Magnetic susceptibility near a phase transition into the current state

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The magnetic susceptibility is found for a system which is unstable relative to electron-hole coupling in the vicinity of the point of transition to the current state. It is shown that exciton modes may yield a diverging paramagnetic contribution to the susceptibility. A functional for the free energy of the system in a magnetic field is set up. The structure of the functional is such that the system cannot be described in terms of the magnetic moment in a local form. The current state therefore cannot be classified within the framework of ordinary magnetic symmetry groups.

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INTRODUCTION

This is a continuation of an earlier study of systems with spontaneous currents.[1,2] The behavior of these systems was previously investigated[2] in the absence of external magnetic fields, and an expression satisfying the continuity equation was obtained for the spontaneous current. Most useful in the derivation of this expression were requirements that follow from the gauge-invariance condition. We investigate below the behavior of systems with spontaneous current in an external magnetic field, obtain the Landau functional for the free energy in an external field, and calculate the orbital magnetic response of the system near a phase-transition point. It will be shown that the gauge-invariance requirement limits substantially the form of the response function, especially that part of the function which is connected with the collective modes of the exciton or zero-sound type.

1. THE HAMILTONIAN

Just as before[2] the problem of the orbital response of a system with electron-hole instability will be investigated within the framework of a two-band Hamiltonian $\hat{H}$ containing interband transitions:

$$\hat{H}=\sum_n\int d^3\mathbf{r}\psi_n^+(\mathbf{r})\varepsilon_n(\mathbf{p})\psi_n(\mathbf{r})+\sum_{n\neq m}\int d^3\mathbf{r}\psi_n^+(\mathbf{r})\mathbf{P}_{nm}\hat{\mathbf{p}}\psi_m(\mathbf{r})$$
$$+{}^1/_2\sum_{n,m}\int d^3\mathbf{r}_1d^3\mathbf{r}_2\psi_n^+(\mathbf{r}_1)\psi_n(\mathbf{r}_1)V(\mathbf{r}_1-\mathbf{r}_2)\psi_m^+(\mathbf{r}_2)\psi_m(\mathbf{r}_2). \quad (1)$$

Here $m,n=1,2$ are the band indices, m_0 is the mass of the free electron; $\mathbf{P}_{nm}=\pi_{nm}/m_0$; $\pi_{nm}=-\langle n|i\nabla|m\rangle$ is the interband matrix element of the momentum operator $\mathbf{p}=-i\nabla$; $|n\rangle$ and $|m\rangle$ are the Bloch amplitudes at the extremum of the n (m) band;

$$\varepsilon_1(\hat{\mathbf{p}})=-\varepsilon_2(\hat{\mathbf{p}})=\hat{\mathbf{p}}^2/2m^*+E_g/2,$$

where E_g is the width of the forbidden band, and m^* is the effective mass of the electron, assumed for simplicity to be isotropic and the same in both bands, apart from the sign. In the case of a semiconductor $E_g>0$, and for a semimetal $E_g<0$. The electromagnetic field is introduced in such a Hamiltonian in a gauge-invariant manner by replacing the operator $\hat{\mathbf{p}}$ by the extended derivative

$$\hat{\mathbf{p}}\rightarrow\hat{\mathbf{p}}+\frac{|e|}{c}\mathbf{A}(\mathbf{r}),$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential of the field. The form of the current operator $\mathbf{j}$, when the Hamiltonian $\hat{H}$ is so written, is defined in obvious fashion:

$$\hat{j}=-\delta\hat{H}/\delta\mathbf{A}. \quad (2)$$

2. GENERAL RESTRICTION ON THE RESPONSE

Before we proceed to specific model calculations, we must consider the general restrictions imposed by the gauge-invariance of the Hamiltonian (1) on the structure of the orbital magnetic response of the system. For example, from the requirement that there be no reaction of the system to a longitudinal vector-potential

$$\mathbf{A}(\mathbf{r})=\sum_\mathbf{q}\mathbf{A}_\mathbf{q}e^{i\mathbf{q}\mathbf{r}}+\text{c.c.}, \quad (3)$$

(when $\mathbf{A}_\mathbf{q}\parallel\mathbf{q}$) there follow in trivial manner the known[3] generalized sum rules for the oscillator strengths.

Further restrictions on the form of the response $\chi'(\mathbf{q})$ to a transverse vector potential ($\mathbf{A}_\mathbf{q}\perp\mathbf{q}$)

$$\mathbf{j}_\mathbf{q}=q_\perp^2\chi'(\mathbf{q})\mathbf{A}_\mathbf{q} \quad (4)$$

can be obtained by analyzing the diagram series for the current $\mathbf{j}_\mathbf{q}$. It must be ascertained first of all whether collective excitations of the zero-sound type (or of an exciton in the investigated case) make an anomalous pole contribution to the orbital susceptibility, and if they do, of what sign?

It can be stated[4] that a complete irreducible vertex of the zero-sound type has in the mean-field approximation and in the momentum representation the form:

$$\Gamma(\mathbf{k},\mathbf{k}',\mathbf{q})=V(\mathbf{k}-\mathbf{k}')+\sum_n D_n^{-1}(\mathbf{q})\varphi_{n\mathbf{q}}^*(\mathbf{k})\varphi_{n\mathbf{q}}(\mathbf{k}'), \quad (5)$$

where $D_n(\mathbf{q})$ depends explicitly on the small momentum transfer $\mathbf{q}$ and contains all the singularities of the zero-sound type. In the case of a semiconductor system with Hamiltonian (1), the quantity $V(\mathbf{k})$ is the bare interaction potential, and $\varphi_n(\mathbf{k})$ is the wave function of an exciton in the n-th excited state. To take Fermi-liquid effect into account it suffices to replace the bare potential $V(\mathbf{k})$ in all the calculations that follow by a vertex

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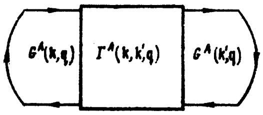
$\Gamma^{(1)}$ that is irreducible in the electron-hole channel[4] and has no singularities of the zero-sound type.

The contribution of the Bose branches of the excitation spectrum of the system to the current $\mathbf{j}_\mathbf{q}$ is obtained by varying the diagram $\Omega(\mathbf{k},\mathbf{k}',\mathbf{q})$ with respect to the vector potential $\mathbf{A}_\mathbf{q}$ (see Fig. 1):

$$\Omega(\mathbf{k},\mathbf{k}',\mathbf{q})=G^A(\mathbf{k},\mathbf{q})\Gamma_A(\mathbf{k},\mathbf{k}',\mathbf{q})G^A(\mathbf{k}',\mathbf{q}),$$

where $G^A(\mathbf{k},\mathbf{q})$ is the complete Green's function of the system in the magnetic field. By virtue of the gauge-invariance of the Hamiltonian (1), the vector potential $\mathbf{A}_\mathbf{q}$ enters in Ω as $\mathbf{q}\to 0$ only in the form



$$G^A(\mathbf{k},\mathbf{q})=G\left(\mathbf{k}+\frac{|e|}{c}\mathbf{A}_\mathbf{q}\right),\quad \Gamma_A(\mathbf{k},\mathbf{k}',\mathbf{q})=\Gamma\left(\mathbf{k}+\frac{|e|}{c}\mathbf{A}_\mathbf{q},\mathbf{k}'+\frac{|e|}{c}\mathbf{A}_\mathbf{q}\right). \tag{6}$$

It follows obviously from (6) that Ω makes no contribution to the current as $\mathbf{A}_\mathbf{q}$ approaches a constant and $\mathbf{q}\to 0$. Indeed, in first-order in the vector potential, the current connected with Γ (5) is

$$\mathbf{j}_\mathbf{q}=-\frac{|e|}{c}\sum_{\mathbf{k},\mathbf{k}'}\left(\frac{\partial}{\partial\mathbf{k}}\mathbf{A}_\mathbf{q}\frac{\partial}{\partial\mathbf{k}'}+\frac{\partial}{\partial\mathbf{k}'}\mathbf{A}_\mathbf{q}\frac{\partial}{\partial\mathbf{k}}\right)\Omega(\mathbf{k},\mathbf{k}',\mathbf{q})=0, \tag{7}$$

because the total derivative is included under the summation sign.

We consider the pole part of (5), which contains the product $\varphi^*_{n\mathbf{q}}(\mathbf{k})\varphi_{n\mathbf{q}}(\mathbf{k}')$. Such a factorized structure of the pole part of the total vertex allows us to make a general statement concerning the sign of the singular contribution of the excitations of the zero-sound type to the response. We note immediately that a singularity remains in the response only when the vector potential is separated from the left and right factors $[G^A(\mathbf{k}',\mathbf{q})\varphi^*_{n\mathbf{q}}(\mathbf{k})$ and $G^A(\mathbf{k}',\mathbf{q})\varphi_{n\mathbf{q}}(\mathbf{k}')$ respectively$]$ simultaneously. Furthermore, a contribution to the response is made only by current terms that are obtained by separating $\mathbf{q}$ likewise from both factors. In the opposite case, when $\mathbf{q}$ is separated from only one of the factors, the other factor remains a total derivative and vanishes upon summation over the momentum, in analogy with (7). Thus, the singular contribution to the current consists of factors, the expansion of each of which in powers of $\mathbf{q}$ begins at least from the term linear in $\mathbf{q}$, while the expression for the current begins with the term quadratic in $\mathbf{q}$. Since the entire product is a squared modulus

$$\left|\sum_\mathbf{k} G(\mathbf{k},\mathbf{q})\varphi_{n\mathbf{q}}(\mathbf{k})\right|^2,$$

the response function should be positive, i.e., paramagnetic. Noting that the momentum transfer enters in the expression for $\Omega(\mathbf{k},\mathbf{k}',\mathbf{q})$ only in the form $\mathbf{k}\pm\mathbf{q}$, $\mathbf{k}'\pm\mathbf{q}$, we can prove the following statement: The singular response to a homogeneous field receives contributions only from zero-sound-type modes that are not symmetrical with respect to the momentum (exciton excitations in states with odd n). The principal symmetrical mode with $n=0$, on the other hand, makes no contribution to the singular response to a homogeneous field.

3. ORBITAL RESPONSE OF AN INTRINSIC SEMICONDUCTOR NEAR THE EXCITON-INSTABILITY POINT

At the temperature $T=0$, a semiconductor with $E_g>0$, described by the Hamiltonian (1), becomes unstable to electron-hole pairing if the width of its forbidden band turns out to be less than the exciton binding energy. This instability can be described within the framework of the approximation of a low-density exciton condensate.[5] To make the equations less cumbersome, it will be assumed from now on that the hybridization parameter $\mathbf{P}\equiv\mathbf{P}_{12}$ in (1) is small, so that a perturbation theory in its terms can be developed, and the only electron-electron interaction left is the interband interaction of the density-density type.

In the absence of an external vector potential ($\mathbf{A}=0$) a dielectric phase transition in such a system is described in standard fashion[6] and is accompanied by the appearance, generally speaking, of an inhomogeneous order parameter Δ:

$$\Delta(\mathbf{r}_1,\mathbf{r}_2)=\sum_\mathbf{k}\Delta(\mathbf{k},\mathbf{R})e^{i\mathbf{k}\mathbf{r}},\quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\quad \mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2.$$

We separate in $\Delta(\mathbf{k},\mathbf{R})$ the symmetrical Δ_s and antisymmetrical Δ_a parts:

$$\Delta(\mathbf{k},\mathbf{R})=\Delta_s(\mathbf{k},\mathbf{R})+\Delta_a(\mathbf{k},\mathbf{R}),\quad \Delta_s(\mathbf{k},\mathbf{R})=\Delta_s(-\mathbf{k},\mathbf{R}),$$
$$\Delta_a(\mathbf{k},\mathbf{R})=-\Delta_a(-\mathbf{k},\mathbf{R})=\Delta_a\mathbf{n}_\mathbf{k}. \tag{8}$$

Here $\mathbf{R}$ is the coordinate of the exciton center, $\Delta(\mathbf{k},\mathbf{R})$ is the Fourier component of the wave function of the exciton with respect to the coordinate of the relative motion of the electron and hole in the pair, and $\mathbf{n}_\mathbf{k}$ is a unit vector in the $\mathbf{k}$ direction.

As shown in Ref. 2, current begins to flow if an inhomogeneous symmetrical imaginary or an antisymmetric real order parameter is produced in the system. If $\Delta(\mathbf{k},\mathbf{R})$ depends weakly on $\mathbf{k}$, the expression for the current becomes

$$\mathbf{j}(\mathbf{R})=\frac{e}{c}\{L\,\mathrm{Re}\,\mathrm{rot}[\mathbf{P}\,\mathrm{grad}\,\Delta_s(\mathbf{R})]+Q\,\mathrm{Im}\,\mathrm{rot}[\mathbf{P}\Delta_a(\mathbf{R})]\} \tag{9}$$

or, changing to Fourier components,

$$\Delta(\mathbf{R})=\sum_\mathbf{q}\Delta_\mathbf{q}e^{i\mathbf{q}\mathbf{R}},$$

we have

$$\mathbf{j}_\mathbf{q}=\frac{e}{c}\{L\,\mathrm{Re}[\mathbf{q}[\mathbf{P}\mathbf{q}]]\Delta_{s\mathbf{q}}+Q\,\mathrm{Im}[\mathbf{q}[\mathbf{P}\Delta_{a\mathbf{q}}]]\}. \tag{9'}$$

The coefficients L and Q were determined earlier[2] and take different forms for a semiconductor and a semimetal. The coefficient Q differs from zero only to the extent that the Fermi surfaces of the electrons and holes are not congruent, for example as a result of doping.

In the nonreconstructed semiconductor phase, owing to the presence of $\mathbf{P}\cdot\mathbf{k}$ hybridization in the Hamiltonian (1), the imaginary antisymmetric component Δ_a of the order parameter differs from zero, and is determined at $\mathbf{A}=0$ from the self-consistency equation, which takes at small $\mathbf{P}$ the form

$$\Delta_a(\mathbf{k})=\int V(\mathbf{k}-\mathbf{k}')\frac{\Delta_a(\mathbf{k}')-\mathbf{P}\mathbf{k}'}{2\varepsilon(\mathbf{k}')}d^3\mathbf{k}'. \tag{10}$$

Turning on the magnetic field changes the order parameter, as a result of which orbital currents can appear in the system. To calculate the orbital response of the system to a static vector potential **A** it is necessary to find, in an approximation linear in **A**, the current **j** induced by the vector potential. The current terms that are linear in the field consist of two groups. The terms of group α determine the current component $\mathbf{j}_\alpha$ which is due to the appearance of the field-induced order parameter Δ^{ind}, and the terms of group β determine that part of the current which would arise only if the field would change the expression for the current (9) with the order parameter remaining unchanged. The corresponding analytical expressions are

$$\mathbf{j}_\alpha=\frac{e}{c}\sum_{\mathbf{k}}\left(-\frac{\partial\varepsilon}{\partial\mathbf{k}}\frac{\mathbf{Pk}-\Delta_0}{4\varepsilon^2}+\frac{\mathbf{P}}{2\varepsilon}\right)\Delta_{\mathbf{q}}^{\mathrm{ind}},$$

$$\mathbf{j}_\beta=\frac{e}{c}\sum_{\mathbf{k}}\left\{-\mathbf{A}_{\mathbf{q}}\frac{\partial}{\partial\mathbf{k}}\left[\frac{\partial\varepsilon}{\partial\mathbf{k}}\frac{(\mathbf{Pk}-\Delta_0)^2}{4\varepsilon^2}-\mathbf{P}\frac{\mathbf{Pk}-\Delta_0}{2\varepsilon}\right]\right\}+\mathbf{j}_\beta', \tag{11}$$

$$\mathbf{j}_\beta'=\frac{e}{c}\sum_{\mathbf{k}}\left\{-\mathbf{A}_{\mathbf{q}}\frac{\partial\Delta_0}{\partial\mathbf{k}}\frac{\partial\varepsilon}{\partial\mathbf{k}}\frac{\mathbf{Pk}-\Delta_0}{2\varepsilon^2}+\mathbf{A}_{\mathbf{q}}\frac{\partial\Delta_0}{\partial\mathbf{k}}\frac{\mathbf{P}}{2\varepsilon}\right\}.$$

The equation for the field-induced order parameter in the approximation linear in **A** is

$$\Delta_{\mathbf{q}}^{\mathrm{ind}}=\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\frac{\Delta_{\mathbf{q}}^{\mathrm{ind}}(\mathbf{k}')}{2\varepsilon(\mathbf{k}')}+\frac{e}{c}\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\left[\frac{\mathbf{PA}_{\mathbf{q}}}{2\varepsilon(\mathbf{k}')}-\left(\mathbf{A}_{\mathbf{q}}\frac{\partial\varepsilon(\mathbf{k}')}{\partial\mathbf{k}'}\right)\frac{\mathbf{Pk}'-\Delta_0}{2\varepsilon^2(\mathbf{k}')}\right]. \tag{12}$$

It is easy to note that the gauge-invariance requirements are satisfied for both (11) and (12). When a gauge field $\mathbf{A}=\mathrm{const}$ is turned on, it follows from (12) that

$$\Delta^{\mathrm{ind}}=-\frac{e}{c}\mathbf{A}\frac{\partial\Delta_0}{\partial\mathbf{k}}, \tag{13}$$

where $\Delta_0=\Delta_a$ is the value of the order parameter at $A=0$ [Eq. (10)]. The result (13) can be obtained also in a simpler manner by noting that the introduction of $\mathbf{A}=\mathrm{const}$ in the Hamiltonian (1) reduces to the replacement $\mathbf{k}\to\mathbf{k}-e\mathbf{A}/c$ in Eq. (10) for Δ_a. Then, according to (10), $\Delta_0(\mathbf{k})$ is replaced by $\Delta_0(\mathbf{k}-e\mathbf{A}/c)$ and Eq. (13) is obtained by simply expanding the quantity $\Delta_0(\mathbf{k}-e\mathbf{A}/c)$ in a series in **A**. Substitution of relation (13) in Eq. (11) for the current leads now to the equality

$$\mathbf{j}=\mathbf{j}_\alpha+\mathbf{j}_\beta=-\left(\frac{e}{c}\right)^2\sum_{\mathbf{k}}\mathbf{A}\nabla_{\mathbf{k}}\left[\nabla_{\mathbf{k}}\varepsilon\frac{(\mathbf{Pk}-\Delta_0)^2}{4\varepsilon^2}-\mathbf{P}\frac{\mathbf{Pk}-\Delta_0}{2\varepsilon}\right]=0, \tag{14}$$

since a total derivative is contained under the summation sign.

With the aid of (11) and (12) we can calculate the orbital susceptibility of an undoped semiconductor at $T=0$ near the dielectric-instability point. Of fundamental interest is the singular (possibly diverging) part of the susceptibility χ'. It is easy to show that a singularity in χ' can be due only to diagrams of group α, which contain the order parameter induced by the magnetic field $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$ The diagrams of group β together with the gauge part (13) of the diagrams of group α are not singular and simply lead to a renormalization of the Landau diamagnetism. It is therefore convenient to represent the induced order parameter Δ^{ind} in the form of a sum of two terms:

$$\Delta_{\mathbf{q}}^{\mathrm{ind}}=-\frac{e}{c}\mathbf{A}_{\mathbf{q}}\nabla_{\mathbf{k}}\Delta_a+\delta_{\mathbf{q}}(\mathbf{k}), \tag{15}$$

where $\delta_{\mathbf{q}}(\mathbf{k})$ is the part proportional to the derivatives of the vector potential (i.e., to the real field **B**), and the first term is due to the gauge requirement (13). Obviously, a singular contribution to χ' can be due only to $\delta_{\mathbf{q}}(\mathbf{k})$. Substituting (15) in (12) we can obtain, accurate to terms of order q^2, the following equation for $\delta_{\mathbf{q}}$:

$$\delta_{\mathbf{q}}(\mathbf{k})=\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\frac{\delta_{\mathbf{q}}(\mathbf{k}')}{\varepsilon_{\mathbf{k}'+\mathbf{q}/2}+\varepsilon_{\mathbf{k}'-\mathbf{q}/2}}-D_{\mathbf{q}}(\mathbf{k}), \tag{16}$$

$$D_{\mathbf{q}}(\mathbf{k})=\frac{e}{c}\left[\frac{\mathbf{q}\times[\mathbf{P}\times\mathbf{q}]}{4m^*}\right]\mathbf{A}_{\mathbf{q}}\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\frac{1}{4\varepsilon^2(\mathbf{k}')}. \tag{17}$$

The function $D_{\mathbf{q}}$ is a source for $\delta_{\mathbf{q}}$ in Eq. (16). It follows therefore from (16) that an inhomogeneous magnetic field induces in an intrinsic semiconductor only a symmetrical [in the sense of (8)] order parameter. It follows from (16) that, in coordinate form, $\delta\sim\mathrm{curl}\,\mathbf{B}$, therefore there is no anomalous response of the system to a uniform magnetic field $\mathbf{B}=\mathrm{const}$ near the exciton-instability point.

A solution of the equation with the source (16) can be obtained in explicit form with the aid of the Green's function of the homogeneous equation (16) at $D_{\mathbf{q}}=0$. It is convenient to make in (16) the substitution

$$(\varepsilon_{\mathbf{k}-\mathbf{q}/2}+\varepsilon_{\mathbf{k}+\mathbf{q}/2})\psi=\delta_{\mathbf{q}}(\mathbf{k}).$$

It follows then from (16) that

$$\frac{k^2}{2\mu}\psi(\mathbf{k})-\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\psi(\mathbf{k}')+\left(E_g+\frac{q^2}{2M}\right)\psi(\mathbf{k})=-D_{\mathbf{q}}(\mathbf{k}),$$

$$\mu=m^*/2,\quad M=2m^*. \tag{18}$$

The Green's function of the homogeneous equation corresponding to (18) can be easily obtained:

$$G=\sum_n\frac{\varphi_n^*(\mathbf{k})\varphi_n(\mathbf{k}')}{E_n+E_g+q^2/2M}, \tag{19}$$

where the functions φ_n are given by the wave eigenfunctions of the exciton and are determined from the Schrödinger equation

$$\frac{k^2}{2\mu}\varphi_n(\mathbf{k})-\sum_{\mathbf{k}'}V(\mathbf{k}-\mathbf{k}')\varphi_n(\mathbf{k}')=E_n\varphi_n(\mathbf{k}). \tag{20}$$

As a result of solution of the inhomogeneous equation (16), we can write (18) in the form

$$\psi_{\mathbf{q}}(\mathbf{k})=\frac{\delta_{\mathbf{q}}(\mathbf{k})}{\varepsilon_{\mathbf{k}-\mathbf{q}/2}+\varepsilon_{\mathbf{k}+\mathbf{q}/2}}=-\int\sum_n\frac{\varphi_n^*(\mathbf{k})\varphi_n(\mathbf{k}')}{E_n+E_g+q^2/2M}D_{\mathbf{q}}(\mathbf{k}')d^3\mathbf{k}'. \tag{21}$$

It is seen from (21) that as the exciton-instability point is approached, the exciton ground-stage energy E_0 approaches the width of the forbidden band, and the value of $\psi_{\mathbf{q}}(\mathbf{k})$, and with it also the parameter $\delta_{\mathbf{q}}(\mathbf{k})$ induced by the magnetic field tends to infinity. Recalling the expression (9) for the current connected with the inhomogeneous imaginary order parameter, and substituting in it the value of $\delta_q(\mathbf{k})$, we find from (21) that the current $\mathbf{j}_q^{ind}$ induced by the magnetic field $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$ is equal to

$$\mathbf{j}_q^{ind}=\left(\frac{e}{c}\right)^2\sum_{\mathbf{k},\mathbf{k}'}\frac{[\mathbf{q}\times[\mathbf{P}\times\mathbf{q}]]}{4\varepsilon^2(\mathbf{k})}\Big\{V(\mathbf{k}-\mathbf{k}')$$

$$+\sum_n\frac{(\mathbf{k}^2/2\mu-E_n)(\mathbf{k}'^2/2\mu-E_n)}{E_n+E_g+q^2/2M}\varphi_n^*(\mathbf{k})\varphi_n(\mathbf{k}')\Big\}\frac{[\mathbf{q}\times[\mathbf{P}\times\mathbf{q}]]}{4\varepsilon^2(\mathbf{k}')}\frac{1}{(4m^*)^2}\mathbf{A}_q. \quad (22)$$

Consequently the susceptibility of the system (its singular part) takes the form

$$\chi_q'=\left(\frac{e}{c}\right)^2[\mathbf{P}^2\mathbf{q}^2-(\mathbf{P}\mathbf{q})^2]\sum_{\mathbf{k},\mathbf{k}',n}\frac{\tilde\varphi_n^*(\mathbf{k})\tilde\varphi_n(\mathbf{k}')}{E_n+E_g+q^2/2M}[4\varepsilon(\mathbf{k})\varepsilon(\mathbf{k}')]^{-2}\frac{1}{(4m^*)^2},$$
$$\tilde\varphi_n(\mathbf{k})=\varphi_n(\mathbf{k})(k^2/2\mu-E_n). \quad (23)$$

It follows therefore from (23) that, in accordance with the general ideas advanced at the beginning of the present article, the singular contribution from the collective modes to the orbital susceptibility is paramagnetic. It must be emphasized that the pole singularity of χ', investigated in this article and due to the possible instability of the collective modes $E_n+E_g+q^2/2M\to 0$, have no bearing whatever on the singular contribution made to χ' by the single-particle excitations,[7] whose spectrum $\varepsilon(\mathbf{k})$ is determined essentially by the spin-orbit interaction. In an undoped semiconductor, according to (23), only the response to an inhomogeneous ($\mathrm{curl}\,\mathbf{B}\neq 0$) field has a paramagnetic divergence (i.e., $\chi_q'\sim q^2$). This distinguishes in principle a system with a spontaneous current from ordinary magnets. One can speak in fact of the divergence of a response of the system to a homogeneous current if the system is placed in a superconducting solenoid with a fixed magnetic flux (i.e., what is specified is $\mathbf{B}$ rather than $\mathbf{H}$).

4. FREE-ENERGY FUNCTIONAL

To demonstrate that a system with a spontaneous current differs from ordinary magnetic systems it is necessary to construct the free-energy functional. This will be done here for a semimetal model [Eq. (1) at $E_g<0$], possibly a doped one. The use of the semimetal model makes it possible to show that the results obtained in the preceding section for the semiconductor model are not applicable to that model only, as well as to investigate the system near the transition temperature.

A free-energy functional for the semimetal model was obtained earlier[2,8] in the absence of external magnetic fields. A corresponding phase diagram was plotted in the $T\mu$ plane, where μ is the chemical potential connected with the doping, and the region where an inhomogeneous current state exists was found. The free energy of the magnetic field is incorporated in the usual manner in the functional F, accurate to terms linear in $\mathbf{A}$, by averaging the operator of the interaction of the electrons with the field in (1) over the thermodynamic Green's functions that contain an arbitrary order parameter $\Delta(r)$ [Eq. (8)]. It turns out as a result that

$$F(\Delta)=F_0(\Delta)-\frac{e}{c}L[\mathbf{P}\,\mathrm{grad}\,\Delta_{\mathrm{Im}}^s]\,\mathrm{rot}\,\mathbf{A}-\frac{e}{c}Q[\mathbf{P}\Delta_{\mathrm{Re}}^a]\,\mathrm{rot}\,\mathbf{A}, \quad (24)$$

$$L=-\frac{1}{6}\frac{k_F^3}{m^*}\frac{T}{\pi}\sum_{n>0}\frac{\omega_n(\omega_n^2-3\mu^2)}{(\omega_n^2+\mu^2)^3},\quad Q=\frac{2}{3}\frac{k_F^3T}{\pi}\sum_{n>0}\frac{\mu\omega_n}{(\omega_n^2+\mu^2)^2}, \quad (25)$$

where $F_0(\Delta)$ is the Landau functional of the system in the absence of a field.

In what follows it suffices to choose $F_0(\Delta)$ in the form

$$F_0(\Delta)=\alpha^s(\Delta_{\mathrm{Im}}^s)^2+\alpha^a(\Delta_{\mathrm{Re}}^a)^2+\beta[(\Delta_{\mathrm{Im}}^s)^2+(\Delta_{\mathrm{Re}}^a)^2]^2$$
$$+\gamma[(\mathrm{grad}\,\Delta_{\mathrm{Im}}^s)^2+(\mathrm{div}\,\Delta_{\mathrm{Re}}^a)^2]+\gamma_1\{[\mathrm{div}\,\mathrm{grad}\,\Delta_{\mathrm{Im}}^s]^2+[\mathrm{grad}\,\mathrm{div}\,\Delta_{\mathrm{Re}}^a]^2\}, \quad (26)$$

where the expressions for the coefficients α, β, γ, and γ_1 should be taken, for example, from the preceding paper.[8]

As expected, variation of the functional (24) with respect to the vector potential leads to a correct [cf. Eq. (9)] expression for the current, and variation of (24) at a specified field $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$ makes it possible to obtain equations for the order parameter Δ^{ind} induced by it. The result is that in the semimetal model

$$\Delta_s^{ind}=D_q^s/\alpha^s,\quad \Delta_a^{ind}=\mathbf{D}_q^a/\alpha^a;$$
$$D_q^s=-\frac{e}{c}L[q^2(\mathbf{P}\mathbf{A}_q)-(\mathbf{P}\mathbf{q})(\mathbf{q}\mathbf{A}_q)],$$
$$\mathbf{D}_q^a=-\frac{e}{c}Q[\mathbf{q}(\mathbf{P}\mathbf{A}_q)-(\mathbf{P}\mathbf{q})\mathbf{A}_q]. \quad (27)$$

It follows from (4), (9), and (27) that at $T>T_c$ the response of the system to the total field $\mathbf{B}_q$ with wave vector $\mathbf{q}$ is

$$\chi'=\chi'^s+\chi'^a=\frac{e^2|\mathbf{P}|^2L^2}{c^2}\frac{q^2}{\alpha^s}+\frac{e^2|\mathbf{P}|^2Q^2}{c^2\alpha^a}. \quad (28)$$

The first term in (28) is completely equivalent to the result (23) for the response obtained in the semiconductor model without doping. The second term differs from zero only in the case of doping ($\mu\neq 0$), is due to the real antisymmetrical component Δ_{Re}^a induced in this case, and is paramagnetic. It should also be noted that this last term in χ' describes the response of the system to a uniform magnetic field $\mathbf{B}=\mathrm{const}$.

When the phase-transition point is approached from above, depending on the sign of γ, either a homogeneous ($\gamma>0$) or an inhomogeneous ($\gamma<0$) phase can be produced. It is easy to verify that if an inhomogeneous phase is produced, the true divergence of the susceptibility will arise only for a magnetic field whose wave vector coincides with the wave vector of the produced phase. At the transition point itself, it is possible to obtain for the denominators in (27) the expression

$$\alpha^{s,a}(T_c)=\gamma_1(q^2-q_0^2)^2,$$

where q_0 is the characteristic vector of the order parameter produced at the transition point, and $\mathbf{q}$ is the wave vector of the field. Below the phase-transition point the temperature behavior of the susceptibility obeys the law characteristic of the self-consistent field approximation.

Finally, attention must be called to the already noted difference between a system with spontaneous current, where the order parameter is the density Δ of the exciton condensate, and magnetic systems in which the order parameter is the local magnetization $\mathbf{M}$. The point is that owing to the relation (9) between the spontaneous current and the order parameter, of the form $\mathbf{j}\sim\mathrm{curl}\,\mathrm{curl}\,\Delta$, it is impossible in the general case to go over in the functional F (24) from the variable Δ to the variable $\mathbf{M}$ and retain the local structure of the functional. In this sense, the symmetry classification of the current state does not coincide with the classification of magnets.

CONCLUSION

It has thus been shown in the present paper that the collective excitations of the exciton type (or of the zero-sound type in a Fermi liquid) cannot lead to anomalies of the diamagnetic sign in the susceptibility near the point of their instability, and only a divergence of the paramagnetic sign in possible. It has been observed that this susceptibility divergence takes place in systems that are unstable to a transition into a state with spontaneous current. It has been noted that the singular part of the susceptibility corresponds only to the response to an inhomogeneous field **B** (curl **B** $\neq 0$). Therefore no divergences of χ' should be observed in a homogeneous field. The structure of the free-energy functional is such that the description of systems with spontaneous currents with the aid of the ordinary magnetic order parameter is impossible in the general case.

It should be noted, finally, that the demonstrated absence of anomalous diamagnetic contribution from the collective modes still does not mean that anomalies of this type are impossible inside the region of the reconstructed phase, since their analysis calls for taking into account dynamic effects in the spirit of the preceding paper.[9]

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