# Diffraction of the surface polaritons by an impedance step in the region of resonance with oscillations in a transition layer

V. M. Agranovich, V. E. Kravtsov, and T. A. Leskova

*Spectroscopy Institute, USSR Academy of Sciences*
(Submitted 20 April 1981)
Zh. Eksp. Teor. Fiz. **81**, 1828–1838 (November 1981)

A theory is developed of diffraction of surface polaritons (SP) by the impedance step formed on depositing on a metal a film of thickness $d \ll \lambda = 2\pi c/\omega$ ($\omega$ is the surface-polariton frequency) in the region of existence of an additional surface wave. In the case under consideration the additional surface wave is a result of the fact that the impedance boundary condition for a metal covered by a film contains a second derivative and is of the form
where $Z_0$ is the impedance of the surface of the pure metal, $\mu = d/\varepsilon_1(\omega)$ and $\varepsilon_1(\omega)$ is the film dielectric constant and vanishes in the region of resonance with the oscillations in the transition layer (film). The amplitudes of the reflected and refracted surface waves, and also the angular distribution of the intensity of the volume radiation produced, are found by applying a factorization technique. An anomalously high efficiency of tranformation of the surface wave energy into the energy of volume radiation near the resonance with oscillations in the transition layer is predicted. Some possibilities of employing the diffraction of SP by impedance steps for experimental investigation of the optical properties of surfaces and thin films are discussed.

PACS numbers: 71.36. + c, 73.90. + f

## 1. INTRODUCTION

Surface electromagnetic waves [surface polaritons (SP)] of the optical band are being more frequently employed at present for the investigation of the optical properties of surfaces and of thin films. The reason is that the SP dispersion is not only determined by the dielectric constants of the contiguous media, but contains information on the properties of the transition layer. The transition layer exerts a particularly strong influence when the frequency $\omega_0$ of the dipole oscillations in the transition layer (or in a thin film deposited on a substrate) lands in the region in which the SP frequencies are restructured. In this case, as shown in Ref. 1, a gap $\Delta$ having a width of the order of $\omega_0(d/\lambda)^{1/2}$ is produced in the SP frequency spectrum ($d$ is the transition-layer thickness and $\lambda = 2\pi c/\omega_0$). An effect of this kind and, in particular, the square-root dependence of $\Delta(d)$, were observed both in the IR band[2] and in the region of the electronic-excitation spectrum.[3]

Recently one of us[1] has shown that at resonance with the oscillations, an additional surface wave is produced in the transition layer besides the gap. In a certain sense, the appearance of the additional surface wave can be regarded as an effect of spatial dispersion in terms of the parameter $kd$, where $k$ is the (two-dimensional) wave vector of the SP. It is known that in three-dimensional crystal optics allowance for the additional wave leads, in particular, to a substantial modification of the Fresnel formulas for the coefficients of the reflection and transmission of radiation through a crystal boundary.[5] It turns out that a similar situation arises also in the problem of reflection and refraction of surface waves by surface-separation lines. An essential feature of the crystal optics of SP is the possibility of transformation of the surface waves into volume ratiation, so that the mathematical description of the problem becomes much more complicated.

In the absence of additional surface waves this problem (that of diffraction of SP by an impedance step) was considered earlier (see, e.g., Ref. 6). It is precisely the theory of the diffraction of SP by an impedance step, with allowance for the additional surface waves, which is the subject of the present article.

To solve the problem we use, just as in Ref. 6, a factorization method and impedance boundary conditions. In our case, however, these boundary conditions contain derivatives of the electric field $E_z$ along the surface. These derivatives play a role similar to that of the spatial-dispersion terms in volume crystal optics. We recall now that in volume crystal optics, when account is taken of spatial dispersion in the region where additional waves exist, it becomes necessary to use the so-called supplementary boundary conditions (SBC). The SBC are necessary here because in most cases the nonlocal dielectric tensor of the medium can be regarded as known only for those points of the medium which are far enough from the boundary. On the other hand, in these very cases (for certain models see Ref. 5), when the tensor $\varepsilon(\mathbf{r}, \mathbf{r}')$ can be obtained for arbitrary points $\mathbf{r}$ and $\mathbf{r}'$, there is no need for the SBC, since the entire information needed to determine the field amplitude is contained already in $\varepsilon(\mathbf{r}, \mathbf{r}')$. In our case, the analog of the material relation is the impedance boundary condition, which is likewise generally speaking unknown in the region of the impedance step for an arbitrary transition layer. The terms with first derivatives of $E$ can be written in the following general form:

$$\mu_1 \frac{\partial E_z}{\partial x} + \frac{\partial}{\partial x}(\mu_2 E_z),$$

 © 1982 American Institute of Physics 

where $\mu_1$ and $\mu_2$ are functions of $x$ such that $\mu_1 + \mu_2 = \mu\theta(x)$ as $|x| \to \infty$. In a certain sense, the problem of choosing the functions $\mu_1$ and $\mu_2$ is similar to that arising in gyrotropy theory.[7] It appears that, just as in Ref. 7, it can be solved on the basis of certain general principles, for example the principle of symmetry of the kinetic coefficients.

In the present article, however, we confine ourselves to a transition-layer model in the form of a thin film having a dielectric constant that is inhomogeneous over the surface. Within the framework of this model it can be shown that $\mu_1 = 0$, and the dependence of $\mu_2$ on the parameters of the film can be found. Thus, the impedance boundary condition turns out to be fully specified, and it is therefore perfectly natural that no supplementary boundary conditions are needed. The problem, as shown below, was reduced to a solution of an integral equation with a kernel that depends on the manner in which the dielectric constant $\varepsilon_1(x)$ varies along the film. We obtain the solution of this equation by a factorization method in the case when $\varepsilon_1(x)$ takes the form of a step. This method makes it possible to find the amplitudes of the reflected and transmitted surface waves, as well as the angular distribution of the radiation diffracted into the vacuum for an arbitrary complex dielectric constant $\varepsilon_1(\omega)$ of the film. The result, however, can be expressed in terms of elementary functions only in the case of weak damping. This is precisely the case considered in detail in the present article.

## 2. GENERAL FORM OF THE SOLUTION BY THE FACTORIZATION METHOD

Let a surface wave of the form

$$H_x = H_z = 0, \quad H_y = H_{00}\exp(ik_0 x - \varkappa_0 z),$$

be incident on the impedance step from the metal-surface side that is not covered by the film. Here $H_\alpha$ is the magnetic field of the SP in vacuum, $k_0 = (\omega^2/c^2 + \varkappa_0^2)^{1/2}$; $\varkappa_0 = \omega/c(-\varepsilon)^{-1/2}$; $\operatorname{Im} k^0 > 0$, $\operatorname{Re}\varkappa_0 > 0$, $\varepsilon$ is the dielectric constant of the metal. We seek the diffracted field in the vacuum in the form:

$$H_x^{(s)} = H_z^{(s)} = 0,$$

$$H_y^{(s)} = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} \exp[-iwx - V(w)z]\frac{F(w)}{V(w) - \varkappa_0}. \tag{1}$$

$$V(w) = (w^2 - \omega^2/c^2)^{1/2}.$$

It is easy to verify that $H^{(s)}$, defined in accordance with (1), satisfies the Maxwell wave equation. We require also that the total field $H$ satisfy on the boundary $z = 0$ the impedance boundary condition

$$\frac{\partial H}{\partial z} + \varkappa_0 H + \frac{\partial}{\partial x}\left[\mu(x)\frac{\partial H}{\partial x}\right] = 0, \quad \mu = d(\varepsilon_1^{-1}(x) - 1). \tag{2}$$

Substituting (1) in (2) and taking the inverse Fourier transform, we obtain the following integral equation for $F(w)$

$$F(w) + \int_{-\infty}^{+\infty} \frac{dw'}{2\pi} F(w') \frac{w' w \tilde{\mu}(w - w')}{V(w') - \varkappa_0} = H_{00} k_0 w \tilde{\mu}(w + k_0),$$

where $\tilde{\mu}(w)$ is the Fourier component of $\mu(x)$; alternately, introducing the function $\Phi(w) = (k_0/w)F(w)$, we have

$$\Phi(w) + \int_{-\infty}^{+\infty} \frac{dw'}{2\pi} \Phi(w') \frac{(w')^2 \tilde{\mu}(w - w')}{V(w') - \varkappa_0} = -H_{00} k_0^2 \tilde{\mu}(w + k_0). \tag{3}$$

If $\mu(x)$ is a step function $\mu(x) = \mu\theta(x)$, using the well-known relation

$$\tilde{\mu}(w) = \frac{i\mu}{w + i\delta} \quad (\delta \to +0),$$

we get from (3)

$$\Phi(w) + \frac{i\mu k_0^2 H_{00}}{w + k_0} = i\mu \int_{-\infty}^{+\infty} \frac{dw'}{2\pi} \Phi(w') \cdot \frac{(w')^2}{[V(w') - \varkappa_0](w' - w - i\delta)}. \tag{4}$$

We put

$$\frac{\mu w^2}{V(w) - \varkappa_0} = \Psi(w) - 1. \tag{5}$$

The function $\Psi(w)$ can be represented as a function of the complex variable $w$ (see, e.g., Ref. 6) by a product of two functions:

$$\Psi(w) = \Psi_+(w)\Psi_-(w),$$

such that $\Psi_+(w)$ is analytic and has no zeroes in the upper complex $w$ plane, and $\Psi_-(w)$ has the same properties in the lower plane. It is easy to verify that the function

$$\Phi(w) = -\frac{i\mu k_0^2 H_{00}}{\Psi_-(-k_0)\Psi_+(w)(w + k_0)} \tag{6}$$

is a solution of (4).

It can be shown in perfect analogy that if a SP of the form

$$H_x = H_z = 0, \quad H_y = H_{0i}\exp(ik_i x - \varkappa_i z),$$

$$\operatorname{Im} k_i < 0, \quad \operatorname{Re}\varkappa_i > 0 \quad (i = 1, 2),$$

where $k_i^2 = \omega^2/c^2 + \varkappa_i^2$ and $\varkappa_i$ satisfies the dispersion equation

$$\mu\varkappa^2 + \varkappa + \mu\omega^2/c^2 - \varkappa_0 = 0 \tag{7}$$

is incident from the side $x > 0$ covered by the film, then the field of the diffracted radiation can be represented in the form

$$H_x^{(s)} = H_z^{(s)} = 0,$$

$$H_y^{(s)} = \int_{-\infty}^{+\infty} \frac{dw}{2\pi} \exp(iwx - V(w)z) \frac{F^{(i)}(w)}{\mu(V(w) - \varkappa_1)(V(w) - \varkappa_2)}, \tag{8}$$

where

$$F^{(i)}(w) = i\frac{\mu k_i H_{0i}\Psi_-(k_i) w \Psi_+(w)}{w - k_i}. \tag{9}$$

## 3. EXPRESSIONS FOR THE SP REFLECTION AND TRANSMISSION COEFFICIENTS, AND THE ANGULAR DISTRIBUTION OF RADIATION DIFFRACTED INTO VACUUM

Let $z = 0$ and $x < 0$ (the metal surface is free of film). Then the integration contour in (1) can be closed through the upper half-plane. The integral of (1) is in this case the sum of the pole contribution and of an integral along an upper-half-plane cut that starts at the branch point $w = \omega/c + i\delta$. It can be shown that at large distances $|x| \gg \varkappa_0^{-2}\omega/c$ the integral along the cut decreases like $|x|^{-1/2}\exp(i\omega|x|/c)$. Thus, it yields an expression for volume radiation produced upon diffraction and gliding along the surface. The pole contribution, on the other hand, is of the form

$$H^{(s)}_{y(pol)}=iF(k_0)k_0^{-1}\varkappa_0\exp(-ik_0x) \quad (10)$$

and obviously describes the reflected surface wave, which we designate $H_0\exp(-ik_0x)$. Taking (6) into account, we then have the following expression for the reflection coefficient:

$$H_0/H_{00}=-\varkappa_0\mu/2\Psi_-(-k_0)\Psi_+(k_0). \quad (11)$$

To find the amplitudes of the transmitted surface waves, we must transform Eq. (1) somewhat. We note that it follows from (7) and (5) that

$$\Psi(w)=\frac{\mu(V(w)-\varkappa_1)(V(w)-\varkappa_2)}{(V(w)-\varkappa_0)},$$

where $\varkappa_{1,2}$ are the solutions of the dispersion Eq. (7). Taking also (6) into account, we transform (1) into

$$\frac{H_y^{(s)}}{H_{00}}=\int_{-\infty}^{+\infty}\frac{dw}{2\pi}\exp(-iwx-V(w)z)\frac{ik_0w\Psi_-(w)}{\Psi_-(-k_0)(w+k_0)(V(w)-\varkappa_1)(V(w)-\varkappa_2)}. \quad (12)$$

At $z=0$ and $x>0$ the integration contour in (12) can be closed through the lower $w$ half-plane. The pole contributions then turn out to be

$$\frac{H^{(s)}_{y(pol)}}{H_{00}}=-\frac{k_0^2\exp(ik_0x)}{(\varkappa_0-\varkappa_1)(\varkappa_0-\varkappa_2)}+\frac{H_1}{H_{00}}\exp(-ik_1x)+\frac{H_2}{H_{00}}\exp(-ik_2x), \quad (13)$$

where the coefficients are given by

$$H_1/H_{00}=k_0\varkappa_1\Psi_-(k_1)/\Psi_-(-k_0)(k_1+k_0)(\varkappa_1-\varkappa_2),$$
$$H_2/H_{00}=k_0\varkappa_2\Psi_-(k_2)/\Psi_-(-k_0)(k_2+k_0)(\varkappa_2-\varkappa_1). \quad (14)$$

It is easy to show that $k_0^2=(\varkappa_0-\varkappa_1)(\varkappa_0-\varkappa_2)$, so that the first of the pole terms cancels out the incident surface wave. The two other terms in (13) describe the normal and additional surface waves. We call attention to the fact that $\mathrm{Re}\,k_1<0$, but $\mathrm{Re}\,k_2>0$. Therefore the energy flux of the normal wave in vacuum, flows in the positive $x$ direction, away from the step. At the same time, the energy flux carried in the vacuum by the additional wave flows in the opposite direction. The reason is that the bulk of the energy flux carried by the additional surface wave is concentrated in the film, and at $\varepsilon_1<0$ the energy fluxes in the film and in the vacuum are oppositely directed in the region where the additional wave exists.

Let now $z$ be large enough, namely

$$z\gg\max(\varkappa_0^{-1},\varkappa_1^{-1},\varkappa_2^{-1}).$$

Calculation of the integral (1) by the saddle-point method yields the following dependence of the amplitude of the volume radiation on the observation angle $\theta$ ($\tan\theta=-x/z$):

$$\frac{H^{(v)}}{H_{00}}=-\frac{\exp(i\omega r/c+i\pi/4)}{(2\pi r\omega/c)^{1/2}}\times\frac{\mu k_0\omega^2c^{-2}\sin\theta\cos\theta}{\Psi_-(-k_0)\Psi_+(\omega c^{-1}\sin\theta)(k_0+\omega c^{-1}\sin\theta)(\varkappa_0+i\omega c^{-1}\cos\theta)}. \quad (15)$$

Similarly, using (8) and (9) we can obtain expressions for the SP reflection and transmission coefficients and for the angular distribution of radiation diffracted into vacuum when a surface wave is incident on the impedance film from the side $x>0$ which is covered by the film. Denoting the amplitudes of the surface waves at $z=x=0$ by $H_0$, $H_1$, and $H_2$ respectively, and the amplitude of the cylindrical wave in vacuum by $H^{(v)}$, we have the following expressions for the ratios of these amplitudes to the amplitude of the incident surface wave ($i=1,2$):

$$\frac{H_0}{H_{0i}}=\frac{\varkappa_0(k_i-k_0)}{(\varkappa_i+\varkappa_0)k_i}\frac{\Psi_-(k_i)}{\Psi_-(-k_0)},\quad \frac{H_i}{H_{0i}}=-\frac{\varkappa_i}{2(\varkappa_j-\varkappa_i)}\Psi_-(k_i)\Psi_+(-k_i),$$
$$\frac{H_i}{H_{0j}}=-\frac{\varkappa_ik_j}{(\varkappa_i-\varkappa_j)(k_i+k_j)}\Psi_-(k_j)\Psi_+(-k_i)\quad(i\neq j), \quad (16)$$
$$\frac{H^{(v)}}{H_{0j}}=\frac{\exp(ir\omega/c+i\pi/4)}{(2\pi r\omega/c)^{1/2}}\times\frac{k_j\omega^2c^{-2}\sin\theta\cos\theta\Psi_-(k_j)\Psi_+(-\omega c^{-1}\sin\theta)}{(k_j+\omega c^{-1}\sin\theta)(\varkappa_1+i\omega c^{-1}\cos\theta)(\varkappa_2+i\omega c^{-1}\cos\theta)}.$$

## 4. ENERGY FLUXES CARRIED BY THE SURFACE POLARITON. ENERGY CONSERVATION LAW

The energy flux carried by the SP is obviously given by

$$W=L_y\int_{-\infty}^{+\infty}S_x(z)\,dz,$$

where $S_x$ is the energy flux density and $L_y$ is the length of the SP excitation region. The energy flux density is determined here by the relation (see, e.g., Ref. 5) $S_x=v_{gr}u(z)$, where $v_{gr}=\partial\omega(k)/\partial k$ is the group velocity of the SP and $u(z)$ is the energy density given by

$$u=\frac{1}{16\pi}\left[\frac{\partial}{\partial\omega}(\omega\varepsilon(\omega))|E|^2+|H|^2\right].$$

The group velocity of the SP can be easily found by differentiating the dispersion equation with respect to $\omega$. Near the resonance, when $|\omega-\omega_0|\ll\omega_0$, the main contribution to $v_{gr}$ is made by the derivative $\partial\mu/\partial\omega=-\mu(\omega-\omega_0)^{-1}$. In this case the expression for the group velocity takes the form

$$v_{gr}^{(i)}=(\varkappa_2-\varkappa_1)|\omega-\omega_0|/\varkappa_i|k_i|,$$

where the index $i=1,2$ corresponds to the normal or to the additional surface wave. The energy density in the film is very high under the indicated conditions, so that

$$\int_{-\infty}^{+\infty}u(z)\,dz\approx U_fd=\frac{1}{16\pi}|H_y|^2\frac{k_i^2c^2\mu}{\omega|\omega-\omega_0|},$$

where $U_f$ is the energy density in the film. Thus, the expression for the energy flux carried by the surface wave is of the form

$$W_i=L_y\frac{c^2}{16\pi\omega}|H_y|^2\frac{|k_i|}{\varkappa_i}\frac{\varkappa_2-\varkappa_1}{\varkappa_2+\varkappa_1}, \quad (17)$$

where $H_y$ is the amplitude of the magnetic field at $z=0$. We note that the same Eq. (17) is obtained if $\mathbf{S}$ is replaced by the usual expression for the Poynting vector $\mathbf{S}=(c/4\pi)\mathbf{E}\times\mathbf{H}$. It is easily seen that as $\mu\to0$, when $\varkappa_2\gg\varkappa_1$, $|k_1|\to k_0$, and $\varkappa_1\to\varkappa_0$ Eq. (17) goes over into the expression for the energy flux of an SP on a clean metal surface, neglecting the penetration of the field into the metal:

$$W_0=L_y\frac{c^2}{16\pi\omega}|H_y|^2\frac{k_0}{\varkappa_0}. \tag{18}$$

To obtain equations for the ratios of the energy fluxes of the reflected and transmitted surface waves to the energy flux of the incident wave, it is necessary to know the expressions for the squared moduli $|H_y|^2$ of the corresponding fields. Equations (11) and (14) as well as (16) make it possible to obtain all the information on the character of the transformation of the surface wave into other surface waves and into volume radiation upon diffraction by an impedance step. However, the functions $\Psi_+$ and $\Psi_-$ in the equations indicated above are expressed in the form of contour integrals which do not reduce generally speaking to any simple elementary or special functions. Nonetheless, in the case of weak damping of the surface waves it is possible to express the moduli $|\Psi_1|$ and $|\Psi_-|$ relatively simply in terms of $\varkappa_i$ and $k_i$. The expressions for the squared moduli of the fields $|H_y|^2$ therefore take likewise a simple form if the damping of the surface waves is neglected. We present here the equations for the energy flux conversion coefficients for precisely this case.

After substituting the squared moduli of the fields $|H_y|^2$ in the expressions for the energy fluxes (17) and (18), we obtain the following equations for the conversion coefficients of the SP energy fluxes:

$$\begin{gathered}
\frac{W_1}{W_{02}}=\frac{W_2}{W_{01}}=4\frac{\varkappa_1}{\varkappa_2}\frac{(k_2+k_0)k_1^2}{(|k_1|+k_0)(k_2+|k_1|)^2},\\
\frac{W_0}{W_{01}}=\frac{W_1}{W_{00}}=4\varkappa_0\varkappa_1\frac{k_1^2}{k_0^2}\frac{(k_2+k_0)^3(k_2-|k_1|)}{(\varkappa_1+\varkappa_0)^2(\varkappa_2+\varkappa_0)^2(k_2+|k_1|)^2},\\
\frac{W_0}{W_{02}}=\frac{W_2}{W_{00}}=4\frac{\varkappa_0}{\varkappa_2}\frac{k_0^2(k_2-|k_1|)}{(k_2-k_0)^2(k_0+|k_1|)},\\
\frac{W_0}{W_{00}}=\frac{\varkappa_0^2k_0^2(k_0+k_2)^2}{(k_0+|k_1|)^2(k_2-k_0)^4},\qquad \frac{W_1}{W_{01}}=\frac{\varkappa_1^2k_1^2(k_2-|k_1|)^2}{(k_0+|k_1|)^2(k_2+|k_1|)^4},\\
\frac{W_2}{W_{02}}=\frac{(k_2+k_0)^2(k_2-|k_1|)^2}{\varkappa_2^2k_2^2},
\end{gathered} \tag{19}$$

where the index 0 corresponds to the SP on the metal surface free of film, the indices 1 and 2 correspond to the normal and additional waves in the presence of the film, and $W_{0i}$ $(i=0,1,2)$ is the energy flux of the surface wave incident on the impedance step. We take particular notice of the fact that the SP energy-flux conversion-coefficient matrix obtained by us turns out to be symmetrical

$$W_i/W_{0j}=W_j/W_{0i}\quad (i\neq j).$$

We consider now the angular distribution of the dependences of the radiation diffracted into the vacuum. The total energy flux $W^{(V)}$ radiated into the vacuum is defined obviously as follows:

$$W^{(V)}=\frac{1}{\pi}\int_{-\pi/2}^{\pi/2}W^{(V)}(\theta)d\theta;\quad W^{(V)}(\theta)=\frac{1}{8}L_y cr|H^{(V)}|^2.$$

Obtaining $|H^{(V)}(\theta)|^2$ from (15) and (16), we get the following expressions for the angular distribution of the volume-radiation intensity:

$$\begin{gathered}
\frac{W^{(V)}}{W_{00}}=\frac{2\varkappa_0k_0^2(k_2+k_0)}{(|k_1|+k_0)(k_2-k_0)^2}\cdot\frac{k_x^2k_z^2(k_2-k_x)}{(|k_1|-k_x)(k_0+k_x)(k_2+k_x)^2(k_0-k_x)^2},\\
\frac{W^{(V)}}{W_{01}}=\frac{2\varkappa_1k_1^2(k_2-|k_1|)}{(|k_1|+k_0)(k_2+|k_1|)^2}\cdot\frac{k_x^2k_z^2(k_2-k_x)}{(|k_1|-k_x)(k_0+k_x)(k_2+k_x)^2(|k_1|+k_x)^2},\\
\frac{W^{(V)}}{W_{02}}=\frac{2(k_2+k_0)(k_2-|k_1|)}{\varkappa_2}\cdot\frac{k_x^2k_z^2}{(k_2-k_x)(k_0+k_x)(|k_1|-k_x)(k_2+k_x)^2}.\\
k_x=-\frac{\omega}{c}\sin\theta,\quad k_z=\frac{\omega}{c}\cos\theta.
\end{gathered} \tag{20}$$

A characteristic difference between the angular distribution (20) and that considered in Ref. 6 for the case of diffraction of SP by the boundary line of two metals with different impedances is that the radiation intensity vanishes in the direction normal to the surface. The reason is that in our case the impedance boundary condition contains derivatives.

In the absence of damping it follows from the energy conservation law that the sum of the fluxes $W_i$ of the surface waves reflected from and passing through the impedance step, and also of the energy flux $W^{(V)}$ of the volume radiation produced as a result of the diffraction, should equal the energy flux of the incident surface wave $W_{0i}$:

$$W_0+W_1+W_2+W^{(V)}=W_{0i}. \tag{21}$$

Using the explicit expression for the SP dispersion law, it can be verified that (21) is identically satisfied in all the cases discussed here.

## 5. DISCUSSION OF RESULTS

All the results reported above are the consequence of the use of the impedance boundary condition (2). It is known that it is valid only if the fields vary very slowly in vacuum: $k_\parallel d\ll 1$ and $k_\parallel\delta\ll 1$, where $\delta=c|\varepsilon|^{-1/2}/\omega$ is the depth of penetration of the field into the metal, and $k_\parallel^{-1}$ is the characteristic length of the variation of the fields in a direction parallel to the surface. An analysis of the solution obtained above for the case of a jumplike change of $\varepsilon_1(x)$ shows that the electric field near the impedance step diverges like $r^{-1/2}$. Therefore the impedance boundary conditions are not valid at small distances $r$ from the impedance step, $r\lesssim\max(d,\delta)\equiv r_0$. At the same time it is clear that if the impedance step is smeared out and the width in the region $\rho$ where $\varepsilon_1(x)$ varies from 1 to $\varepsilon_1(\infty)$ is such that $\rho>r_0$, the boundary conditions (2) are valid on the entire surface $z=0$. Thus, the integral Eq. (3) holds only at $\rho>r_0$.

Let us determine now the conditions under which the smearing of the impedance step by an amount $\rho>r_0$ does not effect significantly the solution of Eq. (3). To this end we represent the integral term in (3) in the form

$$i\int_{-\infty}^{+\infty}\frac{dw'}{2\pi}\Phi(w')\frac{(w')^2(\partial\mu/\partial x)_{w-w'}}{(V(w')-\varkappa_0)(w-w'+i0)}. \tag{22}$$

The quantity $(\partial\mu/\partial x)_{w-w'}$ can be regarded as constant at

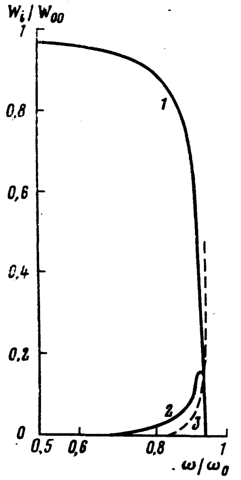


FIG. 1. SP energy-flux conversion coefficients for a silver film ($\hbar\omega_p = 3.8$ eV) of thickness $d = 40$ Å, deposited on the surface of aluminum ($\hbar\omega_p = 15.8$ eV): 1) $W_1/W_{00}$; 2) $w_2/W_{00}$; 3) $W_0/W_{00}$.

$|w - w'| < \rho^{-1}$. Therefore, if the main contribution to (22) is made by $|w'| \ll \rho^{-1}$, than at $|w| \ll \rho^{-1}$ we can set $(\partial\mu/\partial x)_{w-w'} = 1$, which is equivalent to a transition to Eq. (4). An analysis of the solution obtained above for (4) [see (6)] shows that the main contribution to (22) at $(\partial\mu/\partial x)_w = \text{const}$) is made by $|w'| \lesssim \max(k_2, |w|)$. Thus, the transition to the limit as $\rho \to 0$ should be understood in the sense that

$$r_0 = \max(d, \delta) < \rho \ll d|\varepsilon_1|^{-1}. \quad (23)$$

Condition (23) takes account of the fact that $k_2 \lesssim d^{-1}|\varepsilon_1|$. We note that at frequencies that are close to resonance with the oscillations in the transition layer, Eq. (23) is valid for any $\rho > r_0$.

It was already emphasized above that the questions that arise in the presence of impedance boundary conditions with derivatives turned out to be analogous in a certain sense to those that must be discussed in the analysis of reflection and refraction of volume waves by boundaries of gyrotropic media. It is known that in the latter case the kinetic-coefficient symmetry principle has made it necessary to take account in the

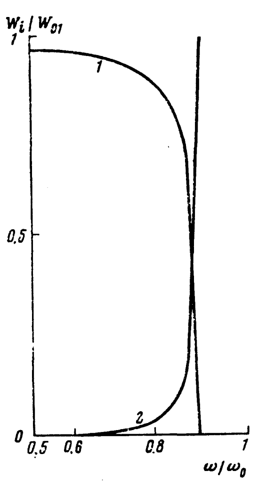


FIG. 2. SP energy-conversion coefficients: 1) $W_0/W_{01}$; 2) $W_2/W_{01}$.

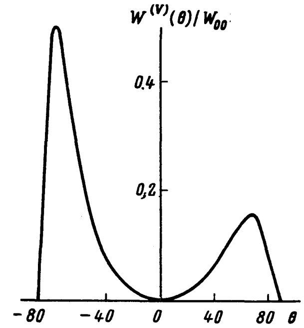


FIG. 3. Energy dependence of volume radiation $W^{(V)}(\theta)/W_{00}$.

boundary conditions a surface current proportional to the gyrotropic constants. In our case, however, when considering the diffraction of waves by an impedance step, the boundary conditions (2) also contain a term proportional to $\partial\mu/\partial x$, but this is equivalent to the appearance of a certain linear rather than surface current. Formally, by a method similar to that used above, we can obtain expressions for the fields without taking this linear current into account. This is equivalent to using in (1) the function $\Phi(w)$ in place of $F(w)$. Then, however, the energy conservation law is not satisfied, and this situation takes place apparently for any model of an impedance step in the region where the SP is at resonance with oscillations in the transition layer.

The results of the calculations for the SP energy fluxes and for the volume radiation are shown in Figs. 1–4. We wish to call attention here primarily to the frequency dependence of the impedance of the volume radiation produced when a surface wave is incident on the step from the clean metal surface. Since the reflection coefficient for this SP is always small $[\lesssim(\sqrt{2}+1)^4|\varepsilon|^{-1}$ as $|\varepsilon| \to \infty]$ and the intensity of the additional surface wave excited under these conditions is small $[\lesssim 4(1+\sqrt{2})|\varepsilon|^{-1/2}]$, the volume-radiation spectrum will contain waves in a narrow spectral interval having a width $g$ of the order of the true (indirect) gap

$$g \approx \pi\omega_0 (d/\lambda)|\varepsilon|^{1/2}.$$

This phenomenon may be the basis for an experimental method of investigating the oscillation spectra in thin films, and is in a certain sense similar to the formation of residual rays upon reflection of white light from ionic crystals.

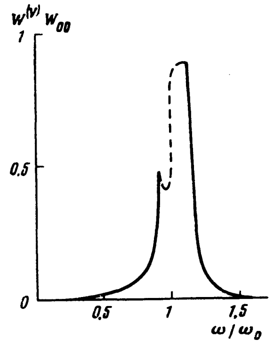


FIG. 4. Dependence of the integrated coefficient $W^{(V)}/W_{00}$ of conversion of SP energy into volume radiation on the frequency in the vicinity of a resonance with the oscillations in the transition layer.

The authors are deeply grateful to L.A. Vaĭnshteĭn and A.V. Popov for very helpful discussions of various problems involving diffraction of surface waves.

[1]V. M. Agranovich and A. G. Mal'shukov, Opt. Comm. **11**, 169 (1974).
[2]C. A. Yakovlev, V. G. Nazin, and G. N. Zhizhin, Opt. Comm. **15**, 293 (1975); Zh. Eksp. Teor. Fiz. **72**, 687 (1977) [Sov. Phys. JETP **45**, 360 (1977)].
[3]T. Lopez-Rios, F. Abeles, and G. Vuye, J. de Phys. **39**, 645 (1978).
[4]V. M. Agranovich, Zh. Eksp. Teor. Fiz. **77**, 1124 (1979) [Sov. Phys. JETP **50**, 567 (1979)].
[5]V. M. Agranovich and V. L. Ginzburg. Kristallooptika s uchetom prostranstvennoĭ dispersii i teoriya éksitonov (Spatial Dispersion in Crystal Optics and the Theory of Excitons), Nauka, 1979 (Wiley, 1967).
[6]L. A. Vaĭnshtein, Teoriya difraktsii i metod faktorizatsii (Diffraction Theory and Factorization Method), Sov. Radio, 1966.
[7]V. M. Agranovich and V. I. Yudson, Opt. Comm. **9**, 58 (1973).

Translated by J. G. Adashko