Nonlinear Landau-Zener effects in the absorption spectra of accelerated atoms

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We consider nonlinear effects in the absorption spectra of atoms and ions subjected to a constant acceleration a. We demonstrate nonlinear saturation of the absorbed power on increase in the interaction V between atoms and the field, governed by the parameter $p = 2\pi V^2/ka$ (k is the wave vector), analogous to the Landau-Zener parameter in the theory of inelastic transitions. We show that a population inversion can appear in an initially uninverted two-level system subjected to an electromagnetic field and acceleration. The effects discussed are important in the kinetics of ionic lasers.

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1. INTRODUCTION

A change in the velocity of an atom interacting with radiation in a plasma or a gas has a considerable influence on the dynamics of absorption of this radiation, absorption spectrum, populations of the excited states, etc. This is due to the fact that one of the main broadening mechanisms is the Doppler effect which causes an effective shift of the frequency of an atomic oscillator by an amount kv ($k=\omega/c$ is the wave vector of radiation whose frequency is ω, and v is the velocity of an atom)—see, for example, Ref. 1.

Deformation of the spectra of atoms subjected to a constant acceleration a was considered by Kol'chenko, Rautian, and Smirnov[2,3] (see also the book by Rautian *et al.*[4]) using the first order of perturbation theory in respect of the interaction V of an atom in the field of an electromagnetic wave. The advantage of the perturbation theory treatment[2–4] is the ability to calculate the effect in question for any type of relaxation and any acceleration, provided V is sufficiently small. We shall go beyond the perturbation theory framework and study new nonlinear effects in the absorption of light caused by acceleration. The essence of these effects can be stated as follows. When the Doppler broadening is sufficiently large, the main contribution to the absorption is made at the resonance points defined by $v=v_\omega=\Delta\omega/k$ ($\Delta\omega=\omega-\omega_0$ is the frequency shift of the absorbed radiation ω relative to the unperturbed frequency of the atomic transition ω_0). Atoms whose velocity is $v<v_\omega$ are accelerated and can reach the resonance value $v=v_\omega$ and then absorb a photon. Here, the Doppler phase shift kv, which acquires a definite direction under the action of acceleration, intersects characteristically a resonance value $kv_\omega=\Delta\omega$. At the resonance intersection point itself the mechanism of the transition responsible for the absorption of light is fully analogous to the mechanism of inelastic Landau–Zener transitions well known from the physics of atomic collisions.[5,6] The parameter p governing the probability of a transition and, consequently, the nonlinear effects in absorption, has a typical Landau–Zener structure: $p=2\pi V^2/ka$. If $p\ll 1$, then the perturbation theory results are obtained,[2–4] whereas for $p\gg 1$ new nonlinear effects of saturation of the absorbed power are observed.

An important consequence of the theory developed below is the feasibility of a population inversion of an initially uninverted two-level system subjected to an electromagnetic field and acceleration. It should be pointed out that the Landau–Zener nonlinearities have been investigated earlier for the collisional-broadening spectra.[7] We should mention particularly the work of Vartanyan, Przhibel'skiĭ, and Chigir',[8] who checked these nonlinearities experimentally using a specially selected light source with a time-dependent emission frequency. We shall consider the characteristics of the Landau–Zener nonlinearities in the spectra of accelerated atoms experiencing the Doppler broadening.

The nonlinear absorption effects are considered in Sec. 2 using the simplest model of a two-level system without relaxation. Relaxation of the levels is introduced in Sec. 3 and a study is made of the relationship between its effects and the effects of saturation in the case of inhomogeneous broadening. Population inversion of atomic levels is considered in Sec. 4 and generalization to the case of unequal relaxation constants of the levels is made in Sec. 5. The range of validity of the effects under discussion is considered in Sec. 6.

2. LANDAU–ZENER NONLINEARITIES IN THE SPECTRA OF A TWO-LEVEL SYSTEM SUBJECTED TO ACCELERATION

The system of equations for the elements of the density matrix of a spatially homogeneous gas of two-level atoms interacting with a resonant traveling electromagnetic wave and subjected to a constant acceleration a has the following form in the steady-state case (Ref. 4, p. 272):

$$\left(\gamma+a\frac{d}{dv}\right)N(v)=\Delta q(v)+2iV(\rho-\rho^*), \quad (2.1)$$

$$\left(\gamma+a\frac{d}{dv}\right)\rho(v)=iVN-i(kv-\Delta\omega)\rho. \quad (2.2)$$

Here, $N(v)=\rho_{mm}(v)-\rho_{nn}(v)$; $\Delta q(v)=(q_m-q_n)W(v)$ is the pumping; $\rho=\rho_{mn}(v)$; $V=d_{mn}E_0/\hbar$ is a matrix element of the dipole interaction of an atom with a wave field E_0; $\Delta\omega=\omega-\omega_{mn}$; k is the wave vector of the wave (it is assumed that the directions of acceleration and wave propagation are the same). The absorbed power is given by

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$$P(\omega)=2\hbar\omega V\int_{-\infty}^{\infty}\operatorname{Im}\rho(v)\,dv. \qquad (2.3)$$

The system (2.1)–(2.2) can be solved by directly expressing all the quantities occurring near it in terms of, for example, Imρ. However, this solution procedure is fairly complex (it is necessary to solve third-order equations with variable coefficients). Therefore, we shall adopt a different solution method: we shall use the fact that in this case the density matrix and amplitude methods are equivalent.[4] With this in mind, we shall reduce the system (2.1)–(2.2) to a system of equations from which the damping γ and the pumping $\Delta q(v)$ are absent.

We shall seek solutions $N(v)$ and $\rho(v)$ of the system (2.1)–(2.2) in the form

$$N(v)=\int_{-\infty}^{\infty}dv_0\,\Delta q(v_0)\exp\left[\frac{\gamma}{a}(v_0-v)\right]\frac{\Theta(v-v_0)}{a}n(v,v_0), \qquad (2.4)$$

$$\rho(v)=\int_{-\infty}^{\infty}dv_0\,\Delta q(v_0)\exp\left[\frac{\gamma}{a}(v_0-v)\right]\frac{\Theta(v-v_0)}{a}r(v,v_0). \qquad (2.5)$$

Here, $\Theta(x)$ is the step function; $\Theta(x)=0$ if $x<0$ and $\Theta(x)=1$ if $x\geqslant 0$; $n(v,v_0)$ and $r(v,v_0)$ are functions satisfying the following systems of equations

$$a\frac{dn}{dv}-2iV(r-r^*)=0,$$
$$a\frac{dr}{dv}+i(kv-\Delta\omega)r-iVn=0 \qquad (2.6)$$

and the initial conditions

$$n(v_0,v_0)=1,\quad r(v_0,v_0)=r^*(v_0,v_0)=0. \qquad (2.7)$$

The system (2.6) with the initial conditions (2.7) is equivalent to the following system describing the amplitudes of the atomic states b_1 and b_2 ($\xi=v-v_\omega$ is used below):

$$ia\frac{d}{d\xi}b_1=-\frac{k\xi}{2}b_1-Vb_2,\quad b_1(\xi_0)=1,$$
$$ia\frac{d}{d\xi}b_2=\frac{k\xi}{2}b_2-Vb_1,\quad b_2(\xi_0)=0, \qquad (2.8)$$

if we assume that

$$n=|b_1|^2-|b_2|^2,\quad r=b_1^*b_2,\quad r^*=b_1b_2^*. \qquad (2.9)$$

The system (2.8) is identical, apart from the notation, with the system obtained in considering the inelastic Landau–Zener transition in the problem of level crossing.[5,6] In fact, the time derivatives become in our case the derivatives with respect to v, and the linearly varying terms become the Doppler frequency shifts kv (or $k\xi$). The level crossing condition in the Landau–Zener theory corresponds clearly to the resonance condition $\Delta\omega=kv$ (or $\xi=0$) in our case.

The system (2.8) does not include the relaxation parameter γ, which occurs only in Eqs. (2.4) and (2.5) in the form of simple damping. In investigating the physical singularities of the problem it would be interesting to consider the case of sufficiently small values of γ (or large values of a), when $\gamma\equiv 0$ can be substituted in Eqs. (2.4) and (2.5) (the criterion for this simplification will be obtained later).

We shall express the absorbed power in terms of the amplitudes $b_{1,2}$. Eliminating b_1 by means of the second equation of the system (2.8), we obtain

$$\operatorname{Im}(b_1^*b_2)=\frac{a}{2V}\frac{d}{dv}|b_2|^2. \qquad (2.10)$$

It follows from Eqs. (2.3)–(2.5) (where we substitute $\gamma=0$), and from Eqs. (2.9) and (2.10)

$$P(\omega)=\Delta q\hbar\omega\int_{-\infty}^{\infty}dv_0\,W(v_0)\int_{v_0}^{\infty}dv\frac{d}{dv}|b_2|^2. \qquad (2.11)$$

Employing the symbol $\langle\ldots\rangle_W$ for the averaging over a Maxwellian distribution $W(v_0)$, we can write down Eq. (2.11) in the form

$$P(\omega)=\hbar\omega\Delta q\langle|b_2(\infty,v_0)|^2\rangle_W. \qquad (2.12)$$

Thus, in the absence of relaxation the absorption in a system of two-level systems is governed by the asymptotic behavior of the amplitude b_2 (or b_1). We shall therefore turn back to the system (2.8). A general solution of the Landau–Zener system of equations can be expressed in terms of confluent hypergeometric functions $\Phi(a,c,z)$. A particular solution satisfying the initial conditions of Eq. (2.8) is as follows[9,10]:

$$b_1=\exp[-(z_0^2+z^2)/2]\{\Phi_1(z)\Phi_2'(z_0)-\Phi_1'(z_0)\Phi_2(z)\},$$
$$b_2=(ak/V^2)^{1/2}\exp[-3\pi i/4-(z_0^2+z^2)/2]$$
$$\times\{\Phi_1'(z)\Phi_2'(z_0)-\Phi_1'(z_0)\Phi_2'(z)\}. \qquad (2.13)$$

Here,

$$\Phi_1(z)\equiv\Phi(-\nu,{}^1/_2,z^2),\quad \Phi_2(z)\equiv z\Phi({}^1/_2-\nu,{}^1/_2,z^2),$$
$$z^2=e^{-i\pi/2}k\xi^2/2a,\quad z_0^2=e^{-i\pi/2}k\xi_0^2/2a,\quad \nu=iV^2/2ak.$$

The expressions (2.12) and (2.13) allow us to solve the problem of the absorption spectrum of accelerated atoms in an arbitrary field. In the limit of weak fields V, we can use the relationship between the functions $\Phi(a,c,z)$ in the limit $\nu\to 0$ with the Fresnel integrals[9] and show that Eqs. (2.11) and (2.13) yield the expression (21.19) in Ref. 4 if all the relaxation constants Γ and Γ_j tend to zero:

$$P(\omega)=2\hbar\omega V^2\Delta q\int_0^{\infty}dt\exp\left[-\left(\frac{k\bar v}{2}\right)^2t^2\right]\frac{\sin(kat^2/2-\Delta\omega t)}{kat}. \qquad (2.14)$$

We shall now obtain an expression for the absorption spectrum in an arbitrary field V. Using the properties of the confluent hypergeometric function,[10] we find that fairly lengthy but simple calculations give

$$|b_2(\infty,v_0)|^2=\frac{\pi}{2}\frac{V^2}{ak}e^{i\pi\nu}\left|\frac{\Phi_2'(z_0)}{\Gamma(1-\nu)}-2z_0\frac{\Phi(1-\nu,{}^3/_2,z_0^2)}{\Gamma({}^1/_2-\nu)}\right|^2. \qquad (2.15)$$

In the case of arbitrary but finite values of the field intensity V and high values of $|(k/2a)^{1/2}(v-v_\omega)|$, we find that

$$|b_2|^2_{(v_0-v_\omega)\to\pm\infty}=\Theta(v_\omega-v_0)(1-e^{-p}),\quad p=2\pi V^2/ak. \qquad (2.16)$$

According to Eq. (2.12), $|b_2|^2$ can be averaged over the Maxwellian distribution $W(v_0)$. The hypergeometric functions in Eq. (2.15) can be replaced with their asymptotic expressions if the width of the region of large changes $\Delta v_{\rm eff}$ is less than the thermal velocity $v_T=(2T/m)^{1/2}$ (T is the absolute temperature and m is the mass of an atom).

The value of $\Delta v_{\rm eff}$ can be estimated from the characteristic scale of the change in the hypergeometric function (compare with Ref. 11):

$$\Delta v_{\rm eff}\approx\begin{cases}(a/k)^{1/2}, & V^2/ak\ll 1\\ V/k, & V^2/ak\gg 1\end{cases}. \qquad (2.17)$$

If $\Delta v_{\rm eff} \ll v_T$, we can assume that $|b_2|^2$ has an abrupt jump at $v_0=v_\omega$; then, applying the asymptotic expression (2.16), we find that the absorbed power is

$$P(\omega)=\hbar\omega\Delta q(1-e^{-p})\int_{-\infty}^{v_\omega} dv_0\, W(v_0). \tag{2.18}$$

If $V^2/ak \ll 1$, Eq. (2.18) reduces to the perturbation theory result given by Eq. (2.14). The nonlinear dependence of the absorbed power on the intensity of light is related to the equalization of the level populations; moreover, in the case of an atomic system without damping there are no average steady state level populations at all. The problem of level populations is considered in detail in the following sections.

3. ALLOWANCE FOR THE RELAXATION OF LEVELS. RELATIONSHIP TO THE THEORY OF INHOMOGENEOUS BROADENING

We shall now allow for the decay of levels ($\gamma \neq 0$) in Eqs. (2.4) and (2.5). Introduction of γ makes it possible to consider a new limiting case of inhomogeneous broadening. It can be deduced both from the stationary solution of the system (2.1)–(2.2), where we substitute $a=0$, and from the nonstationary solution of the system ($a\neq 0$) by going to the limit $a\to 0$, and this is by far no trivial a problem.

The absorbed power is no longer given by Eq. (2.11) but by

$$P(\omega)=\Delta q\hbar\omega\int_{-\infty}^{\infty} dv_0\, W(v_0)\int_{v_0}^{\infty} dv\, \exp\left[\frac{\gamma}{a}(v_0-v)\right]\frac{d}{dv}|b_2|^2. \tag{3.1}$$

The nature of the behavior of the nonlinear effect depends on the ratio of the size $\Delta v_{\rm eff}$ of the region of rapid variation of the exponential function and the amplitude in Eq. (3.1). In case of the exponential function this ratio is

$$\Delta v_{\rm eff}\approx a/\gamma, \tag{3.2}$$

whereas for the amplitude its value has been obtained earlier in Eq. (2.17). We shall show that if the condition

$$\gamma V/ka \gg 1 \tag{3.3}$$

is satisfied, then Eq. (3.1) yields an expression describing inhomogeneous broadening, and if

$$\gamma V/ka \ll 1 \tag{3.4}$$

it describes nonlinear absorption of the (2.18) type.

When the condition (3.3) is satisfied, the argument and the index of the functions Φ_1 and Φ_2 in Eq. (2.13) are large. Employing the appropriate asymptotics of the hypergeometric functions [Eq. (4.5.21) in Ref. 10], we can reduce the expression for b_2 to

$$b_2(\xi,\xi_0)=V\left(\frac{\operatorname{th}\alpha\operatorname{th}\alpha_0}{(k\xi)(k\xi_0)}\right)^{1/2}\{\exp(\nu[h(\alpha_0)-h(\alpha)])-\exp(\nu[h(\alpha)-h(\alpha_0)])\}, \tag{3.5}$$
$$\operatorname{sh}^2\alpha=(k\xi)^2/4V^2,\quad \nu=iV^2/2ak,\quad h(\alpha)=2\alpha+\operatorname{sh}2\alpha.$$

Introducing a new variable $t=(v-v_0)/a$, we can show that if the condition (3.3) is satisfied, the contribution to the integral t is made by a narrow region near ξ_0. Expanding $|b_2|^2$ to the vicinity of the point ξ_0 with the aid of Eq. (3.5), we find that the first nonvanishing term is

$$|b_2|^2=2\frac{V^2}{\Omega_0^2}[1-\cos\Omega_0 t],\quad \Omega_0^2=(k\xi_0)^2+4V^2. \tag{3.6}$$

Hence, integration with respect to t gives

$$P(\omega)=2\hbar\omega\Delta q\int_{-\infty}^{\infty} dv_0\, W(v_0)\frac{V^2}{\gamma^2+4V^2+(\Delta\omega-kv_0)^2}$$

which is identical with the well-known result in the theory of inhomogeneous broadening.[4]

We shall now show that in the opposite case of sufficiently weak relaxation ($\gamma V/ka \ll 1$), Eq. (3.1) describes, like Eq. (2.1), nonlinear absorption. In fact, it follows from Eqs. (2.16)–(2.18) and (3.4) that $|b_2|^2$ is in the form of a step and its derivative is a sharp peak at $v=v_\omega$. We shall take outside the exponential function, which varies slowly at this point, and we shall replace $|b_2|^2$ with the asymptotic expression (2.16). Then, the absorbed power is given by

$$P(\omega)=\hbar\omega\Delta q(1-e^{-p})\int_{-\infty}^{v_\omega} dv_0\, W(v_0)\exp\left[\frac{\gamma}{a}(v_0-v_\omega)\right]. \tag{3.7}$$

Substitution of the variable $v_0=v_\omega+a\tau$ readily shows that, apart from a factor allowing for the nonlinear dependence of the field, Eq. (3.7) is identical with Eq. (21.20) from Ref. 4:

$$P(\omega)=-2\hbar\omega V^2\frac{\Delta q}{\gamma}\bar{K}(\Delta\omega),$$
$$\bar{K}(\Delta\omega)=\pi[1-\Phi(\tilde{z})]\left(\exp\left[\tilde{z}^2-\left(\frac{\Delta\omega}{kv_T}\right)^2\right]\right)\Big/\frac{2ka}{\gamma}, \tag{3.8}$$
$$\tilde{z}=kv_T/(2ka/\gamma)-\Delta\tilde{\omega}/kv_T.$$

Here, $\Phi(\tilde{z})$ is the error function.

In the limit of strong fields, the absorbed power is independent of the field intensity and we shall show later that this saturation is not related to the equalization of the level populations. If $\gamma v_T/a \ll 1$, Eq. (3.7) reduces to Eq. (2.18). In the opposite case, when $\gamma v_T/a \gg 1$, it follows from Eq. (3.7) that

$$P(\omega)=\hbar\omega\Delta q(1-e^{-p})\frac{a}{\gamma kv_T\pi^{1/2}}\exp\left[-\left(\frac{\Delta\omega}{kv_T}\right)^2\right]. \tag{3.9}$$

Thus, if the conditions

$$\gamma V/ka \ll 1,\quad \gamma v_T/a \gg 1,\quad V^2/ka \gtrsim 1 \tag{3.10}$$

are satisfied, the absorbed power is directly proportional to the acceleration of atoms.

4. ADIABATIC INVERSION OF THE POPULATIONS OF ATOMIC LEVELS

We shall consider the question of the level populations by supplementing the system (2.1)–(2.2) with an additional equation for the total population of the levels m and n given by $O(v)=\rho_{mm}(v)+\rho_{nn}(v)$:

$$\left(\gamma+a\frac{d}{dv}\right)O(v)=(q_m+q_n)W(v). \tag{4.1}$$

We shall use

$$\langle\ldots\rangle=\int_{-\infty}^{\infty} dv(\ldots).$$

It follows from Eq. (4.1) that the total integrated population of the levels is

$$\langle O\rangle=(q_m+q_n)/\gamma, \quad (4.2)$$

since $\langle W\rangle=1$.

Integrating the first of the equations in the system (2.1)–(2.2) with respect to v and taking account of Eq. (2.3), we obtain the following power-balance equation:

$$\hbar\omega\gamma\langle N\rangle/2+P(\omega)=\hbar\omega\Delta q/2. \quad (4.3)$$

Here, the individual terms have the following meaning: $\hbar\omega\gamma\langle N\rangle/2$ is the power absorbed by the atoms; $P(\omega)$ is the work done by the field per unit time; $\hbar\omega\Delta q/2$ is the power contributed by pumping. Since

$$\rho_{mm}=(O+N)/2, \quad \rho_{nn}=(O-N)/2,$$

it follows from Eqs. (4.2) and (4.3) that

$$\langle N\rangle=\frac{1}{\gamma}\left[\Delta q-\frac{2p(\omega)}{\hbar\omega}\right], \quad \langle\rho_{mm}\rangle=\frac{1}{\gamma}\left[q_m-\frac{p(\omega)}{\hbar\omega}\right], \quad \langle\rho_{nn}\rangle=\frac{1}{\gamma}\left[q_n+\frac{p(\omega)}{\hbar\omega}\right]. \quad (4.4)$$

The expression for $\langle N\rangle$ can be obtained also directly from Eqs. (2.4) and (3.1) if we bear in mind that $|b_1|^2+|b_2|^2=1$. Using Eq. (3.1) and the previous notation $\langle\ldots\rangle_W$ [see Eq. (2.12)], we find that the integrated difference between the populations is

$$\langle N\rangle=\frac{\Delta q}{\gamma}\left\langle 1-2\int_{v_0}^{\infty}dv\exp\left[\frac{\gamma}{a}(v_0-v)\right]\frac{d}{dv}|b_2(v,v_0)|^2\right\rangle_W. \quad (4.5)$$

Transformations similar to those used in the derivation of Eq. (3.8) reduce (4.5) to

$$\langle N\rangle=\frac{\Delta q}{\gamma}\left\langle 1-2\exp\left[\frac{\gamma}{a}(v_0-v_\omega)\right](1-e^{-p})\Theta(v_\omega-v_0)\right\rangle_W. \quad (4.6)$$

We can easily see that if the conditions

$$p\gg1, \quad \gamma v_T/a\ll1, \quad v_\omega/v_T\gg1 \quad (4.7)$$

are satisfied, the sign of $\langle N\rangle$ is opposite to that of Δq, i.e., the field causes an inversion of the level populations in the atomic subsystem. The unperturbed population of the upper level can be higher or lower than the population of the lower level, depending on the pumping.

The physical meaning of this result is easily understood using the distribution of levels in the Landau–Zener model. In fact, for high values of $V(p\gg1)$ the system moves between "adiabatic" terms (separated at the crossing point by V) terms, which corresponds to transitions with the probability of 1 between the initial noncrossing ("diabatic") terms corresponding to the levels 1 and 2 of the original atom. Consequently, an atom initially entirely in the state 1 finds itself in the vacant state 2. The state 1 is then emptied completely. On the basis of the above discussion, it is natural to call this effect an adiabatic population inversion.

5. GENERALIZATION TO THE CASE OF DIFFERENT RELAXATION TIMES

We shall now consider the case when the levels m and n have different lifetimes: $\gamma_n\neq\gamma_m$. We shall confine ourselves to the case corresponding to direct $m\to n$ decay ($\gamma_{mn}\equiv0$). In fact, following the procedure in Sec. 2, we can reduce the system of equations for the density-matrix elements to a system of equations for the amplitudes b_m and b_n of the atomic states in which the process of relaxation is represented by the following combinations:

$$\Delta\gamma=(\gamma_m-\gamma_n)/2, \quad \gamma=(\gamma_m+\gamma_n)/2. \quad (5.1)$$

We shall seek the solution of this problem in the form of a sum of the solutions corresponding to the pumping of either the level m or the level n. In the system for the amplitudes this corresponds to the following initial conditions:

$$b_m^{(1)}(\xi_0)=1, \quad b_n^{(1)}(\xi_0)=0, \quad b_m^{(2)}(\xi_0)=0, \quad b_n^{(2)}(\xi_0)=1.$$

Then, the total absorbed power $P(\omega)$ is given by

$$P(\omega)=\hbar\omega\left\langle\frac{1}{a}\int_{v_0}^{\infty}dv\exp\left[\frac{\gamma}{a}(v_0-v)\right]\times\{q_m(\gamma-\Delta\gamma)|b_n^{(1)}|^2-q_n(\gamma+\Delta\gamma)|b_m^{(2)}|^2\}\right\rangle_W, \quad (5.2)$$

where q_m and q_n are the rates of pumping of the levels m and n.

In contrast to Sec. 2, the amplitudes b_n and b_m depend on $\Delta\gamma$. As established earlier, new nonlinear effects then appear if the width of the region where the amplitudes vary significantly is much less than a/γ. The width of the transition region $\Delta v_{\rm eff}$ now depends[10] also on $\Delta\gamma$:

$$\Delta v_{\rm eff}\approx\max\left\{\frac{V}{k},\ \left(\frac{a}{k}\right)^{1/2},\ \frac{\Delta\gamma}{k}\left(1+\frac{V^2}{ka}\right)\right\}. \quad (5.3)$$

If we assume that the condition $\Delta v_{\rm eff}\ll a/\gamma$ is satisfied, we can use the asymptotic expressions for $|b_n^{(1)}|$, and $|b_m^{(2)}|$, so that after simple transformations we obtain

$$P(\omega)=\hbar\omega(1-e^{-p})\int_{-\infty}^{v_\omega}dv_0\,W(v_0)\times\left\{q_m\exp\left[\frac{\gamma_m}{a}(v_0-v_\omega)\right]-q_n\exp\left[\frac{\gamma_n}{a}(v_0-v_\omega)\right]\right\}. \quad (5.4)$$

Hence, if $\gamma_n=\gamma_m$, we obtain directly Eq. (3.7). If $\gamma_n\neq\gamma_m$, we can expect an effect in which nonlinear absorption of the radiation occurs in some parts of the spectrum and nonlinear amplification in other parts (see Ref. 4). The necessary condition for this effect is that the level decaying faster should be pumped more rapidly.

6. CONCLUSIONS

The main consequence of the above analysis is the conclusion that acceleration gives rise to new nonlinear saturation effects in the absorbed power governed by the Landau–Zener parameter $p=2\pi V^2/ka$. These effects are manifested in the range $\gamma V/ka\ll1$, since if $\gamma V/ka\gg1$, only the conventional inhomogeneous broadening is observed. One of the manifestations of these effects is an adiabatic inversion of populations in a two-level system subjected to resonant illumination and acceleration.

The effects under discussion are of direct interest for the kinetics of ionic lasers. In fact, if we assume that $V^2/ka\gtrsim1$ and $\gamma V/ka\lesssim1$, we find that the necessary acceleration is $\gamma^2/ka\lesssim1$. Next, assuming that in the case of optical transitions we have $\gamma\sim10^{-8}$ a.u., $k=\omega_0/c\sim10^{-3}$ a.u., we find that $a\sim10^{-13}$ a.u. $\sim10^{12}$ cm/sec^2. For example, in the case of an argon ion laser subjected to an electric field ~10 V/cm the parameter γ^2/ka estimated in accordance with Ref. 2 is $\frac{1}{3}$.

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