Line shape and potential gauge

M. I. Shirokov

Joint Institute for Nuclear Research, Dubna
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A new theoretical formula for the unstable-state excitation probability is given which takes account of the so-called virtual transitions. An improved version of the experiment for measuring the Lamb shift and the spectral line shape is proposed. The gauge-invariance problems encountered in line-shape calculations are discussed and solved in this experiment.

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1. INTRODUCTION

An unstable state is characterized by some energy distribution. It has been successfully measured in Lamb-shift experiments. Microwave radiation of frequency k induces transitions in hydrogen atoms from the metastable $2S$ state into the short-lived $2^2P_{1/2}$ state. The dependence of the number of atoms remaining in the $2S$ state on the difference $k-\varkappa$, where $\varkappa$ is the difference between the energies of the $2S$ and $2P$ levels, is measured. This function will be called the line shape of the $2P$ level.

Lamb[1] has pointed out that the observed line shape agrees with the result of the calculation (performed in the Weisskopf–Wigner approximation) in which the microwave field is described by an external scalar potential equal to $\mathbf{E}\cdot\mathbf{q}$ ($\mathbf{q}$ is the electron coordinate and $\mathbf{E}$ is the microwave electric field). But a similar calculation in which the same field is described by the corresponding nonzero vector potential $\mathbf{A}$ and a zero scalar potential yields another line shape that does not agree with observed shape. Lamb writes: "Of course, the difference between the perturbations $\mathbf{E}\cdot\mathbf{q}$ and $-\mathbf{A}\cdot\mathbf{p}/m$ just corresponds to a gauge transformation under which the theory is known to be invariant, so that both perturbations must lead to the same physical predictions. Nevertheless, a closer examination shows that the usual interpretation of probability amplitudes is valid only in the former gauge..." (i.e., for $\mathbf{E}\cdot\mathbf{q}$).

This problem has been discussed in a number of papers. In Refs. 2–4, the choice of the $\mathbf{E}\cdot\mathbf{q}$ interaction is justified. In Refs. 5–7 it is asserted that the same result is obtained with either interaction, but in fact this was demonstrated for processes described by the S matrix, i.e., processes of the scattering-reaction type. In Sec. 2 of the present paper we show why the measurement of the line shape and the Lamb shift itself should not pertain to this type. We indicate a modified experiment in which they should be measured. This experiment is not described by the S matrix. It can be shown that, in the case of this experiment, the standard calculations yield different theoretical line shapes in different gauges (see Secs. 2 and 4), so that the Lamb problem is not removed by Fried's[5] and Davidovich and Nussenzveig's[6] investigations. A similar problem arises for other observable quantities (see Sec. IV of Ref. 3).

Kuo-Ho Yang[3] has proposed for line-shape calculations a nonstandard method in which the states of the atom are described by the eigenfunctions of a gauge-invariant operator that, generally speaking, does not coincide with the Hamiltonian's free part determined in the usual fashion. This method yields in any potential gauge the same results as the $\mathbf{E}\cdot\mathbf{q}$ interaction. This can be considered to be the solution to the Lamb problem. But Kuo-Ho Yang[3] considered an electron interacting only with an external electromagnetic field.

In Sec. 3 we discuss the general case in which the electromagnetic field is quantized. We consider the form of quantum electrodynamics proposed in Refs. 2, 8–10. We show that the free part H_0 of its Hamiltonian is invariant under any gauge transformations. Also invariant are the eigenfunctions of H_0 that describe the initial and final states and the interaction Hamiltonian H_I. We obtain with the aid of such H_0 and H_I gauge-invariant results for "non-S-Matrix" observables, in particular, for the line shape measurable in the experiment suggested in Sec. 2. In the Coulomb gauge of quantum electrodynamics, for example, these results are obtained by a complex nonstandard method. At the same time, the standard computations yield the same S matrix in any gauge. We show that the $\mathbf{E}\cdot\mathbf{q}$-type interaction arises in the form of the theory under discussion.

In Sec. 4 we propose for line-shape calculations a formula, (20), that takes account of the so-called virtual processes.

The gauge-invariant methods, discussed in Secs. 3 and 4, of computing line shapes predict that the experiment proposed in Sec. 2 should yield the same line shape as the existing experimental techniques based on Lamb-shift measurements (see, for example, Refs. 11 and 12).

2. LINE SHAPE AND GAUGE INVARIANCE OF THE S MATRIX

1. Fried[5] has performed the following Lamb-experiment-related computation. A microwave photon of frequency k falls on an atom in the $2S$ state. The probability amplitude T for transition into the final $1S$-plus-a-Lyman-photon-γ state is computed (as a function of k). The main contribution is made by the "resonance channel" $k+2S\to 2P\to 1S+\gamma$. But other

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intermediate channels, e.g., $k+2S \to 3P \to 1S+\gamma$, which are characterized by strong energy nonconservation in the virtual transitions $k+2S \to 3P$ and $3P \to 1S+\gamma$, are also possible. Denoting the contribution of all such "background channels" by B, we represent T as $R+B$ in the case of the $\mathbf{A}\cdot\mathbf{p}/m$ interaction. In the case of the $\mathbf{E}\cdot\mathbf{q}$ interaction we correspondingly have $T'=R'+B'$. Fried showed (in the first nonzero approximation) that, although $R \neq R'$, $T=T'$. A similar calculation is reported in Ref. 6. It turns out in that case that B' is small, so that $R+B \cong R'$.

The equality $T=T'$ can be regarded as a manifestation of the well-known property of nondependence of the (renormalized) S matrix on the choice of the gauge. Let us emphasize that all the other known demonstrations[5-7] of the equality of the results given by the $\mathbf{A}\cdot\mathbf{p}/m$ and $\mathbf{E}\cdot\mathbf{q}$ interactions pertain to S-matrix processes.

2. But only the resonance term, and not the total amplitude, is directly related to the Lamb shift. In fact it is the level shift of just the $2P$ state that we are interested in. If the "background" is substantial, then the experiment should be modified in such a way that it singles out precisely the resonance term. For example, it is necessary to measure (as a function of k) the number of Lyman photons emitted after the beam of hydrogen atoms gets out of the range of action of the microwave field. Then the contribution of all the nonresonance channels will decrease: in contrast to the resonance contribution, it will not increase in time (the extent of energy nonconservation in the transition $k+2P \to 3P$ is much greater than the width of the $2P$ level corresponding to a lifetime of 1.6×10^{-9} sec).

This "purer" experiment is no longer described by the S matrix, since it provides information about the intermediate phase of the $k+2S \to 1S+\gamma$ process, and not just about the initial and final phases (it can be asserted further than only the delayed γ photons are measured in the experiment). For this experiment, the $\mathbf{A}\cdot\mathbf{p}/m$ and $\mathbf{E}\cdot\mathbf{q}$ interactions give different answers (R and R' respectively).

3. Let us note that, instead of R or R', it is sufficient to compute the $k+2S \to 2P$ transition probability, since the experiment can be set up in such a way that the subsequent decay $2P \to 1S+\gamma$ will be registered with probability equal to 1. It is precisely the $k+2S \to 2P$ transition probability that was computed by Lamb, and is computed by Newton *et al.* in Sec. 4.2 of Ref. 11. A new formula for its computation will be proposed in Sec. 4.

3. QUANTUM ELECTRODYNAMICS WITHOUT A GAUGE GROUP

A new form of quantum electrodynamics applicable to localized charges has been proposed by Power and Zienau,[2] Wooley,[8] Babiker *et al.*,[9] and the present author.[10] We show in Ref. 10 that it can be regarded as another gauging of electrodynamics (along with the Coulomb gauge, for example), with the condition (8) (see below) as the gauge condition. In this section we show that this condition is so strong that this form of the theory actually does not contain a gauge group, and therefore there should be no grounds for the appearance of difficulties with the gauge noninvariance of H_0 and H_I.

We shall limit ourselves to the consideration of the simple case of one spinless electron interacting with a quantized electromagnetic field. The generalization to the case of second-quantized Dirac electrons offers no difficulties (see Ref. 10).

1. The new gauge is obtained from the Coulomb gauge by means of the canonical transformation $O'=S^+OS$ of the Coulomb gauge operators O with the following operator S:

$$S=\exp\left[(-ie)\int_0^q [d\xi_x A_{\perp x}(\xi)+d\xi_y A_{\perp y}(\xi)+d\xi_z A_{\perp z}(\xi)]\right]. \tag{1}$$

The integral in (1) is evaluated along the straight line joining q to the coordinate origin, which is located at the center of the potential W binding the electron. The Hamiltonian of the Coulomb gauge in the case under consideration has the form

$$H=[\mathbf{p}-e\mathbf{A}_\perp(\mathbf{q})]^2/2m+W(\mathbf{q})+{}^1/_2\int d^3x[\mathbf{E}_\perp^2(\mathbf{x})+\mathbf{H}^2(\mathbf{x})]. \tag{2}$$

We have

$$q'=S^+qS=q, \quad A_\perp'=A_\perp, \quad H'=H, \tag{3}$$

$$\mathbf{p}'=S^+\mathbf{p}S=\mathbf{p}+e\nabla\Lambda(\mathbf{q}), \tag{4}$$

$$\Lambda=-\int_0^q d\boldsymbol{\xi}\cdot\mathbf{A}_\perp(\boldsymbol{\xi}). \tag{5}$$

Using (3) and (4), we obtain

$$\mathbf{p}-e\mathbf{A}_\perp(\mathbf{q})=\mathbf{p}'-e[\mathbf{A}_\perp(\mathbf{q})+\nabla\Lambda(\mathbf{q})]\equiv\mathbf{p}'-e\mathbf{a}(\mathbf{q}). \tag{6}$$

Here $\mathbf{a}=\mathbf{A}_\perp+\nabla\Lambda$ can be regarded as a new potential. Its expression in terms of the magnetic-field operator is found in Refs. 8 and 10:

$$\mathbf{a}(\mathbf{x})=\mathbf{A}_\perp(\mathbf{x})+\nabla\Lambda(\mathbf{x})=-\int_0^1 \alpha\, d\alpha[\mathbf{x}\times\mathbf{H}(\alpha\mathbf{x})]. \tag{7}$$

As can be seen from (4) and (7), the operators $\mathbf{p}'$ and $\mathbf{a}$ are connected with $\mathbf{p}$ and $\mathbf{A}_\perp$ by an operator gauge transformation. The potential $\mathbf{A}_\perp$ satisfies the gauge condition $\operatorname{div}\mathbf{A}_\perp=0$. The following condition can be imposed on the potential $\mathbf{a}$:

$$\int_0^x d\boldsymbol{\xi}\cdot\mathbf{a}(\boldsymbol{\xi})=0 \text{ for all } \mathbf{x}. \tag{8}$$

It is not difficult to verify that the mean part of (7) satisfies the condition (8). The imposition of the conditon (8) forbids the inverse transition to the Coulomb gauge with the aid of the gauge transformation that is the inverse of (4), (7), since $\mathbf{A}_\perp$ does not satisfy (8).

The Hamiltonian (2) expressed in terms of the new, primed operators is given in Ref. 10:

$$H=(\mathbf{p}'-e\mathbf{a})^2/2m+W(\mathbf{q}')-e\int_0^{q'} d\boldsymbol{\xi}\cdot\mathbf{E}_\perp'(\boldsymbol{\xi})$$

$$+{}^1/_2\int d^3x(\mathbf{E}_\perp'^2+\mathbf{H}^2)+2^{-1}e^2\sum_{m,n}\int_0^{q'} d\xi_m\int_0^{q'} d\xi_n'\delta_{mn}^{tr}(\boldsymbol{\xi}-\boldsymbol{\xi}'). \tag{9}$$

Here $\mathbf{E}_\perp' = S^+\mathbf{E}_\perp S \neq \mathbf{E}_\perp$ (see Ref. 10) and δ^{tr} is the "transverse part of the δ function." For a discussion of the last (divergent) term in (9), see Sec. 4.2 of Ref. 2.

2. The theory described by the Hamiltonian (2) is invariant under the gauge transformation

$$\mathbf{A}_\perp \to \mathbf{A}_\perp + \nabla\chi, \quad \mathbf{p} \to \mathbf{p} + e\nabla\chi. \tag{10}$$

Here χ should be a harmonic function, i.e., $\nabla\cdot\nabla\chi = 0$, since $\operatorname{div}\mathbf{A}_\perp = 0$. If we require that the potentials $\mathbf{A}_\perp$ and $\mathbf{A}_\perp + \nabla\chi$ decrease at infinity (we are, of course, talking about their matrix elements), then $\nabla\chi \to 0$ at infinity. If $\Delta\chi = 0$ everywhere, then $\nabla\chi = 0$ also everywhere, and the group (10) does not exist. But we are considering here physical systems located in a finite volume V (e.g., within the confines of a laboratory). If (10) exists only for points $\mathbf{x}$ belonging to the portion V of all space, then $\Delta\chi$ does not vanish everywhere, and therefore $\nabla\chi$ can decrease at infinity without vanishing identically. For example, an infinite current-carrying solenoid induces a nonzero potential $\mathbf{A}_\perp \sim \nabla\chi$ in the surrounding space. Another example is the "quasi-gauge" transformation (26) in Ref. 10.

Let us show that even this gauge freedom is not possessed by the potential $\mathbf{a}$ and the new electron momentum $\mathbf{p}'$. Let us try to write

$$\tilde{\mathbf{a}}(\mathbf{x}) = \mathbf{a}(\mathbf{x}) + \nabla\chi'(\mathbf{x}), \quad \mathbf{x}\in V. \tag{11}$$

Since (8) should be valid for $\tilde{\mathbf{a}}$ and $\mathbf{a}$, we have

$$0 = \int_0^{\mathbf{q}} d\boldsymbol{\xi}\cdot\nabla\chi'(\boldsymbol{\xi}) = \chi'(\mathbf{q}) - \chi'(0),$$

if V encompasses the entire region V_e in which the electron is localized. It turns out that $\chi'(\mathbf{q})$ is a constant, and $\nabla\chi' = 0$.

It can be shown that the gauge transformation (11) with $\nabla\chi' \neq 0$ can occur only if V does not encompass V_e: $V_e \cup V \neq V$, $V_e \cap V \neq V_e$. But such a transformation is accompanied by changes in the observables. For example, let (11) be induced by a solenoid whose magnetic field H is equal to zero outside V [it follows from (7) that $\mathbf{a}$ should have the form of a gradient wherever $H = 0$]. In the case under consideration the region of localization of H should intersect V_e [otherwise, (11) would be valid for V_e, and then $\nabla\chi' = 0$]. Such a field H leads, for example, to the observed splitting of the levels.

We use the designation "quasi-gauge" for those transformations of operators which have the form of ordinary gauge transformations only in a certain (simply-connected) portion of V of all space. The volume V should encompass the entire physical system under consideration. Then the transformations in question will not be accompanied by changes in any observables measured inside V, and will be indistinguishable from ordinary gauge transformations in the region V.

The gauge invariance of $\mathbf{p}'$ follows also from the invariance of the right-hand side of (4) under (10). Let us note that, under (10), we have $\Lambda(\mathbf{q}) \to \Lambda(\mathbf{q}) - \chi(\mathbf{q}) + \chi(0)$. (Of course, we would be more precise if, remaining within the framework of the new gauge, we talked about the absence of gauge freedom for $\mathbf{p}'$.) Also invariant under any (both ordinary and quasi-) gauge transformations are the operator $H_e' = \mathbf{p}'^2/2m + W$, its eigenfunctions (describing the atomic states in the new gauge), and the interaction Hamiltonian.

3. Finally, we can, using (4), now give in the original Coulomb gauge the operator corresponding to $H_e' = \mathbf{p}'^2/2m + W(\mathbf{q})$. But H_e' does not commute with

$$\int d^3x(\mathbf{E}_\perp^2 + \mathbf{H}^2),$$

as well as with the photon-number operator. As a consequence, we cannot describe even the simplest, "atom unexcited, no photon" state [even if we manage to find the eigenfunctions of the operator $(\mathbf{p} - e\nabla\Lambda)^2/2m + W$]. We also have to modify the description of the photons. If we are able to do everything correctly, then we shall obtain the same result that is obtained in the new gauge in the standard fashion.

4. Let us consider (9) in the dipole approximation, which corresponds to the replacement of $\mathbf{A}_\perp(\mathbf{q})$ in (2) by $\mathbf{A}_\perp(0)$. Setting $\mathbf{q} = 0$ in (6), and using the equality $\mathbf{a}(0) = 0$, which follows from (7), we obtain

$$\mathbf{p} - e\mathbf{A}_\perp(0) = \mathbf{p}'. \tag{12}$$

Thus, in this approximation the interaction of the electron with the photons is described by only the third term in (9):

$$-e\int_0^{\mathbf{q}'} d\boldsymbol{\xi}\cdot\mathbf{E}_\perp'(\boldsymbol{\xi}) \approx -e\mathbf{E}_\perp'(0)\mathbf{q}'$$

(the $\mathbf{E}\cdot\mathbf{q}$ interaction). The transition to the dipole approximation is expounded in greater detail in the preprint of the present paper.[13]

4. PROBABILITY FOR ELECTROMAGNETIC EXCITATION OF AN ATOM

1. Let us to begin with suppose that we have at $t = 0$ an atom in the ground state and one photon with energy k (state Φ_k). The amplitude of the probability of finding at the moment t the state Φ_n, i.e., the "atom in n-th excited state, no photons" state, is equal to

$$A_{nk} = \langle\Phi_n|U(t, 0)|\Phi_k\rangle.$$

The probability for excitation of the atom is usually defined as $|A_{nk}|^2$. But $|A_{nk}|^2$ is only a part of the total probability of finding the atom in the n-th state:

$$P_{nk} = \sum_\nu |\langle\Phi_{n\nu}|U(t,0)|\Phi_k\rangle|^2. \tag{13}$$

Here we sum over all the states of the type "atom in n-th state, arbitrary number ν of photons with arbitrary polarizations ε and momenta." It is precisely (13) that corresponds to the experiment in which only the excitation of the atom (and no accompanying photon) is detected. Naturally, the energy in the majority of the $\Phi_k \to \Phi_{n\nu}$ transitions in (13) is not conserved if by energy we mean the eigenvalue of the free part of the Hamiltonian. As is well known, the proba-

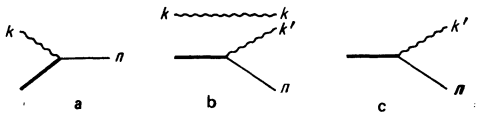

FIG. 1.

bility of such "virtual transitions" is not equal to zero for finite t. In particular, the atom can be excited, and, moreover, can emit a photon with any momentum.

Let us note that the just described process occurs whether or not the photon k is present at the beginning. An expression of the type (13) is not equal to zero even if as the initial state we take the lowest state Φ_0, i.e., the "atom in ground state and no photons" state, instead of Φ_k. Therefore, the observable probability for excitation of the atom should be associated with only that of (13) which stems from a physical cause: the presence of a photon in the initial state. We define this part as

$$W_{nk}(t)=\sum_{\nu}|\langle\Phi_{n\nu}|U(t,0)|\Phi_k\rangle|^2-\sum_{\nu}|\langle\Psi_{n\nu}|U(t,0)|\Phi_0\rangle|^2. \quad (14)$$

From P_{nk} we have subtracted the "theoretical background," i.e., the probability for "causeless" transitions. Let us emphase that, in (14), the state Φ_k, like Φ_0, should be normalized to unity.

In the lowest-order perturbation theory the matrix elements of the first sum in (14) are represented by the diagrams in Figs. 1a and 1b, while the matrix elements of the second sum (the subtrahend) are represented by the diagram in Fig. 1c. In these diagrams the ground state of the electron in the atom is represented by a heavy line; the excited state, by a thin line.

Similar definitions involving "background" subtraction are proposed for other cases in Refs. 14–17.

Let us note that the virtual transitions are in no way taken into account in the Weisskopf–Wigner approximation. These transitions may not occur if the states of the atom and the field are described in a special fashion.[6] Then (14) is equal to

$$|\langle\Phi_n|U(t,0)|\Phi_k\rangle|^2=|A_{nk}|^2.$$

2. We shall represent (13) as the average over the state Φ_k of the Heisenberg operator for the number of electrons in the n-th state:

$$\mathcal{N}_n(t)=U^+(t,0)\mathcal{N}_nU(t,0).$$

The latter may be found not by determining the operator $U(t,0)$ (it may not exist[19]), but by solving the equations for the Heisenberg operators of the theory.

We shall need a second-quantized description of the nonrelativistic electron. The Hamiltonian (2) corresponds to the single-electron sector of the Hamiltonian $\int d^3xH(\mathbf{x})$, where

$$H(\mathbf{x})=\psi^+(\mathbf{x})[(-i\nabla-e\mathbf{A}_\perp)^2/2m+W(\mathbf{x})]\psi(\mathbf{x})+[\mathbf{E}_\perp^2(\mathbf{x})+\mathbf{H}^2(\mathbf{x})]/2$$

(see, for example, Ref. 19). The operator $\psi(\mathbf{x})$ is expanded in terms of the eigenfunctions of $H_e=\mathbf{p}^2/2m+W(\mathbf{q})$:

$$\psi(\mathbf{x})=S_n\varphi_n(x)\alpha_n, \quad (15)$$

where S_n denotes summation and /or integration over the spectrum of H_e. In terms of the electron and photon creation operators, α_n^+ and $a^+(\mathbf{k},\varepsilon)$ respectively, the states Φ_k, Φ_n, and Φ_0 in (14) can be written as:

$$\Phi_k=\alpha_0^+a^+(\mathbf{k},\varepsilon)\Omega_0,\quad \Phi_n=\alpha_n^+\Omega_0,\quad \Phi_0=\alpha_0^+\Omega_0. \quad (16)$$

Here α_0^+ creates an electron in the ground state and Ω_0 is the particle-free state.

Let us consider the electron-number operator $N_n=\alpha_n^+\alpha_n$ for the n-th state. Let us expand $\alpha_n^+\alpha_n$ in terms of the operators Π_N of projection onto states with a definite number N of electrons at the n-th level. There can be in these states electrons at other levels, as well as an arbitrary number of photons. We have

$$\alpha_n^+\alpha_n=0\cdot\Pi_0+1\cdot\Pi_1+2\cdot\Pi_2+\dots; \quad (17)$$

$$\Pi_1=\sum_{\nu=0}^{\infty}\int dk_1\dots\int dk_\nu\sum_{\varepsilon_1,\dots,\varepsilon_\nu}|\Phi_{n\nu}\rangle\langle\Phi_{n\nu}|+\dots, \quad (18)$$

$$|\Phi_{n\nu}\rangle\equiv\alpha_n^+a^+(\mathbf{k}_1,\varepsilon_1)\dots a^+(\mathbf{k}_\nu,\varepsilon_\nu)\Omega_0.$$

Only the single-electron part Π_1 of the operator has been written out. If our nonrelativistic spinless electrons obey the Fermi statistics, then there cannot be two electrons in the same state, and then $\Pi_2=0$. In view of (17) and (18), we have

$$\langle U(t,0)\Phi_k|\alpha_n^+\alpha_n|U(t,0)\Phi_k\rangle=\sum_{\nu}\langle U(t,0)\Phi_k|\Phi_{n\nu}\rangle\langle\Phi_{n\nu}|U(t,0)\Phi_k\rangle=P_{nk}. \quad (19)$$

We have used the fact that $U(t,0)\Phi_k$, like Φ_k, is a single-electron state; therefore, only Π_1 from (17), and what is more only that part of Π_1 which has been written out in (18), makes a contribution to (19).

Let us introduce the Heisenberg operators $\alpha_n(t)=U^+(t,0)\alpha_nU(t,0)$, and rewrite (14) in the form

$$W_{nk}(t)=\langle\Phi_k|\alpha_n^+(t)\alpha_n(t)|\Phi_k\rangle-\langle\Phi_0|\alpha_n^+(t)\alpha_n(t)|\Phi_0\rangle. \quad (20)$$

In this definition, the $\alpha_n^+(\alpha_n)$ can be the creation (annihilation) operators for the second-quantized Dirac electrons.

3. In the new gauge the atomic states are described by the eigenfunctions $\varphi_n'(\mathbf{x})$ of the operator $H_e'=p'^2/2m+W$. Although this operator is not equal to $H_e=\mathbf{p}^2/2m+W$, the $\varphi_n'(\mathbf{x})$ analytically coincide with the eigenfunctions of H_e if we choose the same canonical representation, $-i\nabla$, for $\mathbf{p}'$ and $\mathbf{p}$ in the two gauges. Similarly, we can choose identical representations for the photon operators $a(\mathbf{k},\varepsilon)$ and $a'(\mathbf{k},\varepsilon)$ as well. Thus, Φ_k and Φ_0 will be described in like manner in the new and Coulomb gauges. But the interaction terms in (2) and (9) will be dissimilar. Therefore, the probability, W_{nk}', computed in the new gauge can differ from the probability, W_{nk}, computed in the Coulomb gauge. These probabilities have been computed in the exactly soluble model of an oscillator electron dipolarly interacting with an electromagnetic field.[13] It turns out that $W_{kn}'\neq W_{nk}$, although the level shifts and level widths are found to have identical values in the two gauges. Specifically, W_{nk}' differs from W_{nk} by the factor $k^2/\varkappa^2$, where $\varkappa$ is the difference between the energies of the excited and ground states of the atom [see the formulas

(50) and (53) in Ref. 2]. The importance of this discrepancy for the determination of the Lamb shift has already been discussed by Power and Zienau[2] and Fried.[5]

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