

Energy spectrum of quasiparticles in a one-dimensional disordered system with an arbitrary scattering potential

V. N. Prigodin

I. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
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The one-electron state density is calculated by summing all the perturbation-theory diagrams in the case of nonoverlapping impurities with arbitrary scattering potential. The appearance of state-density peaks corresponding to resonant scattering by impurity complexes is predicted on the basis of the derived equations.

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1. INTRODUCTION

One-dimensional disordered systems are among the most investigated models. A very important result was the development, by Berezinskii,¹ of a method of determining the kinetics of a quantum particle in such a system. This method has subsequently yielded many other results for one-dimensional disordered systems.²⁻⁵

In this paper we use Berezinskii's method to calculate the one-electron state density. It must be noted that Berezinskii's technique has already been used to investigate the state density^{3,4} in connection with a study of Dyson singularities in a model in which the impurities are arranged in a lattice.⁶ Just as in Berezinskii's paper, diagrams of a special type, which were found to oscillate slowly when umklapp processes are taken into account, were summed. In the present problem, the positions of the impurities are perfectly random and we sum all the diagrams.

The question itself of the state density of a one-dimensional disordered system was investigated in many studies.⁷ These, however, always dealt with particular cases of the scattering potential. The most investigated among them is the δ -function method (the Frisch-Lloyd method). We have solved this problem for an arbitrary scattering potential, and were therefore able to predict new singularities in the behavior of the state density of a one-dimensional disordered system.

2. DERIVATION OF THE FUNDAMENTAL EQUATIONS

We consider an electron situated in the field of randomly located centers. We calculate the state density

$$\rho(\epsilon) = -\pi^{-1} \text{Im} \langle G(x, x+0) | \epsilon + i0 \rangle, \quad (1)$$

where $\langle \rangle$ means averaging over the impurity positions, carried out in the usual cross technique. A typical diagram for $G(x, x+0)$ is shown in Fig. 1. The Berezinskii technique of calculating such diagrams is well described in Refs. 3 and 4. The impurity lines must be first gathered into total forward and backward scattering amplitudes f_+ and f_- , in which we include, following Ref. 4, the factor i/v . In the general case it is necessary to distinguish in the backscattering amplitude f_- between f_- and \bar{f}_- , depending on the direction from which the electron is incident on the scattering potential.

We define the right-hand side R_m as the sum of diagrams located to the right of the point x . Considering a small displacement of the point x , we obtain for $R_m(x)$ in analogy with Refs. 3 and 4 (c is the density and p is the Fermi momentum)

$$-dR_m(x)/dx = c \sum_{m' \geq 0} W_{mm'} R_{m'}(x) e^{2ipx(m-m')} - cR_m(x), \quad (2)$$

$$W_{mm'} = \sum_k C_m^k C_{m'-1}^{k+m'-m} (1+f_+)^{2m-2k} (f_-)^k (f_-)^{k+m'-m}. \quad (3)$$

The function (3) is illustrated in Fig. 2 (see also Ref. 4). The combinatorial factors in (3) result from allowance for all possible methods of connecting the impurity lines when m , m' , and k are given. The substitution

$$R_m(x) = e^{2ipxm} R_m$$

reduces Eq. (2) to the form

$$-2ipmR_m = c \sum_{m'} W_{mm'} R_{m'} - cR_m. \quad (4)$$

If impurities of a different type are encountered, it is necessary to include in (3) an additional averaging over their distribution, with the corresponding weight $P(f)$. For overlapping impurities there appears in (3) a contribution of fourth order in the density. It will, however, be small relative to the parameter $\tau_0 c$, where τ_0 is the characteristic radius of the impurity potential. This contribution can be taken into account in principle by introducing the amplitude of scattering by a potential made up of two impurities, and then averaging the scattering over the distance between the centers.

The state density (1) is expressed in terms of R_m in the form

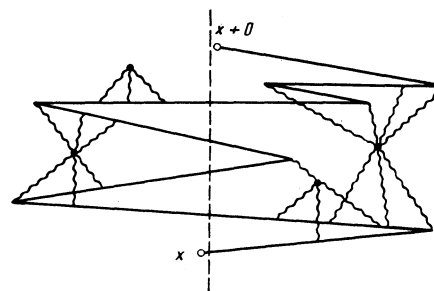


FIG. 1. Typical diagram for the Green's function $G(x, x+0)$.

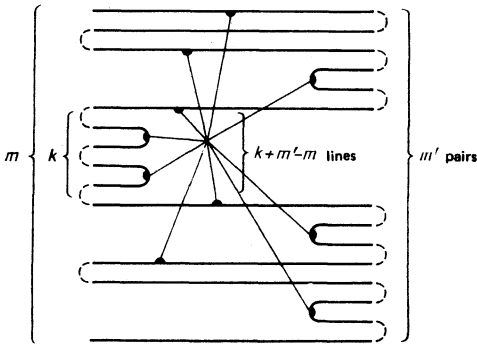


FIG. 2. General term of the sum for the transition probability $W_{mm'}$ in the case of impurity scattering [see (3)].

$$\rho(\varepsilon) = \text{Re} \left[\frac{1}{\pi\nu} \sum_{\pm} (R_m + R_{m\pm})^2 \right]. \quad (5)$$

Equations (4) and (5) solve completely the state-density problem. The solution of Eq. (4) must be sought with the boundary condition $R_0 = 1$.

Let us recall some general properties of the scattering amplitude. For an arbitrary scattering potential, the amplitudes f_+ and f_- or \bar{f}_- are connected by the relation

$$|1+f_+|^2 + |f_-|^2 = 1, \quad (6)$$

which states the law of conservation of the number of particles or the unitarity condition. There is one other unitarity relation that connects all three amplitudes:

$$-(\bar{f}_- + f_+^*) = f_+ f_-^* + \bar{f}_- f_+^*. \quad (7)$$

The entire scattering process is thus characterized by three parameters. We shall consider next the case of a symmetrical potential, for which $f_- = \bar{f}_-$, and it follows from (6) and (7) that⁴

$$(1+f_+)^2 = f_-^2 - f_-/f_+^*.$$

Consequently, the total scattering amplitude can be described in this case by two parameters, which can be conveniently chosen in the form

$$f = f_- = -i|\gamma|^{1/2} e^{-i\alpha}, \quad 1+f_+ = (1-\gamma)^{1/2} e^{-i\alpha}, \quad (8)$$

where γ is the coefficient of reflection from an individual impurity

3. TRANSITION TO FUNCTIONAL EQUATIONS

We change over in (4) from a difference to an integro-differential equation. We write for $C_{m'-1}^{k+m'-m}$ the following integral representation⁵:

$$C_{m'-1}^{k+m'-m} = \frac{1}{2\pi i} \oint \frac{dz}{z} \frac{z^{m'}}{(z-1)^{m-k}}, \quad (9)$$

where the integration with respect to z is along a circle with a radius larger than unity, in the positive direction. Substituting (9) in (3), summing over k , and making the change of variables $\xi = f^{1/2} z$, we rewrite Eq. (4) in the form

$$-2ikmR_m = \sum_{m' \geq 0} W_{mm'} R_{m'} - R_m, \quad (10)$$

$$W_{mm'} = \frac{1}{2\pi i} \oint \frac{d\xi}{\xi} \xi^{m'} \delta^m, \quad |\xi| > \gamma^{1/2}, \quad (11)$$

$$\delta = \frac{f}{f'} \frac{f'\xi - 1}{\xi - f}, \quad k = \frac{p}{c}.$$

We note that we have above $|\delta| \leq 1$ at $|\xi| \geq 1$.

We introduce for R_m a generating function defined by

$$R(z) = \sum_{m \geq 0} z^m R_m. \quad (12)$$

The equation for $R(z)$ is obtained from (10) and (11):

$$2ikz \frac{dR}{dz} = R - \frac{1}{2\pi i} \oint \frac{d\xi}{\xi} \frac{(\xi-f)R(\xi)}{(1-zf)\xi + f(z-f)/f'}. \quad (13)$$

In the foregoing $|z| = 1$ and the integration with respect to ξ is along a circle of radius $|\xi| = 1 + 0$.

According to the definition (12), the function $R(z)$ is analytic in the unit circle. Therefore the contribution to the integral in the right-hand side of (13) comes from the two poles of the integrand. We obtain as a result

$$-2ikz \frac{dR}{dz} = \frac{(1-\gamma)z}{(1-zf)(z-f')} R \left(\frac{f}{f'} \frac{f-z}{1-zf} \right) - R(z) + \frac{f}{f-z}. \quad (14)$$

The right-hand side of (5) is more conveniently written with the aid of the function P_m defined by

$$P_m = R_m, \quad m \geq 0; \quad P_m = R_{-m}^*, \quad m < 0.$$

The equation for the generating function corresponding to P_m

$$P(z) = \sum_{m=-\infty}^{\infty} z^m P_m = R(z) + R^*(1/z) - 1 \quad (15)$$

can be obtained from (14):

$$-2ikz \frac{dP}{dz} = \frac{(1-\gamma)z}{(1-zf)(z-f')} P \left(\frac{f}{f'} \frac{f-z}{1-zf} \right) - P. \quad (16)$$

Equation (16) should be supplemented by the normalization condition

$$\oint \frac{dz}{z} P(z) = 1, \quad |z| = 1,$$

which is a variant of the boundary condition (10). The expression for $\rho(\varepsilon)$ can be rewritten with the aid of (15) in the form

$$\rho(\varepsilon) = \frac{1}{2\pi\nu} \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z^2} (1+z)^2 P(z) P \left(\frac{1}{z} \right).$$

The variable z in (16) is defined on the unit circle. We now express (16) in terms of a real variable. We define

$$z = \frac{1+is}{1-is}, \quad \Phi(s) = \frac{(1+z)^2}{4\pi z} P(z); \quad (17)$$

$$k \frac{d(1+s^2)\Phi}{ds} = \Phi - \frac{1+ab}{(1+bs)^2} \Phi \left(\frac{s-a}{1+bs} \right), \quad (18)$$

$$a = \frac{\sin \alpha + \gamma^{1/2}}{\cos \alpha}, \quad b = \frac{\sin \alpha - \gamma^{1/2}}{\cos \alpha}.$$

The expression for $\rho(\varepsilon)$ and the normalizations in the variables (17) take the form

$$\rho = \frac{2}{\nu} \int_{-\infty}^{\infty} ds \Phi(s) \Phi(-s), \quad \int_{-\infty}^{\infty} ds \Phi(s) = 1. \quad (19)$$

Equations (17)–(19) constitute one of the main results of the present paper. Further analysis of (17) calls for invoking the concrete dependences of γ and α on the energy. We confine ourselves hereafter to general remarks concerning the solution of (17) and point to certain new facts that follow from (17).

4. STATE-DENSITY PEAKS

Equation (17) has the simplest structure if $\gamma^{1/2} = \sin \alpha$:

$$k \frac{d}{ds} [(1+s^2)\Phi] = \Phi - \Phi \left[s \pm 2 \left(\frac{\gamma}{1-\gamma} \right)^{1/2} \right]. \quad (20)$$

The condition $\gamma^{1/2} = \sin \alpha$ is satisfied, in particular, in the Frisch-Lloyd model. In this case

$$\gamma = u_0^2 / (u_0^2 + v^2), \quad (21)$$

where v is the particle velocity and u_0 is the strength of the δ -function potential. Equation (20) contains two scales with respect to the variable s . An analytic solution of (20) can be obtained by considering different parameter regions in which limiting relations are established between these two scales. Equation (20) can in this case be reduced to a differential one.

If $\gamma^{1/2} \neq \sin \alpha$, Eq. (17) can also be reduced to the form (20). We introduce

$$F = \frac{\lambda}{2} \left(1 + \frac{b}{a} s^2 \right) \Phi(s), \quad (22)$$

$$\lambda = |a/b|^{1/2}, \quad u = \ln \lambda. \quad (23)$$

At $\sin^2 \alpha < \gamma$ we make the change of variables

$$t = \ln \frac{\lambda + s}{\lambda - s}, \quad s = \lambda \operatorname{th} \frac{t}{2}.$$

In the new variables, Eq. (17) takes the form

$$2k \frac{d}{dt} [(\operatorname{ch} t \operatorname{ch} u + \operatorname{sh} u) F] = F - F(t - t_0), \quad (24)$$

$$\rho = \frac{4}{\lambda v} \int_{\Gamma} dt \operatorname{ch}^2 \frac{t}{2} F(t) F(-t), \quad \int_{\Gamma} F(t) dt = 1, \quad (25)$$

where $t_0 = 2 \operatorname{arctanh}(a/\lambda)$. The integration in (25) is in the positive direction along a contour made up of the straight lines $\operatorname{Im} t = 0$ and $\operatorname{Im} t = \pi$. The present equation (24) contains already three scales with respect to the variable t , and its analysis is more difficult.

At $\sin^2 \alpha > \gamma$ we make the following change of variables:

$$\varphi = \frac{1}{i} \ln \frac{\lambda - is}{\lambda + is}, \quad s = -\lambda \operatorname{tg} \frac{\varphi}{2}.$$

For F given in (22), Eq. (17) can then be rewritten in the form

$$2k \frac{d}{d\varphi} [(\cos \varphi \operatorname{sh} u - \operatorname{ch} u) F] = F - F(\varphi - \varphi_0), \quad (26)$$

$$\rho = \frac{4}{\lambda v} \int_0^{2\pi} d\varphi \cos^2 \frac{\varphi}{2} F(\varphi) F(-\varphi), \quad \int_0^{2\pi} d\varphi F(\varphi) = 1, \quad (27)$$

where $\varphi_0 = -2 \operatorname{arctan}(a/\lambda)$. Equation (26) is of a form typical of the theory of resonant phenomena.⁶ If φ_0 has a value close to a rational fraction M/N of 2π , the harmonics that are multiples of N should be enhanced in F . This can be detected, for example, by solving (26) by

successive approximations in the case of small values of γ . According to (23), the small parameter in (26) is then $\sinh u$.

The resonance effect manifests itself more clearly, however, in the Fourier representation⁷:

$$F_n = \frac{1}{2\pi} \int_0^{2\pi} e^{-in\varphi} F(\varphi) d\varphi, \quad F(\varphi) = \sum_{-\infty}^{\infty} e^{in\varphi} F_n.$$

The Fourier-transformed Eq. (26) takes the form

$$F_n = \frac{ink \operatorname{sh} u}{1 - e^{-in\varphi} + 2ikn \operatorname{ch} u} (F_{n-1} + F_{n+1}), \quad \rho = \frac{2\pi}{\lambda v} \sum_{-\infty}^{\infty} (F_n + F_{n+1})^2, \quad (28)$$

where $F_0 = 1/2\pi$. If $\gamma \ll 1$, Eq. (28) can be solved by iteration. It is seen from (28) that at small $k = p/c \ll 1$ the individual harmonics become enhanced if $\varphi_0(\varepsilon) \approx 2\pi M/N$. The resonance effects can manifest themselves in the state density in the form of peaks of finite height.

We can propose a general scheme for solving (16). Let $\varphi_0 = 2\pi M/N + \Delta$, where $\Delta \ll 1$. Since the value of M is immaterial here, we put $M = 1$. We define

$$F_n = F[\varphi - 2\pi(n-1)/N], \quad (29)$$

where $n = 1, 2, \dots, N$ and $0 \leq \varphi \leq 2\pi/N$. With the aid of (29) we can rewrite (26) in matrix form:

$$\frac{d}{d\varphi} \begin{pmatrix} h_1 & -\Delta & 0 \\ & h_2 & \ddots \\ & & \ddots & -\Delta \\ 0 & & & h_N \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ & 1 & \ddots \\ 0 & & \ddots & -1 \\ -1 & 0 & & 1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix}, \quad (30)$$

$$h_n = 2k[\operatorname{sh} u \cos(\varphi - 2\pi(n-1)/N) - \operatorname{ch} u],$$

which should be supplemented by the condition for the continuity of F and normalization. It is possible to change over to similar equations, but now of infinite order, in (21) and (24), too. The solution (30) can in principle be written also in the general case. It is impossible, however, to carry the calculations through to conclusion in this case. We shall therefore study in detail only the case $\varphi_0 = \pi$.

The value $\varphi_0 = \pi$ corresponds according to (23) to the choice $\alpha = \pi/2$. The system of equations (30) takes in this case, after diagonalization, the form

$$\pm (1 + \gamma^{1/2} \cos \varphi) \frac{d}{d\varphi} (1 - \gamma^{1/2} \cos \varphi) F_{1,2} = F_{1,2} - C; \quad (31)$$

here $F_1 = F(\varphi)$ and $F_2 = F(-\varphi)$, where $0 \leq \varphi \leq \pi$. The solution of Eqs. (31) can be represented in the form

$$F_{1,2} = \Phi_{1,2} / (1 - \gamma^{1/2} \cos \varphi),$$

$$\Phi_1 = C_1 - C_1 \frac{(1-\gamma)^{1/2}}{2k} \int_{-\pi}^{\varphi} \frac{\exp[\psi(\varphi+\varphi') - \psi(\varphi)]}{1 - \gamma^{1/2} \cos(\varphi+\varphi')} d\varphi', \quad (32)$$

$$\Phi_2 = C_2 + C_2 \frac{(1-\gamma)^{1/2}}{2k} \int_{-\pi}^{\varphi} \frac{\exp[-\psi(\varphi+\varphi') + \psi(\varphi)]}{1 - \gamma^{1/2} \cos(\varphi+\varphi')} d\varphi',$$

$$\psi(\varphi) = \frac{1}{k} \operatorname{arctg} \left(\frac{\operatorname{tg} \varphi}{(1-\gamma)^{1/2}} \right),$$

$$C = \frac{(1-\gamma)^{1/2}}{2k} (1 + e^{-\pi/k}) C_1 = \frac{(1-\gamma)^{1/2}}{2k} (1 + e^{\pi/k}) C_2. \quad (33)$$

The intermediate integration constants were chosen

above to satisfy the continuity of F ; $\Phi_2(\pi) = \Phi_1(\pi)$, $\Phi_2(0) = \Phi_1(0)$. The integrand in (32) and (33) has a singularity as $\gamma \rightarrow 1$. It can be separated by making the change of variables

$$\operatorname{tg} \beta = \operatorname{tg} \varphi / (1 - \gamma)^{1/2}. \quad (34)$$

In the new variables, Eqs. (32) and (33) take the form

$$\Phi_{1,2}(\beta) = C(1 - \gamma)^{1/2} \left(1 + e^{-\pi/k} - \frac{\gamma^{1/2}}{k} \Psi(\pm \beta) \right), \quad (35)$$

$$\Psi(\beta) = \int_0^\pi \frac{\cos(\beta + \beta')}{[1 - \gamma \sin^2(\beta + \beta')]^{1/2}} e^{-\beta'/k} d\beta'.$$

With allowance for (34), Eqs. (27) can be rewritten as follows:

$$\rho = \frac{4C^2}{v} \left\{ \pi(1 + e^{-\pi/k})^2 - 2 \frac{\gamma^{1/2}}{k} (1 + e^{-\pi/k}) \int_0^\pi d\beta \cos \beta \frac{1 + \gamma^{1/2} - 2\gamma \sin^2 \beta}{(1 - \gamma \sin^2 \beta)^{1/2}} \Psi(\beta) \right. \\ \left. + 2 \frac{\gamma}{k^2} \int_0^\pi d\beta (1 + \gamma^{1/2} - 2\gamma \sin^2 \beta) \Psi(\beta) \Psi(-\beta) \right\}, \quad (36)$$

$$1 = C \left\{ 2\pi(1 + e^{-\pi/k}) - \frac{2\gamma}{k} \int_0^\pi d\beta \frac{\cos \beta}{(1 - \gamma \sin^2 \beta)^{1/2}} \Psi(\beta) \right\}. \quad (37)$$

Substituting (35) in (36) and carrying out partial integration, we arrive ultimately at the following equations:

$$\rho = \frac{4C^2}{v} \left\{ \pi(1 + e^{-\pi/k})^2 - 2 \frac{\gamma^{1/2}}{k} (1 + e^{-\pi/k}) \int_0^\pi d\beta e^{-\beta/k} [(1 + \gamma^{1/2}) I_0(\beta) - 2\gamma I_1(\beta)] \right. \\ \left. + 2 \frac{\gamma}{k^2} e^{-\pi/k} \int_0^\pi d\beta \operatorname{ch} \frac{\pi - \beta}{k} [\beta I_0(\beta) + \gamma^{1/2} \sin \beta I_1(\beta)] \right\}, \quad (38)$$

$$1 = C \left\{ 2\pi(1 + e^{-\pi/k}) - \frac{2\gamma}{k} \int_0^\pi d\beta e^{-\beta/k} I_0(\beta) \right\};$$

$$I_0(\beta) = \int_0^\pi d\theta \frac{\cos \theta \cos(\theta + \beta)}{(1 - \gamma \sin^2 \theta)^{1/2} [1 - \gamma \sin^2(\theta + \beta)]^{1/2}}, \quad (39)$$

$$I_1(\beta) = \int_0^\pi d\theta \sin^2 \theta \frac{\cos \theta \cos(\theta + \beta)}{(1 - \gamma \sin^2 \theta)^{1/2} [1 - \gamma \sin^2(\theta + \beta)]^{1/2}}, \quad (40)$$

$$I_2(\beta) = \int_0^\pi d\theta \frac{\cos 2\theta \cos(\theta + \beta/2) \cos(\theta - \beta/2)}{[1 - \gamma \sin^2(\theta + \beta/2)]^{1/2} [1 - \gamma \sin^2(\theta - \beta/2)]^{1/2}}. \quad (41)$$

The integrals (39)–(41) are not singular. From (38)–(41) we obtain the following expressions for $\rho(\varepsilon)$ in two limiting cases:

$$\rho(\gamma=0) = \frac{1}{\pi v}, \quad \rho(\gamma=1) = \frac{\pi c^2}{4p^2 v \operatorname{sh}^2(\pi c/2p)}.$$

In (38), interest attaches to the expression for $\rho(\varepsilon)$ in the region of small k . We note that in this case, at $k \ll 1$, the main contribution to (38) comes from integration with respect to β near zero. Although the integrals (39) and (40) cannot be expanded here in small powers of β , since the integrand becomes nonanalytic as $\gamma \rightarrow 1$, they can be estimated. For example, $I_0(\beta) = I_0(0) - J_0(\beta)$, where $J_0(\beta) \leq \beta$. The main contribution to (38) from $I_0(\beta)$ comes from integration of $I_0(0)$. The contribution from $J_0(\beta)$ is then small in terms of the parameter k . As a result we have

$$\rho = \frac{1}{\pi v} \left[\frac{(1 - \gamma)^{1/2}}{1 + \gamma^{1/2}} + O(k(1 - \gamma)^{1/2}; e^{-\pi/k}) \right]. \quad (42)$$

It is known,⁶ for example, from the Frisch-Lloyd model that the state density in the fluctuation region decreases exponentially. It follows from (42) that the peaks deduced above in this region may turn out to be fully observable. If γ is chosen as in the Frisch-Lloyd model (21), we find from (42) that the amplitude of the peak remains finite also as $\varepsilon \rightarrow 0$

$$\rho \rightarrow 1/2\pi u_0.$$

5. CONCLUSION

We note in conclusion that the state-density peaks observed above differ entirely in character from the Dyson singularities.³⁻⁶ The latter are due entirely to the fact that the distance between the impurities, while haphazard, is not always a multiple of a certain constant. As a result, states with wavelengths that are multiples of this constant will be preferred. In our problem the impurities are perfectly randomly disposed, and the singularities observed are due to resonant scattering by impurity complexes.

The state-density peak considered above ($\varphi_0 = \pi$) corresponds to the appearance of a resonance level in the field of a complex of two closely lying impurities. Indeed, the amplitudes of the scattering by the potential barrier made up of two impurities located at the points $x = 0$ and $x = l$ are equal to

$$1 + f_+^{(2)} = (1 - \gamma) \frac{e^{-2i\alpha}}{1 + \gamma e^{-2i\alpha + 2ipl}} \quad (43)$$

$$f_-^{(2)} = -i\gamma^{1/2} \frac{e^{-i\alpha}(1 + e^{-2i\alpha + 2ipl})}{1 + \gamma e^{-2i\alpha + 2ipl}}.$$

From (43) follows the existence of a virtual level in the field of two closely lying impurities¹⁾ ($pl \ll 1$). The position of the level corresponds to $\alpha(E_0) = \pi/2$. Putting

$$\alpha = \frac{\pi}{2} + \frac{\lambda}{2\gamma_0} (\varepsilon - \varepsilon_0),$$

we have

$$1 + f_+^{(2)} = - \frac{1 - \gamma_0}{1 - \gamma_0 + i\lambda(\varepsilon - \varepsilon_0)}. \quad (44)$$

It follows from (43) and (44), as well as from the exact (28) and (38), that the amplitude of the corresponding state-density peak depends on the values of the parameters $1 - \gamma_0$ and $k_0 = p_0/c$. The first of them determines the width of the virtual level, while the parameter k_0 specifies the statistical weight of the corresponding impurity complex, since the resonance level appears in complexes in which the distance between the impurities is small: $l \ll 1/p_0$. On the other hand, l cannot be less than the impurity-potential radius. Thus, the corresponding peaks may turn out to be noticeable only in the fluctuation region of the spectrum. We note that for a particle with energy $\varepsilon = \varepsilon_0$ such a complex turns out to be transparent.

Writing (43) at $pl \ll 1$ in the form

$$1 + f_+^{(2)} = (1 - \Gamma)^{1/2} e^{-i\beta}, \quad f_-^{(2)} = -i\Gamma^{1/2} e^{-i\beta}, \quad (45)$$

$$\Gamma^{1/2} = \frac{2\gamma^{1/2} \cos \alpha}{(1 + \gamma^2 + 2\gamma \cos \alpha)^{1/2}}, \quad \operatorname{tg} \beta = \frac{\sin 2\alpha}{\cos 2\alpha + \gamma},$$

we note that at $\cos 2\alpha = -\gamma$ the phase factors in (45) co-

incide with the corresponding phase factors $1+f_+$ and f_- at $\alpha = \pi/2$. In this case, therefore, the resonance level appears in the field of a complex having already four closely lying impurities. The condition $\cos 2\alpha = -\gamma$ corresponds according to (26) to the appearance of the next peak in the state density, with $\varphi_0 = \pi/2$.

¹⁾This situation can be easily visualized. If the potential containing the specified virtual level is spatially broken up into individual potentials, a virtual level in the given energy region will then exist already in each of these potentials. Only by expanding these potentials together do we reproduce the given level.

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