Concerning the nature of the 1/f noise

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It is shown that the spectra with frequency dependence of the form $1/f$, which are observed in measurements of low-frequency noise, can be the result of action on the analyzer by the spectrum of the stationary infralow-frequency noise (ILF noise), whose correlation time is much longer than the analyzer reciprocal bandwidth. Expressions are obtained for the instrumental function of the spectrum analyzer used in analog and digital methods of measurements, and for the output signal when ILF noise acts on the analyer. The measured signal is proportional in this case to the mean square and not to the spectral density of the noise, and does not reflect in any way the dependence of the latter on the frequency. The reliability of the available experimental data on the $1/f$ noise are discussed. It is concluded that it is impossible at present to draw a definite conclusion that fluctuations with a spectrum of the form $1/f$ exist in nature. The experimental results of Voss and Clarke [Phys. Rev. **B13**, 556 (1976)] on $1/f$ noise of metallic films are analyzed and a quantitative and qualitative explanation, connected with equilbrium fluctuatins of the temperature, is proposed within the framework of the theory of ILF noise. A theoretical expression is also obtained for the Hooge constant and it is shown that it depends not only on the parameters of the material and on the temperature, but also on the instrumental function of the analyzer.

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The low-frequency flicker noise with spectrum of the form $1/f^n$ has to this day found no generally recognized explanation that would be as universal as the very fact of the existence of this noise in a tremendous number of physical (and not only physical) systems (see, e.g., the reviews by Kogan[1] and Bell[2]). The theoretical work in this field reduces mainly to searches for physical models in which there might be realized formal mathematical constructions that yield a spectrum of the $1/f$ type in a wide but finite frequency band. After the initial attempts to observe the low-frequency limit of the existence of the $1/f$ noise,[3,4] which led to observation of this noise in semiconductors down to frequencies $\sim 10^{-6}$ Hz,[5] the attention in the experimental research became focused on the determination of the factors that influence the appearance, magnitude, and exact form of the $1/f$ noise spectrum. As a result, an empirical formula was proposed for the spectral density:

$$G(f) \sim \alpha / N_c f, \quad \alpha \approx 2 \cdot 10^{-3}, \tag{1}$$

which holds true with order-of-magnitude accuracy for volume noise in metals in semiconductors,[6] and strong evidence was found in favor of the equilibrium temperature fluctuations as the cause of the $1/f$ noise in metallic films.[7] These experiments have shown, however, that the magnitude of the $1/f$ noise can be estimated, but did not explain its spectral dependence.

In the present paper we wish to call attention to the fact that under certain conditions, which are quite probable in measurements of low-frequency noise, the output signal of the spectrum analyzer can vary with the tuning frequency in accordance with the $1/f$ law, even if a stationary noise whose spectral density does not have this form is applied to the input of the analyzer.

1. ANALYSIS OF MEASUREMENT METHODS

Analog measurements. In the case of the simplest spectrum analyzer, consisting of a narrow-band filter with a transfer characteristic $H(\omega_0, t)$, a linear broadband amplifier with a gain $A(\omega_0)$, a square-law detector, and an integrator with an averaging time T, the output signal $\overline{x^2}(t, \omega_0)$ as a function of the input process $y(t)$ is given by

$$\overline{x^2}(t, \omega_0) = A^2(\omega_0) \frac{1}{T} \int_{t-T}^{t} dt' \iint_{-\infty}^{\infty} dt_1 dt_2 H(t'-t_1) H(t'-t_2) y(t_1) y(t_2). \tag{2}$$

If $y(t)$ is a stationary random process with zero mean value and spectral density $G_y(\omega)$, then the mean value of the output signal can be represented in the form

$$\langle \overline{x^2}(\omega_0) \rangle = A^2(\omega_0) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |K(\omega_0, \omega)|^2 G_y(\omega), \tag{3}$$

where

$$K(\omega_0, \omega) = \int_{-\infty}^{\infty} dt H(\omega_0, t) e^{i\omega t}$$

is the frequency characteristic of the filter. The dependence of the gain $A(\omega_0)$ on the tuning frequency ω_0 is determined by the choice of the analyzer calibration. If the calibration is against a harmonic signal, when $y(t) = \cos \omega_e t$, we have

$$A_h^2(\omega_0) = 2 |K(\omega_0, \omega_e)|^{-2}, \quad \omega_0 = \omega_e, \tag{4a}$$

and in the case of calibration against white noise, $G_y(\omega) = \text{const}$,

$$A_w^2(\omega_0) = \left[\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |K(\omega_0, \omega)|^2 \right]^{-1} = [2 |K(\omega_0, \omega_0)|^2 \Delta]^{-1}. \tag{4b}$$

The second identity in (4b) is simultaneously the definition of the equivalent noise band Δ of the analyzer. In the case (4a) the output signal has the dimension of power in the band Δ, and in the case (4b) $\langle \overline{x^2}(\omega_0) \rangle$ has directly the dimension of spectral density, so that we shall denote it by $G^{\text{out}}(\omega_0)$. Spectra are always measured with a narrow-band filter, when the figure of merit

$$Q = f_0 / \Delta \gg 1, \quad f_0 = \omega_0 / 2\pi. \tag{5}$$

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The inequality (5) is usually regarded as sufficient (see, e.g., Ref. 8, Sec. 5.2–3) to satisfy the criterion that the variation of $G_y(\omega)$ be slow over the interval Δ. It is then possible to take $G_y(\pm\omega_0)$ outside the integral in (3), and in the case of the calibration (4b) we have simply

$$G^{out}(\omega_0)=G_y(\omega_0). \tag{6}$$

The condition (5), however, is necessary but not sufficient for the validity of (6). Its insufficiency, as will be shown, can manifest itself in measurements of the high-frequency wing of the spectral density of the low-frequency (slow) fluctuations.

For the sake of clarity, we consider an analyzer with the simplest resonant circuit, described by the equation

$$\ddot{x}(t)+\gamma\dot{x}(t)+\omega_0^2x(t)=y(t). \tag{7}$$

We obtain accordingly

$$H(t-t')=e^{-\gamma(t-t')/2}\frac{\sin[\omega_1(t-t')]}{\omega_1}\Theta(t-t'),$$

$$|K(\omega_0,\omega)|^2=[(\omega^2-\omega_0^2)^2+\omega^2\gamma^2]^{-1}, \tag{8}$$

$$A_h^2(\omega_0)=2\gamma^2\omega_1^2,\quad A_w^2(\omega_0)=2\gamma\omega_0^2,\quad \Delta=\gamma/4,\quad \omega_1=\omega_0^2-\gamma^2/4.$$

Let the input signal $y(t)$ have a spectral density of the form

$$G_y(\omega)=\langle y^2\rangle\frac{2\nu}{\omega^2+\nu^2}. \tag{9}$$

Substituting (8) and (9) in (3), we obtain

$$G^{out}(\omega_0)=2\langle y^2\rangle\nu\frac{\omega_0^2(1+\gamma/\nu)+\nu^2-\gamma^2}{(\omega_0^2+\nu^2)^2-\nu^2\gamma^2}. \tag{10}$$

It is easily seen that the condition (5) that the analyzer be narrow-band (in our case it takes the form $\pi\gamma/2\omega_0 \ll 1$) still does not determine uniquely the $G^{out}(\omega_0)$ dependence. It is also important to know the relation between γ and ν, i.e., between the widths of the input spectrum and the resonant filter. Indeed, at $\gamma/\nu\ll 1$, i.e., in the case of a relatively narrower filter, we have the usual result

$$G^{out}(\omega_0)=\langle y^2\rangle\frac{2\nu}{\omega_0^2+\nu^2}\equiv G_y(\omega_0). \tag{11}$$

In the other limiting case, however, when $\gamma/\nu\gg 1$, we obtain

$$G^{out}(\omega_0)=\langle y^2\rangle\frac{2\gamma}{\omega_0^2}\equiv\langle y^2\rangle\frac{8\Delta}{\omega_0^2}. \tag{12}$$

From this it follows directly that if the measurements are carried out in this situation at a constant Q, i.e., under the condition $\gamma(\omega_0)/\omega_0=$ const, then the output signal will imitate the presence of noise with a spectrum $1/f$ at the input of the analyzer. The general form of the frequency dependence of the output signal (10), under conditions of measurements with constant Q, is shown in the figure for the case $\gamma/\omega_0=0.05$. It is clearly seen that although at high frequencies ($\omega>\nu$) the true spectral density of the noise (9) is everywhere proportional to $1/\omega^2$ and the condition (5) is satisfied, all this is still not enough to let the region of the maximum $|K(\omega_0,\omega)|^2$ to make the main contribution to the integral in (3). Conversely, as soon as the filter bandwidth γ becomes larger than the width of the input spectrum ν, a transition from the relation (11) to (12) takes place quite rapidly. In other words, the output signal of the analyzer ceases to yield information on the form of the spectrum of the input and begins to vary with frequency in accordance with the $1/f$ law.

If the measurements are carried out at a constant bandwidth, then it follows from (12) that at $\gamma/\nu\ll 1$ the output signal will imitate the presence of noise with $1/f^2$ spectrum at the input of the analyzer. Although in the particular example considered by us this dependence coincides with the high-frequency asymptotic form of (9), it follows nevertheless from (12) that the measurement does not yield the "correct" spectrum in this case.

Let us verify that the conclusion (based on a concrete example) that false observation is possible of a $1/f$ spectrum in measurements with constant Q, or of a $1/f^n$ spectrum in measurements with a constant bandwidth Δ, is not the consequence of the frequency characteristic (8) chosen above for the analyzer and of the chosen spectral noise density (9), but a manifestation of the general laws of spectroscopy. To this end it is necessary to turn to the initial equations (2) and (3). Unfortunately, although the spectral analysis of stationary random processes is by its very character a spectroscopic measurement method, Eq. (3) does not have a form usual for spectroscopy, when the measured output spectrum is connected with the input spectrum via the instrumental function and a convolution integral with respect to the frequencies (see, e.g., Ref. 9). It is easy, however, to carry out the necessary transformations that lead to the desired result.

It is seen from (8) that the transfer characteristic of a resonant filter can be represented as a product of two factors. The time dependence of one of them is rapid and is determined by the frequency to which the filter is tuned, and the other factor depends slowly on the time and describes the buildup of an oscillation in the resonant circuit, with a characteristic time constant inversely proportional to the bandwidth. This property

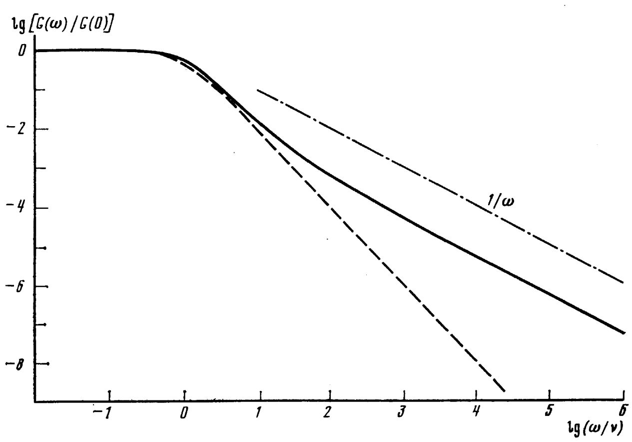

FIG. 1. Dependence, calculated from formula (10), of the output signal of the analyzer on the tuning frequency under the condition $\Delta/f=$ const (solid line). The dashed line is the spectral density of the noise at the analyzer input.

is common to filters whose frequency characteristics have in the complex plane isolated poles that are close to the real axis (see, e.g., Ref. 8, Sec. 2.2–5), so that for the transfer characteristics of narrow-band filters we can use the following approximate representation[8]:

$$H(t)=H_0(t)\cos[\omega_0 t+\varphi(t)], \tag{13}$$

where the envelope $H_0(t)$ is a slowly varying function that decreases at infinity, and the phase φ is frequently just a constant. From this we easily obtain

$$|K(\omega_0,\omega)|^2=\frac{1}{4}[|K_0(\omega_0+\omega)|^2+|K_0(\omega_0-\omega)|^2]$$
$$+\frac{1}{2}\mathrm{Re}[e^{2i\varphi}K_0(\omega+\omega_0)K^*(\omega-\omega_0)], \tag{14}$$

where

$$K_0(\omega)=\int_{-\infty}^{\infty} dt H_0(t)e^{i\omega t}. \tag{15}$$

This function has all the properties of the frequency characteristic of a physically realizable filter, but in contrast to $|K(\omega)|^2$ it has a maximum at zero.

Let now $H_0(t)$ belong to a single-parameter family of functions and let $1/\Delta_1$ be the characteristic time scale that determines both the width and the rate of decrease of $H_0(t)$ and is such that $\omega_0/\Delta_1 \gg 1$. Then $H_0(t)\equiv H_0(t\Delta_1)$, and

$$K_0(\omega)=\frac{1}{\Delta_1}K_1\left(\frac{\omega}{\Delta_1}\right). \tag{16}$$

Substituting (14)–(16) in (3), using the fact that $G_y(\omega)$ is even, and using the definition (4b), we obtain

$$G^{out}(\omega_0)=\frac{1}{\Delta}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left\{\left|\tilde{K}_1\left(\frac{\omega_0-\omega}{\Delta_1}\right)\right|^2\right.$$
$$\left.+\mathrm{Re}\left[e^{2i\varphi}\tilde{K}_1\left(\frac{\omega_0+\omega}{\Delta_1}\right)\tilde{K}_1^*\left(\frac{\omega_0-\omega}{\Delta_1}\right)\right]\right\}G_y(\omega), \tag{17}$$

where $\tilde{K}_1(\omega)=K_1(\omega)/K_1(0)$. At $\omega\neq 0$ the second term in the curly brackets of (17) can always be neglected, after which we obtain the sought expression

$$G^{out}(\omega_0)=\frac{1}{\Delta}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left|\tilde{K}_1\left(\frac{\omega_0-\omega}{\Delta_1}\right)\right|^2 G_y(\omega), \tag{18}$$

in which $|K_1((\omega_0-\omega)/\Delta_1)|^2$ plays the role of the instrumental function referred to unity at the maximum. It remains to show that Δ and Δ_1 are proportional. Substituting (14) and (16) in the definition of Δ (4b), we have

$$\Delta=\Delta_1\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}|\tilde{K}_1(\lambda-\lambda_0)|^2=\Delta_1\cdot\mathrm{const}. \tag{19}$$

The constant in (19) does not depend on the resonant frequency ω_0, but can vary with the type of filter.

It is now easy to obtain the conclusions we need, by analyzing Eq. (18) and using customary spectroscopy concepts. It is known that if the instrumental function is narrower than the investigated spectrum, then the measurement result is

$$G^{out}(\omega_0)=G_y(\omega_0). \tag{20}$$

On the other hand if the investigated spectrum has a narrower contour than the instrumental function, then the result of the measurements will be the contour of the instrumental function

$$G^{out}(\omega_0)=\frac{1}{\Delta}\left|\tilde{K}_1\left(\frac{\omega_0-\omega_m}{\Delta_1}\right)\right|^2\langle y^2\rangle, \tag{21}$$

where ω_m is the frequency at which the maximum of $G_y(\omega)$ takes place. This result is a consequence of the fact that both the instrumental function and the investigated spectral density belong to the family of δ-like functions, since they satisfy the conditions of conservation of normalization and non-negativity when the parameters that characterize their width are varied.

It is now obvious from (21) that when the spectral density of the noise is narrower than $|\tilde{K}_1(\omega)|^2$, and the noise frequency is low enough to satisfy the condition $\omega_m \ll \omega_0$ [we shall use in such cases the term "infralow-frequency" (ILF) noise], then in constant-Q measurements the output signal has a dependence of the $1/f$ type:

$$G^{out}(\omega_0)=\frac{Q}{f_0}|\tilde{K}_1(2\pi Q)|^2\langle y^2\rangle,\quad f_0=\frac{\omega_0}{2\pi}. \tag{22}$$

On the other hand, if for the same relations between the noise spectrum and the instrumental function the measurements are carried at a constant bandwidth Δ, then the output signal has the frequency dependence of the far wing of the instrumental function, which likewise has as a rule the form $1/f^n$.

Thus, we have found that the $1/f$ dependence at the output of a spectrum analyzer, which is typical of low-frequency flicker noise, does not necessarily reflect the true behavior of the spectral density of the noise at the input, but can nevertheless appear regularly in measurements of ILF noise.

Some refinements are necessary if $\omega_m=0$. It is then necessary to use (17) instead of (18). This, as can be seen directly, does not change the conclusion that the $1/f$ law is strictly followed in measurements of ILF noise under conditions of constant Q, but can make somewhat less regular the behavior of the output signal in measurements with constant bandwidth. For example, the exponent n may begin to vary slowly with frequency. In general, as can be seen from (3), if Δ does not depend on ω_0 and ILF noise is present at the input of the analyzer, then the output signal is proportional to $|K(\omega_0,0)|^2$, and should always be a decreasing function of the frequency at $\Delta/\omega_0 \ll 1$. The derivation of (17) and of its consequences will be needed to prove that under conditions of "incorrect" measurements it is possible to observe alternating-sign variation and even a strict $1/f$ law.

This raises the following question: what in fact will be observed if a $1/f$ noise is actually applied to the input of the analyzer? Since for a stationary noise no such behavior of $G_y(\omega)$ is possible on the entire frequency axis, we use a model spectrum, which is obtained if the relaxation frequencies ν have a continuous distribution in a certain interval $\nu_1 \leq \nu \leq \nu_2$ in accordance with the law $g(\nu)\propto d\nu/\nu$ (see, e.g., Ref. 10). Integrating Eq. (10) with the indicated distribution function, we find that so long as $\gamma/\nu_2<1$, the quantity $G^{out}(\omega_0)$ represents correctly the frequency variation of $G_y(\omega)$.

Upon violation of this condition, however, which can naturally take place only in the frequency region $\omega \gg \nu_2$, we have again a gradual transition of $G^{out}(\omega_0)$ from a $1/f^2$ dependence to a $1/f$ dependence, similar to that shown in the figure, although the considered model spectral density $G_y(\omega)$ is known to be proportional to $1/f^2$ in this region.

Digital reduction of the measurements. In the case of digital method, the initial object is a temporal realization of the noise, measured at discrete points in equal steps τ_0 in time and of finite duration T_0. The measured set of points is then used to estimate the correlation function of the noise, followed by an estimate of the noise spectrum with the aid of the discrete Fourier transformation (DFT). As an alternative, the DFT can be directly applied to the measured realizations and the spectrum estimated in the form of the square of the modulus of the coefficients of the DFT. In some respect the analysis of digital methods, from our point of view, is even simpler, since the analog of formula (18) is already known in the literature (see, e.g., Ref. 11, Sec. 6.3.5). In our notation, the corresponding expression is of the form

$$G^{out}(\omega)=T_0\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}W(\omega-\omega')G_y(\omega'), \tag{23}$$

where $W(\omega)$ is the spectral window. Broadly extended windows can be written in the form (Ref. 11, Chap. 7)

$$W(\omega)\propto\left[\frac{\sin(\omega T_0/2n)}{\omega T_0/2n}\right]^n, \tag{24}$$

where $n=1,2,4$ corresponds respectively to a rectangular, Bartlett, and Parzen window. Comparing (23), (24) with (18), (21) we can easily verify that in the case of digital reduction of ILF noise measurements, a false observation of the spectrum of the $1/f^n$ type will take place. Such measurements are always carried out with a constant bandwidth ($\sim 1/T_0$), at least in each of several frequency intervals into which the entire investigated band is subdivided. As a result of effective "averaging" of the resultant curve over all the subbands, a spectrum of the type $1/f^n$ can be obtained with $1 \leq n \leq 4$, depending on the chosen window. Indeed, in digital reduction the value of n is most frequently larger than unity and is not an integer. For example, $n \approx 1.3$ and $1 < n \leq 1.4$ were obtained in Refs. 5 and 7, respectively.

In addition to the foregoing, there is one other effect that distorts the "pure" $1/f^n$ dependence in measurements of ILF noise. This effect is connected with the specific features of the DFT and is not taken into account by expression (24). To explain its character and to obtain a result that is correct over all the frequency band, we present a brief derivation of Eq. (23).

Assume we have an estimate of the correlation function $C_y(k\tau_0)$ in $2N$ discrete points ($k=-N, -N+1, \ldots, N-1$). By definition (Ref. 11, Chap. 7)

$$G^{out}(\omega)=\tau_0\sum_{k=-N}^{N-1}w(k\tau_0)C_y(k\tau_0)e^{i\omega k\tau_0}, \tag{25}$$

where $w(t)$ is the correlation window. Representing the correlation function in the form

$$C_y(k\tau_0)=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}G_y(\omega')e^{-i\omega' k\tau_0} \tag{26}$$

and substituting in (25), we obtain

$$G^{out}(\omega)=T_0\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}W(\omega-\omega')G_y(\omega'), \tag{27}$$

where $T_0=2N\tau_0$ is the total measurement time, and the spectral window is defined by the expression

$$W(\omega-\omega')=\frac{1}{2N}\sum_{k=-N}^{N-1}w(k\tau_0)e^{ik\tau_0(\omega-\omega')}. \tag{28}$$

In the case, e.g., of a Bartlett window

$$w(k\tau_0)=1-|k|\tau_0/N\tau_0 \tag{29}$$

and the summation in (28) yields

$$W(\omega)=\frac{\sin^2(N\omega\tau_0/2)}{2N^2\sin^2(\omega\tau_0/2)}. \tag{30}$$

A comparison of (30) with (24) shows that the use of the DFT led to replacement of $N\omega\tau_0/2$ in the denominator by $N\sin(\omega\tau_0/2)$. So long as the frequency ω is small compared with the Nyquist frequency, defined by the condition $\omega_N=\pi/\tau_0$, the continuous and discrete Fourier transformations yield practically identical expressions for the instrumental function (spectral window). As a rule, however, the calculations are carried out for frequencies in the interval $\pi/N\tau_0 \leq \omega \leq \omega_N$. In this case the frequency dependence of $G^{out}(\omega_0)$ in the presence of ILF noise is described in accordance with (27) and (30) by the formula

$$G^{out}(\omega)=\frac{T_0}{2}\frac{\sin^2(N\omega\tau_0/2)}{N^2\sin^2(\omega\tau_0/2)}\langle y^2\rangle, \tag{31}$$

from which it is obvious that a smooth transition takes place from a dependence $1/f^2$ on the low-frequency edge of the spectrum to saturation on its high-frequency edge. This behavior of the spectra can actually be observed on the figures in Refs. 5 and 7. When an attempt is made to approximate formula (31) by an $1/f^n$ law, this effect can also lead to non-integer values of n, in this case between 1 and 2. The frequent oscillations predicted by formula (1) should be most pronounced only on the low-frequency end of the band, where their period is comparable with the frequency ω. We note that in Ref. 7 there is mention of a "steplike character" of the obtained spectral estimates precisely at low frequencies. Generally speaking, the period of these oscillations coincides with the bandwidth of the spectral window (see the analogous analysis of the instrumental function of a diffraction grating in Ref. 12), and their appearance should depend on the choice of the set of values $\{\omega_j\}$, for which G^{out} is calculated, on the variance of the estimate of G^{out}, i.e., on the "noise" along the Y axis, on the random scatter of the instants of the readings, and on the accuracy with which the calculations are performed (the "noise" along the X axis).

It should be noted that in the literature on spectral analysis there is no unanimity with respect to the justification of the use of formula (25) for the estimate of the spectrum, and correspondingly different recommendations are made concerning the choice of the set of discrete frequencies $\{\omega_j\}$ (cf., e.g., Sec. 7.1.1 in Ref. 11

and Sec. 7.3.3 in Ref. 13). It appears that a more consistent approach should be based here on the idea of interpolating the functions by trigonometric polynomials, which will make it possible to indicate exactly the ratios of the DFT coefficients and the initial function, as well as permits a unique choice of the set $\{\omega_j\}$ and an estimate of the errors. An investigation of these questions, however, is outside the scope of the present paper. We indicate only that in our opinion it is more correct to calculate $G^{\text{out}}(\omega)$ at points $\omega_j \equiv 2\pi f_j = 2\pi j/T_0$, where $j = 0, 1, \dots, N$. The even harmonics occur then at the minima of the function (31), and the odd at the maxima. Owing to the finite noise level in the estimate of the spectral density, the zeros will not be emphasized, whereas the maxima remain practically unchanged. This reasoning enables us to replace, in the analysis that follows, $\sin^2(N\omega\tau_0/2)$ by its mean value.

2. ANALYSIS OF THE KNOWN EXPERIMENTS

Since the questions concerning interpretation of spectral measurements of low-frequency noise, considered in the present paper, have up to now apparently attracted no attention, the extensive literature on measurements of the $1/f$ noise contains highly skimpy information on the experimental conditions and on the methods of the data reduction in that part which is of interest here. The published recommendations on measurement methods[11,13,14] do not make it possible to exclude the possibility of an erroneous interpretation of the observed spectra of the $1/f^n$ type. At the present time, it seems to us that two alternatives can be offered: 1) some part of the available experimental results does indeed offer evidence of the presence of $1/f$ noise, and 2) there is no such noise in nature at all. Until a thorough analysis of the available results is made (which most likely can be made only by the authors of the works themselves), or until new experiments are made in which the necessary control measurements that follow from the present paper are taken, it will apparently be necessary for the time being to assume that there is no convincing proof of the existence of "genuine" $1/f$ noise. However, even if the second alternative is assumed, an explanation must be found for the really observed dependences of the $1/f^n$ type, and, in particular, the presently prevailing opinion that the lower limit of the $1/f$ spectra has at present not been reached. Without claiming to present a final solution, we advance certain leading arguments.

As follows from (21) and (31), the presence of ILF noise at the input of the analyzer makes the output signal proportional to the mean square of the fluctuations and to the value of the instrumental function, and inversely proportional to the analyzer band width. No information on the true ILF noise spectrum can be obtained under these conditions, since the noise turns out to be completely integrated. However, the ILF noise should satisfy the condition that its correlation time is much longer than the time of establishment of the transient processes in the spectrum analyzer in analog measurements, or the duration of the realization in digital measurements. One should therefore consider as physical mechanisms those that can have very long correlation times. All of them, however, had as a rule not a $1/f$ spectrum, but one of the relaxation type (9) and were therefore rejected. Our reasoning is not subject to this restriction. The noise spectrum, provided it is connected with a sufficiently slow process, can be arbitrary (but, of course, integrable), including also of type (9). This allows us, when explaining electrical noise, to turn to models in which it is assumed that the presence of the current noise is connected with fluctuations produced in the sample conductivity σ by equilibrium fluctuations of the thermodynamic parameters (of the temperature T and of the number N of the carriers), i.e.,

$$\delta U = -I\left(\frac{\partial\sigma}{\partial T}\delta T + \frac{\partial\sigma}{\partial N}\delta N\right)\Big/\sigma^2, \tag{32}$$

where U is the voltage and I is the current through the sample.

Temperature fluctuations can appear in temperature-sensitive materials, e.g., in the case of volume noise in semiconductors or in metallic films. In the case of contact and surface noise, fluctuations in the number of carriers can lead to fluctuations of the charge in the capacitance of barrier structures, and modulate by the same token the height of the barrier. In this case neither type of fluctuation can in principle be removed, and their mean square depends only on the temperature, on the volume of the sample, and on the specific characteristics e.g.,

$$\langle\delta T^2\rangle = k_B T^2/c_V V, \tag{33}$$

where c_V is the specific heat and V is the volume of the sample. For a selected volume we can only vary the spectrum of the temperature fluctuations

$$G_T(\omega) = \langle\delta T^2\rangle\tau/(1+\omega^2\tau^2), \tag{34}$$

by varying the conditions of energy exchange between the system and the ambient, and by the same token varying τ. The better the sample is insulated against heat exchange with the ambient, the longer the time of establishment of the temperature and the lower the frequency of the temperature fluctuations. It should be noted that in all attempts to measure $1/f$ noise down to very low frequencies the samples are always kept in a thermostat, and as a rule the time required for the temperature to assume the study value apparently exceeds the measurement time and exceeds all the more the reciprocal bandwidth of the analyzer, so that to decrease the variance of the measurement results one always tends to satisfy the condition $(T_0\Delta)^{1/2} \gg 1$. This may explain why there is no lower limit of the $1/f^n$ dependence in such measurements. It can also be assumed that in those cases when the $1/f$ dependence gave way to a low-frequency plateau, such measurements were simply discarded as not relevant to the $1/f$-noise problem. We indicate incidentally that in Ref. 15 a case is noted, in which a transition from a $1/f^n$ dependence that increases towards lower frequencies was observed when the rate of heat exchange of the sample was increased (τ was decreased).

It follows from the foregoing, in our opinion, that on the basis of the formulated notion concerning the ILF noise it is possible to construct an incontrovertible picture wherein the main features of the experimental results on $1/f$ noise are attributed to the presence of ILF noise. The final solution of the question, however, must undoubtedly be provided by experiment. We indicate only that among the candidates for slow processes that can appear in strain-sensitive structures, one can possibly include also tidal variations of the force of gravity. The variation of the force of gravity is of the order of $\delta g/g \sim 10^{-7}$, which is readily measurable, and these forces are likewise impossible to eliminate in practice. Their spectrum is rich in harmonics concentrated around the semidiurnal, diurnal, monthly, annual, nine-year and 18-year cycles, and also contain a considerable fluctuating component (see, e.g., Refs. 16 and 17). Finally, inclusion into consideration of slow fluctuations near phase transitions and in nonequilibrium systems, as well as of the variations of the geophysical parameters extends even more the group of possible slow processes.

It is of interest to attempt to analyze quantitatively the experimental results of Clarke and Voss[7] from the point of view of the possible presence of ILF noise in their measurements. The authors of Ref. 7 have become convinced that their measurements point to equilibrium fluctuations of the temperature as the cause of the low-frequency noise in thin metallic films. They have therefore assumed that the spectral density of the voltage fluctuations on the films is proportional to $\langle \delta T^2 \rangle$, and presented all the data that make it possible to calculate the contribution of such fluctuations to the current noise. Voss and Clarke, however, thought it necessary to multiply $\langle \delta T^2 \rangle$ by a certain frequency dependent factor, which would yield a dependence of the $1/f$ form in the measured frequency region. They settled ultimately on a model spectrum chosen after analyzing the process of heat diffusion in a thin film, with allowance for its geometry, and wrote

$$G_U(\omega)/U^2 = \beta^2 \langle \delta T^2 \rangle / [3 + 2\ln(l/w)] f, \qquad (35)$$

where $\beta = R^{-1} dR/dT$, R is the resistance of the film, l and w are its length and width [cf. (34)].

From our point of view, taking into account the data given in Ref. 7 on the reduction procedure, we can find that in this case a formula of type (31) should be valid, and namely (G^+ is the spectral density in the positive frequencies)

$$\frac{G_U^{+\,out}(\omega)}{U^2} = \frac{2 \cdot 0.5}{\pi^2 M f \tau_0} \frac{\beta^2 \langle \delta T^2 \rangle}{f}, \quad f = \frac{\omega}{2\pi} > 0, \qquad (36)$$

where $\sin^2(M\omega\tau_0/2)$ is replaced by 0.5 and M is the number of counts. $(M\tau_0)^{-1}$ is the width Δ of the spectral window [cf. formula (6) in Ref. 12], so that we have from (36)

$$\frac{G_U^{+\,out}(\omega)}{U^2} = \frac{1}{\pi^2 (f/\Delta)} \frac{\beta^2 T^2}{3 N_a f}, \qquad (37)$$

where N_a is the number of atoms in the volume of the film and is obtained from the classical expression for the heat capacity $3N_a k_B$ [see (33)].

Formula (37) can be applied to a calculation of noise in two films, Bi and Au, for which geometrical dimensions are given in Ref. 7 (see Figs. 1 and 2 of Ref. 7), and quantitative data are available on the noise measured at 10 Hz. It is necessary only to find $\Delta = (M\tau_0)^{-1}$ by using the information given in Ref. 7. Assuming the total number of points in the realization to be $M = 1024$, and recognizing that in the case of Bi the high-frequency limit of the measured band is 10^3 Hz, we obtain $1/2\tau_0 = 10^3$ Hz, whence $\Delta_{Bi} = 2$ Hz. For the Au film we estimate Δ by choosing a value of $1/M\tau_0$ equal to the limiting frequency of the measurement band on Fig. 2 of Ref. 7, which yields $\Delta_{Au} = 1$ Hz. Obtaining for $N_a = VN_a$ the values $1.2 \times 10^{-10} \times 2.84 \times 10^{22}$ and $1.25 \times 10^{-10} \times 6 \times 10^{22}$ for the Bi and Au films, respectively, and substituting in (37) all these numbers together with the experimentally measured[7] values $\beta_{Bi} = -2.9 \times 10^{-3}$ K^{-1} and $\beta_{Au} = 1.2 \times 10^{-3}$ K^{-1}, we obtain

$$G_{Bi}^{+out}\ (10\,\text{Hz})/U^2 = 15 \cdot 10^{-17}\ \text{Hz}^{-1},\ G_{Au}^{+out}\ (10\,\text{Hz})/U^2 = 0.58 \cdot 10^{-17}\ \text{Hz}^{-1}. \qquad (38)$$

The values measured in Ref. 7 are 13×10^{-16} Hz^{-1} (Bi) and 0.6×10^{-16} Hz^{-1} (Au), while the values calculated there, using the model spectrum (35), are respectively 9.3×10^{-16} and 0.76×10^{-16} Hz^{-1}. As seen from a comparison of these data with (38), our analysis makes it possible to estimate quantitatively[1)] the measured noise in metallic films by means of formula (37), expressing this noise in terms of the mean squared thermodynamic fluctuations of the temperature and the instrumental function. We have introduced here no additional complications whatever, unlike in Ref. 7, where a model spectrum was used to explain the frequency dependence of the measured signal. In addition to this quantitative estimate, formula (37) yields also a frequency dependence that is closer to experiment ($\sim 1/f^2$) than formula (35). The authors of Ref. 7 attempted to attribute the $1/f^2$ spectrum in their measurements to the presence of a slow monotonic drift. Actually, this mechanism can lead to a discontinuity in the realization at the edges of the interval when the realization is periodically continued (this continuation is in fact carried out when the DFT is used), and consequently also to the $1/f^2$ frequency dependence of the power spectrum. However, the discontinuities of the first kind in the realization should lead to a $\sim 1/f^2$ dependence on the high-frequency and of the measurement range, and not on the low-frequency one as in experiment (see Fig. 1 of Ref. 7). The difference noted by Voss and Clarke between the spectra of the response to a step perturbation and to a δ pulse can also be understood by turning to formula (27) [or (2)]. In the former case we have at the input of the analyzer a signal whose spectrum is concentrated in the region of very low frequencies, and in the latter a signal with almost uniform frequency spectrum of the power. Since in the former case the output signal is given again by (31), in which only $\langle y^2 \rangle$ should be replaced by the total power of the action, it becomes perfectly obvious why the signal measured in this case had the same frequency dependence as the measured output spectrum following the action of an ILF noise, while no such correlation was observed in the response to a δ pulse.

All the foregoing makes it quite probable that ILF noise due to equilibrium fluctuations of the temperature was present in the experiments of Voss and Clarke.[7]

One more deduction can be drawn from (37), namely a symbolic expression for the Hooge constant α [see (1)]. It is obvious, that for the considered case of the noise in metallic films in the experiments of Voss and Clarke

$$\alpha=\frac{\beta^2T^2}{3\pi^2(f/\Delta)}. \qquad (39)$$

An expression of the same type for α (but four times larger) will hold if the measurements are carried out with an analog spectrum analyzer having a Lorentz contour, as follows from (12). Using the values of β from Ref. 7 and putting $T=300$ K and $(f/\Delta)\approx 10$, we obtain from (39) the following values:

Film material:	Cu	Ag	Au	Sn	Bi
$\alpha\cdot 10^3$:	4.4	3.7	0.44	3.9	2.55

It is seen that the calculated values of α correlate satisfactorily with the "mean-statistical" value 2×10^{-3} obtained empirically by Hooge.[6] It must be emphasized, however, that according to (39) α depends not only on the sample parameters and on the temperature, but also on the chosen method of the spectral analysis and resolution. It is also obvious that from the point of view of the ILF-noise premises, the dilemma posed by Voss and Clarke, whether N_a is the number of atoms or the number of free carriers, can be differently resolved, depending on the nature of the source of the fluctuations.

3. CONCLUSIONS

It appears that one of the consequences of the present paper may be a complication of the measurement procedure on account of the need for monitoring the "physical content" of the measured signal. To avoid as much as possible an investigation of the far wing of the frequency characteristic of the analyzer, we can propose, as a test, to place at its input a low-pass filter whose cutoff frequency lies in the investigated region. Then the input to the analyzer will be a random process with a spectral density not $G_y(\omega)$, but $G_y(\omega)|z(\omega)|^2$. If these operations alter the frequency dependence of the output signal in the region of the frequency cutoff, this means that in the absence of the low-pass filter the analyzer yields the spectral density of the input process. In the case of ILF noise, on the other hand, the insertion of a low-pass filter should not lead to a change in the frequency dependence of the output signal.

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[1)]The cause of the discrepancy with the experiment may be the fact that the calibration chosen by us (4b) does not correspond to the one actually employed (on which there is no information in Ref. 7), or else the possible presence in such samples of additional noise of uncertain origin (see Ref. 18).

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