Parametric excitation of spin waves in the antiferromagnet $FeBO_3$

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(Submitted 28 May 1981)
Zh. Eksp. Teor. Fiz. **81**, 1913–1924 (November 1981)

The parametric excitation of spin waves in the easy-plane antiferromagnet $FeBO_3$ by the method of parallel pumping at a frequency $\nu_p = 35.4$ GHz has been studied. Experiments were carried out in the temperature range $T = 1.2$ to 120 K. It was found that the excitation process is of the hard type. The temperature dependence of the spin wave relaxation frequency, $\Delta\nu_{1k}$, was measured and it was shown that at low temperatures scattering by impurities is the main relaxation mechanism. The inhomogeneous exchange constant could be determined by observing the size effect, and is $\alpha_\parallel = 8.7\times10^{-2}$ Oe · cm. An anomaly in the above-threshold susceptibility χ'' was found which is evidently related to the developement of secondary instability of the parametric spin waves. Oscillations of the power absorbed by the specimen were observed at high pumping levels ($h/h_{c2} \gg 1$), and their frequency increased with increasing h/h_{c2}.

PACS numbers: 75.30.Ds, 75.50.Ee

1. INTRODUCTION

The study of parametric excitation of spin waves provides extensive information about the properties of the elementary excitations in magnetically ordered crystals. A number of such experiments have been carried out in recent years on antiferromagnetics (see Refs. 1 to 5). However, for a long time parametric excitation of spin waves was only successful in antiferromagnetics containing Mn^{2+} magnetic ions in an S state ($CsMnF_3$, $MnCO_3$, $CsMnCl_3$). It was not clear why it was not possible to excite spin waves under similar experimental conditions in antiferromagnets containing other magnetic ions, for example Fe^{3+} ions also in an S state. We were interested in studying parametric excitation of spin waves in antiferromagnets with other magnetic ions, for example $FeBO_3$, α-Fe_2O_3, $CoCO_3$, which besides a similar magnetic structure, differ appreciably in a number of their magnetic parameters, such as the magnitude of the exchange field and the spin–orbit and magnetoelastic interaction, so that the influence of the latter on the process of parametric excitation and on the properties of the spin waves could be elucidated.

The present work is devoted to a study of parametric excitation of spin waves in $FeBO_3$. Due to its high Néel temperature, transparency in the visible part of the spectrum, large magnetostriction and its strong Dzyaloshinskiĭ interaction, $FeBO_3$ has become one of the major objects of magnetic investigations.

The structure of $FeBO_3$ is described by the D^6_{3d} space group symmetry. The rhombohedral elementary cell with $a_0 = 5.520$ Å and $\alpha = 49.54°$ (in the hexagonal representation $a'_0 = 4.626$ Å, $c'_0 = 14.493$ Å) contains two molecular units.[6-8] Neutron studies[9] showed that $FeBO_3$ becomes a weak ferromagnet below $T_N = 348$ K, in which the magnetic moments of the sublattices lie in a plane perpendicular to the C_3 axis.

The spin-wave spectrum of a two-sublattice antiferromagnet with an easy-plane type of anisotropy consists of two branches. If a static magnetic field **H** lies in the basal plane of the crystal, then in the continuous medium approximation ($d^{-1} \ll |\mathbf{k}| \ll \pi/a_0$ where d is the specimen size) the spin wave spectrum is represented by the following equations:[10-12]

$$(\nu_{1k}/\gamma)^2 = H(H+H_D) + H_\Delta^2 + 36H_A^6H_E\cos 6\varphi + \alpha_\parallel^2k_\parallel^2 + \alpha_\perp^2k_\perp^2 + H_{dip}^2, \quad (1)$$

$$(\nu_{2k}/\gamma)^2 = 2H_AH_E + H_D(H+H_D) + \alpha_\parallel^2k_\parallel^2 + \alpha_\perp^2k_\perp^2, \quad (2)$$

where ν_{1k} and $\mathbf{k}$ are the frequency and wave vector of the spin waves; γ is the gyromagnetic ratio, H_E is the exchange field; H_D is the Dzyaloshinskiĭ field, H_A is the uniaxial anisotropy field; H_A^6 is the hexagonal anisotropy field in the basal plane; H_Δ^2 is the gap in the spectrum produced by the magneto–elastic and hyperfine interaction; $\alpha_\parallel$ and $\alpha_\perp$ are inhomogeneous exchange constants (the suffices $\parallel$ and $\perp$ indicate the direction of $\mathbf{k}$ relative to the principal C_3 axis); H_{dip}^2 is the gap produced by the spin wave field. In an orthogonal coordinate system for which $0z \parallel C_3$ and $0x \parallel \mathbf{H}$, and with the conditions that the inhomogeneous exchange is isotropic ($\alpha_\parallel = \alpha_\perp = \alpha$) and that $4\pi\chi_\perp \ll 1$ ($\chi_\perp = l_0/2H_E$ is the perpendicular susceptibility), we obtain[13]

$$H_{dip}^2 = 4\pi\chi_\perp[H(H+H_D)+\alpha^2k^2]\left[\frac{k_z^2}{k^2} + \frac{(H+H_D)^2}{H(H+H_D)+\alpha^2k^2}\frac{k_y^2}{k^2}\right]. \quad (3)$$

There is no simple expression for H_{dip}^2 in the case of anisotropic inhomogeneous exchange for an arbitrary direction of the wave vector $\mathbf{k}$, however when $\mathbf{k}$ is directed along one of the coordinate axes, the expression for H_{dip}^2 is obtained by substituting in Eq. (3) the corresponding component α_i.

Eqs. (1) and (2) also determine the homogeneous resonance spectrum ($k = 0$) if the demagnetizing field, dependent on the shape of the specimen, is included in them instead of H_{dip}^2. However, calculations show that the effect of this on the AFMR spectrum can be neglected.

The constants which determine the spin wave spectrum can be measured by various methods (static magnetic measurements, nuclear and electron magnetic resonance, acoustic resonance, Mössbauer effect, neutron scattering, magnetooptical studies). $FeBO_3$ has been studied by all the methods mentioned. The results of

 0038-5646/81/111013-07$02.40

TABLE I.

Method	$H_E \cdot 10^{-3}$, kOe	H_D, kOe	H_Δ^2, kOe^2	H_A^6, Oe	ν_{20}, GHz	γ, GHz/kOe
Static	4.1 [19] 2.6 [20] 3.6 [21. 22]	100 [19] 85±7 [20]		<1 [20]		2.94 [20]
NMR	3.4 [20] 3 [23]					
Acoustic resonance			6±2 [24] 0.15 (300 K) [25]			
Mössbauer	3.04 [15]	93 [15]			480 [15]	
AFMR		107 [22] 101 [26] 92.7 * [26]	 4.9 [26] 1.1 (300 K) [27]	<1 [22] $0.7-3.7\cdot10^{-2}$ [28]	 465 [26]	2.81 [22] 2.80 [26]
Two-magnon light scattering	3.07 [29] 3.07 [17]					
Brillouin scattering	3.26 [16]	107 [16]	0.3 (300 K) [27]			

*This value of H_D was determined from the $\nu_{20}(H)$ dependence, while other values were derived from the $\nu_{10}(H)$ dependence.

the existing work agree in that the temperature dependences of the sublattice magnetizations M and of the effective fields H_E, H_D and H_A are represented to high accuracy by the Brillouin function $B_{5/2}(T)$ (Refs. 9 and 14) and that the exchange field is determined by nearest neighbor interaction.[15-17] It follows from the latter that $\alpha_\parallel = 1/6H_E c_0'$ and $\alpha_\perp = 6^{-1/2} H_E a_0'$ (Ref. 18). This enables the results obtained by different methods and at different temperatures to be compared by reducing them to $T = 0$. The results of such an analysis are shown in Table I. It can be seen from the table that the constants of the spectrum agree well among themselves, except for H_Δ^2 and H_A^6, which as the authors pointed out, vary from specimen to specimen.

2. METHOD AND SPECIMENS

Parametric excitation of spin waves was investigated with a direct amplification spectrometer, using the method described by Meixner *et al.*[29] A high Q ($\approx 10^4$) cylindrical resonator was used designed for the H_{012} oscillation mode. The specimen was secured to the bottom of the resonator at an antinode of a microwave field h with the help of a cigarette paper pocket, the fields H and h lay in the basal plane of the crystal. This method of fixing avoided elastic strains in the specimen on cooling, which influence the spin wave spectrum in $FeBO_3$ noticeably.

The microwave cw magnetron oscillator of frequency 35.4 GHz and power 10 W operated in a long pulse mode, achieved by modulating its anode voltage. The pulse duration was varied from 0.1 to 1 ms with a repetition frequency 50 Hz.

Measurements were made over the temperature range from 1.2 to 120 K. For $T > 4.2$ the resonator with the specimen was placed in helium vapor; for $T \leq 4.2$ the resonator was filled with liquid helium. The temperature was measured with a semiconductor resistance thermometer to an accuracy no worse than 2%.

The microwave pulse, after traversing the resonator with the specimen, was observed on an oscilloscope screen. The dependence of pulse amplitude on static magnetic field H was recorded with an x–y recorder. The microwave power delivered to the resonator was measured with a thermistor bridge. The field h at the specimen was calculated from the power and the resonator parameters with an absolute accuracy of 20%, but the relative accuracy was considerably better, being ~3%.

The specimens were transparent green-colored plates with natural faces, having transverse dimensions 2×2 mm and thickness between 0.08 and 0.5 mm. The plane of the plate coincided with the crystal basal plane.[1)]

3. EXPERIMENTAL RESULTS

B. Relaxation of spin waves

1. The experiments showed that starting at some threshold level h_{c1} of the microwave field at the specimen, absorption arises in it at small static fields $H < H_0$. Figure 1 shows the absorption curves for different amplitudes of the microwave field h on the specimen. One is thus observing the usual picture for parametric excitation of spin waves. The range of field H in which absorption is observed corresponds to excitation of spin waves in the first branch of the spectrum with frequency $\nu_{1k} = \nu_p/2$, where ν_p is the pumping frequency, and according to Eq. (1) the field H_0 must correspond to $k = 0$. Unfortunately it is not possible to verify this condition by substituting known values into Eq. (1) because, as already noted, the parameters H_Δ^2 and H_A^6, which have an appreciable influence on the spectrum, change from specimen to specimen and moreover depend on elastic deformations. It is even possible that they are not uniform throughout the specimen, which seems to be indicated by the absence of a sharp absorption edge on the high field side.

Study of the kinetics of the excitation process showed that at low temperatures the absorption in a specimen starts abruptly at a value of the field $h = h_{c1}$ and also ends abruptly at a smaller field $h = h_{c2}$. The excitation process is thus of the hard type, as in $MnCO_3$ and $CsMnF_3$.[30-32] The value of the hardness $(h_{c1} - h_{c2})/h_{c1}$ decreases with increasing temperature and magnetic field as in Refs. 30 to 32. Its maximum value at $T = 1.2$ is ≈ 0.8.

We should note that we observed one absorption jump at the lowest temperature. As the temperature in-

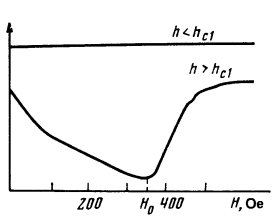

FIG. 1. Microwave power-absorption curves at $T = 1.2$ K.; H_0 is calculated according to Eq. (1) for $k = 0$ with $H_\Delta^2 = 5$ kOe^2, $H_A^6 = 0$.

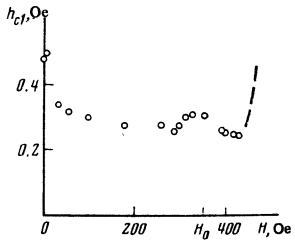

FIG. 2. Dependence of threshold field h_{c1} on the magnitude of the static field H at $T = 4.2$ K; H_0 as in Fig. 1.

creased its magnitude decreased and starting at $T \approx 4$ K, the jump broke up into several steps corresponding to different values of h_{c1}. This is evidently due to nonuniform excitation conditions in the specimen which are more appreciable the lower the value of the hardness. In what follows we shall understand by h_{c1} the minimum value of the threshold exciting field.

Figure 2 shows the experimental threshold field variation $h_{c1}(H)$ at $T = 4.2$ K. It can be seen that for not too small H, h_{c1} depends weakly on field. Experiments showed that such a form of $h_{c1}(H)$ dependence is maintained at higher temperatures.

Ozhogin[33] has discussed the theoretical problem of parametric excitation of spin waves in antiferromagnetics with easy plane type of anisotropy and weak ferromagnetism. For the case of $\nu_p \ll \nu_{20}$ and $\mathbf{H} \parallel \mathbf{h} \perp C_3$ he obtained the following expression for the threshold field:

$$h_c = \frac{\nu_p \Delta\nu_{1k}}{\gamma^2(2H+H_D)}, \qquad (4)$$

where $\Delta\nu_{1k}$ is the relaxation frequency of the excited waves.

We calculated the value of $\Delta\nu_{1k}$ from the measured h_{c1} using Eq. (4). Figure 3 shows the temperature dependence of $\Delta\nu_{1k}$ for $H = 120$ Oe. It can be seen from Fig. 3 that a peak occurs at $T = 18$ K superimposed on the background of a relaxation monotonically increasing with temperature.

We shall now discuss the nature of the relaxation in $FeBO_3$. Relaxation of the parametrically excited spin waves is determined by their interaction with thermal spin waves and with other elementary excitations in the

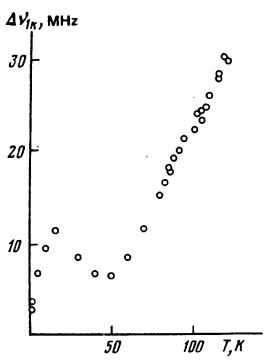

FIG. 3. Temperature dependence of the relaxation frequency $\Delta\nu_{1k}(T)$ for $H = 120$ Oe ($k \approx 0.5 \times 10^5$ cm^{-1}).

specimen (phonons and nuclear spin waves), by so-called intrinsic relaxation processes, and also by extrinsic processes, scattering of spin waves by defects (crystal boundaries, impurity atoms, and dislocations). It is usually assumed that, to a first approximation, relaxation produced by extrinsic processes is temperature independent. This is justifiable if there are no defects with levels separated in energy by the order of k_BT. The strong temperature dependence of the relaxation observed in our experiments would therefore appear to indicate that in our case intrinsic relaxation processes play an appreciable part. Relaxation determined by three-magnon processes $\Delta\nu_{3m}$ was calculated by Sobolev.[34] In our case his expression for $\Delta\nu_{3m}$ can be written in the form

$$\Delta\nu_{3m} = \frac{(\mu H)^2(\mu H_E/S)(T/\Theta_N)^3}{32\pi^5\hbar^3\nu_p(\nu_p^2-4\nu_{10}^2)^{1/2}}\left[1-\exp\left(-\frac{2\pi\hbar\nu_{1k}}{k_BT}\right)\right]\int_{x_-}^{x_+} e^{-x}x^2dx; \qquad (5)$$

$$x_\pm = \frac{2\pi\hbar\nu_{20}^2}{k_BT[\nu_p \mp (\nu_p^2-4\nu_{10}^2)^{1/2}]}, \quad \mu = 2\mu_B = 1.86\cdot 10^{-20}\ \text{erg}\cdot\text{G}^{-1},$$

where $\Theta_N = \hbar s/k_B v_0^{1/3}$, $s = 2\pi\alpha_\parallel\gamma = 1.4\times 10^6$ cm$\cdot$ s^{-1} is the limiting propagation velocity of spin waves,[16] v_0 is the volume of a unit cell and $S = 5/2$.

Calculations show that $\Delta\nu_{3m}$ evaluated by Eq. (5) should be 0.05 MHz at $T = 120$ K, i.e., two orders of magnitude smaller than the observed value.

According to Bar'yakhtar *et al.*,[35] relaxation determined by processes with more magnons participating should be an order of magnitude smaller in $FeBO_3$. We can evidently neglect relaxation produced by interaction of electronic with nuclear spin waves in our case, since in unenriched crystals (Fe^{57} content ~2%) the nuclear magnetization is negligibly small even at $T = 1.2$ K.

If the velocity of sound c is less than the limiting spin wave velocity, s, the relaxation determined by magnon–phonon interaction is governed by two processes: the combination of a parametric magnon with a thermal phonon and the decay of a parametric magnon into a magnon and a phonon or into two phonons. The relaxation determined by the combination process has been calculated by Lutovinov *et al.*[36] The total relaxation frequency due to magnon–phonon interaction, for $2\pi\nu_{10}/\rho \ll sk \ll \delta k_BT/2\hbar$ ($\delta = s/c$) is[2)]

$$\Delta\nu_{\text{m-ph}} = \frac{4}{\pi^2\delta^2(2+\delta)}\frac{\Theta}{Mc^2}\frac{T}{\Theta_N}, \qquad (6)$$

where $M = \rho v_0$ is the mass of a unit cell, $\Theta = v_0\beta_i$ is the characteristic magnetoelastic interaction energy and β_i is the appropriate component of the magnetostriction tensor. We used the existing values of the constants at $T = 77$ K to calculate $\Delta\nu_{\text{m-ph}}$: $c = 4.71\times 10^5$ cm$\cdot$s^{-1} (Ref. 16) and $\Theta = 1.3\times 10^{-13}$ erg (Ref. 24). The value obtained, $\Delta\nu_{\text{m-ph}} = 6.8$ MHz agrees with the experimental values of $\Delta\nu_{1k}$ in its order of magnitude. However, Eq. (5) cannot describe the results obtained since $\Delta\nu_{1k}$ is not proportional to T.

Other relaxation processes involving phonons have not at present been studied theoretically so that the question of the nature of intrinsic relaxation in $FeBO_3$ remains unresolved.

2. Peaks in the temperature dependence of the relaxation frequency, similar to those observed by us at $T = 18$ K (see Fig. 3), and apparently of the same nature, were noted earlier by Wolfe *et al.*[37] and by LeCraw *et al.*[38] who studied the line width of antiferromagnetic resonance in $FeBO_3$. According to existing ideas, they are related to the presence of impurity levels in the specimens investigated which lead to "slow" relaxation of spin waves. The theory of such processes[39] indicates that the maximum attenuation occurs when the condition $2\pi\nu_{1k}\tau = 1$ is fulfilled, where τ is the relaxation time for the ion. In crystals containing Fe^{3+} ions such levels are usually associated with the presence of Fe^{2+} ion impurities, which occur in crystals on the introduction of any impurities with a valency greater than three. If this is so, then one can determine the activation energy E_0 of the impurity level according to the results of Refs. 37 and 38 ($\nu_{10} = 34.5$ GHz) and our result ($\nu_{1k} = 17.2$ GHz). Assuming that $\tau = \tau_0 \exp(-E_0/k_BT)$, we obtain the value $E_0/k_B \approx 20$ K. Such a magnitude for the activation energy is characteristic for electron migration processes $Fe^{2+} \rightleftarrows Fe^{3+}$ (Ref. 40).

The experimental results thus again confirm that the relaxation of spin waves in crystals containing Fe^{3+} ions is very sensitive to the presence of impurities, especially at low temperatures. The absence of sufficiently pure crystals was evidently the reason for the unsuccessful experiments on parametric excitation of spin waves in $FeBO_3$. We note in passing that the high value of the hardness observed by us at low temperatures and the simultaneous observation of the impurity contribution to the relaxation, seems to confirm the suggestion of the impurity nature of the hardness made by us earlier.[30]

3. Scattering of spin waves at the crystal boundaries is another extrinsic relaxation process. This problem has been considered theoretically for two limiting cases. For totally absorbing boundaries[41]

$$\Delta\nu = [\Delta\nu_\infty^2 + \Delta\nu_b^2]^{1/2} \tag{7}$$

and in the case of completely reflecting boundaries[42]

$$\Delta\nu = [\Delta\nu_\infty(\Delta\nu_\infty + \Delta\nu_b)]^{1/2}, \tag{8}$$

where $\Delta\nu_\infty$ is the relaxation in an infinite specimen,

$$\Delta\nu_b = \frac{\beta}{\pi}\frac{v}{d} = \frac{2\beta}{d}\frac{\partial\nu_{1k}}{\partial k} \tag{9}$$

is the relaxation determined by scattering at the boundaries of a specimen of size d, v is the group velocity of spin waves and β a numerical coefficient of order unity. We obtain from Eq. (9) using Eq. (1)

$$\Delta\nu_b = (4\alpha_i\beta\gamma^2/\nu_p d)\,[(\nu_p/2\gamma)^2 - H(H+H_D)]^{1/2}. \tag{10}$$

Calculations carried out according to Eq. (10) for $\alpha_{\parallel} = 7.8\times10^{-2}$ Oe·cm (Ref. 16), $d = 0.5$ mm, $\beta = 1$ and $H = 120$ Oe give a values $\Delta\nu_b = 8.4$ MHz. At the lowest temperatures ($T = 1.2$ K) the minimum value of $\Delta\nu_{1k}$ we observed was ≈ 2.8 MHz. From the fact that $\Delta\nu_b$ is appreciably greater than $\Delta\nu_{1k}$ it follows that the excited spin waves are well reflected from the crystal boundaries. Calculation carried out using Eq. (8) gives $\Delta\nu_\infty \approx 0.85$ MHz at $T = 1.2$ K, i.e., the mean free path $l = v/\pi\Delta\nu_\infty \approx 0.5$ cm.

Size effect

It was observed that at $T < 4.2$ K for a certain field range $H_1 < H < H_0$ (H_0 is the field corresponding to excitation of spin waves with $k = 0$) the dependence of the power transmitted through the resonator containing the specimen on the static field for h slightly exceeding the threshold field h_{c1} is not a monotonic function, but has resonance type dips at certain values of H. Figure 4 shows an example of a section of an absorption curve. As the temperature decreases the field range within which the non-monotonic behavior is observed broadens (H_1 decreases) down to zero fields, while the position of each individual dip stays the same.

It was shown in the previous section that spin waves are reflected at the specimen boundaries. Since the mean free path l becomes greater than the transverse dimensions of the specimen at low temperatures, it is natural to explain the observed phenomena by an additional condition imposed on the spin waves by the specimen boundaries. These conditions must lead to resonance of spin waves directed perpendicular to the plane of the plate, i.e., along the principal crystal axis ($|\mathbf{k}| = k_z$), for which

$$N\lambda/2 = d, \tag{11}$$

where d is the plate thickness, $\lambda = 2\pi/k$ is the spin wave wavelength and N is an integer. In our experiment spin waves with $k \sim 10^4$ to 10^5 cm^{-1} were excited, which corresponds to $N \sim 10^3$.

It follows from Eqs. (1) and (11) that the interval ΔH between neighboring dips should be determined by the relation

$$\Delta H^2 = \left(\frac{2\pi\alpha_i}{d}\right)^2 \frac{H_0(H_0+H_D) - H(H+H_D)}{(2H+H_D)^2}. \tag{12}$$

In our case $H \ll H_D$ and Eq. (12) can be simplified:

$$\Delta H^2 = (2\pi\alpha_i/d)^2 \frac{H_0 - H}{H_D}, \tag{13}$$

where α_i is the appropriate component of α. Since d is the specimen dimension along the principal axis, $\alpha_{\parallel}$ must be taken for α_i in Eq. (13).

Measurements were made on three specimens with thickness $d = 0.5$, 0.49 and 0.08 mm. Figure 5 shows the experimental results on a $(\Delta Hd)^2$ vs H plot. The linear relation obtained confirms the proposal made.

The inhomogeneous exchange constant $\alpha_{\parallel}$ can be determined from the experimental results. The value obtained $\alpha_{\parallel} = (8.82\times10^{-2} \pm 5\%)$ Oe·cm agrees with the value $\alpha_{\parallel} = 7.8\times10^{-2}$ Oe·cm determined by Jantz *et al.*[16] from light scattering at $T = 77$ K and evaluated for $T = 4.2$ K.

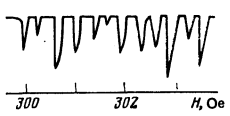

FIG. 4. Chart recorder trace of part of the absorption curve for h slightly (~1 db) exceeding h_{c1} at $T = 1.2$ K.

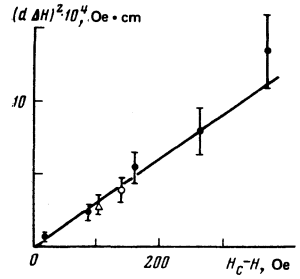

FIG. 5. Dependence of the spacing between dips ΔH on $H_0 - H$. ●$-d = 0.5$, Δ$-d = 0.49$, ○$-d = 0.08$ mm, $T = 1.2$ K.

It follows from our experiments that spin waves in $FeBO_3$ are excited with a discrete set of wave vectors, at any rate for fields exceeding the threshold by small amounts. This fact may, evidently, change existing views on the above-threshold state. As has been shown,[43] if the spectrum of excited waves is continuous, then the above-threshold state is determined by a phase mechanism of limiting the wave amplitude. This statement may be invalid for a discrete spectrum.

A similar effect was observed earlier in parametric excitation of spin waves only for the ferrimagnetic yttrium iron garnet (YIG) (Ref. 44).

Secondary instability

When the field h at the specimen exceeded some limiting value $h_c^* > h_{c1}$ which depended on temperature and on the magnitude and direction in the basal plane of the static field H, we found an "inverse discontinuity" on the pulse traversing the resonator, corresponding to a reduction in the power attenuation. A typical oscilloscope trace of a pulse for $h > h_c^*$ is shown in Fig. 6 (the splitting corresponding to parametric excitation of spin waves is not visible on this trace because the specimen is far beyond the excitation threshold: $h_c^*/h_{c1} \sim 5$ to 7 dB). The time t from the start of the pulse to the inverse discontinuity decreased with increasing h/h_c^*.

It can be suggested from the observed behavior that the spin wave system develops a secondary instability. Such an instability can be related to an avalanche-type growth in amplitude A_2 of the secondary waves into which the parametric spin waves decompose when their amplitude reaches a critical value A_c. During the time t there is then a buildup of amplitude A_2 to the value at which the inverse effect of the secondary waves on the parametric waves becomes noticeable.

The $h_c^*(T)$ dependence is shown in Fig. 7. Experiment showed that the ratio h_c^*/h_{c1} is about the same over the whole temperature range, except in the region of the impurity peak, and is ~2.5.

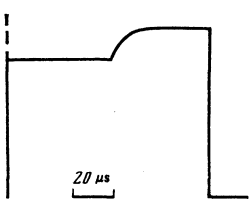

FIG. 6. Oscilloscope trace of microwave pulse traversing the resonator with the specimen for $h > h_c^*$.

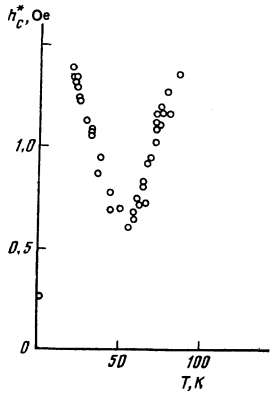

FIG. 7. Temperature dependence of h_c^*, $H = 120$ Oe.

We carried out the following experiment to determine the relaxation time of the secondary waves. Two rectangular microwave pulses of the same amplitude were supplied to the resonator containing the specimen, and their duration and the time interval between them Δt could be varied; the v.h.f. field at the specimen slightly exceeded h_c^*. For a sufficiently large Δt inverse discontinuities in absorption were observed for both pulses passing through the resonator after the same time from the start of the pulses, $t_1(h) = t_2(h)$. As Δt decreased, the time t_2 started to decrease. The $t_2(\Delta t)$ relation for two values of h is shown in Fig. 8. When the duration of the first pulse was less than $t_1(h)$, i.e., when the secondary wave could not build up, the effect of the first pulse on t_2 was not observed. It follows from this experiment that the relaxation time of the secondary waves is tens of μs, i.e., almost two orders of magnitude greater than the relaxation time of parametric spin waves.

The properties of above-threshold susceptibility connected with the buildup of secondary waves was observed in parametric excitation of spin waves in YIG.[45] It was shown that such short lived secondary waves in YIG are spin waves with frequency and wave vector corresponding to the "bottom" of the wave band. In YIG such spin waves do not, essentially, interact with the pumping microwave so that they cannot be excited directly, in spite of their smaller relaxation times.

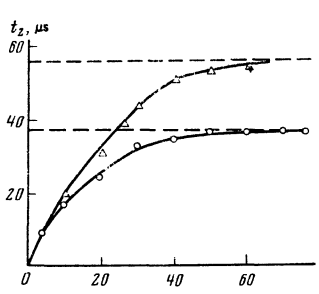

FIG. 8. Dependence of the time from the start of the pulse to the inverse discontinuity on the second pulse, t_2, on the time interval between the pulses Δt, $H = 120$ Oe, $T = 70$ K, ○$-h = h_1$, Δ$-h = h_2$, $h_2 > h_1$.

In our case, according to present ideas, all spin waves, independently of the direction of their wave vector are related in the same way with the pumping field, and since we did not observe a sharp reduction in the threshold field h_{c1} in fields $H \approx H_0$ corresponding to $k \approx 0$, we think it more probable that the secondary waves are sound waves. The following fact also supports this suggestion. When the specimen was only contained in the paper pocket, we observed sinusoidal oscillations on the pulse passing through the resonator starting immediately after the inverse discontinuity. The frequency of these oscillations (~2 MHz) for a specimen with thickness $d = 0.5$ mm was very stable. On attaching the specimen to the bottom of the resonator at one point, the whole picture stayed unchanged but the oscillations disappeared. We consider that these oscillations correspond to excitation of normal modes of elastic vibrations of the specimen which occurs on increasing the amplitude of long wavelength secondary sound waves.

B. Above-threshold oscillations

At helium temperatures and for $(h/h_{c2})^2 \approx 10$ we observed the onset of sinusoidal oscillations in the level of power absorption on the pulse traversing the resonator. These oscillations appear at a given power after the inverse discontinuity and differed from those mentioned in the previous section in that their frequency depended on the magnitude of the microwave field at the specimen. Since these oscillations are a property of the above-threshold state (when the part of the relaxation determining the hard nature of the spin wave excitation has already been cut off), the state of the system should be characterized by the ratio h/h_{c2}. The conditions for the oscillations to arise depended on the directions of the static magnetic field in the basal plane. Figure 9 shows a typical experimental dependence of the square of the oscillation frequency Ω on h/h_{c2}.

Free oscillations of attenuated power beyond the parametric excitation threshold were observed in YIG. A description of this phenomenon and its theory have been given in detail.[46] According to this theory, the dependence of the oscillation frequency on the microwave field at the specimen is given by

$$\Omega^2 = \left| \frac{2T+S}{S} \right| [(h/h_c)^2 - 1] \pi^2 \Delta\nu_k^2, \qquad (14)$$

where T and S are coefficients in the Hamiltonian describing four-magnon interactions. The condition for self-excitation of free oscillations is $(2T+S)/S < 0$.

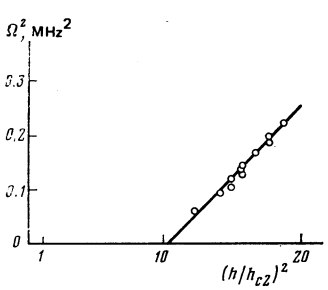

FIG. 9. Dependence of the square of the frequency, Ω, of above threshold oscillations on the square of the v.h.f. field h at the specimen, $H = 240$ Oe, $T = 1.2$ K.

To a first approximation $T = S$ (Ref. 43) in easyplane antiferromagnetics and the conditions for self-excitation are not satisfied.

In spite of the fact that the square of the frequency of the oscillations we observed depends on the square of the microwave field h, as predicted by Eq. (14), we consider that their nature is different from free oscillations. The following fact suggests this: the oscillation frequency Ω does not go to zero for $h = h_{c2}$ and the relaxation frequency $\Delta\nu_{1k}$ determined from Eq. (14) turns out to be 30 times smaller than $\Delta\nu_\infty$.

The authors are very grateful to P.L. Kapitza for his interest in the work, to A.S. Borovik-Romanov for valuable advice, to V.S. Lutovinov and A.I. Smirnov for useful discussions and to V.S. Voronin for help in carrying out the experiments.

[1)]The authors are grateful to L. V. Velikov and Yu. M. Bun'kov for providing the $FeBO_3$ single crystals.
[2)]The authors are grateful to V. S. Lutovinov for informing them of the results of his calculations before their publication.

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Translated by R. Berman