Excitation of capillary waves on the surface of a liquid metal bordering with an unstable plasma

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The dispersion of coupled capillary waves on the surface of a liquid metal that borders with an unstable plasma is investigated for the first time ever. It was assumed that a Pierce electron-beam instability develops in the plasma. It is shown that the coupled short-wave perturbations develop at much weaker electric fields than in the vacuum case considered earlier by Tonks [Phys. Rev. **48**, 562 (1935)] and Frenkel' [Phys. Zs. Sowietunion **8**, 675 (1935)]. The capillary plasma waves have an aperiodic instability. The critical electron-beam currents, the values of the wave vector in the excitation region, and the instability growth rate are estimated. The results explain qualitatively the experimental data of Gabovich and Poritskiĭ [JETP Lett. **33**, 304 (1981)], who were the first to investigate capillary-wave excitation on the surface of a liquid cathode in a plasma atmosphere. A new mechanism is proposed for the anomalous electron emission of islands of metallic films [P. G. Borzyak and Yu. A. Kulyupin, Electronic Processes in Metallic Island Films (in Russian), Naukova dumka, Kiev, 1980] under the assumption that the emission centers are liquid drops of the metal. The electron beam produced in such structures causes the buildup of capillary wave and enhancement of the emission.

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1. It is known that the surface of a liquid metal is unstable in a sufficiently strong electric field applied along the normal to the surface.[1-3] The dispersion relation that describes the instability of gravitational-capillary waves[3] can be written in the form

$$\omega^2=\frac{\alpha k^3}{\rho}+gk-\frac{kE_0E'}{4\pi\rho\xi'}\bigg|_{z=0}, \tag{1}$$

where α is the surface-tension coefficient of the liquid metal, ρ is the density of the liquid, g is the free-fall acceleration, k is the wave vector directed along the surface, E_0 is the electric field intensity on the metal surface ($z=0$, $E_0 \parallel z$), $E'=-\partial\varphi'/\partial z$ is the perturbed electric field in the space surrounding the metal, and $\xi'=\xi\exp(-i\omega t+ikx)$ is a small displacement of the liquid along the normal. Relation (1) was obtained from the balance of the pressures on the perturbed surface of the metal. Small displacements of the liquid (ξ') distort the potential on the surface:

$$\varphi|_{z=0}=\varphi_0-E_0\xi'=-E_0\xi'. \tag{2}$$

Since the potential on the metal surface is fixed (zero), this distortion should be offset by perturbation (φ') of the potential of the space over the liquid surface. From this we obtain the boundary condition[3]

$$\varphi'|_{z=0}=E_0\xi'. \tag{3}$$

If the metal surface borders on vacuum, then $\varphi' = E_0\xi'\exp(-kz)$ and the dispersion relation (1) takes the form first obtained by Frenkel':

$$\omega^2=\alpha k^3/\rho+gk-k^2E_0^2/4\pi\rho. \tag{4}$$

In the derivation of (4), no account was taken of the viscous damping, i.e., it was assumed that

$$(\alpha k^3/\rho)^{1/2},\quad kE_0/(4\pi\rho)^{1/2}\gg 2\nu k^2, \tag{5}$$

where $\nu=\eta/\rho$ and η is the viscosity coefficient.

It is seen from (4) that the short-wave perturbations (we shall disregard hereafter the gravitational component) are excited in sufficiently strong electric fields

$$E_0^2>4\pi\alpha k. \tag{6}$$

The instability is aperiodic. If $k=10^3$ cm^{-1}, the criterion (6) is satisfied for liquid copper ($\alpha=1.35\times10^3$ erg/cm^2) if $E_0>1.2\times10^6$ V/cm. The conditions (5) are taken well satisfied ($\nu=3.78\times10^{-3}$ cm^2/sec). The values of the constants α and ν are given for the melting temperature of copper ($T\approx1085$° C).

Notice should be taken in this connection of the unusually effective experiments of Gabovich and Poritskiĭ,[4] who observed (on liquid copper) waves with the indicated wave vectors, but for electric fields $\approx10^5$ V/cm near the surface. In these experiments "a stream of dense highly ionized hydrogen plasma was used to melt the surface and to produce near it strong electric fields".[4]

It is natural to assume that when the liquid cathode borders on a plasma, or more accurately on a cathode layer, the short-wave perturbations can be excited on the surface of the liquid metal in weaker electric fields, since instabilities that are the cause of the electric-field perturbation E' [Eq. (1)] appear in the layers. From this follows the possible appearance of coupled capillary-plasma waves.

To check on this assumption, we have considered the case when a Pierce instability[5] of the electron beam appears in the layer that separates the liquid cathode from the plasma. This instability is produced near the cathode because of the secondary electron emission from the cathode bombarded by the plasma ions. There is no doubt that this is not the only instability that can develop in the layer, and we do not claim a quantitative description of the experiments of Gabovich and Poritskiĭ. The choice of just this model was based on the fact that the Pierce instability is typical of bounded systems, that it is also aperiodic, and mainly that it is actually observed in beam-plasma discharges.[6] We note also that capillary-plasma waves have never been considered before and that the model character of the calculations presented below has been verified.

2. In contrast to the usual Pierce instability for longitudinal perturbations of the density and of the potential, we shall consider the case of "oblique" perturbations, having a transverse wave vector k (otherwise the boundary condition (3) is not satisfied). To simplify the calculations we consider a case when the density and velocity of the electrons in the unperturbed beam are constant (n_0, v_0). Such a model is valid if the space charge of the electrons is compensated by the ions, something hardly realizable at each point of the layer. Nonetheless, we consider just this case in view of the great difficulties involved in the solution of the inhomogeneous problem.

The initial equations that describe the plasma instability are[5]

$$-i\omega v_{1z}+v_0\frac{dv_{1z}}{dz}=\frac{e}{m}\frac{d\varphi_1}{dz},\quad -i\omega v_{1x}+v_0\frac{dv_{1x}}{dz}=ik\frac{e}{m}\varphi_1,$$
$$-i\omega n_1+v_0\frac{dn_1}{dz}+n_0\left(\frac{dv_{1z}}{dz}+ikv_{1x}\right)=0,\quad \frac{d^2\varphi_1}{dz^2}-k^2\varphi_1=4\pi e n_1, \tag{7}$$

where e and m are the charge and mass of the electron.

The perturbations of other quantities were chosen in the form $A'=A_1(z)\exp(-i\omega t+ikx)$. For the case of a plane wave $A_1\sim\exp(ik_\parallel z)$ the dispersion relation obtained with the aid of (7) has no unstable roots for ω.

The solutions of (7) are of the form

$$\varphi_1=C_1\,\mathrm{sh}\,kz+C_2\,\mathrm{ch}\,kz+C_3e^{r_1z}+C_4e^{r_2z},$$
$$n_1=\frac{1}{4\pi e}[C_3(r_1^2-k^2)e^{r_1z}+C_4(r_2^2-k^2)e^{r_2z}],$$
$$v_{1z}=C_5e^{i\omega z/v_0}+\frac{ek}{m}\Big[C_1\frac{i\omega\,\mathrm{ch}\,kz+kv_0\,\mathrm{sh}\,kz}{k^2v_0^2+\omega^2}+C_2\frac{i\omega\,\mathrm{sh}\,kz+kv_0\,\mathrm{ch}\,kz}{k^2v_0^2+\omega^2}$$
$$-\frac{i}{\omega_p}(C_3r_1e^{r_1z}-C_4r_2e^{r_2z})\Big], \tag{8}$$
$$v_{1x}=-\frac{\omega}{kv_0}C_5e^{i\omega z/v_0}+\frac{ike}{m}\Big[\frac{i\omega\,\mathrm{sh}\,kz+kv_0\,\mathrm{ch}\,kz}{k^2v_0^2+\omega^2}C_1$$
$$+\frac{i\omega\,\mathrm{ch}\,kz+kv_0\,\mathrm{sh}\,kz}{k^2v_0^2+\omega^2}C_2-\frac{i}{k\omega_p}(C_3e^{r_1z}-C_4e^{r_2z})\Big],$$

where

$$r_1=\frac{i}{v_0}(\omega+\omega_p),\quad r_2=\frac{i}{v_0}(\omega-\omega_p),\quad \omega_p^2=\frac{4\pi e^2n_0}{m}.$$

The boundary conditions (liquid cathode) are

$$n_1,\ v_{1z},\ v_{1x}|_{z=0}=0,\quad \varphi_1|_{z=0}=E_0\xi,\quad \varphi_1|_{z=L}=0, \tag{9}$$

where L is the boundary of the layer in which the beam passes. Under the conditions of the experiments of Ref. 4, the dimension of the cathode layer is estimated by the authors at $L\approx10^{-2}$ cm. In the case of the one-dimensional Pierce problem ($k=0$), the constant C_5 is identically equal to zero.

With the aid of (8) and (9) it is easy to obtain an expression for $d\varphi_1/dz|_{z=0}$. Substitution of this expression in (1) yields the dispersion equation for the coupled capillary-plasma waves:

$$\Omega^2=\tilde\alpha\tilde k^3-\tilde E_0^2\tilde k^2\frac{\mathrm{ch}\,\tilde k}{\mathrm{sh}\,\tilde k}[(\beta^2+\Omega^2+\tilde k^2)^2-4\beta^2\Omega^2]$$
$$\times\Big\{(\Omega^2-\beta^2+\tilde k^2)(\Omega^2+\tilde k^2)+2\beta^2\tilde k^2+\frac{\beta\tilde k}{\mathrm{sh}\,\tilde k}[(\Omega^2+\beta^2+\tilde k^2)$$
$$\times e^{i\Omega}\sin\beta+2i\Omega\beta(e^{i\Omega}\cos\beta-\mathrm{ch}\,\tilde k)]\Big\}^{-1}, \tag{10}$$

where $\Omega=\omega L/v_0$, $\beta=\omega_pL/v_0$, $\tilde k=kL$, $\tilde\alpha=\alpha/\rho v_0^2L$, $\tilde E_0^2$ $E_0^2/4\pi\rho v_0^2$.

We consider now the limiting transitions to the known results. As $\beta\to0$ (the vacuum case) the dispersion takes the form

$$\omega^2=\frac{\alpha k^3}{\rho}-\frac{k^2E_0^2}{4\pi\rho}\frac{\mathrm{ch}\,kL}{\mathrm{sh}\,kL}. \tag{11}$$

This expression coincides with (4) in the limit $kL\gg1$. In (11) we took into account the presence of a conducting screen located over the surface of the liquid. In this case both solutions [$\sim\exp(-kz)$ and $\exp(kz)$] must be taken into account in the solution for the vacuum potential and it is this which leads to the dispersion (11). The vanishing of the denominator of (10) corresponds exactly to the Pierce dispersion if we put $\tilde k=0$. Let us consider the Pierce dispersion for "oblique" perturbations:

$$(\Omega^2+\tilde k^2)(\Omega^2-\beta^2+\tilde k^2)+2\beta^2\tilde k^2+\frac{\tilde k\beta}{\mathrm{sh}\,\tilde k}[\sin\beta e^{i\Omega}(\Omega^2+\beta^2+\tilde k^2)$$
$$+2i\Omega\beta(e^{i\Omega}\cos\beta-\mathrm{ch}\,\tilde k)]=0. \tag{12}$$

Putting $-i\Omega=\gamma\ll1$ (near the instability threshold), we can obtain

$$\gamma=-\frac{(\beta^2+\tilde k^2)(\beta\sin\beta+\tilde k\,\mathrm{sh}\,\tilde k)}{\beta[2\beta(\mathrm{ch}\,\tilde k-\cos\beta)-\sin\beta(\beta^2+\tilde k^2)]}. \tag{13}$$

At the excitation threshold ($\gamma=0$) we have

$$\beta\sin\beta+\tilde k\,\mathrm{sh}\,\tilde k=0, \tag{14}$$

from which it follows that excitation of short-wave perturbations ($\tilde k>1$) is possible at

$$\beta_n=3\pi/2+2\pi n,\quad n=0,\ 1,\ 2\ldots. \tag{15}$$

The threshold values of the wave vector ($\tilde k_n$) at which the growth rate of the Pierce instability is zero are equal, for the first three models, to $\tilde k_0=1.73$; $\tilde k_1=2.27$; $\tilde k_2=2.59$. It is precisely near these values of the wave vectors that coupled waves should be excited, since the growth rate of the capillary waves is also zero ($\tilde\alpha\tilde k_n>E_0^2\cosh\tilde k_n/\sinh\tilde k_n$).

We investigate now the dispersion (10) near the values (15) of β. Putting $\Omega\ll1$, we obtain

$$\Omega^2=\tilde\alpha\tilde k^3-\tilde E_0^2\tilde k\frac{(\beta_n^2+\tilde k^2)\,\mathrm{ch}\,\tilde k}{\tilde k\,\mathrm{sh}\,\tilde k-\beta_n-i\beta_n\Omega(1+2\beta_n(\tilde k^2+\beta_n^2)^{-1}\mathrm{ch}\,\tilde k)}. \tag{16}$$

As seen from (10) and (16), no coupling of the capillary waves with the Pierce space-charge waves takes place if $E_0=0$. It is easy to show with the aid of (16) that an aperiodic instability of the coupled wave (the growth rate of these waves is zero at $E_0=0$) appears in the wave-vector interval

$$\bar{k}_n \leq \bar{k} \leq \bar{k}_n + \Delta k,$$

where

$$\Delta k \approx \frac{E_0^2}{\bar{\alpha}} \frac{1+\beta_n^2/\bar{k}_n^2}{\bar{k}_n + \operatorname{th} \bar{k}_n} \quad \left(\frac{E_0^2}{\bar{\alpha}} \ll 1\right). \tag{17}$$

As a rule $\bar{E}_0^2 \ll \bar{\alpha}$ (for copper at $v_0 = 10^8$ cm/sec, $E_0 = 10^5$ V/cm, and $L = 10^{-2}$ cm we have $\bar{\alpha} = 1.52 \cdot 10^{-12}$, $\bar{E}_0^2 = 8.9 \cdot 10^{-14}$). The maximum of the growth rate of the coupled waves is reached near $\bar{k} = \bar{k}_n$ and is equal to

$$\gamma_m = (E_0^2 A)^{1/2}, \quad A = \bar{k}_n \frac{\operatorname{ch} \bar{k}_n (\bar{k}_n^2 + \beta_n^2)}{\beta_n (1 + 2\beta_n (\bar{k}_n^2 + \beta_n^2)^{-1} \operatorname{ch} \bar{k}_n)}. \tag{18}$$

Expression (18) is valid at

$$\gamma_m \gg (\bar{\alpha} \bar{k}_n^2)^{1/2}, \quad 2\bar{\nu} \bar{k}_n^2 \quad (\bar{\nu} = \nu / L v_0).$$

For the zeroth mode, at the parameters indicated above, we have $\Delta k = 0.2$; $\gamma_m = 10^{-4} (\omega_i = 10^6 \text{ sec}^{-1})$.

Thus, excitation of short-wave perturbations on the surface of a liquid metal is possible at beam-current values

$$j_n = \beta_n v_0^2 m / 4\pi e L^2 \quad (j_0 \approx 10 \text{ A/cm}^2). \tag{19}$$

The electric field does not play the decisive role if the foregoing conditions are satisfied.

It should be noted that even in the absence of the Pierce instability ($\sin\beta > 0$) the electron beam weakens significantly the Tonks criterion (6). Thus, at $\beta = 0.9\pi$, the capillary waves are excited up to $\bar{k} \approx 0.68$, the maximum of the growth rate is reached at the point $\bar{k} \approx 0.4$ and is equal to $\gamma = 4 \times 10^{-7}$. According to (6), at the same values of the initial parameters, we have $\gamma > 0$ only at $\bar{k} < 0.06$. This effect is due to a regrouping of the beam in the electric field, because of the distortion of the metal surface. At $E_0 < 0$ (in our geometry, the field is directed from the boundary of the layer to the cathode) the electrons are bunched in the vicinity of the troughs of the capillary structure and an uncompensated positive charge is produced in the region of the crests and amplifies the initial field. If $E_0 > 0$, the electrons are gathered in the region of the crests and the field is likewise amplified. With the aid of (8) it is easy to verify that the sign of the perturbed electric field on the metal surface (at $\xi' > 0$) always coincides with the sign of E_0.

3. The results above permit a new approach to the explanation of the anomalous electron emission produced when current flows in island-type metal films.[7] The individual islands in such films have an emissivity greatly exceeding the equilibrium value (at the melting temperature of the lattice).

It is known that the emissivity of liquid cathodes increases greatly when microinhomogeneities (in particular, capillary waves) are excited on their surfaces.[8] If it is assumed that the emission centers in the films are liquid drops of the metal, it is easy to show that capillary waves can exist in them even at drop dimensions $\approx 10^{-6} - 10^{-5}$ cm, since the capillary frequency greatly exceeds the damping decrement due to the viscosity. The electric fields near the emitting centers, however, are relatively weak,[1)] $\approx 10^5$ V/cm (Ref. 7), and the criterion (6) does not hold for such short wavelengths. Since the distance between the emitting center and the neighboring "cold" islands is small ($L \approx 10^{-6}$ cm, much less than the electron mean free path), the electrons form a beam that can be subject to Pierce instability at high current densities. The experimental conditions in the case of islands agree well with the Pierce theoretical model of a bounded electron beam whose charge is compensated by immobile ions (by the ion matrix of the metal). The Pierce instability leads to a buildup of capillary-plasma waves on the surfaces of the liquid islands and to enhancement of the emission. As a rule, the emission centers are located in the current channels with larger current density,[9] $\approx 10^6 - 10^7$ A/cm^2. According to (19), for the zeroth mode we have $j_0 \approx 10^6$ A/cm^2 ($L = 10^{-6}$ cm, $v_0 = 10^7$ cm/sec). If the dimension of the emission center is relatively large ($R \gg L$), a shortwave ($\lambda \approx 4L$) structure of capillary plasma waves is produced on its surface, and it is this which enhances the emission.

We have thus shown in this paper that a plasma atmosphere facilitates greatly the excitation of shortwave capillary perturbations on the surface of a liquid metal even in relatively weak electric fields.

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[1)]The voltage drop in the region of the emitting centers is higher than in other sections of the film.

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