Plasma turbulence and dissipation of a strong electromagnetic wave in the vicinity of the $n_c/4$ resonance

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(Submitted 19 June 1981)
Zh. Eksp. Teor. Fiz. **82**, 125–138 (January 1982)

We study plasma turbulence excited by two-plasmon decay of a strong electromagnetic wave. We construct an analytical model which determines the main turbulence characteristics—the effective collision frequency and the energy of the plasma oscillations for different values of the amplitude of the electromagnetic pumping and of the "mismatch" of the resonance $\omega = 2\omega_p$. Computer simulation of the turbulence shows a satisfactory agreement with the qualitative analytical model.

PACS numbers: 52.35.Ra, 52.35.Hr, 52.40.Db

1. The question of the mechanisms for the absorption of a high-frequency field in a plasma is a central problem for the problem of laser thermonuclear fursion. The complexity of this problem is caused by the fact that a number of competing mechanisms are possible—besides the usual collisional absorption, the anomalous absorption mechanisms based upon decay instabilities play an important, and for long-wavelength lasers, a dominant role. The main zones for the anomalous absorption are the vicinity of the critical density $n_c = m\omega^2/4\pi e^2$, where the frequency of the electromagnetic radiation is close to the plasma frequency, and the region $n \approx n_c/4$, of one-fourth of the critical density, i.e., $\omega \approx 2\omega_p$ (see Fig. 1). In the first of these zones the dissipation of the electromagnetic energy is connected with the creation of plasma turbulence through the decay of the electromagnetic wave into a plasma wave and a sound wave, while in the second zone the decay of the electromagnetic wave into two plasma waves plays the analogous role. The intensity of the anomalous absorption in the vicinity of the critical density is appreciably higher, but the electromagnetic wave traverses first the $n_c/4$ region, and a study of the anomalous absorption in that region is also of considerable interest. The first studies of the anomalous absorption in the vicinity of $n_c/4$ were made in Ref. 1. A qualitative picture was constructed of the plasma turbulence, taking into account the convective carrying away of oscillations in an inhomogeneous plasma, and the absorption of plasma oscillations by electrons and the "growth of tails" on their distribution function was studied. Later papers[2,3] were devoted basically to a more detailed study of the mechanism for convective carrying away and, using this as a basis, thresholds and growth rates for parametric instabilities in the inhomogeneous plasma of a deuterium-tritium target were obtained.

The effective collision frequency which determines the rate of dissipation of electromagnetic energy was obtained by Bychenkov, Silin, and Tikhonchuk[4] in the framework of the weak turbulence approximation for the plasma oscillations. This approximation is justified in those cases where the convective carrying away of the oscillations to the rarefied plasma region is important; this leads to a spectral transfer of the energy of the oscillations to the region of small phase velocities and, as a consequence to the appearance of resonance absorption by the electrons. However, in those cases where the convective carrying away is unimportant it is impossible to construct a logically consistent picture of the absorption of the plasma oscillations without including the modulational instability and the ideas of strong turbulence. The first step in that direction was taken by Chen and Liu,[5] who considered the formation of a soliton as the result of the decay of the pumping electromagnetic wave into two plasma oscillations and the subsequent modulational instability of these oscillations. The plasma oscillation frequency shift $\Delta\omega_p$ connected with the modulation of the density in the soliton leads to the stabilization of the three-wave parametric instability in the case when $\Delta\omega_p \sim \gamma_d$ (γ_d is the growth rate of the instability considered). After that, using the maximum amplitude of the plasma oscillations determined in this way the effective collision frequency for an electromagnetic wave was evaluated in that paper. The inconsistency of approach just discussed is connected with the fact that, essentially, there is in Ref. 5 no equilibrium picture of plasma turbulence with a continuous outflow of electromagnetic energy into the plasma oscillations. They considered instead the dynamic process of a saturation of the parametric instability on account of the shift, due to formation of an isolated soliton, of the real part of the frequency. The introduction of the effective collision frequency, i.e., the imaginary part of the frequency, is incorrect for such a process. Much more justified seems to us to be the approach used by Ribenchik to the calculation of the effective collision frequency.[6] Starting from the general ideas of strong Langmuir turbulence developed in Ref. 7, the process for the outflow of the electromagnetic energy into plasma turbulence can be considered to be the result of decay in the me-

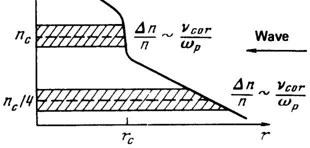

FIG. 1. Density profile of a plasma target.

 0038-5646/82/010074-08$04.00

dium, with random phase jumps produced when the waves decay. Rubenchik obtained in that way an estimate accurate to a numerical coefficient of order unity, $\nu_{eff}\sim\omega_p T/mc^2$, which is the same as the expression for ν_{eff} obtained in the framework of weak turbulence.

The fact that at this moment there exist both a qualitative analytical theory of strong Langmuir turbulence[7,8] and numerical models of that turbulence[9,10] enables us to construct a more detailed picture of the anomalous absorption, based upon strong turbulence, in the vicinity of $n_c/4$. This is just the subject of the present paper. Its plan is the following. In the second section we write down the basic equations of the paper and carry out a qualitative study of plasma turbulence initiated by electromagnetic pumping under exact resonance conditions $\omega=2\omega_p$. In the third section we give the results of a numerical simulation of this turbulence. In the fourth section we study analytically and numerically the turbulence when there is a "detuning" of the $\omega=2\omega_p$ resonance. This enables us to estimate the width of the turbulent region. The analysis in the first four sections pertains to the case of a uniform plasma. We consider in the fifth section a real plasma of a deuterium-tritium target, we obtain the conditions for neglecting the convective carrying away of plasma oscillations, and we estimate the coefficient of absorption of an electromagnetic wave at different values of its amplitude.

2. We assume that as a result of the competition between the influx of energy in the incident electromagnetic wave and the dissipation of energy in plasma turbulence a stationary distribution of the amplitudes of the electromagnetic field is established

$$\mathbf{E}^{tr}={}^1/_2\mathbf{E}_0(\mathbf{r})e^{-i\omega_0 t}+\text{c.c.},\quad \mathbf{H}^{tr}={}^1/_2\mathbf{H}_0(\mathbf{r})e^{-i\omega_0 t}+\text{c.c.} \tag{1}$$

When we consider plasma oscillations initiated by an electromagnetic wave it is convenient to separate the fast time-dependence and to write all quantities that characterize the plasma oscillations in the form

$$\mathbf{v}^l={}^1/_2\mathbf{v}(t,\mathbf{r})e^{-i\omega_p t}+\text{c.c.},\quad \delta n^l={}^1/_2 n'(t,\mathbf{r})e^{-i\omega_p t}+\text{c.c.},$$
$$\mathbf{E}^l={}^1/_2\mathbf{E}(t,\mathbf{r})e^{-i\omega_p t}+\text{c.c.}, \tag{2}$$

where ω_p is the plasma frequency reckoned from the unperturbed density n_0: $\omega_p^2=4\pi e^2 n_0/m$.

Accurate to terms of first order in the parameter

$$\left|\frac{1}{\omega_p}\frac{\partial\ln v}{\partial t}\right|\ll 1$$

the equation for the velocity of the electrons in the plasma oscillations can be written in the following way:

$$-i\omega_p\mathbf{v}=-\frac{e}{m}\mathbf{E}-\frac{e}{im\omega_p}\frac{\partial\mathbf{E}}{\partial t}+\frac{3T_e}{4\pi e n_0 m}\nabla\operatorname{div}\mathbf{E}$$
$$-\frac{e^2}{2m^2\omega_0\omega_p}\{(\mathbf{E}_0\nabla)\mathbf{E}^*+(\mathbf{E}^*\nabla)\mathbf{E}_0\}e^{-i\Omega t}. \tag{3}$$

We have used here the notation $\Omega=\omega_0-2\omega_p$. When we evaluated the nonlinear term $(\mathbf{v}_0\cdot\nabla)\mathbf{v}+(\mathbf{v}\cdot\nabla)\mathbf{v}_0$ we restricted ourselves to the main approximation in the above indicated small parameter in the formula for the electron velocity $\mathbf{v}=e\mathbf{E}/im\omega_p$ in the plasma oscillations and we substituted the electron velocity in the field of the electromagnetic wave in the form

$$\mathbf{v}_0=e\mathbf{E}_0/im\omega_0.$$

With the same accuracy we have the following equation for the high-frequency density variations:

$$-i\omega_p n'+\frac{\partial n'}{\partial t}+\operatorname{div}[(n_0+\delta n)\mathbf{v}]-\frac{n_0 e^2 e^{-i\Omega t}}{2im^2\omega_p^2\omega_0}\operatorname{div}(\mathbf{E}_0\operatorname{div}\mathbf{E}^*)=0. \tag{4}$$

In this equation $\delta n(t,\mathbf{r})$ is the amplitude of the slow quasi-neutral density variations caused by the high-frequency pressure force. Eliminating from Eqs. (3) and (4) the electron velocity $\mathbf{v}$ and using the Poisson equation

$$n'=-\frac{1}{4\pi e}\operatorname{div}\mathbf{E}$$

to express n', we get the following equation for $\mathbf{E}$:

$$-2i\omega_p\frac{\partial}{\partial t}\operatorname{div}\mathbf{E}+\frac{4\pi e^2\delta n}{m}\operatorname{div}\mathbf{E}-\frac{3T_e}{m}\operatorname{div}\nabla\operatorname{div}\mathbf{E}$$
$$+\frac{e\omega_p}{2m\omega_0}\operatorname{div}\{(\mathbf{E}_0\nabla)\mathbf{E}^*+(\mathbf{E}^*\nabla)\mathbf{E}_0-\mathbf{E}_0\operatorname{div}\mathbf{E}^*\}e^{-i\Omega t}=0.$$

We can easily simplify the nonlinear term in this equation using the well known vector-analysis formula

$$\operatorname{rot}[\mathbf{a}\times\mathbf{b}]=\mathbf{a}\operatorname{div}\mathbf{b}-\mathbf{b}\operatorname{div}\mathbf{a}+(\mathbf{b}\nabla)\mathbf{a}-(\mathbf{a}\nabla)\mathbf{b}.$$

Taking also into account the possibility of collisionless damping of the plasma mode we write the final equation for $\mathbf{E}$ as follows:

$$\operatorname{div}\left\{\frac{2}{i\omega_p}\left(\frac{\partial\mathbf{E}}{\partial t}+\hat{\Gamma}\mathbf{E}\right)-\frac{3T_e}{m\omega_p^2}\nabla\operatorname{div}\mathbf{E}\right.$$
$$\left.+\frac{\delta n}{2n_0}\mathbf{E}+\frac{e}{2m\omega_0\omega_p}(\mathbf{E}^*\nabla)\mathbf{E}_0 e^{-i\Omega t}\right\}=0. \tag{5}$$

The term $\hat{\Gamma}\mathbf{E}$, which describes the Landau damping of the plasma oscillations, has the form of a convoluted integral operator:

$$\hat{\Gamma}\mathbf{E}=\int\Gamma(\mathbf{r}-\mathbf{r}')\mathbf{E}(t,\mathbf{r}')d\mathbf{r}',$$

where the Fourier transform of the function $\Gamma(\mathbf{r})$ serves as the Landau damping rate for the Fourier-harmonics of the electrical field:

$$\Gamma(\mathbf{r})=\frac{1}{(2\pi)^3}\int\Gamma_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}}d\mathbf{k}.$$

For a numerical simulation of plasma turbulence on the basis of Eq. (5) we shall assume that the distribution function of the particles at resonance with the oscillations remains Maxwellian (see Ref. 10 for the justification of this procedure). In that case

$$\Gamma_{\mathbf{k}}=\left(\frac{\pi}{8}\right)^{1/2}\omega_p\frac{1}{k^3r_D^3}\exp\left\{-\frac{3}{2}-\frac{1}{2k^2r_D^2}\right\}, \tag{5'}$$

$r_D=(T_e/4\pi e^2 n_0)^{1/2}$ is the electron Debye radius.

The equation for the slow quasi-neutral density variations $\delta n(t,\mathbf{r})$ has, as is usual in plasma turbulence, the form of the sound equation with a driving force due to high-frequency pressure produced by the plasma oscillations:

$$\frac{\partial^2\delta n}{\partial t^2}+\hat{\gamma}\frac{\partial\delta n}{\partial t}-\frac{T_e+T_i}{M}\Delta\delta n=\frac{1}{16\pi M}\nabla^2|E|^2. \tag{6}$$

The terms $\hat{\gamma}\delta n/\partial t$ describes the resonance damping of the low-frequency mode:

$$\hat{\gamma}\delta n=\int\gamma(\mathbf{r}-\mathbf{r}')\delta n(\mathbf{r}')d\mathbf{r}', \quad \gamma(\mathbf{r})=\frac{1}{(2\pi)^3}\int\gamma_{\mathbf{k}}e^{i\mathbf{kr}}d\mathbf{k}.$$

We use for the damping rate $\gamma_{\mathbf{k}}$ in this paper the following model formula:

$$\gamma_{\mathbf{k}}=\left(\frac{\pi}{8}\right)^{1/2}\omega_p\left[\frac{m}{M}kr_D+\left(\frac{m}{M}\right)^{1/2}kr_D\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right]. \quad (6')$$

The first term in this formula is the electron contribution to the damping of the low-frequency mode, and the second is the damping by resonant ions which is important in a plasma with hot ions $T_i \gtrsim T_e$.

When we neglect the nonlinearity of the plasma mode, the set of Eqs. (5), (6) describes the decay of the long wavelength ($\mathbf{k}^t\to 0$) electromagnetic quantum into two plasmons with wave numbers $kr_D=(\Omega/3\omega_p)^{1/2}$, which fly off in opposite directions $\mathbf{k}$ and $-\mathbf{k}$. The growth rate of the corresponding parametric instability is equal to

$$\gamma_d=\left(\frac{3E_0^2}{64\pi n_0 mc^2}\right)^{1/2}\omega_p\sin\theta\cos\theta, \quad (7)$$

where we have substituted the wave number of the electromagnetic pumping $k_0=3^{1/2}\omega_0/2c$, θ is the angle between the wave vectors of the plasmon $\mathbf{k}$ and of the electromagnetic quantum $\mathbf{k}_0$, while the maximum growth rate is reached for $\theta=\pi/4$.

The buildup of energy in the plasma oscillations leads to the appearance of the modulational instability and to an associated channel of short-wavelength Langmuir energy transfer by the collapsing cavitons. Due to the balance between the influx of energy into the turbulence caused by the decay of the pump and the absorption of the short-wavelength oscillations by resonance electrons, a quasi-stationary state of turbulence is established for the description of which we shall follow the representation developed in Refs. 7, 8.

We write the electric field of the oscillations in the form

$$\mathbf{E}=\sum_{\mathbf{k}}\mathbf{E}_{\mathbf{k}}\exp(-i\Phi_{\mathbf{k}}(t)+i\mathbf{kr}), \quad (8)$$

$\Phi_{\mathbf{k}}(t)$ are the random phase of the plasmons whose correlation function

$$\langle\exp(i\Phi_{\mathbf{k}}(t))\exp(-i\Phi_{\mathbf{k}'}(t'))\rangle=\delta_{\mathbf{kk}'}\exp(-\nu_{cor}|t-t'|) \quad (8')$$

corresponds to phase jumps occurring with a frequency ν_{cor}. The mechanism for phase mixing is the scattering of plasmons by long-wavelength density fluctuations due to the modulational instability.

For the Fourier amplitude $\mathbf{E}_{\mathbf{k}}(t)$ of the expansion (8) we have from (5) the following equation:

$$\mathbf{E}_{\mathbf{k}}(t)=\frac{e}{2m\omega_0}\int_0^t dt'\mathbf{E}_{-\mathbf{k}}^*(t')\frac{(\mathbf{k}_0\mathbf{k})(\mathbf{E}_0\mathbf{k})}{k^2}\exp\{-i\Omega t'+i\Phi_{\mathbf{k}}(t')+i\Phi_{-\mathbf{k}}(t')\}. \quad (9)$$

Using this relation we get easily from (5) an equation for the change with time of the energy $W=\Sigma_{\mathbf{k}}|E_{\mathbf{k}}|^2/8\pi$ of the plasma oscillations:

$$\frac{dW}{dt}\approx\frac{e^2}{2m^2\omega_0^2}\sum_{\mathbf{k}}\frac{(\mathbf{k}_0\mathbf{k})^2(\mathbf{E}_0\mathbf{k})^2}{k^4}\frac{|E_{\mathbf{k}}|^2}{8\pi}\times\frac{2\nu_{cor}}{\Omega^2+4\nu_{cor}^{*2}}\approx\frac{e^2k_0^2E_0^2}{4m^2\omega_0^2\nu_{cor}^*}\overline{\sin^2\theta\cos^2\theta}W. \quad (10)$$

To obtain this equation we used the following correlation function for the phases which arise upon the decay if the plasmons

$$\langle\exp\{i\Phi_{\mathbf{k}}(t')+i\Phi_{-\mathbf{k}}(t')\}\exp\{-i\Phi_{\mathbf{k}}(t)-i\Phi_{-\mathbf{k}}(t)\}\rangle=\exp\{-\nu_{cor}^*|t-t'|\}.$$

Moreover, we substituted $\Omega\lesssim 2\nu_{cor}^*$ in the last formula, and the bar indicates an average over the spectrum of the Langmuir oscillations. Following Ref. 7 we put $\nu_{cor}^*\sim\alpha\omega_p W/n_0T$, where α is a numerical coefficient. We then get from (10) a formula for the power absorbed by the plasma turbulence from the electromagnetic pump in the vicinity of the $n_c/4$ resonance:

$$\frac{dW}{dt}\approx\frac{3}{8\alpha}\frac{T}{mc^2}\overline{\sin^2\theta\cos^2\theta}\omega_p\frac{E_0^2}{8\pi}.$$

This formula corresponds to the following effective collision frequency when $\omega\approx 2\omega_p$:

$$\nu_{eff}\approx\frac{3}{8\alpha}\omega_p\frac{T}{mc^2}\overline{\sin^2\theta\cos^2\theta}. \quad (11)$$

One can easily give an interpretation to the result. In the state of advanced turbulence, as we have already noted above, plasmons produced by the electromagnetic pump undergo random phase jumps with a frequency $\sim\nu_{cor}^*\gg\gamma_d$. According to Zakharov and Zaslavskiĭ[11] in that case the growth rate of the parametric instability decreases to a value γ_d^2/ν_{cor}^* which, as one can see easily, agrees with Eq. (10).

The energy changed into plasma turbulence as a result of the parametric instability of the pump is localized in the cavitons and is transferred following the collapse of the cavitons to the short-wavelength absorption region. The plasmon energy flux into the collapsing cavitons is equal to $\gamma_{mod}W$, where $\gamma_{mod}=\omega_p(mW/Mn_0T)^{1/2}$ is the growth rate of the modulational instability. From the condition for energy balance

$$\gamma_{mod}W=\nu_{eff}E_0^2/8\pi \quad (12)$$

we get an equation for the energy of the plasma oscillations as a function of the amplitude of the electromagnetic pump:

$$W=\left(\frac{3}{32\alpha}\frac{T}{mc^2}\frac{E_0^2}{8\pi}\right)^{2/3}\left(n_0T\frac{M}{m}\right)^{1/3}. \quad (13)$$

3. Plasma turbulence initiated by a pump of frequency $\omega_0\approx 2\omega_p$ has also been studied by numerical methods. A one-dimensional model of plasma turbulence has been studied and the wave vector of the pump has been chosen at an angle $\theta=\pi/4$ to the direction along which the turbulence was developing (the x-axis). We took in that case for the pump field in the form

$$E_x=-\frac{E_0}{\sqrt{2}}\exp\left(i\frac{k_0}{\sqrt{2}}x+i\frac{k_0}{\sqrt{2}}y\right),$$
$$E_y=\frac{E_0}{\sqrt{2}}\exp\left(i\frac{k_0}{\sqrt{2}}x+i\frac{k_0}{\sqrt{2}}y\right), \quad (14)$$
$$k_0=3^{1/2}\omega_p/c.$$

The electric field of the plasma oscillations is expressed in terms of a potential $\mathbf{E}=-\nabla\psi$. We can also easily separate the dependence on the y-coordinate in the plasma oscillations:

$$\psi=\psi(x)\exp\left(i\frac{k_0}{2\sqrt{2}}y\right).$$

We use for the change to dimensionless quantities in the initial set of Eqs. (5), (6) the usual scales for measuring time, coordinates, electric field, and density variations:

$$t=3M/\omega_p m,\quad x=3(M/m)^{1/2}r_D,$$
$$E=\left(\frac{16\pi}{3}n_0T\frac{m}{M}\right)^{1/2},\quad \delta n/n_0=m/3M,$$

and we can then write the equations in the following form:

$$\frac{2}{i}\left(\frac{\partial^2}{\partial x^2}-\frac{\varkappa_0^2}{8}\right)\left(\frac{\partial\psi}{\partial t}+\hat{\Gamma}\psi\right)-\left(\frac{\partial^2}{\partial x^2}-\frac{\varkappa_0^2}{8}\right)^2\psi+\left(\frac{\partial}{\partial x}n\frac{\partial\psi}{\partial x}-\frac{\varkappa_0^2}{8}n\psi\right)$$
$$+\frac{a}{i}\exp\left\{i\frac{\varkappa_0 x}{\sqrt{2}}-i\delta_0 t\right\}\left(\frac{\partial^2}{\partial x^2}+\frac{\varkappa_0^2}{8}\right)\psi^*=0, \qquad (15)$$

$$\frac{\partial^2 n}{\partial t^2}+\hat{\gamma}\frac{\partial n}{\partial t}-\frac{\partial^2 n}{\partial x^2}=\frac{\partial^2}{\partial x^2}\left\{\left|\frac{\partial\psi}{\partial x}\right|^2+\frac{\varkappa_0^2}{8}|\psi|^2\right\}, \qquad (16)$$

where

$$\varkappa_0=3\left(\frac{3T}{mc^2}\frac{M}{m}\right)^{1/2},\quad \delta_0=\frac{\omega_0-2\omega_p}{3\omega_p}\frac{M}{m}.$$

The damping rates and the parameter a are in dimensionless variables equal to

$$\Gamma_\varkappa=81\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{M}{m}\right)^{5/2}\exp\left\{-\frac{3}{2}-\frac{9M}{2m\varkappa^2}\right\},$$
$$a=\frac{3M}{2m}\left(\frac{3E_0^2}{16\pi n_0 mc^2}\right)^{1/2},$$
$$\gamma_\varkappa=\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{m}{M}\right)^{1/2}\varkappa\left[1+\left(\frac{M}{m}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{2T_i}\right)\right].$$

As usual in linear theory we consider the decay of the electromagnetic pump determined by Eq. (14) into two Langmuir satellites, the electrostatic potential of which we write in the form

$$\psi(x)=\psi_1\exp\left\{i\left(\varkappa+\frac{\varkappa_0}{2\sqrt{2}}\right)x-i\left(\omega+\frac{\delta_0}{2}\right)t\right\}$$
$$+\psi_2\exp\left\{-i\left(\varkappa-\frac{\varkappa_0}{2\sqrt{2}}\right)x+i\left(\omega-\frac{\delta_0}{2}\right)t\right\}.$$

The dispersion equation for the parameteric instability is obtained from Eqs. (15), (16) by the standard method of the linear theory and leads to the following formula for the growth rate of the parametric instability:

$$\gamma_d=\frac{1}{2}\left\{a^2\frac{(\varkappa^4-\varkappa^2\varkappa_0^2/2)}{(\varkappa^4+\varkappa_0^4/16)}-\left[\delta_0-\left(\varkappa^2+\frac{\varkappa_0^2}{4}\right)\right]^2\right\}^{1/2}. \qquad (17)$$

It follows from (17) that instability is possible only when $\varkappa>\varkappa_0/\sqrt{2}$. For given $\varkappa$ the maximum growth rate of the parametric instability is reached for "mismatches" $\delta_0\approx\varkappa^2+\varkappa_0^2/4$. When $\varkappa\gg\varkappa_0$ the maximum growth rate of the instability $\gamma_d=a/2$, which agrees with (7) at $\theta=\pi/4$. The set of Eqs. (15), (16) describing plasma turbulence was integrated numerically with initial conditions corresponding to random noise field and density distributions. The pump amplitude was

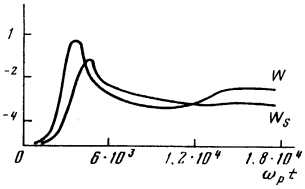

FIG. 2. Time dependences of W/nT and W_s/nT at a pump $E_0^2/8\pi n_0 T=2\times10^{-2}$ and a "mismatch" $\Omega=2\times10^{-2}\,\omega_p$.

varied in the range

$$2\cdot10^{-2}\leqslant E_0^2/8\pi n_0T\leqslant 2.5\cdot10^{-1},$$

the "mismatch" $\Omega=\omega_0-2\omega_p$ in the range $0\leqslant\Omega/\omega_p\leqslant0.17$ the parameter $T/mc^2=3\times10^{-3}$. In that case $\varkappa_0=12$. The method of solution is described in Ref. 12. We now discuss the results.

Figures 2 and 3 illustrate for the values $E_0^2/8\pi n_0 T=2\times10^{-2}$, $\delta_0=\varkappa_0^2$ the dynamics of plasma turbulence. We show in Fig. 2 the time-dependence of the energy of the plasma W and of the low-frequency modes of oscillations $W_s=\frac{1}{2}n_0T\langle\delta n^2/n_0^2\rangle$. In the initial stage the energy of the plasma oscillations grows exponentially with time with the parametric-instability growth rate γ_d. The maximum level of W depends on the actual stabilization mechanism of the parametric instability (nonlinear decay, outflow of the oscillations from the instability zone, and so on). These mechanisms were not taken into account in Eqs. (15), (16) and the limitation of the growth is connected solely with the development of the modulational instability of the plasma oscillations. Two consequences of the modulational instability are important for the stabilization—the decrease in the growth rate of the decay instability to a value $\gamma_d^2/\nu_{\rm cor}\sim W_s^{1/2}$ as a consequence of the random jumps in the phases of the plasmons, and the short-wavelength transfer of the energy of the plasma oscillations. The short-wavelength transfer of the oscillations leads to a damping of the plasma and after that of the low-frequency modes, and in final reckoning a quasi-stationary state is established in which the pumping of energy into turbulence is compensated by the absorption by particles.

In the numerical simulation we observed the appearance of a quasi-stationary state only in a plasma with hot ions $T_i\approx T_e$. When $T_i\ll T_e$ the low-frequency mode is slowly damped [see (6′)] and the high level of the ion-sound oscillations that arise in the initial stage is maintained during the whole time of the calculation and fully suppresses the parametric instability of the pump. The quasi-stationary turbulence state which arises

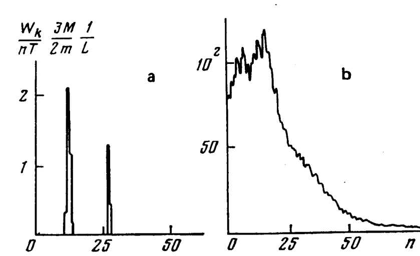

FIG. 3. Plasma turbulence spectra at $E_0^2/8\pi n_0T=2\times10^{-2}$ and $\Omega/\omega_p\approx2\times10^{-2}$: a) initial stage, $t=1.8\times10^3\omega_p^{-1}$; b) quasistationary turbulence, $t=2\times10^4\omega_p^{-1}$, $L=L_0/3(M/m)^{1/2}$, $L=1600\,r_D$ is the length of the turbulence interval, and $k=(n/6)(m/M)^{1/2}r_D^{-1}$.

when $T_i \approx T_e$ is in fact independent of the dynamics of the instability in the initial stage; by introducing in that stage a model damping of the plasma oscillations we changed W_{max} by about an order of magnitude, but the basic characteristics of the quasi-stationary turbulence remained unchanged.

We show in Fig. 3 the dynamics of the spectra of Langmuir turbulence, which confirms the qualitative picture given above. When $\delta_0 = \varkappa_0^2$ the maximum growth rate is reached for $\varkappa^2 = 3\varkappa_0^2/4$, which corresponds to the Langmuir-satellites wave numbers

$$\varkappa_1 \approx \left[\left(\frac{3}{2}\right)^{1/2} + \frac{1}{2}\right]\frac{\varkappa_0}{\sqrt{2}} \quad (n_1=28),$$

$$\varkappa_2 \approx \left[\left(\frac{3}{2}\right)^{1/2} - \frac{1}{2}\right]\frac{\varkappa_0}{\sqrt{2}} \quad (n_2=12).$$

The spectrum of the plasma oscillations in the initial stage [see Fig. 3(a)] is accordingly localized close to the two satellites with $n_1 = 28$ and $n_2 = 12$. The modulational instability leads in the course of time to the short-wavelength transfer of plasma oscillations. Figure 3(b) refers to quasi-stationary turbulence; the spectrum shown in that figure is completely analogous to the Langmuir spectra initiated with a pump $\omega_0 = \omega_p$ (cf. Ref. 10). One can clearly trace in the spectrum an inertial interval with a power-law spectral density behavior as function of the wave number $W_k \propto k^{-3/2}$; this interval corresponds to short-wavelength transfer of plasma oscillations by cavitons that collapse in the supersonic regime-solitons (for details see Ref. 10).

Finally, we show in Fig. 4 the pump-amplitude dependence of the main integrated characteristics of the quasi-stationary turbulence—the energy of the plasma oscillations and the effective collision frequency determined from the energy-balance condition

$$\nu_{eff} = 2\sum_k \Gamma_k |E_k|^2 / E_0^2. \tag{18}$$

The solid curves are obtained from the analytical Eqs. (11), (12) with $\alpha \approx 0.1$ and $\theta = \pi/4$, and the points are the results of a numerical simulation which thus quantitatively confirms the analytical model of plasma turbulence developed in Sec. 2.

4. To solve the problem of the absorption of an electromagnetic wave in the inhomogeneous plasma of a deuterium-tritium target it is necessary to know not only the effective collision frequency, but also the width of the anomalous absorption zone. The latter is determined by the evolution of the plasma turbulence excited by the electromagnetic pump when the "mismatch" $\Omega = \omega_0 - 2\omega_p$ increases which is unavoidable because of the spatial inhomogeneity of the plasma. The present section is devoted to a consideration of this problem.

From the analysis of the dispersion Eq. (17) given in the preceding section which describes the parametric instability of the electromagnetic pump it follows that taking into account that the pump wave number is finite ceases to be important for "mismatches" $\Omega \gg 3k_0^2 r_D^2 \omega_p$. In that case the pumping leads to an excitation of plasma oscillations with oppositely directed wave vector k and −k, with $kr_D \approx (\Omega/3\omega_p)^{1/2}$. The initial spectrum of the plasma oscillations thus turns out in this case to be symmetric with respect to the point $k = 0$. However, its further evolution is not all different from the one described in the previous section—if the modulational instability is taken into account the spectrum broadens to $kr_D \approx (W/3n_0T)^{1/2}$ and after that the short-wavelength transfer caused by the collapse is switched on as well as the resonance absorption of the oscillations by the electrons.

The natural width of the $\omega = 2\omega_p$ resonance is caused by the random jumps in the phase of the plasmons which are produced, $\Omega \lesssim 2\nu_{cor}$. Therefore the picture of the turbulence described above is retained up to "mismatches":

$$\Omega \leqslant \Omega_* = 3k_*^2 r_D^2 \omega_p \approx \omega_p \left(\frac{3T}{32\alpha mc^2}\frac{E_0^2}{8\pi n_0 T}\right)^{2/3}\left(\frac{M}{m}\right)^{1/3}, \tag{19}$$

where for the evaluation of ν_{cor} we substituted the energy of the plasma oscillations from Eq. (13). For large "mismatches", as in plasma turbulence initiated by pumping at a frequency $\omega = \omega_p$,[8-10] an important role is assumed by the stage-by-stage transfer of plasma oscillations, which is caused either by decay of a plasmon into a plasmon with a longer wavelength and ion sound ($l \to l' + s$) or (in an isothermal plasma) by induced scattering of plasmons by the ions in the plasma ($l \to l' + i$).

We consider the case $T_e \gg T_i$ and assume that the amplitude of the Langmuir satellites E_l satisfies the condition

$$E_l^2/16\pi n_0 T \ll (m/M)^{1/2} kr_D.$$

In this case the growth rate of the three-wave decay $l \to l' + s$ which leads to the appearance of the satellite Langmuir spectrum is equal to

$$\gamma_l = \omega_p \left[\frac{E_l^2}{64\pi n_0 T} kr_D \left(\frac{m}{M}\right)^{1/2}\right]^{1/2}. \tag{20}$$

The interval of the stagewise transfer is $\Delta k = (2/3r_D)(m/M)^{1/2}$, and the amplitude of the satellites is determined from the condition that the growth rate of the two-plasmon decay of the electromagnetic wave γ_d equal the growth rate of the stagewise decay γ_l (see Refs. 9, 12):

$$E_l^2 \approx \frac{3}{4} E_0^2 \frac{T}{mc^2 (m/M)^{1/2} kr_D}.$$

The total energy of the plasma oscillations produced by the stagewise decay in weak turbulence is equal to

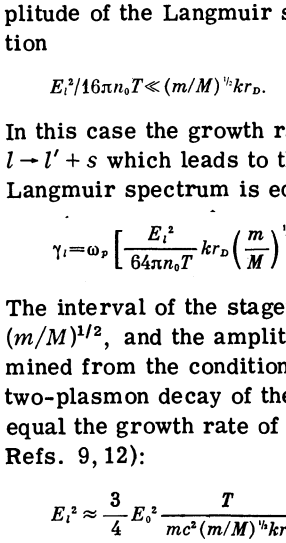

FIG. 4. W (curve 1) and ν_{eff} (curve 2) as functions of the amplitude of the electromagnetic pump E_0^2.

$$W_{WT}\approx\frac{E_0^2}{16\pi}\frac{T}{mc^2}\frac{M}{m}. \quad (21)$$

For relatively small "mismatches" satisfying the condition

$$\Omega_*<\Omega<\omega_p W_{WT}/n_0T,$$

the spectrum of the Langmuir oscillations produced by the stagewise transfer is modulationally unstable. This means that the level of oscillations W_{WT} is not reached and the energy of the oscillations is frozen at the threshold of the modulational instability:

$$W\approx n_0T\Omega/\omega_p. \quad (22)$$

Under these conditions the free development of the modulational instability is impossible as otherwise the fast spectral transfer connected with the modulational instability would lead to a relatively low level W determined by Eq. (13), which would turn out at the large mismatch values which we consider [the opposite condition to (19)] to be clearly below the threshold for the modulational instability. In other respects, however, the picture remains the usual one for strong turbulence—modulational instability in the wave-number range $kr_D\approx(\Omega/3\omega_p)^{1/2}$, the formation of collapsing cavitons, and resonance absorption by electrons of short-wavelength oscillations. This kind of dyanmics of the modulational instability is maintained up to "mismatches"

$$\Omega_1\approx\omega_p\frac{E_0^2}{16\pi n_0T}\frac{M}{m}\frac{T}{mc^2}. \quad (23)$$

For large "mismatches" the spectrum of the plasma oscillations which occur in weak turbulence in the wave-number region $k>k_1$ is modulationally stable. The spectral transfer of plasma oscillations under these conditions behaves as if it were two-stage, in complete analogy with the problem of the dynamics of the Langmuir spectrum excited by an electromagnetic pump at the plasma frequency.[9,10] In the first stage a long-wavelength transfer of oscillations takes place along the weak turbulence channel. After that, when the wave numbers

$$k_{min}\approx\frac{E_l^2}{36\pi n_0T}\left(\frac{M}{m}\right)^{1/2}\frac{1}{r_D}\ll\frac{k_1^2}{k_{min}},\qquad k_{min}\ll k_1,$$

are reached, the modulational instability and a collapse are switched on and guarantee an energy flux to the short wavelength region and resonance absorption of this energy by the electrons in the plasma.

We now estimate the quantity ν_{eff} under conditions when the long-wavelength spectral transfer along the weak turbulence channel is important. The frequency ν_{cor} which determines the speed of the plasmon phase mixing is in that case given by the relation

$$\nu_{cor}\approx\omega_p\sum_{\mathbf{k}}\frac{1}{3k^2r_D^2}\frac{\delta n_\mathbf{k}^2}{n_0^2}\approx\frac{\omega_p}{3k^2r_D^2}\frac{W_s}{n_0T}, \quad (24)$$

W_s is the energy of the ion-sound oscillations which appear in the decay chain. According to Ref. 13 $W_s=(\omega_s/\omega_p)W$ [$\omega_s=\omega_p(m/M)^{1/2}kr_D$ is the sound mode frequency] and we then get from (24)

$$\nu_{cor}\approx\omega_p\frac{E_0^2}{48\pi n_0T}\frac{T}{mc^2}\left(\frac{M}{m}\right)^{1/2}\frac{1}{kr_D}. \quad (25)$$

The energy flux of the plasma oscillations that arise in the stage-wise decays can be written in the form

$$I=\frac{\gamma_d^2}{\nu_{cor}}\frac{E_l^2}{8\pi}$$

and using Eqs. (7) and (25) one can easily show that it corresponds to an effective collision frequency which is the same order of magnitude as the estimate (11) obtained in the strong turbulence framework:

$$\nu_{eff}^{WT}\approx\frac{27}{64}\omega_p\frac{T}{mc^2}. \quad (26)$$

When the "mismatch" Ω increases the plasma oscillations which originate from the pump shift to the shorter-wavelength region and the Landau damping rate increases for them. The maximum values of the "mismatch" for which it is possible in general to excite plasma turbulence by an electromagnetic pump are determined from the condition for the stabilization of the parametric instability of the pump by Landau damping of the plasma oscillations. This condition has the form[13]

$$\Gamma(k_a)\approx\gamma_d,\qquad 3k_a^2r_D^2=\Omega_{max}/\omega_p. \quad (27)$$

A numerical simulation of plasma turbulence for large values of the "mismatch" Ω confirms on the whole the qualitative picture given above.

We show in Fig. 5 the dyanmics of the spectra W_k of the plasma and W_{sk} of the sound-wave turbulence, obtained by the numerical simulation for a pumpmg amplitude $E_0^2/8\pi n_0T=2\times10^{-2}$ and a "mismatch" $\Omega=7\times10^{-2}\omega_p$. In the initial stage Langmuir satellites are excited with $n_1=32$ and $n_2=48$ in agreement with the linear theory expounded above. In the further development the chain of $l\to l'+s$ decays leads to the formation of a satellite spectrum which is typical of weak turbulence with a transfer step

$$\Delta k=(2/3r_D)(m/M)^{1/2}\qquad(\Delta n=4),$$

where, as usual in weak turbulence, the energy flux in the stagewise decays is in the direction of the long-wavelength part of the spectrum. One can clearly trace in Fig. 5(b) two sections in the Langmuir oscillation spectrum corresponding to a chain of satellites arising in the decay of the oscillation with $n_1=32$ and a chain with an appreciably lower amplitude arising as the result of the decay of the oscillation with $n_2=48$. From the condition for the dacay of a Langmuir quantum $l_\mathbf{k}\to l_{\mathbf{k}'}+s$:

$$^3/_2(k^2-k'^2)r_D^2\approx|\mathbf{k}-\mathbf{k}'|(m/M)^{1/2}r_D$$

it follows that for plasma oscillations with $kr_D\gg(m/M)^{1/2}$ the $l\to l'+s$ decay in the one-dimensional case is possible only for plasmons with oppositely directed wave vectors that lead to the generation of an ion-sound oscillation with a wave number $k_s=k+k'\approx2k$. In the initial stage of the instability, therefore, when the energy of the Langmuir oscillations is localized in the satellites $n_1=32$ and $n_2=48$, the parametric interac-

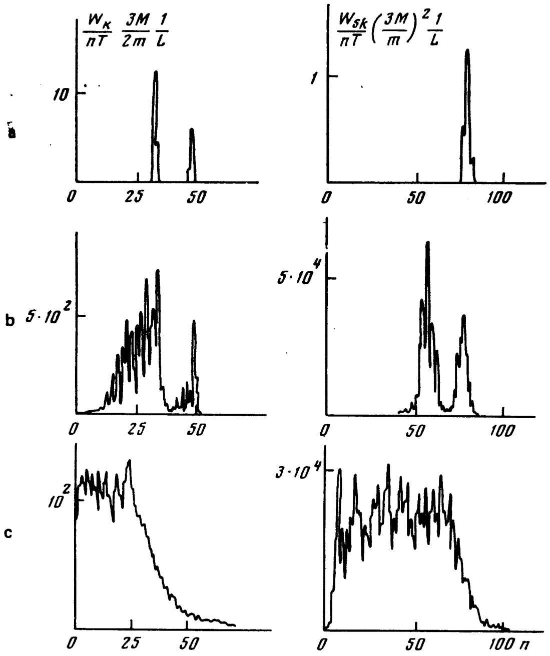

FIG. 5. Dynamics of the W_k and W_{sk} spectra for $E_0^2/8\pi n_0 T = 2\times10^{-2}$, $\Omega = 7\times10^{-2}\,\omega_p$; a) $t = 3.6\times10^3\,\omega_p^{-1}$, b) $t = 5\times10^3\,\omega_p^{-1}$, c) $t = 1.9\times10^4\,\omega_p^{-1}$.

tion of the waves leads to the appearance of sound oscillations with $n_s = 80$. After that in the weak turbulence stage the spectrum of the ion-sound oscillations which occur is localized in the region of the doubled harmonics of the plasma spectrum so that $k_s > 2k_{l,\min}$. The later development of the modulational instability of the plasma oscillations and the formation of collapsing caviton-solitons leads, on the one hand, to the appearance of the harmonics of the low-frequency mode with small n, and, on the other hand, to the transfer of the plasmon energy to the short-wavelength part of the spectrum ($n > 50$), where the plasmons are effectively absorbed by particles [Fig. 5(c)].

The dependence of the energy W of the plasma oscillations on the "mismatch" Ω obtained in the numerical simulation for a value $E_0^2/8\pi n_0 T \approx 2\times10^{-2}$ is plotted by open circles on the analytical curve (Fig. 6); the numerical results agree with the analytical ones. The Ω-dependence of ν_{eff} in the numerical simulation is shown in Fig. 7. For small Ω ($\Omega \leq 2\times10^{-2}\omega_p \approx \Omega_*$) the quantity ν_{eff} is independent of the "mismatch," while for large Ω, corresponding to weak turbulence, ν_{eff} slowly decreases with increasing "mismatch" and, finally, for a "mismatch" value $\Omega \approx 0.15\omega_p$, corresponding according to (27) to a stabilization of the para-

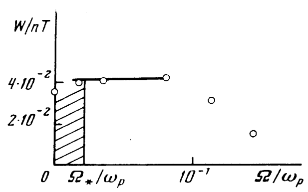

FIG. 6. W as function of the "mismatch" Ω for $E_0^2/8\pi n_0 T = 2\times10^{-2}$.

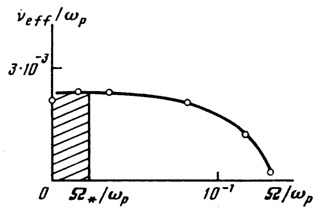

FIG. 7. ν_{eff} as function of the "mismatch" Ω for $E_0^2/8\pi n_0 T = 2\times10^{-2}$.

metric instability of the pump, the quantity ν_{eff} steeply decreases by about two orders of magnitude.

5. Although the analysis in the preceding sections pertained to an idealized situation of a homogeneous plasma, one can use the results of those sections to calculate the coefficient for the absorption of an electromagnetic wave in a real plasma target. For a nonuniform plasma, of primary importance is the nature of the parametric instability that leads to the excitation of plasma oscillations. The two-plasmon instability of the electromagnetic wave in the weak turbulence framework can be either an absolute or a convective (drift) instability. However, an absolute instability corresponding to the presence of turning points for the plasmons which appear in the instability is possible only in a rather narrow range near the resonance point $\omega = 2\omega_p$ with a width $\Delta\omega_p \sim \gamma_d$.[14] An absolute two-plasmon instability can therefore not lead to any appreciable absorption of the electromagnetic wave in the vicinity of one-quarter of the critical density. The conclusion by Chen and Liu[5] about the smallness of the absorption at the resonance $\omega = 2\omega_p$, notwithstanding the relatively large effective collision frequency, was based just upon the fact that the absorption of the electromagnetic energy in that paper was connected with an absolute two-plasmon instability. In actual fact, however, the main contribution to the anomalous absorption in the vicinity of $n_c/4$ come from a convective two-plasmon instability and the absorption turns out to be very appreciable.[15] The amplification coefficient for the plasma oscillations when we take into account that they are convectively carried away from the resonance region turns out to be equal to[1]

$$K = \frac{\pi}{24}\frac{\omega_p^2 r_c}{kc^2}\frac{E_0^2}{8\pi n_0 T} \qquad (28)$$

(r_c is the critical radius of the plasma target).

From the condition $K > \ln\Lambda$ ($\ln\Lambda$ is the Coulomb logarithm) for reaching a nonlinear level by the plasma oscillations we can estimate the width of the absorption region

$$\Delta r \approx 3r_c k^2 r_D^2 \approx \frac{T}{mc^2}\frac{\omega_p^2 r_c^3}{c^2}\left(\frac{E_0^2}{96\pi \ln\Lambda n_0 T}\right)^2, \qquad (29)$$

as well as the coefficient $\varkappa$ for the absorption of the electromagnetic wave, defining it as the ratio of the power dissipated in plasma turbulence to the energy flux in the wave incident upon the plasma:

$$\varkappa = \int \nu_{eff}\frac{E_0^2}{8\pi}dr \Big/ \frac{c}{4\pi}E_0^2.$$

Substituting for ν_{eff} the order of magnitude (26) and for the width of the absorption region Δr the value from (29) we get the following absorption coefficient which increases very rapidly ($\propto E_0^4$) with increasing amplitude of the electromagnetic wave:

$$\varkappa = \frac{\omega_p{}^3 r_c{}^3}{2c^3}\left(\frac{E_0{}^2}{96\pi \ln \Lambda n_0 mc^2}\right)^2. \qquad (30)$$

Such an absorption of the electromagnetic wave occurs for sufficiently small values of its amplitude when the width of the absorption region (29) determined from the condition that the convective carrying away of the plasma oscillations can be neglected remains less than the width Δr_1 determined from the condition for the two-plasmon instability by Landau damping: $\Delta r_1 = 3k_a^2 r_D^2 r_c$, where k_a is determined from Eq. (27). The corresponding condition on the wave amplitude has the form

$$\frac{E_0{}^2}{8\pi n_0 T} < 8 \ln \Lambda \frac{mc^2}{T} \frac{k_a r_D{}^2}{r_c}. \qquad (31)$$

For large amplitudes the width of the anomalous absorption region $\Delta r = \Delta r_1$ and we get the following estimate for the absorption coefficient:

$$\varkappa \approx \frac{3}{2} \frac{\omega_p r_c}{c} \frac{T}{mc^2} k_a{}^2 r_D{}^2. \qquad (32)$$

Assuming for simplicity that the distribution function of the resonance electrons which absorb the plasma oscillations remains Maxwellian we get from (27) the following approximate equation for k_a:

$$k_a{}^2 r_D{}^2 \approx \frac{1}{2} \ln^{-1}\left[\frac{c}{v_0} \ln \frac{c}{v_0}\right]. \qquad (33)$$

We note that up to a logarithmic factor the estimate (32) for the absorption coefficient agrees with the formula obtained in Ref. 15 in the weak turbulence theory framework. Equation (33) gives an upper bound of the width of the resonance region. In a real plasma target there occur "tails" of accelerated electrons on the electron distribution function. In particular, these can be electrons accelerated by powerful plasma turbulence in the vicinity of the critical density and moving to the periphery of the target. Taking the "tails" into account leads to a shift of the resonance absorption of the oscillations by the electrons to smaller values of k and as a consequence to a decrease in the width of the resonance region and in the absorption coefficient $\varkappa$.

Formula (32) for the absorption coefficient $\varkappa$ remains valid up to electromagnetic field amplitudes

$$\frac{E_0{}^2}{16\pi n_0 T} \approx 3k_a{}^2 r_D{}^2 \frac{4}{1+(M/2m)(T/mc^2)}.$$

For large amplitudes the modulation of the plasma density by the ponderomotive force of the high-frequency pressure produced by the simultaneous action of the electromagnetic wave and the plasma oscillations upsets the resonance coupling between the plasma and the electromagnetic oscillations and, as a consequence, decreases the coefficient of anomalous absorption.

The general conclusion from the analysis given in this paper is that the maximum absorption of the electromagnetic wave in the vicinity of $n_c/4$, given by Eq. (32), may turn out to be very appreciable and the contribution from that region must also be taken into account also in the total absorption balance. Taking such factors as electron "tails" and the modification of the electron density by the electromagnetic field pressure into account may lower the absorption coefficient. However, allowance for these factors requires special considerations that depend on the actual conditions of the experiment and is therefore outside the scope of the present paper.

The authors are grateful to V. P. Silin and V. T. Tikhonchuk for useful remarks.

[1]A. A. Galeev, V. N. Oraevskiĭ, and R. Z. Sagdeev, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 194 (1972) [JETP Lett. **16**, 136 (1972)].
[2]A. A. Galeev, G. Laval, T. O'Neil, M.N. Rosenbluth, and R. Z. Sagdeev, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 48 (1973) [JETP Lett. **17**, 35 (1973)].
[3]A. A. Galeev, G. Laval, T. O'Neil, M. N. Rosenbluth, and R. Z. Sagdeev, Zh. Eksp. Teor. Fiz. **65**, 973 (1973) [Sov. Phys. JETP **38**, 482 (1974)].
[4]V. B. Bychenkov, V. P. Silin, and V. T. Tikhonchuk, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 309 (1977) [JETP Lett. **26**, 197 (1977)].
[5]H. H. Chen and C. S. Liu, Phys. Rev. Lett. **39**, 881 (1977).
[6]A. M. Rubenchik, Proc. XIII Internat. Conf. Phen. Ionized Gases, Vol. II, p. 887 (1977).
[7]A. A. Galeev, R. Z. Sagdeev, Yu. S. Sigov, V. D. Shapiro, and V. I. Shevchenko, Fiz. Plazmy **1**, 10 (1975) [Sov. J. Plasma Phys. **1**, 5 (1975)].
[8]A. A. Galeev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Zh. Eksp. Teor. Fiz. **73**, 1352 (1977) [Sov. Phys. JETP **46**, 711 (1977)].
[9]G. A. Gorbushina, L. M. Degtyarev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Odnomernaya turbulentnost' lengmyurovskikh voln (One-dimensional turbulence of Langmuir waves) Preprint of the Inst. Appl. Math. No 17, 1978.
[10]L. M. Degtyarev, R. Z. Sagdeev, G. I. Solov'ev, V. D. Shapiro, and V. I. Shevchenko, Fiz. Plazmy **6**, 485 (1980) [Sov. J. Plasma Phys. **6**, 263 (1980)].
[11]V. E. Zakharov and G. M. Zaslavskiĭ, ZhTF **34**, 212 (1964) [*sic*].
[12]V. P. Silin and V. T. Tikhonchuk, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 504 (1978) [JETP Lett. **27**, 474 (1978)].
[13]A. A. Galeev and R. Z. Sagdeev, Nucl. Fusion **13**, 603 (1973).
[14]V. P. Silin and A. N. Starodub, Zh. Eksp. Teor. Fiz. **66**, 176 (1974) [Sov. Phys. JETP **39**, 82 (1974)].
[15]A. N. Erokhin and V. P. Silin, Short Commun. Phys., Lebedev Inst. Phys., Acad. Sc. USSR No. 12, pp. 42-46 (1977).

Translated by D.ter Haar