Damping of thermomagnetic waves in bismuth

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The real and imaginary parts of the wave vector of thermomagnetic waves were measured as functions of the frequency. The dependence of the real part was found to be practically linear, whereas the imaginary part is practically independent of frequency at low frequencies and then increases noticeably with increasing frequency. These results can be explained within the framework of a theory that takes into account the finite dimensions of the sample. The electric resistance of the investigated samples could be estimated from the results of the experiment.

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We have previously[1] reported observation of thermomagnetic waves (TMW) which are electromagnetic waves propagating in a conducting medium in which a temperature gradient exists. These waves were predicted theoretically by L. E. Gurevich[2] and then were investigated by him and co-workers in greater detail.[3,4] The real part of the TMW wave vector was investigated in Ref. 5. Simultaneous measurement of the static Nernst-Ettingshausen coefficient of the same samples has made it possible to establish a satisfactory qualitative and quantitative agreement between experiment and theory.

We report here an experimental investigation of the damping of thermomagnetic waves. These investigations are of interest from the point of view of checking on the theory and make it possible in principle to study the relaxation of quasiparticles.

EXPERIMENTAL PROCEDURE

We used in the experiments V2 and V5 bismuth samples having a ratio 600 of the resistivities measured at 293 K and 4.2 K and respective thicknesses 1.3 and 0.75 cm. The shapes of the samples and the method of producing the temperature gradient were described in Ref. 5. Just as in that reference, the traveling wave was excited by a transmitting coil wound on the sample near the "hot end" and connected to the output of a low-frequency oscillator. Stepwise scanning of the frequency was used, in the range 20–2000 Hz. The amplitudes and phases of the signal received in two receiving coils placed along the sample were measured with a quadrature lock-in detector.

Since the distribution of the wave vector in space is described by the factor $\exp(i\mathbf{k}\cdot\mathbf{r}) \equiv \exp(-\mathbf{k}_2\cdot\mathbf{r})\exp(-i\mathbf{k}_1\cdot\mathbf{r})$, where $\mathbf{k} \equiv \mathbf{k}_1 + i\mathbf{k}_2$ (the real and imaginary parts of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) were determined from the formulas

$$k_1=(\varphi_2-\varphi_1)/l,\quad k_2=(-1/l)\ln(U_2/U_1),$$

where φ_1, φ_2, U_1, and U_2 are the phases and amplitudes of the signal in the near and far receiving coils, and l is the distance between them. This procedure made it possible to measure the absolute damping and exclude the instrumental factors determined by the dependences of the amplitude and phase on the frequency in the receiving end transmitting channels. The use of lock-in detectors has made it possible to reduce substantially the influence of the amplifier input noise on the measured phase and amplitude. The temperature was measured with a carbon resistance thermometer secured to the lateral surface of the sample between the receiving coils. The upper and middle parts of the sample, as described in Ref. 5, were thermally insulated by a multilayer teflon film. This apparently did not exclude the possibility of penetration of superfluid helium under the teflon film and could lead to a certain error in the determination of the true temperature of the sample.

EXPERIMENTAL RESULTS

Figures 1 and 2 show the dependences of the real and imaginary parts of the wave vector on the frequency at different temperatures and values of the temperature gradient in the sample.

The frequency dependences of the real part of the

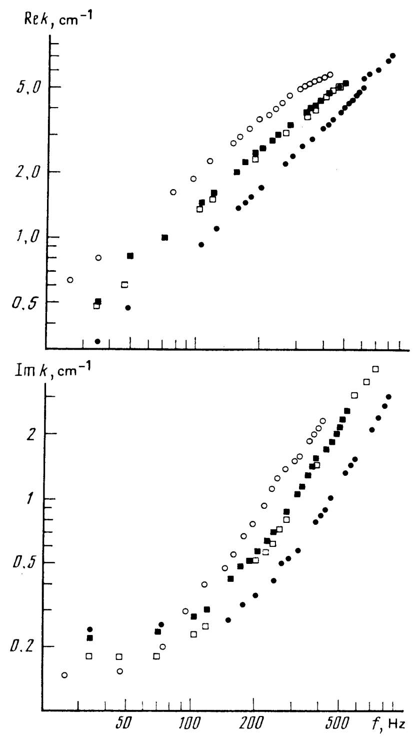

FIG. 1. Frequency dependences of the real and imaginary parts of the wave vector for sample V2 at the following temperatures: ○) 1.83, □) 2.03, ■) 2.08, ●) 2.44 K.

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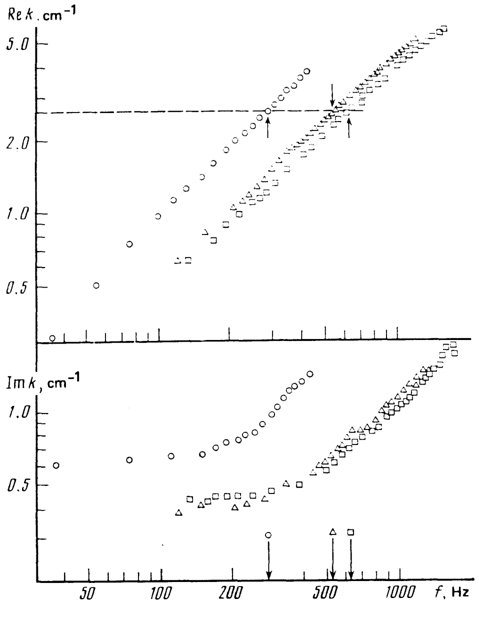

FIG. 2. Frequency dependences of the real and imaginary parts of the wave vector for the sample V5 at the following temperatures: ○) 1.94, △) 2.27, □) 2.53 K. The arrows show the frequencies ω_0 at which Re$\boldsymbol{k} = 1/a$.

wave vector are close to linear, so that for each of the presented curves it is possible to determine the wave phase velocity v, which is practically independent of the frequency but is determined by the temperature and by the temperature gradient in the sample. The imaginary part of the wave vector depends little on the frequency at low frequencies, but the imaginary part of the wave vector increases when the frequency rises above a certain threshold. The frequencies starting with which Imk increases with frequency are different for different temperature gradients, but correspond to approximately the same value of the real part of the wave vector, and consequently to the same wavelength, ~5.5 and ~3 cm for samples V2 and V5, respectively. The ratio of these values is close to the ratio of the transverse dimensions of the samples themselves, giving grounds for assuming that the "break" on the Im$k(\omega)$ plot is due to the effect of the finite dimensions of the samples on the wave damping, inasmuch as the theory of homogeneous wave in an infinite medium yields the relation Im$k \propto \omega^2$ as $\omega \to 0$.

THEORY

The dispersion law of thermomagnetic waves in an infinite medium is obtained by substituting the currents and the fields in the form

$$\mathbf{j}=\mathbf{j}'e^{i(\mathbf{kr}-\omega t)},\quad \mathbf{E}=\mathbf{E}'e^{i(\mathbf{kr}-\omega t)},\quad \mathbf{H}=\mathbf{H}'e^{i(\mathbf{kr}-\omega t)} \tag{1}$$

into the system of equations

$$\mathrm{rot}\,\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},\quad \mathrm{rot}\,\mathbf{H}=\frac{4\pi}{c}\mathbf{j},\quad \mathrm{div}\,\mathbf{H}=0,\quad \mathbf{E}=\rho\mathbf{j}+\alpha_1[\nabla T\times\mathbf{H}] \tag{2}$$

(α_1 is the Nernst-Ettingshausen coefficient), and takes in accord with Ref. 3 the form

$$\omega+\alpha_1 c(\nabla T,\mathbf{k})+i\rho c^2k^2/4\pi=0, \tag{3}$$

with $\mathbf{j}'$, $\mathbf{E}'$, and $\mathbf{H}'$ independent of the coordinates.

When thermomagnetic waves propagate in a bounded medium, the amplitudes of the fields and of the current depend on the coordinate perpendicular to the surface (the radial direction in the case of the cylinder and the normal direction in the case of a plate). The procedure of finding the dispersion law of the electromagnetic waves propagating in a plate and in a cylinder is widely known and has been described in the literature (see, e.g., Ref. 6), and we shall therefore not dwell on it in detail. The dispersion relation is the condition for the compatibility of the solutions of Maxwell's equations in the medium and outside it (in vacuum), and of the continuity of the corresponding field components on the surface. Inasmuch as there are no surface currents or charges in our case, all the field components are continuous.

In accord with the experimental conditions, we have considered the waveguide TE mode ($\mathbf{E}'\perp\mathbf{k}$). In the case of a cylinder, we have assumed that the electric field of the wave has only the φ component different from zero and that the field components are independent of the angle φ. In the case of a plate we have assumed that the only nonzero $\mathbf{E}'$ components are those parallel to the surface and antisymmetrical with respect to the coordinate parallel to the normal to the surface.

Taking all the foregoing into account, the dispersion law takes in a plate the form

$$z\,\mathrm{ctg}\,z+y=0, \tag{4}$$

and in a cylinder

$$zJ_0(z)K_1(y)+yK_0(y)J_1(z)=0, \tag{5}$$

where $J_{0,1}$ are Bessel functions, $K_{0,1}$ are modified Hankel functions, and in both cases z is given by the expression

$$z^2+y^2+i\gamma y-ix=0, \tag{6}$$

where a is half the plate thickness or else the cylinder radius, $x=4\pi a^2\omega/\rho c^2$ is the relative frequency, $y=ak$ is the relative wave vector, and $\gamma=4\pi a|\alpha_1\nabla T|/\rho c$ is the reduced temperature gradient.

In accordance with the experimental condition, we have assumed the frequency to be real and the wave vector complex. Since our samples differed considerably in shape from a right circular cylinder, and the solution of the system (5) in (6) in the complex domain is a rather cumbersome computational task, we confine ourselves to a numerical solution of the system (4) and (6) for a plate. We were mainly interested in the solutions for weakly damped waves (Im$k <$ Rek), since it is precisely such waves that can be observed in experiment.

The dependences of the real and imaginary parts of the wave vector on the frequency, plotted from the calculation results, are shown in Fig. 3 in relative coordinates. The figure shows also the calculated damping of the homogeneous waves in an infinite medium. These results are the solution of the dispersion equation (6), which is equivalent to (3) at $z=0$.

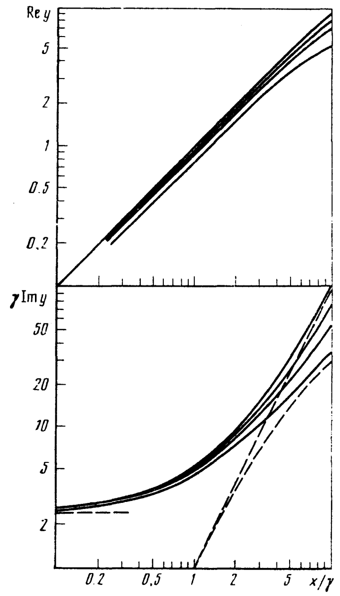

FIG. 3. Results of calculation of the dispersion law. The solid curves correspond to the values of the parameter γ (reading downward) 100, 31, 18, and 10. The horizontal dashed line is the asymptotic value of γ Imy as $x/\gamma \to 0$. The dashed curves are plots of γ Imy calculated from the damping of the homogeneous waves at $\gamma = 100$ (upper curve) and $\gamma = 10$ (lower curve) in an infinite medium.

The calculation and analysis of the system (4), (6), show that as $x/\gamma \to 0$ the damping tends to a constant: $\gamma\,\mathrm{Im}y \to \pi^2/4$, whereas for an infinite medium $\gamma\,\mathrm{Im}y \to (x/\gamma)^2$. At sufficiently high frequencies, when $x \gg \gamma^2$, $\mathrm{Re}y \approx \mathrm{Im}y \approx (x/2)^{1/2}$ and does not depend on the temperature gradient. This is the case of ordinary skin damping. It is seen from the plot that the $\mathrm{Im}y \propto x^2$ dependence does not manifest itself at low temperature gradients ($\gamma = 10$), being hindered at low temperatures by the waveguide effect and by the fact that the condition

$$1 \ll x/\gamma \ll \gamma$$

is not satisfied in the entire frequency region.

DISCUSSION OF RESULTS

A comparison of the experimental and theoretical $\mathrm{Im}k(\omega)$ dependences shows them to be in qualitative agreement. To compare the experiments with the theory quantitatively, we have plotted the curves of Fig. 3 on tracing paper, and aligned the aggregate of the experimental points with the theoretical curves. As seen from the upper part of Fig. 3, the waveguide effects hardly manifest itself in the frequency dependence of the real part of the wave vector, and $\mathrm{Re}y \equiv a\,\mathrm{Re}k \approx x/\gamma$ in the region of low frequencies ($x/\gamma \lesssim 1$). This circumstance has made it possible, knowing the thickness of the sample, to determine the wave vector $\mathrm{Re}k_0 = 1/a$ from the $\mathrm{Re}k(\omega)$ plots of Figs. 1 and 2, and determine the frequency ω_0 at which $x/\gamma \approx a\,\mathrm{Re}k = 1$. Examples of this construction are shown in Fig. 2. The experimental plot and the theoretical curve [drawn in the same (logarithmic) scale] were matched in the horizontal direction by aligning the values of the abscissa $x/\gamma = 1$ of the calculated plot with the abscissa ω_0 obtained by the indicated construction, while in the vertical direction this was done by relatively shifting the plots until best visual agreement was obtained.

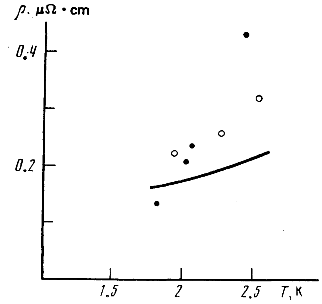

FIG. 4. Resistivity values calculated from the results of measurements of the damping of the thermomagnetic waves for samples V2 (•) and V5 (○), as well as the temperature dependence of the resistivity taken from Ref. 7 (solid curve).

The possible error due to the ambiguity of this alignment does not exceed 10% in our opinion.

By aligning in this manner the experimental and theoretical plots we determined the ratio $\gamma\,\mathrm{Im}y/\mathrm{Im}k$ (which is equal to the ordinate of $\gamma\,\mathrm{Im}y$ at $\mathrm{Im}k = 1$) and by determining the resistivity ρ with the aid of the relation

$$\gamma\,\mathrm{Im}\,y/\mathrm{Im}\,k = 4\pi a^2\omega_0/\rho c^2\,\mathrm{Re}\,k_0.$$

The resistivity values determined in this manner are shown in Fig. 4. For comparison, the solid curve shows the dependence of the resistivity of a bismuth sample having a resistivity ratio 680 in the same temperature interval.[7] It is seen that the resistivities of the investigated samples, estimated from the damping of the thermomagnetic waves, are of the correct order of magnitude and exhibit a qualitatively correct behavior (increase of resistivity with temperature). The main causes of the observed discrepancies between theory and experiment are, in our opinion, the fact that the sample shape was not the same as the one for which the calculation was made, and that the true sample temperature was inaccurately determined.

Summarizing, it can be stated that on the whole the observed results of the measurement of the damping of the thermomagnetic waves can be described quite satisfactorily, both qualitatively and quantitatively, within the framework of a theory that takes into account the finite dimensions of the samples. This makes it possible to use thermomagnetic waves for the investigation of carrier relaxation processes.

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