change of the resonator radius), and β is a coefficient ($\beta>0$ for the situation shown in the figure). The voltage on the ith resonator (relative to ground) can be written in the form

$$u_i=u_0+\delta u_i, \qquad (19)$$

with $u_0\gg\delta u_i$.

Assume that a modulated beam of fast particles with transverse dimension R' much smaller than both R_0 and the characteristic lengths of the excited waves, travels along the system axis.[4)] Let $R_0^{-1}\gg z^{-1}\gg k$; We write down the linearized equations

$$
\begin{gathered}
n_0m_a\frac{\partial v}{\partial t}=-\varkappa T_0\frac{\partial\delta n}{\partial r}-n_0\varkappa\frac{\partial\delta T}{\partial r},\\
\frac{\partial\delta n}{\partial t}+\frac{n_0}{r}\frac{\partial rv}{\partial r}=0\,^{5)},\\
\frac{3}{2}n_0\varkappa\frac{\partial T}{\partial t}=-\frac{n_0\varkappa T_0}{r}\frac{\partial rv}{\partial r}+n_0N\sigma v_0\mathscr{E}_0-Q, \qquad (20)\\
v(R_0;z;t)=\frac{\partial}{\partial t}\delta R \quad (v(0;z;t)=0),\\
\rho'\frac{\partial^2\delta R}{\partial t^2}=-q\delta R+\frac{\beta u_0\delta u}{2\pi R_0\Delta z}+\frac{5}{3}\varkappa T_0\delta n|_{r=R_0};
\end{gathered}
$$

$$C_0\frac{\partial^2\delta u}{\partial t^2}=\frac{(\Delta z)^2}{L}\frac{\partial^2\delta u}{\partial z^2}-\beta u_0\frac{\partial^2\delta R}{\partial t^2}+|e|v_0\Delta z\frac{\partial^2}{\partial t\partial z}\int_0^{R_0}2\pi rN\,dr. \qquad (21)$$

We are considering here a monatomic gas, m_a is the mass of the atoms, v is the velocity (directed along the radius), Δz is the distance between the centers of the resonators (the gap between the resonators is assumed negligibly small), ρ' is the mass per unit surface of the

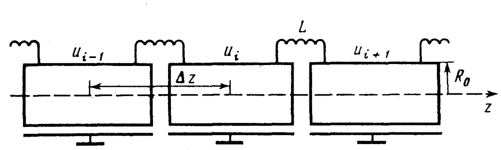

FIG. 1.

membrane, and q is the elastic coefficient per unit surface.

We choose L and C_0 to satisfy the condition

$$v_0^2LC_0=\Delta z^2. \quad (22)$$

The propagation velocity of the electric signal along the chain of resonators becomes then equal to the beam velocity. Equation (21) is greatly simplified and can now already be solved:

$$\delta R=-\frac{|e|\Delta z}{\beta u_0}\int_0^{R_0}2\pi rNdr. \quad (23)$$

We assume that k is chosen close to the resonance of the nth mode. Solving in this case (20) and (21), and taking (23) into account, we obtain

$$\eta_{arc}=-\frac{8\Delta z v_n^2|e|}{3\beta u_0R_0^3J_0(\nu_n)}\left[\frac{3}{5}\frac{(kv_0)^2m_a}{\varkappa T_0}-\frac{\nu_n^2}{R_0^2}\right]^{-1}\times\left(\int_0^{R_0}2\pi r\bar{N}dr\right)^2\left(\int_0^{R_0}2\pi r\bar{N}dr\right)^{-1}. \quad (24)$$

Here ν_n satisfies (17). To avoid large overvoltages in the system (which are shifted in phase relative to the beam-modulation phase in such a way that their contribution to η_{arc} is zero and they are therefore not taken into account there), it is necessary to choose q from the condition

$$q\approx(kv_0)^2\rho'+\frac{5}{3}\varkappa T_0\left[\frac{9(kv_0)^2n_0m_aR_0}{40\varkappa T_0}-\frac{2n_0}{R_0}-\frac{36}{25R_0^2J_0(\nu_n)}\left(\frac{3}{5}\frac{k^2v_0^2m_a}{\varkappa T_0}-\frac{\nu_n^2}{R_0^2}\right)^{-1}\frac{(kv_0)^4n_0m_a^2R_0^3\nu_n^{-2}J_2(\nu_n)}{(\varkappa T_0)^2}\right]. \quad (25)$$

We consider at the same time also a simplier system with one acoustic resonator, in which the returned energy is used to feed the accelerator that produces the beam. Let now the inductance L, previously joining the resonator, be connected to ground through a capacitance $C'\gg C_0$. The beam is pumped by the returned energy when it passes through the capacitor C_2 ($C_2\ll C_0$), one electrode of which is connected to the resonator and the other to the ground. The distance l between C_2 and the resonator is chosen to satisfy the condition $kl=\pi/2$ needed for optimal matching of the phase of the oscillations in C_2 and in the resonator.

In place of (21) we have in this case

$$\delta u=-LC_0\frac{d^2\delta u}{dt^2}-L\beta u_0\frac{d^2\delta R}{dt^2}+Lv_0|e|\frac{d}{dt}\int_0^{R_0}2\pi rN|_{z=0}dr. \quad (26)$$

Choosing in analogy with (22) L and C_0 from the condition

$$LC_0=(kv_0)^{-2}, \quad (27)$$

we obtained in analogy with (23)

$$\delta R=\frac{i|e|}{k\beta u_0}\int_0^{R_0}2\pi rN|_{z=0}dr. \quad (28)$$

If we use (28) and take into account the condition for kl, then the obtained η_{arc} is determined by Eq. (24), in which Δz must be replaced by k^{-1}.

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1) The question of detecting a particle by means of the acoustic wave produced by it as it passes through a medium is considered in Refs. 5–7.

2) Autorecuperation can in principle improve substantially the estimates given in Ref. 8; it is of interest to be able to obtain a nonnegative energy yield (see the start of the article by Vysotskiĭ *et al.*[9]) when the energy released in the reaction (with appropriate use of the reaction products) becomes comparable with the lost energy.

3) In this case we have in mind the fact that the beam-particle velocity v_0 is much larger than the thermal velocity v_{T_e} of the electrons in the plasma.

4) We are not considering a number of technical questions such as transverse focusing of the beam or the differential evacuation of the gas from the gaps between the resonators, the latter needed to prevent breakdowns.

5) This is a continuity equation without sources; in the case of greater interest from the point of view of obtaining high η_{arc}, it makes sense (besides optimizing the beam modulation) to introduce cold gas from the outside into the region near the axis of the resonators (and simultaneously providing for removal of the gas), so as to produce density and temperature profiles that are the optimal for autorecuperation.

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