Contribution to the theory of the transverse galvanomagnetic effects in semiconductors

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A divergence-free conductivity of an electron gas in a quantizing magnetic field is found under the assumption that the mixing of the Landau levels by scatterers of finite but otherwise arbitrary radii is slight. The cases of different scatterer potentials are investigated: the screened Coulomb potential and small-radius centers. The temperature and field dependences of the transverse conductivity $\sigma(T, H)$ are found for these potentials. It is shown that at low temperatures the transverse conductivity in the case of the screened Coulomb potential is due to the resonance scattering of the electrons. It is found that even in the Born approximation the Adams–Holstein formula [J. Phys. Chem. Solids **10**, 254 (1959)] for the case of the screened Coulomb potential of the scatterers contains a logarithmic error. It is shown that for centers of small radii the temperature and field dependences of the transverse conductivity provide direct information about the energy spectrum of the impurity in the quantizing magnetic field. For scatterers of zero radius the temperature and field dependences of the transverse conductivity coincide with the results obtained by Skobov [Sov. Phys. JETP **11**, 941 (1960)]. The admissibility of the use of the Titeica formula [Ann. Phys. (Leipzig) **5**, 129 (1935)] in the case of a non-Born interaction between the electrons and the scatterers is demonstrated.

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1. The transverse conductivity is usually computed in the Born approximation from the interaction of the current carriers with the scatterers. In this case the conductivity in a quantizing magnetic field exhibits a characteristic logarithmic divergence at small energies,[1] which is due to the high density of electron states at these energies [$g(\varepsilon)\sim\varepsilon^{-1/2}\to\infty$ as $\varepsilon\to 0$], and which can be eliminated by introducing some cutoff procedure. Gurevich and Firsov[2] have shown that in the case of the electron–phonon scattering mechanism the diverging integrals can naturally be truncated at the characteristic phonon energy. Magarill and Savvinykh[3] have shown that allowance for the nonlinear dependence of σ on the electric field E also results in the elimination of the divergence. Skobov[4] has noted that in the case of small-radius scatterers [i.e., scatterers whose potentials are δ functions with respect to the parameter al^{-1}, where $l=(c\hbar/eH)^{1/2}$ is the magnetic length] the divergence is automatically eliminated by forgoing perturbation theory.

We derive below a formula for the transverse conductivity of noninteracting electrons scattered by centers of finite but otherwise arbitrary radius a in a quantizing magnetic field under the following assumptions: a) the mixing of the Landau levels by an individual center $U(|\mathbf{r}|)$ is slight; b) $\hbar\tau^{-1}\ll T$, where τ is the characteristic relaxation time of the electron momentum; c) the scatterer concentration n is low: $na^3\ll 1$. Let us emphasize that the potential U is not assumed to be small compared to the characteristic energy of the longitudinal motion of an electron: $\varepsilon\sim T$. It is demonstrated that there is no divergence[1] in the case of centers with arbitrary, but finite radius. The temperature and field dependences of the transverse conductivity are investigated for the case of the screened Coulomb potential of the impurity and the case of shallow ($U\lesssim\hbar^2/m^*a^2$, where m^* is the effective electron mass) centers of small radius ($a<l$).

It is shown that processes in which the electrons are scattered by screened Coulomb centers can be divided into two types: resonant and nonresonant. At temperatures $T\to 0$ the dominant contribution to the transverse conductivity is made by the resonant electron scattering processes, the contribution of the nonresonance scattering processes being small. Further, as we shall show, even in the Born approximation the Adams–Holstein[1] formula for the case of the screened Coulomb potential of the scatterers contains a logarithmic error.

For scatterers of zero radius the dependence $\sigma(T,H)$ coincides with Skobov's[4] result, differing only in the renormalization of the interaction constant. In the case of centers of small, but nonzero radius the formulas obtained allow the determination of the energy spectrum of the neutral impurities in semiconductors from the dependence of the conductivity on temperature and the magnetic-field intensity.

Titeica's[5] intuitive formula is often used to compute the transverse conductivity in a quantizing magnetic field. Adams and Holstein[1] have demonstrated its validity in the Born approximation in the carrier–scatterer interaction strength. We shall show by direct calculation that, in the approximation of slight mixing of the Landau levels by a single scatterer, this formula is valid for arbitrary temperatures, and not only in the Born approximation in the carrier–scatterer interaction strength.

2. Let us consider a system of noninteracting electrons located in a quantizing magnetic field $\mathbf{H}\parallel z$, in the field

$$V(\mathbf{r})=\sum_a U(|\mathbf{r}-\mathbf{R}_a|)$$

of randomly disposed centers, and in a weak electric field $\mathbf{E}\parallel y$ ($eEl/\hbar\omega_H\ll 1$).

The diagonal element of the transverse-conductivity

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tensor is given by Kubo's formula[6]:

$$\sigma_{yy}=\frac{e^2}{2\pi VT}\left\langle \mathrm{Re}\left[\mathrm{Sp}\,\hat{\rho}\hat{v}_y\int_{-\infty}^{+\infty}dE\frac{1}{E-\hat{H}-i\delta}\hat{v}_y\frac{1}{E-\hat{H}+i\delta}\right]\right\rangle \quad (1)$$

where $\hat{H}$ is the total Hamiltonian of the system in the absence of an electric field, V is the volume of the semiconductor, $\hat{v}_y$ is the electron-velocity operator, and $\hat{\rho}$ is the electron density matrix. The sign $\langle\ldots\rangle$ denotes averaging over the disposition of the centers.

In the approximation $\hbar\tau^{-1}\ll T\ll\hbar\omega_H$ the current in the system is the product of the volume-averaged current from a single scatterer and the number of scatterers in the volume. In determining the average current from a single scatterer, we should replace the Hamiltonian $\hat{H}$ by the Hamiltonian $\hat{H}_0$ for the single-center problem. Then in the expression obtained from (1) we should take the trace over the complete system of functions for the single-center problem. Since the matrix elements of the electron-velocity operator $\hat{v}_y$ are nonzero only for the transitions $N\to N\pm1$ between the Landau bands, we construct the wave function for the single-center problem with allowance for the assumption a): we solve the Schrödinger equation in the $N=0$ band, neglecting the other bands, and then mix in the first (i.e., $N=1$) band with the aid of perturbation theory. For an axially symmetric gauge of the magnetic field, the electron state is characterized by the numbers[7] p_z, n, and m (p_z is the momentum component along H; n and m are the radial and azimuthal quantum numbers). Carrying out the indicated calculations, we obtain[8]

$$\sigma_{yy}=4\pi^2e^2nl^4\sum_{m=0}^{\infty}(m+1)\int_{-\infty}^{+\infty}dp_zdp\left(-\frac{\partial\rho}{\partial\varepsilon_p}\right)\delta(\varepsilon_{p_z}-\varepsilon_p)\left|\int_{-\infty}^{+\infty}dz\chi_{m+1}^{p_z*}(z)\right.$$
$$\left.\times[U_m(z)-U_{m+1}(z)]\chi_m^p(z)\right|^2, \quad (2)$$
$$U_m(z)=\langle R_{0m}(\rho)U(\rho;z)R_{0m}(\rho)\rangle.$$

the R_{nm} are the radial wave functions of the electron,[7] and χ_m^p is the solution to the one-dimensional problem of the scattering of an electron with momentum p by the potential $U_m(z)$. In deriving (2), we used the relation for the matrix element of the potential U between the $N=0$ and $N=+1$ bands:

$$U_{01}^{m}(z)=(m+1)^{1/2}[U_{|m|}(z)-U_{|m|+1}(z)].$$

It is convenient to write the formula (2) in terms of the probability amplitudes for transmission (D_m^p) through, and reflection (R_m^p) from, the one-dimensional potential $U_m(z)$ of an electron with momentum p. After simple transformations, (2) reduces to

$$\sigma_{yy}=\frac{4\pi e^2nn_e}{T(2\pi m^*T)^{1/2}}\sum_{m=0}^{\infty}(m+1)\int_0^{\infty}e^{-\mathcal{E}/T}d\varepsilon\,\mathrm{Re}[1-R_m^pR_{m+1}^{p*}-D_m^pD_{m+1}^{p*}]. \quad (3)$$

Here n_e is the electron concentration and ε is the energy of the longitudinal motion of an electron, the spectrum of this energy being assumed to be quadratic.

The formula (3) is valid in the case of the Boltzmann statistics for electrons, to which we shall limit ourselves below.

3. Let us compute with the aid of (3) the transverse conductivity due to the scattering of the carriers by centers whose potential has a long-range Coulomb character. The subsequent analysis will show that the long-range interaction in the case of the purely Coulomb potential results in the appearance of a logarithmic divergence in (3) when the summation over m is performed; therefore, we shall from the very beginning consider the screened Coulomb potential

$$U(r)=-\frac{e^2}{\varkappa r}\exp\left(-\frac{r}{a}\right), \quad (4)$$

where $\varkappa$ is the static permittivity and a is the screening distance.

Analytic expressions can be obtained for the transverse conductivity in the limiting cases of large and small values of the dimensionless parameter $\Delta=T(\hbar/m^*a^2)^{-1}$. Let us begin the analysis with the high-temperature case:

$$\Delta\gg1. \quad (5)$$

Besides (5), we shall assume that the cyclotron energy is high compared to the Bohr energy, which conforms to the condition for the mixing of the Landau levels to be slight, and that the screening distance is large compared to the Bohr radius a_B ($a_B=\hbar^2\varkappa/m^*e^2$):

$$a\gg a_B\gg l. \quad (6)$$

On account of the inequality (6) and the divergence that arises on performing the summation over m in (3) for the purely Coulomb potential, it is clear that the dominant contribution to (3) is made by the terms with $m\gg1$, i.e., the coefficients R_m and D_m in (3) change little in the transition $m\to m+1$. Therefore, let us replace the summation over m by integration, and expand the coefficients R_{m+1} and D_{m+1} in powers of the difference $(m+1)-m$ up to terms of second order in smallness inclusive. After this, (3) assumes the form

$$\sigma_{yy}=\frac{2\pi e^2nn_el^4}{T(2\pi m^*T)^{1/2}}\int_0^{\infty}mdm\int_0^{\infty}e^{-\mathcal{E}/T}d\varepsilon\left[\left|\frac{dR_m^p}{dm}\right|^2+\left|\frac{dD_m^p}{dm}\right|^2\right]. \quad (7)$$

We shall, in computing the coefficients R_m and D_m, take into account the fact that the dominant contribution to the transverse conductivity is made by the electron states with angular momentum components $m\gg1$. For large m the potential $U_m(z)$ obtained from (4) can be written as

$$U_m(z)=-\frac{e^2}{\varkappa}\frac{\exp[-(2ml^2+z^2)^{1/2}/a]}{(2ml^2+z^2)^{1/2}}. \quad (8)$$

If we make in (7) the change of integration variable $m\to ml^2$, we can easily see from (7) and (8) that the dependence of the transverse conductivity on the magnetic-field intensity in the case in which (6) is satisfied has one and the same form in the entire temperature region:

$$\sigma_{yy}\sim H^{-2}. \quad (9)$$

For the function $\chi_m(z)$ in the dimensionless Coulomb units of a_B and $\varepsilon_B=m^*e^4/\hbar^2\varkappa^2$, we have the equation

$$\chi_m''(\xi)+\left\{\varepsilon_\varepsilon+2\frac{\exp[-(\xi^2+\xi_m^2)^{1/2}/\xi_{max}]}{(\xi^2+\xi_m^2)^{1/2}}\right\}\chi_m(\xi)=0,$$
$$\xi=z/a_B,\quad \varepsilon_\varepsilon=\varepsilon/\varepsilon_B,\quad \xi_m=(2m)^{1/2}l/a_B,\quad \xi_{max}=a/a_B. \quad (10)$$

On the basis of the quasiclassicality parameter for $\xi_m\sim\xi_{max}$:

$$\left|\frac{d\chi}{dz}\right| \leqslant \left|\frac{\hbar^2}{2m^*a^2\varepsilon}\right|^{1/2} \sim \left[\frac{\hbar^2}{2m^*a^2T}\right]^{1/2} \ll 1, \tag{11}$$

we find in accordance with the inequality (5) that the quasiclassical approximation is applicable on the entire z axis. In this case, as can easily be shown, the coefficient $R_m = 0$ (there is no reflection), while the coefficient $D_m = \exp[iS(\xi_m;\varepsilon)]$, where

$$S(\xi_m;\varepsilon) = \int_{-\infty}^{+\infty} \left\{\left[\frac{2\exp[-(\xi^2+\xi_m^2)^{1/2}/\xi_{max}]}{(\xi^2+\xi_m^2)^{1/2}} + \varepsilon\right]^{1/2} - \varepsilon^{1/2}\right\} d\xi. \tag{12}$$

Replacing the differentiation with respect to, and integration over, m in (7) respectively by differentiation and integration with respect to ξ_m, we have in the approximation (11) the following formula:

$$\sigma_{yy} = \frac{2\pi e^2 n n_c l^4}{(2\pi m^*)^{1/2} T^{3/2}} \varepsilon_B \int_0^\infty e^{-\varepsilon\varepsilon_B/T} d\varepsilon \int_0^\infty \xi_m d\xi_m \left(\frac{dS(\xi_m;\varepsilon)}{d\xi_m}\right)^2. \tag{13}$$

Let us consider particular cases of (13). At temperatures low compared to the Bohr energy (i.e., for $T \ll \varepsilon_B$), the inequality $\varepsilon \ll 1$ can be assumed to be fulfilled in the entire effective integration domain. Then $S(\xi_m;0) \sim \xi_m^{1/2}$, and

$$\sigma_{yy} \sim a/H^2T^{1/2}. \tag{14}$$

In the opposite limiting case of high temperatures (i.e., for $T \gg \varepsilon_B$), the phase $S(\xi_m;\varepsilon)$, (12), has the following form:

$$S(\xi_m;\varepsilon) = \frac{1}{\varepsilon^{1/2}} \int_{-\infty}^{+\infty} d\xi \frac{\exp[-(\xi^2+\xi_m^2)^{1/2}/\xi_{max}]}{(\xi^2+\xi_m^2)^{1/2}}. \tag{15}$$

If $2(\xi^2+\xi_m^2)^{-1/2} \leqslant \varepsilon$, i.e., if the point $(\xi_m;\varepsilon)$ in the plane of the variables $(\xi_m;\varepsilon)$ is located to the right of the hyperbola $\varepsilon = 2/\xi_m$ (see Fig. 1), then

$$S(\xi_m;\varepsilon) = \frac{2}{\varepsilon^{1/2}} K_0\left(\frac{\xi_m}{\xi_{max}}\right); \tag{16}$$

here K_0 is a MacDonald function. To estimate the transverse conductivity, let us replace the phase S in (13) by (16) and the MacDonald function by its asymptotic value,[9] i.e., let us replace the phase S in (13) by

$$S \approx \frac{2}{\varepsilon^{1/2}} \ln\frac{\xi_m}{\xi_{max}}.$$

Then the integration in (13) yields

$$\sigma_{yy} \sim H^{-2}T^{-3/2} \int_0^\infty e^{-\varepsilon\varepsilon_B/T} d\varepsilon \int_0^\infty \xi_m d\xi_m \left(\frac{dS(\xi_m;\varepsilon)}{d\xi_m}\right)^2$$

$$\approx \int_{\varepsilon_{min}}^{\varepsilon_{max}} d\varepsilon \int_{\xi_{min}}^{\xi_{max}} \xi_m d\xi_m \frac{4}{\varepsilon\xi_m^2} = H^{-2}T^{-3/2} \ln\frac{\varepsilon_{max}}{\varepsilon_{min}} \ln\frac{\xi_{max}}{\xi_{min}}$$

$$= H^{-2}T^{-3/2} \ln^2\frac{T}{2\varepsilon_B}\frac{a}{a_B}, \quad \sigma_{yy}(a\to\infty)\to\infty, \tag{17}$$

$$\varepsilon_{min} = 2/\xi_{max}, \quad \varepsilon_{max} = T/\varepsilon_B, \quad \xi_{min} = 2\varepsilon_B/T.$$

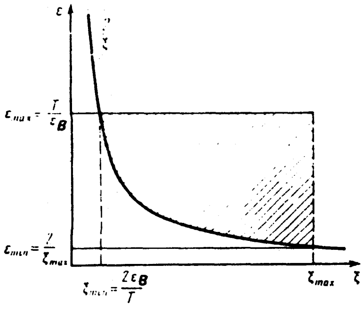

FIG. 1. Integration domain for the Born approximation.

In deriving the last formula, we performed the integration over the region indicated in Fig. 1.

The formula obtained for the transverse conductivity by Adams and Holstein[1] in the Born approximation in the interaction with the Coulomb centers at $T \gg \varepsilon_B$ has [with the same coefficient of proportionality that occurs in our formula (17)] the form

$$\sigma_{yy} \sim H^{-2}T^{-3/2} \ln\frac{\varepsilon_{min}}{T} \ln\frac{E_s}{\hbar\omega_H} \sim H^{-2}T^{-3/2} \ln\frac{\varepsilon'_{min}}{T} \ln\frac{a}{l},$$

where $E_s = \hbar^2/m^*a^2$ and ε'_{min} is some minimum energy at which the logarithmically diverging integral is truncated. In Adams and Holstein's formula the minimum energy ε'_{min} is not determined, and the ξ and ε integrations are performed independently, which, as shown above [see (17)], is inadmissible and leads to an incorrect choice of ξ_{min}.

Let us now consider the opposite—to (5)—limiting case of low temperatures:

$$\Delta \ll 1. \tag{18}$$

As we shall show below, when this inequality is fulfilled, the transverse conductivity is essentially due to the resonance scattering of the slow particle by a deep potential. It is convenient to choose the dimensionless variables slightly differently:

$$\xi = z/a, \quad \varepsilon_e = \varepsilon[\hbar^2/m^*a^2]^{-1}.$$

The quasiclassicality parameter K increases from $K \sim (a_B/a)^{1/2} \ll 1$ at $|\xi| \lesssim a_B/a$ through a value of the order of unity at $|\xi| \sim \ln(a/a_B)$ to $K \sim \varepsilon_e^{-1/2} \gg 1$ at

$$\ln\frac{a}{a_B} < \xi < \ln\frac{a}{a_B} + \frac{3}{2}\ln\frac{1}{\varepsilon_e},$$

and then decreases back to $K \ll 1$ (see Fig. 2). In other words, the motion of the particle has a quasiclassical character in the regions I, III, and V, and is not quasiclassical in the regions II and IV (see Fig. 2).

The electron energy in the regions II and IV can be neglected in the Schrödinger equation. Further, the phase accumulated by the wave function in these regions is small compared to unity. As for the dependences of the amplitudes of the coefficients of transmis-

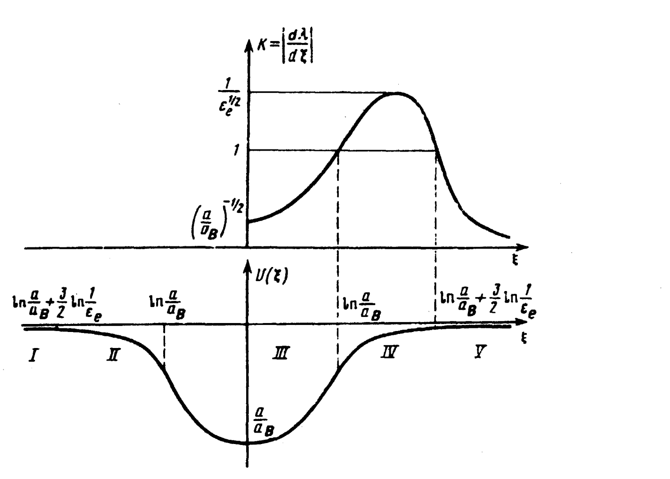

FIG. 2. The potential and the quasiclassicality parameter: I) and V) regions of free motion; III) region of quasiclassical motion; II) and IV) reflection regions.

sion through, and reflection from, a deep potential of the type existing in the regions II and IV, they are insensitive to the form of the potential in these regions; thus, for example, regardless of the dependence on the specific shape of the potential, the amplitude D_m of the transmission coefficient is proportional to the square root of the electron energy.[10] Taking the foregoing into account, and joining the solutions in the regions I–V, we have for the amplitudes of the transmission and reflection coefficients the expressions

$$D_m=\frac{4\varepsilon_e^{1/2}\exp(iS_m)}{(\varepsilon_e+\beta_m)^{1/2}[1+\varepsilon_e^{1/2}/(\varepsilon_e+\beta_m)^{1/2}]^2}[1-g^2(\varepsilon_e;\beta_m)\exp(2iS_m)]^{-1},$$
$$R_m=[\exp(2iS_m)-1]g_m(\varepsilon_e;\beta_m)[1-g^2(\varepsilon_e;\beta_m)\exp(2iS_m)]^{-1}. \quad (19)$$

Here S_m is the phase accumulated by an electron between the nonquasiclassicality region:

$$S_m(\xi_0^m;\xi_m)=\int_{-\xi_0^m}^{\xi_0^m}[U(\xi;\xi_m)]^{1/2}d\xi,\quad U(\xi_0^m;\xi_m)=\beta_m,$$
$$U(\xi;\xi_m)=\frac{2\exp[-(\xi^2+\xi_m^2)^{1/2}]}{(\xi^2+\xi_m^2)^{1/2}}\frac{a}{a_B},\quad \xi_m^2=\frac{2ml^2}{a^2},$$
$$g_m(\varepsilon_e;\beta_m)=[(\beta_m+\varepsilon_e)^{1/2}-\varepsilon_e^{1/2}]/[(\beta_m+\varepsilon_e)^{1/2}+\varepsilon_e^{1/2}]. \quad (20)$$

The constant β_m in (19) has been chosen so as to obtain the correct value for the magnitude of the area of the energy barrier.

We can, allowing for the rapid decrease of the one-dimensional potential $U_m(\xi)$ with increasing ξ, write the scattering phase roughly in the form

$$S_m(\xi_0^m;\xi_m)\approx S_m(\infty;\xi_m)$$
$$=\left(\frac{2a}{a_B}\right)^{1/2}\int_{-\infty}^{+\infty}d\xi\frac{\exp[-1/2(\xi^2+\xi_m^2)^{1/2}]}{(\xi^2+\xi_m^2)^{1/4}}\equiv S_m(\xi_m). \quad (21)$$

Recognizing that the inequality $\varepsilon_e \leqslant \beta$ is also fulfilled, we obtain the following approximate expressions for the amplitudes of the transmission and reflection coefficients:

$$D_m(\xi_m;\varepsilon)=\frac{4\pi\varepsilon_e^{1/2}\exp[iS_m(\xi_m)]}{1-\exp[2iS_m(\xi_m)](1-4\pi\varepsilon_e^{1/2})}$$
$$R_m(\xi_m;\varepsilon)=\frac{(1-2\pi\varepsilon_e^{1/2})[\exp(2iS_m(\xi_m))-1]}{1-\exp[2iS_m(\xi_m)](1-4\pi\varepsilon_e^{1/2})}. \quad (22)$$

The resonant character of the dependence of the transmission and reflection coefficients on the phase $S_m(\xi_m)$ can clearly be seen from these formulas. The effective depth and width of the one-dimensional potential $U_m(z)$ depend on the value of the angular momentum component m; therefore, the scattering of an electron with given energy will, depending on m, have a resonant or a nonresonant character. Let us estimate the contribution made to the transverse conductivity by those terms with different m which respectively describe the resonant and nonresonant scatterings. Resonance sets in at $2S_m(\xi_m^n)=2\pi n$. Hence we have for the number of resonant terms in the sum over m the expression

$$N_m^{\rm res}=\frac{1}{\pi}\left(\frac{2a}{a_B}\right)^{1/2}\int_{-\infty}^{+\infty}d\xi\frac{e^{-\xi/2}}{|\xi|^{1/2}}=4\pi^{-1/2}\left(\frac{a}{a_B}\right)^{1/2}. \quad (23)$$

For the maximum values of $dR_m/d\xi_m$ and $dD_m/d\xi_m$ in the case of resonant scattering we obtain from (15)

$$\left.\frac{dR_m}{d\xi_m}\right|_{\rm res}\sim\left.\frac{dD_m}{d\xi_m}\right|_{\rm res}\sim\frac{1}{\varepsilon_e^{1/2}}S_m'(\xi_m)\sim\frac{1}{\varepsilon_e^{1/2}}\left(\frac{a}{a_B}\right)^{1/2}, \quad (24)$$

while for the nonresonant scattering we have

$$\left.\frac{dR_m}{d\xi_m}\right|_{\rm nonres}\sim\left(\frac{a}{a_B}\right)^{1/2}. \quad (25)$$

To find the width of the resonance peak (23), let us consider the denominators in the formulas (22) near the resonance point ξ_m^n. Expanding $S_m(\xi_m)$ in powers of the difference $\Delta\xi_m^n=\xi_m-\xi_m^n$, we have

$$S_m(\xi_m)=4\pi\varepsilon_e^{1/2}-2iS_m'(\xi_m^n)\Delta\xi_m^n,$$

from which we obtain for the peak width the expression

$$(\Delta\xi_m^n)_{\rm res}\sim\frac{\varepsilon_e^{1/2}}{S_m(\xi_m^n)}\sim\varepsilon_e^{1/2}\left(\frac{a_B}{a}\right)^{1/2}. \quad (26)$$

From the formulas (24) and (26) we find that the contribution of the resonant scattering to the integral over m in (7), which determines the transverse conductivity, is a quantity of the order of

$$I_{\rm res}=\Delta\xi_m^nN_m^{\rm res}\left(\frac{dR_m}{d\xi_m}\right)^2_{\rm res}\sim\frac{a}{a_B}\frac{1}{\varepsilon_e^{1/2}}. \quad (27)$$

At the same time, the nonresonant part of the integral (7) is a quantity of the order of

$$I_{\rm nonres}=(dR_m/d\xi_m)^2_{\rm nonres}\sim a/a_B. \quad (28)$$

Comparing (27) and (28), we find that the contribution to the transverse conductivity of the nonresonant scattering is small compared to the contribution of the states arising from the resonance scattering in the ratio

$$I_{\rm nonres}/I_{\rm res}\sim\varepsilon_e^{1/2}\ll 1. \quad (29)$$

In deriving the last estimates (27)–(29), as well as the formula (7) from (2), we expanded the amplitudes of the transmission and reflection coefficients in powers of the small quantity m^{-1}, which presupposed the smallness of the derivatives:

$$dR_m/dm\ll 1,\quad dD_m/dm\ll 1. \quad (30)$$

It is not difficult to show that for the case under consideration by us the inequalities (30) are equivalent to the inequality

$$\varepsilon_e^{-1/2}(a_B/a)^{1/2}\ll 1. \quad (31)$$

Furthermore, in computing the resonance component of the transverse conductivity, we assumed that each resonance peak contains a large number of terms with different m. This is valid when the following condition is fulfilled:

$$(a/l)^2\varepsilon_e^{1/2}\gg 1. \quad (32)$$

Taking account of the smallness of the contribution to the transverse conductivity of the terms describing the nonresonant scattering, and retaining in (7) only the resonance terms, we obtain $\sigma_{yy}\propto H^{-2}T^{-2}$.

4. Let us now compute with the aid of the formula (3) the transverse conductivity due to the scattering of the carriers by centers of small radius, i.e., with radius $a\ll l$. In the case in which the potential of an individual center is a delta function about al^{-1}, and is not very deep (i.e., $U\lesssim\hbar^2/m^*a^2$), setting $U_{m\geq 1}=0$, and substituting a plane wave for χ_1, we arrive at the formula

$$\sigma_{yy}\propto\alpha^2\int_0^\infty\frac{d\varepsilon}{\varepsilon}\left(-\frac{\partial\rho(\varepsilon)}{\partial\varepsilon}\right)\frac{1}{1+1/2\alpha^2\hbar\omega_H/\varepsilon},$$

which differs from the formula obtained by Skobov[4] only in the value of the dimensionless interaction constant α. In the approximation of slight mixing of the Landau levels $\alpha = f_B(0)/l$ (Ref. 11), whereas for the exact solution to the scattering problem in a magnetic field with summation over all the upper Landau bands $\alpha = f/l$ (Ref. 12). Here $f_B(0)$ and f are respectively the Born and the exact amplitudes of the zero-angle scattering of an electron with zero energy by a single scatterer.

Let us again note the physical meaning of the last formula. Standing under the integral sign is the product of the following quantities: the densities, $\varepsilon^{-1/2}$ and $\varepsilon^{-1/2}$, of the initial and final electron states, the coefficient $(1+\frac{1}{2}\alpha^2\hbar\omega_H/\varepsilon)^{-1}$ of penetration of an electron through the one-dimensional potential, and the distribution function density $(-\partial\rho/\partial\varepsilon)$.

If we do not assume the potential of an individual impurity to be a delta function with respect to the parameter al^{-1}, then, finding the corresponding wave functions of the one-dimensional problem with the aid of the procedure employed in Ref. 11, and substituting them into (2), we arrive at the following formula for the transverse conductivity:

$$\sigma_{yy} = \frac{\pi e^2 n h^2 l^2}{2m^*} \sum_{m=0}^{\infty} (m+1)(\alpha_m - \alpha_{m+1})^2 \int_0^{\infty} \frac{d\varepsilon}{\varepsilon}\left(-\frac{\partial\rho}{\partial\varepsilon}\right) \frac{1}{(1+\alpha_m^2\hbar\omega_H/2\varepsilon)} \times \frac{1}{(1+\alpha_{m+1}^2\hbar\omega_H/2\varepsilon)}, \tag{33}$$

$$\alpha_m = \frac{f_B}{l}\left(\frac{a}{l}\right)^{2m} \quad \text{for} \quad l \gg a; \quad \alpha_0 \equiv \alpha.$$

The formula (33) reflects, in particular, the following general circumstance: for a potential of arbitrary, but finite radius $a \lesssim l$, it follows from the matching conditions that, in the general case of low energies $\varepsilon \to 0$, the wave function χ_m is proportional to the square root of the longitudinal electron energy (i.e., $\chi_m \propto \varepsilon^{1/2}$) in that range of action of the potential $U_m(z)$ which contributes to the transverse conductivity (2). This leads, for a potential of finite radius, to the elimination of the divergence found in Ref. 1. In its turn, this circumstance indicates that all the temperature and magnetic-field dependences predicted by the Born-approximation theory[1] for the transverse conductivity will be different in the low-temperature region. The latter circumstance is, apparently, of considerable experimental interest, in view of the possibility of studying the energy spectrum of impurities in a quantizing magnetic field by investigating the dependence of σ on H and T. Let us demonstrate this in the particular case of the formula (33) for the transverse conductivity for shallow impurities, assuming that the potential of each individual center also has a sufficiently small radius $a < l$. In this case $\alpha_m > \alpha_{m+1}$, and we can limit ourselves in (33) to the first (i.e., $m = 0$) term, it being, however, necessary to retain both factors with α_0 and α_1 in the denominator of the integrand in the formula. Assuming, as before, that the distribution function of the electrons is a Boltzmann distribution, we obtain different H and T dependences for σ in the various temperature regions:

$$\sigma_{yy} \propto T^{-3/2}\ln\frac{T}{H^2}, \quad T \gg \frac{\alpha_0^2}{2}\hbar\omega_H$$

(see Ref. 1),

$$\sigma_{yy} \propto H^{-2}T^{-1/2}, \quad \frac{\alpha_0^2}{2}\hbar\omega_H \gg T \gg \frac{\alpha_1^2}{2}\hbar\omega_H$$

(see Ref. 2),

$$\sigma_{yy} \propto H^{-6}T^{1/2}, \quad T \ll \frac{\alpha_1^2}{2}\hbar\omega_H.$$

It is shown in Ref. 12 that the energies $-\frac{1}{2}\alpha_m^2\hbar\omega_H$ are in fact the bound-state energies of an electron with a given m in the field of an isolated attracting shallow center. Thus, the temperature and field dependences of the transverse conductivity provide direct information about the energy spectrum of the attracting center in a quantizing magnetic field.

Let us again note that, in contrast to Skobov's[4] formula, the integrand in (33) contains the product of the coefficients of penetration of an electron through the one-dimensional barriers $U_{m=0}(z)$ and $U_{m=-1}(z)$.

5. Let us consider the question of the admissibility of the use of Titeica's[5] formula for the computation of the transverse conductivity. Adams and Holstein have proved the correctness of Ref. 5 in the Born approximation in the interaction of an electron with the scatterers. According to Titeica[5] (see also Ref. 13), the transverse conductivity in the ultraquantum limit

$$\sigma_{yy} \propto \int_0^{\infty} d\varepsilon \int_{-\infty}^{+\infty} dy_0 dy_0' \left(-\frac{\partial\rho}{\partial\varepsilon}\right)(y_0 - y_0')^2 W_{y_0\to y_0'}, \tag{34}$$

where y_0 and y_0' are the Landau numbers characterizing the position of the center of the electron orbit[7] and $W_{y_0 \to y_0'}$ is the probability for transition of an electron from y_0 to y_0' during a collision with a scatterer. In the Born approximation this probability is computed with the Landau functions of the zeroth band.[1]

For a scattering potential of arbitrary shape it turns out to be possible to carry out the integration over the Landau numbers y_0 and y_0' in (34) in the Born approximation. The expression for σ_{yy} thus obtained for a spherically symmetric potential $U(r)$ can be represented in the form of a series in m, (2), if the functions χ_m and χ_{m+1} in our formula (2) are replaced by plane waves, which corresponds to the Born approximation,[1] which is valid when the following inequalities are satisfied:

$$U_m(z) \ll \hbar\omega_H, \tag{35a}$$

$$U_m(z) \ll T. \tag{35b}$$

Our formula (2) requires the satisfaction of only the inequality (35a). If (35a) is valid (but the relation between U and T is arbitrary), then the electron transition probability $W_{y_0 \to y_0'}$ in (34) can be expressed in terms of the functions χ_m of the one-dimensional problem and the Landau functions.[14] The integration over the numbers y_0 and y_0' in (34) reduces the Titeica formula (34) to the following expression:

$$\sigma_{yy} = 4\pi^3 e^2 n l^4 \sum_{m=0}^{\infty} (m+1) \int_{-\infty}^{+\infty} dp_z dp \left(-\frac{\partial\rho}{\partial\varepsilon_p}\right)\delta(\varepsilon_{p_z} - \varepsilon_p) \left| \int_{-\infty}^{+\infty} dz [\chi_m^{p_z}(z) U_m(z) - \chi_{m+1}^{p_z}(z) U_{m+1}(z)] \frac{e^{-ipz/\hbar}}{(2\pi\hbar)^{1/2}} \right|^2. \tag{36}$$

The formulas (2) and (36) are identical, a fact which can easily be verified by, for example, going over to the scattering operators,[15] and then expressing (36) in terms of the coefficients R_m and D_m.

6. The situation with a quantizing magnetic field and slight mixing of the Landau levels by the scatterers is realized in semiconductors owing to the small effective mass of the carriers and the large permittivity values. For the typical III–V semiconductor InSb the corresponding parameters in the case of shallow impurities have the following values: $a_B = 10^{-5}$ cm, $U = 10^{-3}$ eV $= \varepsilon_B$, $m^* = 0.013m$, and $\varkappa = 10$. For $H = 6\times 10^4$ G, we have $U/\hbar\omega_H = 0.03$, $l = 10^{-6}$ cm, and $a_B/l = 10$. For shallow impurities in Ge and a magnetic field of the same intensity, oriented along the principal axis of one of the energy ellipsoids, $a_B = 0.5\times 10^{-6}$ cm, $a_B/l = 0.5$, and $U/\hbar\omega_H \approx 1$, i.e., the situation is significantly worse.

Let us now discuss the case in which the transverse conductivity is due to the scattering of the electrons by ionized impurities [see the formulas (5)–(32)]. Normally, the transverse conductivity is measured by measuring the $\sigma(H)$ dependence at a fixed sample temperature.[16] As we have shown [see (9)], when the inequalities $a \gg a_B \gg l$ are fulfilled, the dependence is the same at all temperatures: $\sigma(H) \sim H^{-2}$, but the temperature dependence of σ takes different forms. The character of the temperature dependence $\sigma(T)$ is quite sensitive to the value of the parameter $\Delta = T(\hbar^2/m^* a^2)^{-1}$. For $\Delta \gg 1$ the conductivity is given by the formula (14) or (17), according as the parameter T/ε_B is greater or smaller than unity. For $\Delta \ll 1$ the transverse conductivity σ_{yy} is given by the formula given in Sec. 3. The parameter Δ is determined by the value of the screening distance a for an individual impurity.

If the screening is due to the thermal electrons thrown into the conduction band, then the screening distance can be computed, using the method of Eleonskiĭ *et al.*[17] The condition for the applicability of the linear-screening theory has in this case the form[17]

$$T \gg U\eta^{1/3}, \quad \eta = n_e a_B^3 \ll 1. \qquad (37)$$

But using the expression for a (Ref. 17), we can write the condition $a \gg a_B$ as

$$T \gg 8\pi U\eta, \qquad (38)$$

and the condition $\Delta \ll 1$ as

$$T \ll 2(4\pi)^{1/2} U\eta^{1/2}, \qquad (39)$$

which contradicts (37). Thus, when the impurity is shielded by thermal electrons, the situation $\Delta \gg 1$, (5), is realized, and the transverse conductivity $\sigma(T)$ is described by the formulas (14) and (17).

In slightly doped semiconductors with a small degree of compensation, i.e., with $K_0 \ll 1$, the screening can be realized as the result of the electrostatic interaction between the 0 and 2 complexes.[18] In this case the screening distance at low temperatures is determined only by the concentration of the impurities and the degree of compensation[18]:

$$a = 0.58 N_A^{-1/3} K_0^{-1/6}, \qquad (40)$$

which allows us to realize the condition $\Delta \ll 1$. But, experimentally, this case is significantly more complicated, since the theory of linear screening of a large-scale potential in the case of a small degree of compensation is valid at very small degrees of compensation,[18] i.e., when

$$K_0 \ll 1/400.$$

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