Temperature dependence of the critical current of superconductor–semiconductor–superconductor junctions

L. G. Aslamazov and M. V. Fistul'

Moscow Institute of Steel and Alloys

(Submitted 14 January 1980)
Zh. Eksp. Teor. Fiz. **81**, 382–397 (July 1981)

The temperature dependence of the critical current of superconductor–semiconductor–superconductor junctions is investigated at various densities of the semiconductor doping impurities. The tunnel resistance of the junction is determined in the nondegenerate case and it is shown that this temperature dependence is the same as for an ordinary tunnel element. If the semiconductor is degenerate, the critical current first increases exponentially with decreasing temperature, and then quadratically (at a large electron mean free path) or logarithmically (in the dirty case). In the intermediate impurity-density region the temperature dependence of the critical current is determined by the fluctuations of the bottom of the conduction band in the semiconductor.

PACS numbers: 74.50. + r, 74.40. + k

1. INTRODUCTION

Superconductor-semiconductor-superconductor (S-Sm-S) are intensively investigated of late.[1,2,3] The properties of these junctions depend essentially on the density of the free carriers in the semiconductor: at low density they are similar to ordinary Josephson S-I-S (superconductor-insulator-superconductor) elements, and at high density they are close to superconductor-normal metal-superconductor (S-N-S) junctions.

The free carriers in the semiconductor can be produced both by introducing impurities (doping) or by eliminating the junctions. In the latter case, after turning off the light there remain in the semiconductor (usually, CdS) long-lived conduction electrons, while the holes are captured in traps.[2] By varying the irradiation time (or the impurity density) it is possible to vary the density of the free carriers in the semiconductor and thereby influence the effective transparency of the barrier through which the superconducting electrons tunnel. The properties of the junction (critical current, current-voltage characteristic, and others) are altered in this manner, and this uncovers new possibilities for using the junctions in cryoelectronics.

Greatest interest attaches to the region of intermediate carrier densities, when the barrier is already quite low and the thickness of the semiconductor layer can greatly exceed the distances between the atoms (this makes these junctions stable), and at the same time the junction has a large normal resistance, a factor of importance in practical utilization.

The theory of the Josephson effect in S-Sm-S junctions is at present only in the initial development stage. To interpret the experimental data, use was made previously of theoretical results obtained for S-I-S and S-N-S junctions.[1-3] A microscopic description of the properties of S-Sm-S junctions, capable of determining the critical current of the junction in a wide range of free-carrier density at temperatures close to the critical temperature of the superconductors, was proposed in our preceding paper.[4] In the present paper, on the basis of a microscopic approach, we obtain the temperature dependence of the critical current of an S-Sm-S junction at various free-carrier-densities in the semiconductor.

2. GENERAL EXPRESSION FOR THE JUNCTION CRITICAL CURRENT

The current density j is expressed in terms of the Green's function $G_\omega(\mathrm{r},\mathrm{r}')$ of the system in accordance with the formula[5]

$$j=\frac{ie}{m}T\sum_\omega\left(\frac{\partial}{\partial z'}-\frac{\partial}{\partial z}\right)G_\omega(\mathbf{r},\mathbf{r}')\,|_{\mathbf{r}=\mathbf{r}'},\tag{1}$$

where z is the coordinate perpendicular to the plane of the junction, j is the projection of the current density on this direction, and $\omega = (2n+1)\pi T$ is the Matsubara frequency over which the summation is carried out.

The Green's function is determined from the Gor'kov equations,[5] whose integral form is

$$G_\omega(\mathbf{r},\mathbf{r}')=G_\omega{}^n(\mathbf{r},\mathbf{r}')-\int G_\omega{}^n(\mathbf{r},\mathbf{r}_1)\Delta(\mathbf{r}_1)F_\omega{}^+(\mathbf{r}_1,\mathbf{r}')\,d^3\mathbf{r}_1,\tag{2}$$

$$F_\omega{}^+(\mathbf{r},\mathbf{r}')=\int G_{-\omega}{}^n(\mathbf{r},\mathbf{r}_2)\Delta^*(\mathbf{r}_2)G_\omega(\mathbf{r}_2,\mathbf{r}')\,d^3\mathbf{r}_2,\tag{3}$$

where $G_\omega^n(\mathrm{r},\mathrm{r}')$ is the Green's function of the system in the normal state, and $F_\omega^+(\mathrm{r},\mathrm{r}')$ is the anomalous Green's function. The order parameter $\Delta(\mathrm{r})$ is determined by the formula

$$\Delta(\mathbf{r})=|\lambda|T\sum_\omega F_\omega{}^+(\mathbf{r},\mathbf{r}),\tag{4}$$

and the electron-photon interaction constant λ, and hence also the order parameter Δ in the semiconductor, will be assumed equal to zero.

In the S-Sm-S junction there appear near the boundaries charged interfaces (Schottky barriers) which lead to a bending of the bottom of the conduction band, with a curvature described by the potential $V(z)$. To calculate the current in such systems it is convenient therefore to use a Green's function that depends on the longitudinal coordinates z and z', and change over to the momentum representation with respect to the transverse coordinates ρ and ρ'.

0038-5646/81/070206-08$02.40

The solution of Eqs. (2) and (3) for the Green's function G_ω can be expressed in terms of the Green's function of the system in the normal state in the form of a series

$$G_\omega(\mathbf{p},\mathbf{p}';z,z')=G_\omega{}^n(\mathbf{p},\mathbf{p}';z,z')+\sum_{l=1}^{\infty}(-1)^l\int dz_1\ldots dz_{2l}\int d^2\mathbf{p}_1\ldots d^2\mathbf{p}_{2l}$$
$$\times\int d^2\mathbf{p}_1'\ldots d^2\mathbf{p}_{2l}'\Delta(\mathbf{p}_1-\mathbf{p}_1';z_1)\Delta^*(\mathbf{p}_2-\mathbf{p}_2';z_2)\Delta(\mathbf{p}_{2l-1}-\mathbf{p}_{2l-1}';z_{2l-1})$$
$$\times\Delta^*(\mathbf{p}_{2l}-\mathbf{p}_{2l}';z_{2l})G_\omega{}^n(\mathbf{p},\mathbf{p}_1;z,z_1)G_{-\omega}{}^n(\mathbf{p}_1',\mathbf{p}_2;z_1,z_2)\ldots$$
$$\ldots G_{-\omega}{}^n(\mathbf{p}_{2l-1}',\mathbf{p}_{2l};z_{2l-1},z_{2l})G_\omega{}^n(\mathbf{p}_{2l}',\mathbf{p}';z_{2l},z'). \qquad (5)$$

Substituting (5) in Eq. (1) for the total superconducting current through the junction, we obtain

$$J_s=\int jd^2\rho=\frac{ie}{m}T\sum_\omega\sum_{l=1}^{\infty}(-1)^l\int dz_1\ldots dz_{2l}\int d^2\mathbf{p}_1\ldots d^2\mathbf{p}_{2l}\int d^2\mathbf{p}_1'\ldots$$
$$\ldots d^2\mathbf{p}_{2l}'\Delta(\mathbf{p}_1-\mathbf{p}_1';z_1)\Delta^*(\mathbf{p}_2-\mathbf{p}_2';z_2)\ldots\Delta(\mathbf{p}_{2l-1}-\mathbf{p}_{2l-1}';z_{2l-1})$$
$$\times\Delta^*(\mathbf{p}_{2l}-\mathbf{p}_{2l}';z_{2l})G_{-\omega}{}^n(\mathbf{p}_1',\mathbf{p}_2;z_1,z_2)G_\omega{}^n(\mathbf{p}_2',\mathbf{p}_3;z_2,z_3)\ldots$$
$$\ldots G_{-\omega}{}^n(\mathbf{p}_{2l-1}',\mathbf{p}_{2l};z_{2l-1},z_{2l})\mathscr{L}, \qquad (6)$$

$$\mathscr{L}=\int d^2\mathbf{p}\left(\frac{\partial}{\partial z'}-\frac{\partial}{\partial z}\right)G_\omega{}^n(\mathbf{p},\mathbf{p}_1;z,z_1)G_\omega{}^n(\mathbf{p}_{2l}',\mathbf{p};z_{2l},z')|_{z=z'}. \qquad (7)$$

As seen from (7), the differential operator acts only on the product of two Green's functions, and this product must be calculated.

The Green's function of the system in the normal state is obtained from the equation

$$\left[i\omega-\frac{p^2}{2m}+\mu-V(z)+\frac{1}{2m}\frac{\partial^2}{\partial z^2}\right]G_\omega{}^n(\mathbf{p},\mathbf{p}_1;z,z_1)$$
$$-\frac{1}{(2\pi)^2}\int V_{imp}(z;\mathbf{p}-\mathbf{p}')G_\omega{}^n(\mathbf{p},\mathbf{p}_1;z,z_1)d^2\mathbf{p}'=\delta(\mathbf{p}-\mathbf{p}_1)\delta(z-z_1), \qquad (8)$$

where the term with the impurity potential $V_{imp}(z;\mathrm{p})$ has been separated. The potential $V(z)$, which describes the bending of the bottom of the band in the semiconductor, does not depend on the coordinate in the plane of the junction; μ is the chemical potential. The solution of (8) can be expressed in terms of the Green's function $G_\omega^n(\mathrm{p};z,z_1)$ without impurities, in the form of the series

$$G_\omega{}^n(\mathbf{p},\mathbf{p}_1;z,z_1)=G_\omega{}^n(\mathbf{p};z,z_1)\delta(\mathbf{p}-\mathbf{p}_1)$$
$$+\sum_{k=1}^{\infty}\frac{1}{(2\pi)^{2k}}\int dz_1'\ldots dz_k'\int d^2\mathbf{p}_1'\ldots d^2\mathbf{p}_k'V_{imp}(z_1';\mathbf{p}-\mathbf{p}_1')\ldots$$
$$\ldots V_{imp}(z_k';\mathbf{p}_{k-1}'-\mathbf{p}_k')G_\omega{}^n(\mathbf{p};z,z_1')G_\omega{}^n(p_1';z_1',z_2')\ldots$$
$$\ldots G_\omega{}^n(\mathbf{p}_k';z_k',z_1)\delta(\mathbf{p}_1-\mathbf{p}_k'). \qquad (9)$$

Substituting (9) in (7), we easily see that to find $\mathscr{L}$ we must calculate expressions of the type

$$N=\left(\frac{\partial}{\partial z'}-\frac{\partial}{\partial z}\right)G_\omega{}^n(\mathbf{p}_1;z,z_1)G_\omega{}^n(\mathbf{p}_1;z_2,z')|_{z=z'}, \qquad (10)$$

where z_1 and z_2 lie in the superconducting regions $z_{1,2}<-a$ and $z_{1,2}>a$ [only these regions contribute to expression (6) for J_s, for otherwise $\Delta(z_{1,2})=0$ in accordance with formula (4)]. The point z at which the superconducting current is calculated is situated inside the junction: $-a<z<a$.

Expression (10) can be obtained by writing down the Green's function of the system without impurities in terms of two linearly independent solutions of the equation

$$\left[i\omega-\frac{p^2}{2m}+\mu-V(z)+\frac{1}{2m}\frac{d^2}{dz^2}\right]\psi(z)=0 \qquad (11)$$

in accordance with the formula

$$G_\omega{}^n(\mathbf{p};z,z_1)=\begin{cases}W^{-1}\psi_1(z)\psi_2(z_1), & z<z_1\\ W^{-1}\psi_2(z)\psi_1(z_1), & z>z_1,\end{cases} \qquad (12)$$

where

$$W=(1/2m)[\psi_2'(z)\psi_1(z)-\psi_1'(z)\psi_2(z)]$$

is the Wronskian of Eq. (11). As a result we get

$$N=mG_\omega{}^n(\mathbf{p}_1,z_2,z_1)[\operatorname{sign}z_1-\operatorname{sign}z_2]. \qquad (13)$$

The expression for $\mathscr{L}$ does not depend on z and it is convenient to calculate it as $z\to a$:

$$\mathscr{L}=mG_\omega{}^n(\mathbf{p}_{2l}',\mathbf{p}_1;z_{2l},z_1)[\operatorname{sign}z_1-\operatorname{sign}z_{2l}]. \qquad (14)$$

Substituting (14) in (6) and using expression (5), as well as the property

$$G_\omega{}^n(\mathbf{p},\mathbf{p}';z,z')=G_\omega{}^n(-\mathbf{p}',-\mathbf{p};z',z), \qquad (15)$$

we obtain

$$J_s=-ieT\sum_\omega\int dz_1\,dz_2\int d^2\mathbf{p}_1\,d^2\mathbf{p}_2\,d^2\mathbf{p}_1'd^2\mathbf{p}_2'G_\omega{}^n(\mathbf{p}_1,\mathbf{p}_2;z_1,z_2)G_{-\omega}(\mathbf{p}_2',\mathbf{p}_1';z_2,z_1)$$
$$\times\Delta(\mathbf{p}_1'-\mathbf{p}_1;z_1)\Delta^*(\mathbf{p}_2-\mathbf{p}_2';z_2)[\operatorname{sign}z_1-\operatorname{sign}z_2]. \qquad (16)$$

A similar formula was obtained by Kulik and Gorbonosov[6] for the case of a one-dimensional δ-function barrier without impurities.

When the current is determined for the case of a weak mutual influence of the superconductors (this condition is usually satisfied for S-Sm-S junctions), formula (16) allows us to regard the order parameter in the superconductor regions as constant. Allowance for the coordinate-dependent corrections to Δ adds terms of higher order to the current, since quantity $G_\omega^n(z_1,z_2)G_{-\omega}(z_2z_1)$ in the integrand of (16) is small.

3. CRITICAL CURRENT OF JUNCTION IN THE PURE CASE

We consider first the case when the mean free path of the electrons in the semiconductor l is large compared with the coherence length ξ. This condition can be satisfied even in the case of appreciable doping of the semiconductor (the mean free path in a semiconductor is usually larger than the distances between impurities), and makes it possible, when finding the superconducting current, to neglect scattering by impurities. In this case the Green's functions depend only on the difference $\rho-\rho'$ of the transverse coordinates and we obtain from (16) in the momentum representation

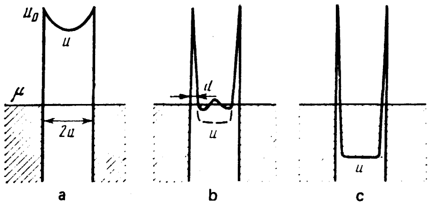

FIG. 1. Band structure of superconductor— semiconductor—superconductor junction at various densities of the free carriers in the semiconductor.

$$j=\frac{iem}{\pi}T\sum_{\omega}\int d\zeta\int_{-\infty}^{-a}dz_1\int_{a}^{\infty}dz_2G_{-\omega}{}^{n}(\zeta;z_1,z_2)\,[\Delta(z_1)\Delta^*(z_2)G_\omega(\zeta;z_2,z_1)$$
$$-\Delta(z_2)\Delta^*(z_1)G_\omega(\zeta;z_1,z_2)],\qquad(17)$$

where $\zeta=p^2/2m-\mu$ is the transverse-motion energy reckoned from the level of the chemical potential.

To find $G_\omega(\zeta;z_1,z_2)$ we use the Gor'kov equations in matrix form[5]:

$$\begin{pmatrix} i\omega-\zeta-V(z_1)+\dfrac{1}{2m}\dfrac{d^2}{dz_1{}^2} & \Delta(z_1) \\ -\Delta^*(z_1) & -i\omega+\dfrac{1}{2m}\dfrac{d^2}{dz_1{}^2}-\zeta-V(z_1)\end{pmatrix}\begin{pmatrix}G_\omega(z_1,z_2)\\ F_\omega{}^+(z_1,z_2)\end{pmatrix}=\begin{pmatrix}\delta(z_1-z_2)\\ 0\end{pmatrix}.\qquad(18)$$

The solution of this system can be sought in the form[7]

$$\begin{pmatrix}G_\omega(z_1,z_2)\\ F_\omega{}^+(z_1,z_2)\end{pmatrix}=\begin{cases}\hat u_1(z_1)f_1(z_2)+\hat u_2(z_1)f_2(z_2), & z_1>z_2\\ \hat u_3(z_1)f_3(z_2)+\hat u_4(z_1)f_4(z_2), & z_1<z_2\end{cases},\qquad(19)$$

where $\hat u_{1,2,3,4}$ are linearly independent solutions of the corresponding homogeneous system [the system (18) without the right-hand side], while $f_{1,2,3,4}$ are obtained from the conditions for the continuity of the functions

$$G_\omega(z_1,z_2),\quad F_\omega{}^+(z_1,z_2),\quad \frac{\partial}{\partial z_1}F_\omega{}^+(z_1,z_2)$$

at $z_1=z_2$ and from the jump of the derivative

$$\frac{\partial}{\partial z_1}G_\omega(z_1,z_2)\Big|_{z_1=z_2-0}^{z_1=z_2+0}=2m.\qquad(20)$$

The solutions $\hat u$ of the homogeneous system can be obtained under certain assumptions concerning the form of the potential $V(z)$. This potential depends substantially on the impurity density in the semiconductor.

At low densities, the thickness of the barrier layer[8] is large compared with the thickness of the semiconductor layer, and all the free electrons go off from the semiconductor into the superconductors. As a result, the semiconductor layer turns out to be uniformly charged and, solving the Poisson equation, we obtain for the dependence of the potential on the longitudinal coordinate the expression

$$V(z)=U_0+A(z^2-a^2),\quad A=2\pi e^2N/\varkappa,\quad N=N_d-N_a,\qquad(21)$$

where N_d and N_a are the densities of the donors and acceptors in the semiconductor, $\varkappa$ is the dielectric constant, and U_0 determines the value of the potential at the interfaces between the semiconductor and the superconductors (see Fig. 1, case a). Formula (21) is valid up to densities

$$N_0\sim\frac{\varkappa(U_0-\mu)}{e^2a^2}\sim\frac{U_0-\mu}{E_0}\frac{1}{a^2a_B},\qquad(22)$$

when the thickness of the barrier layer becomes comparable with the thickness of the semiconductor layer (a_B is the Bohr radius of the impurity, and E_0 is the ionization energy of the impurity).

At densities $N\gg N_0$, only thin semiconductor layers with dimension d near the interfaces (Schottky barriers) are charged, and in the rest of its bulk the semiconductor is electrically neutral. Then the potential $V(z)$ takes the form

$$V(z)=\begin{cases}U+A(|z|+d-a)^2; & a-d<|z|<a\\ U; & |z|<a-d,\end{cases}$$
$$d=[(U_0-U)/A]^{1/2},\qquad(23)$$

where U is the position of the bottom of the band in the neutral semiconductor. Depending on the density of the free carriers in the semiconductor, it can be either nondegenerate or degenerate (see Fig. 1, case c).

To find the solution of the system (18) it is necessary also to know the dependence of the order parameter $\Delta(z)$ on the longitudinal coordinate z. Inasmuch as the penetrability of the Schottky barriers remains small up to densities on the order of the Avogadro number, we shall assume that the superconductors influence each other little, and the value of the order parameter at $z<-a$ is equal to the constant value $|\Delta_1|\exp(i\chi_1)$, and at $z>a$ it equals correspondingly to $|\Delta_2|\exp(i\chi_2)$. In the semiconductor region $-a<z<a$ the value of the order parameter is zero, since it is assumed that there is no interaction in the semiconductor [Eq. (4)].

In the semiconductor region $-a<z<a$ one can find quasiclassical solutions of the homogeneous system (18) (the de Broglie wavelength of the electron is much less than the thickness of the barrier layer). Matching these quasiclassical solutions to plane waves in the superconducting regions, we obtain four linearly independent solutions of the homogeneous system, and with their aid, using (19), we obtain the Green's function of the system (for the case of interest to us, when the coordinates z_1 and z_2 lie in the superconducting regions). For the critical current j_c of the junction we obtain in this case from (17)

$$j_c=\frac{-8|\Delta_1||\Delta_2|eT}{\pi}\sum_{\omega>0}\int_{-\mu}^{\infty}\frac{d\zeta}{\zeta}\frac{p_+(a)p_-(a)}{(\omega^2+|\Delta_1|^2)^{1/2}(\omega^2+|\Delta_2|^2)^{1/2}}$$
$$\times\exp\left\{i\int_{-a}^{a}[p_+(z)+p_-(z)]dz\right\},$$
$$p_\pm=\{2m[\pm i\omega-\zeta-V(z)]\}^{1/2}.\qquad(24)$$

The branch of the square root in (24) is chosen such that the imaginary part of the root is positive. In the derivation of (24) we used the fact that the integral with respect to ζ in (17) receives its principal contribution from the region close to the value $\zeta=-\mu$ (the electrons passing with maximum probability are those having a longitudinal momentum), and the significant role in the sum is played by the values $\omega\sim T$.

The critical current determined by (24) depends substantially on the position of the chemical potential μ relative to the bottom of the conduction band U, which is determined by the density of the free carriers in the semiconductor. The free-carrier density n is given in turn by the time of illumination of the semiconductor or by the degree of its doping. Several characteristic situations are possible in this case.

a) The critical current is determined by the tunneling of the electrons through the charged layer of the semiconductor. At impurity densities $N\ll N_0$, where N_0 is determined by formula (22), the layer of the semiconductor is uniformly charged, and the potential $V(z)$ is obtained from the Poisson equation in accordance with formula (21). In this case we get for the critical current from (24)

$$j_c=\frac{2^{3/2}em^{1/2}(U_0-\mu)^{3/2}}{\pi a\mu}\exp\left[-2^{5/2}m^{1/2}a(U_0-\mu)^{1/2}\left(1-\frac{\alpha}{3}\right)\right]I(\Delta_1,\Delta_2),$$
$$\alpha=2\pi\varkappa^{-1}(U_0-\mu)^{-1}e^2a^2N\ll1; \tag{25}$$
$$I(\Delta_1,\Delta_2)=|\Delta_1||\Delta_2|T\sum_\omega[(\omega^2+|\Delta_1|^2)(\omega^2+|\Delta_2|^2)]^{-1/2};$$
$$I(\Delta,\Delta)=\frac{|\Delta|}{2}\operatorname{th}\frac{|\Delta|}{2T}.$$

In the derivation of (25) we used the fact that the quantity $U_0-\mu$ is large compared with the temperature.

This formula can be expressed in terms of the tunnel resistance of the junction (just as for the S–I–S junction):

$$j_c=\frac{\pi}{R_\text{T}}I(\Delta_1,\Delta_2),\quad \frac{1}{R_\text{T}}=\frac{e^2}{\pi}\int dE\frac{\partial f}{\partial E}\int\frac{d^2k_\parallel}{(2\pi)^2}D^2(E,k_\parallel), \tag{26}$$

where $D^2(E,k_\parallel)$ is the transparency of the barrier to an electron with total energy E and momentum $k_\parallel$ in the plane of the junction, and $f(E)$ is the Fermi distribution function of the electron energies.[9] Thus, the increase of the critical current with increasing impurity density in the region $N\ll N_0$ is due to the increase of the effective transparency of the barrier.

b) The critical current is determined by the tunneling of the electrons through the Schottky barriers and through the neutral layer of the semiconductor.

At a density $N\gg N_0$, Schottky barriers appear near the interfaces, and the semiconductor is in the main electrically neutral.

In this case, fluctuations of the bottom of the conduction bands set in, and the value of U cannot be regarded as constant.[10] However, so long as the impurity density or the irradiation time is still not too large, the free-carrier density in the semiconductor is low and the average position of the bottom of the conduction band lies much higher than the level of the chemical potential. The effect of the fluctuations on the critical current is then insignificant, and by using expression (23) for the potential we obtain from (24) the critical current

$$j_c=\frac{2^{3/2}em^{1/2}(U_0-\mu)(\overline{U}-\mu)^{1/2}}{\pi a\mu}\exp\left[-2^{5/2}m^{1/2}a(\overline{U}-\mu)^{1/2}\right.$$
$$\left.-2^{3/2}m^{1/2}d(U_0-\mu)^{1/2}-2^{3/2}m^{1/2}\frac{\overline{U}-\mu}{A^{1/2}}\ln\frac{dA^{1/2}+(U_0-\mu)^{1/2}}{(\overline{U}-\mu)^{1/2}}\right]I(\Delta_1,\Delta_2). \tag{27}$$

The last term in the argument of the exponential in (27) is small (it becomes significant only at a very high degree of doping, when the impurity density is of the order of the Avogadro number).

Corresponding to (27) is also the tunnel expression (26), where the resistance R_T is determined by the tunneling both through the Schottky barrier and through the neutral layer of the semiconductor, in which the average level of the bottom of the band lies above the chemical-potential level. A similar formula was obtained by Alfeev and Kolesnikov[11] by starting from the assumption that the tunnel expression (25), previously derived only for tunnel junctions, is valid also for S–Sm–S junctions.

We note that in this density region an important role can be assumed in the semiconductor by hopping conduction.[10] If the length of the hop, however, exceeds the thickness of the semiconductor layer, this charge-transport mechanism can be disregarded.

c) The critical current is determined by the tunneling through the Schottky barriers and by the loss of coherence in the neutral semiconductor.

With increasing free-carrier density, the average level of the bottom of the band, relative to the chemical-potential level, is lowered and the fluctuations of the bottom of the conduction band becomes substantial. These fluctuations are due to the large-scale potential[10] and therefore, if the thickness of the semiconductor layer in the S–Sm–S junction is too large, it can be assumed that the position of the bottom of the conduction band changes only in the transverse directions, and U is constant along the z coordinate. The critical current is then determined from the formula

$$j_c=\int dUj_c(U)\mathscr{F}(U), \tag{28}$$

where $\mathscr{F}(U)$ is the distribution function of the random potential U. It turns out in this case that the main contribution to the current is made by deep fluctuations, when the bottom of the conduction band U is much lower than the level of the chemical potential μ. For these fluctuations, the screening of the excess impurity density by electrons is nonlinear, so that the function $\mathscr{F}(U)$ is not Gaussian and can be expressed in the form

$$\mathscr{F}(U)=F_0\exp\left[-\left(\frac{\mu-U}{U_f}\right)^\alpha\right], \tag{29}$$

where U_f is a typical value of the potential fluctuation. In the case of an uncorrelated distribution of the impurities we obtain $\alpha=5/2$ (the estimates are given at the end of the article).

Using formula (28), we obtain for the critical current

$$j_c=A_1F_0T|\Delta_1||\Delta_2|\sum_{\omega>0}\int_0^\infty dU\exp\left[-4am^{1/2}(-U+\sqrt{U^2+\omega^2})^{1/2}\right.$$
$$\left.-\left(\frac{U}{U_f}\right)^\alpha\right](\omega^2+|\Delta_1|^2)^{-1/2}(\omega^2+|\Delta_2|^2)^{-1/2},$$
$$A_1=\frac{2^{7/2}em^{1/2}(U_0-\mu)^{3/2}}{\pi\mu d}\ln^{-1}\left(\frac{U_0-\mu}{U_f}\right)\exp[-2^{3/2}m^{1/2}d(U_0-\mu)^{1/2}]. \tag{30}$$

The significant values of U in (30) are large compared with U_f (in this formula U is reckoned from the chemical-potential level). These fluctuations are rare (the second term in the argument of the exponential is large) but the semiconductor here is degenerate ($T\ll U_f$) and the critical current is substantially increased (the first term in the argument of the exponential increases).

The essential values of ω in (30) are small compared with U, so that the argument of the exponential can be expanded in powers of ω and only the first term need be retained. As a result we get

$$j_c=A_1U_fF_0|\Delta_1||\Delta_2|T\sum_{\omega>0}\left(\frac{\omega}{\pi T_1^*}\right)^{2/(2\alpha+1)}(\omega^2+|\Delta_1|^2)^{-1/2}(\omega^2+|\Delta_2|^2)^{-1/2}$$
$$\times\int_0^\infty dz\exp\left[-\left(\frac{\omega}{\pi T_1^*}\right)^{2\alpha/(2\alpha+1)}\left(z^\alpha+\frac{2}{z^{1/2}}\right)\right], \tag{31}$$

where the following change of variables was made in the integral with respect to U:

$$U=zU_f(\omega/\pi T_1^*)^{2/(2\alpha+1)},$$

and the characteristic temperature is $T^*_1=U_f^{1/2}/\pi a(2m)^{1/2}$. In the derivation of (31) it was assumed that $\max[T, T^*_1]\ll U_f$, for otherwise it is impossible to expand the argument of the exponential in powers of ω (this condition is usually satisfied even in weakly compensated semiconductors).

Expression (31) has different asymptotic forms, depending on the relations between the parameters T, T^*_1, and Δ. At $T^*_1\ll T$, only the first term is significant in the sum over ω in (31), and the integral with respect to z can be calculated by the saddle-point method. As a result we obtain for the critical current

$$j_c=\frac{2\pi^{1/2}}{[\alpha(2\alpha+1)]^{1/2}}A_1U_fF_0|\Delta_1||\Delta_2|T\left(\frac{T}{\alpha T_1^*}\right)^{(2-\alpha)/(2\alpha+1)}(\pi^2T^2+|\Delta_1|^2)^{-1/2}$$
$$\times(\pi^2T^2+|\Delta_2|^2)^{-1/2}\exp\left[-f\left(\frac{T}{T_1^*}\right)^{2\alpha/(2\alpha+1)}\right], \tag{32}$$
$$f=(2\alpha+1)(1/\alpha)^{2\alpha/(2\alpha+1)}.$$

The exponential damping of the critical current with increasing thickness of the junction has a simple physical explanation. The electrons passing through the semiconductor lose their coherence at distances on the order of ξ, and cannot produce a superconducting current at thicknesses $a\gg\xi$. The value of ξ is determined by the velocity v of the electrons with energy $\sim U_f$, reckoned from the bottom of the conduction band:

$$\xi\sim v/T\sim(2U_f/m)^{1/2}/T.$$

At low temperatures $T\ll T^*_1\ll\Delta$, to find the asymptotic form of the critical current it is necessary to calculate the sum of the exponentials in (31). As a result we get

$$j_c=\frac{\Gamma(3/2\alpha)}{4\alpha}A_1U_fF_0T_1^*\left[1-\frac{2}{3}\frac{\Gamma(1/2\alpha)}{\Gamma(3/2\alpha)}\left(\frac{T}{T_1^*}\right)^2\right]. \tag{33}$$

Thus, the critical current decreases quadratically with increasing temperature. We note that in the low-temperature region the values $U\sim U_f$ are significant, so that the asymptotic formula (29) for $\mathscr{F}(U)$ can strictly speaking not be used to calculate the current. The form of the distribution function of the random potential, however, influences only the coefficient of the quadratic term in (33).

We note also that for short junctions the condition $T^*_1\gg\Delta$ may be satisfied. In this case we find from (31) that the temperature dependence of j_c is determined by the pre-exponential expression and is the same in the entire temperature region as for the tunnel junctions.

We consider finally the region of large electron densities, when the bottom of the conduction band in the semiconductor is much lower than the chemical-potential level and the fluctuations of the bottom of the band are no longer significant in the calculation of the critical current. Then, using (24), we obtain

$$j_c=A_2|\Delta_1||\Delta_2|T\sum_{\omega>0}(\omega^2+|\Delta_1|^2)^{-1/2}(\omega^2+|\Delta_2|^2)^{-1/2}\exp\left(-\frac{\omega}{\pi T_1}\right),$$
$$A_2=\frac{2^{7/2}em^{1/2}(U_0-\mu)^{3/2}}{\pi\mu d}\ln^{-1}\left(\frac{U_0-\mu}{\mu-\overline{U}}\right)\exp[-2^{3/2}m^{1/2}d(U_0-\mu)^{1/2}], \tag{34}$$

where the characteristic temperature is

$$T_1=(\mu-\overline{U})^{1/2}/2^{3/2}\pi m^{1/2}a,\qquad \mu-\overline{U}=(3\pi^2)^{2/3}n^{2/3}/2m.$$

We present also the corresponding asymptotic expressions for the critical currents

$$j_c=A_2|\Delta_1||\Delta_2|T(\pi^2T^2+|\Delta_1|^2)^{-1/2}$$
$$\times(\pi^2T^2+|\Delta_2|^2)^{-1/2}\exp(-T/T_1);\quad T\gg T_1, \tag{35a}$$
$$j_c=\tfrac{1}{2}A_2T_1\left[1-\tfrac{1}{6}\left(\frac{T}{T_1}\right)^2\right];\quad T\ll T_1\ll\Delta, \tag{35b}$$
$$j_c=\tfrac{1}{2}A_2I(\Delta_1,\Delta_2);\quad T_1\gg\Delta. \tag{35c}$$

The decrease of the argument of the exponential in (35a) with increasing free-carrier density is due to the increase of the electron velocity and correspondingly of the coherence length.

4. CRITICAL CURRENT IN THE CASE OF A DIRTY SEMICONDUCTOR

At considerable impurity density, the electron mean free path l in the semiconductor can become smaller than the coherence length ξ. In this case, when calculating the superconducting current through the junction, it is necessary already to take into account the scattering by the impurities. To find the critical current it is again convenient to use formula (16), which must be averaged over the position of the impurities. In the case of weak mutual influence of the superconductors, we have

$$J_s=-ieT\sum_\omega\int_{-\infty}^{-a}dz_1\int_a^\infty dz_2\int d^2p_1d^2p_2[\Delta_1\Delta_2^*\overline{G_\omega^n(\mathbf{p}_1,\mathbf{p}_2,z_1,z_2)G_{-\omega}(\mathbf{p}_2,\mathbf{p}_1;z_2,z_1)}$$
$$-\Delta_2\Delta_1^*\overline{G_\omega^n(\mathbf{p}_2,\mathbf{p}_1;z_2,z_1)G_{-\omega}(\mathbf{p}_1,\mathbf{p}_2;z_1,z_2)}], \tag{36}$$

where the bar denotes averaging over the impurity position.

The calculations that follow are for the case of a degenerate semiconductor, inasmuch as even in the intermediate density region, when the chemical potential lies near the average position of the bottom of the conduction band, the largest contribution to the current, just as in the pure case, is made by the deep fluctuations of the bottom of the band (in the nondegenerate case, the impurity density is low and the electron mean free path usually exceeds ξ).

Using the Gor'kov equations, just as in the theory of superconducting alloys,[5] we can average the product of the Green's functions that enter in (36). We then obtain the system of integral equations

$$\begin{aligned}
\Pi_1(\mathbf{p}_1;z_1,z_2)&=\overline{G}_\omega^n(\mathbf{p}_1;z_1,z_2)\overline{G}_{-\omega}(\mathbf{p}_1;z_2,z_1)\\
&+\frac{1}{2p_0\tau_{tr}}\int_{-a}^{a}dz_0'\overline{G}_\omega^n(\mathbf{p}_1;z_1,z_0')\overline{G}_{-\omega}(\mathbf{p}_1;z_0',z_1)\int d^2\mathbf{p}_2\Pi_1(\mathbf{p}_2;z_0',z_2)\\
&-\frac{1}{2p_0\tau_{tr}}\int_{-a}^{a}dz_0'\overline{G}_\omega^n(\mathbf{p}_1;z_1,z_0')\overline{F}_{-\omega}(\mathbf{p}_1;z_0',z_1)\int d^2\mathbf{p}_2\Pi_2(\mathbf{p}_2;z_0',z_2),\\
\Pi_2(\mathbf{p}_1;z_1,z_2)&=\overline{G}_\omega^n(\mathbf{p}_1;z_1,z_2)\overline{F_{-\omega}^+}(\mathbf{p}_1;z_2,z_1)\\
&+\frac{1}{2p_0\tau_{tr}}\int_{-a}dz_0'\overline{G_\omega^n}(\mathbf{p}_1;z_1,z_0')\overline{G}_{-\omega}(\mathbf{p}_1;z_0',z_1)\int d^2\mathbf{p}_2\Pi_2(\mathbf{p}_2;z_0',z_2)\\
&+\frac{1}{2p_0\tau_{tr}}\int_{-a}dz_0'\overline{G_\omega^n}(\mathbf{p}_1;z_1,z_0')\overline{F_{-\omega}^+}(\mathbf{p}_1;z_0',z_1)\int d^2\mathbf{p}_2\Pi_1(\mathbf{p}_2;z_0',z_2),
\end{aligned} \tag{37}$$

where Π_1 and Π_2 are defined by the formulas

$$\Pi_1(\mathbf{p}_1;z_1,z_2)=\frac{1}{S}\int d^2\mathbf{p}_2\overline{G_\omega{}^n(\mathbf{p}_1,\mathbf{p}_2;z_1,z_2)\,G_{-\omega}(\mathbf{p}_2,\mathbf{p}_1;z_2,z_1)},$$
$$\Pi_2(\mathbf{p}_1;z_1,z_2)=\frac{1}{S}\int d^2\mathbf{p}_2\overline{G_\omega{}^n(\mathbf{p}_1,\mathbf{p}_2;z_1,z_2)\,F_{-\omega}{}^+(\mathbf{p}_2,\mathbf{p}_1;z_2,z_1)}, \tag{38}$$

S is the area of the junction, $p_0=[2m(\mu-U)]^{1/2}$ is the electron momentum on the Fermi level, and τ_{tr} is the transport time of scattering by impurities. In the derivation of the system (37) we used the fact that the order parameter differs from zero only in the superconducting regions, and the potential $V(z)0$, on the contrary, only in the semiconductor region, and also the fact that there are no impurities inside the Schottky barriers.

For the averaged single-particle Green's functions in (37) we have the expressions

$$\bar{G}_\omega(\mathbf{p}_1;z_1,z_2)=G_\omega(\mathbf{p}_1;z_1,z_2)$$
$$\times\exp\left[\leftarrow\frac{(z_1+a)\,\mathrm{sign}\,\omega}{2v(\zeta)\tau_{tr}}\right];\quad -a<z_1<a,\quad z_2<-a,$$
$$\bar{G}_\omega(\mathbf{p}_1;z_1,z_2)=G_\omega(\mathbf{p}_1;z_1,z_2)$$
$$\times\exp\left[-\frac{|z_1-z_2|\,\mathrm{sign}\,\omega}{2v(\zeta)\tau_{tr}}\right];\quad -a<z_1<a,\quad -a<z_2<a, \tag{39}$$
$$\bar{G}_\omega(\mathbf{p}_1;z_1,z_2)=G_\omega(\mathbf{p}_1;z_1,z_2)$$
$$\times\exp\left[-\frac{(a-z_1)\,\mathrm{sign}\,\omega}{2v(\zeta)\tau_{tr}}\right];\quad -a<z_1<a,\quad z_2>a,$$

where $v(\zeta)=[2(i\omega-\zeta-U)/m]^{1/2}$, and $G_\omega(\mathbf{p}_1;z_1,z_2)$ is the Green's function of the system without impurities.

In the case of low transparency of the Schottky barriers, the quantity $\overline{F}_{-\omega}(\mathrm{p}_1;z'_0,z_1)$ is small for both coordinates lying in the semiconductor region, and $\underline{G}_\omega(\mathbf{p}_1;z'_0,z_1)$ can be regarded as coinciding with $\overline{G}^n_\omega(p_1;z'_0,z_1)$. Therefore, at $-a<z_1<a$ the first equation of the system (37) can be easily solved (the last term in this equation is small):

$$\Pi_1(z_1,z_2)=\sum_{k=-\infty}^{\infty}\int_{-a}^{a}dz_0\frac{\overline{G}_\omega{}^n(\mathbf{p}_1;z_0,z_2)\,\overline{G}_{-\omega}(\mathbf{p}_1;z_2,z_0)}{4a[\tau_{tr}|\omega|+(\pi k/2a)^2v_0{}^2\tau_{tr}{}^2/6]}$$
$$\times\exp\left[\frac{i\pi k}{2a}(z_0-z_1)\right],\quad v_0=[2(\mu-U)/m]^{1/2}. \tag{40}$$

Substituting (40) in the system (37), we can obtain Π_1 for z_1 and z_2 lying in the superconducting regions, and then use (36) to calculate the critical current

$$j_c=B\frac{v_0}{2a}|\Delta_1||\Delta_2|T\sum_{\omega>0}$$
$$\times\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{(\omega^2+|\Delta_1|^2)^{1/2}(\omega^2+|\Delta_2|^2)^{1/2}}\cdot\left[\omega+\frac{1}{6}\left(\frac{\pi k}{2a}\right)^2v_0{}^2\tau_{tr}\right]^{-1}, \tag{41}$$
$$B(\mu-U)=\frac{4e(U_0-\mu)}{\pi\mu}\exp[-2^{3/2}m^{1/2}d(U_0-\mu)^{1/2}]\left(d\ln\frac{U_0-\mu}{\mu-U}\right)^{-2}.$$

The expressions obtained for the current must be averaged, just as in the pure case, over the positions of the bottom of the conduction band. In the case when the chemical potential lies near the average position of the bottom of the conduction band, we obtain with the aid of (28) and (29) the following expression for the critical current of the junction:

$$j_c=B_1U_fF_0|\Delta_1||\Delta_2|T\sum_{\omega>0}\left(\frac{6}{\omega\tau_{tr}}\right)^{1/2}(\omega^2+|\Delta_1|^2)^{-1/2}$$
$$\times(\omega^2+|\Delta_2|^2)^{-1/2}\int_0^\infty dz\exp(-z^\alpha)\,\mathrm{sh}^{-1}[2z^{-1/2}(\omega/\pi T_2{}^*)^{1/2}], \tag{42}$$
$$T_2{}^*=U_f\tau_{tr}/3\pi ma^2,\quad B_1=B(U_f).$$

The asymptotic forms of this expression are given by

$$j_c=\frac{2\pi^{1/2}B_1U_fF_0}{[\alpha(2\alpha+1)]^{1/2}}|\Delta_1||\Delta_2|(\pi^2T^2+|\Delta_1|^2)^{-1/2}$$
$$\times(\pi^2T^2+|\Delta_2|^2)^{-1/2}\left(\frac{6T}{\pi\tau_{tr}}\right)^{1/2}$$
$$\times\left(\frac{\alpha^2T_2{}^*}{T}\right)^{(\alpha-2)/4\alpha+2}\exp\left[-f\left(\frac{T}{T_2{}^*}\right)^{\alpha/(2\alpha+1)}\right];\quad T\gg T_2{}^*, \tag{43}$$
$$j_c=\frac{\Gamma(3/2\alpha)}{2\alpha}B_1U_fF_0\left(\frac{6T_2{}^*}{\pi\tau_{tr}}\right)^{1/2}\ln\frac{T_2{}^*}{T};\quad T\ll T_2{}^*\ll\Delta.$$

We note that the logarithmic growth of the critical current as $T\to 0$ is limited by the condition of weak mutual influence of the superconductors, at which formula (43) is valid.

For short junctions, the relation $T^*_2\gg\Delta$ can be satisfied. In this case we obtain

$$j_c=\frac{\Gamma(3/2\alpha)}{2\alpha}B_1U_fF_0\left(\frac{6T_2{}^*}{\pi\tau_{tr}}\right)^{1/2}I(\Delta_1,\Delta_2),$$
$$I(\Delta_1,\Delta_2)=\pi|\Delta_1||\Delta_2|T\sum_{\omega>0}\omega^{-1}(\omega^2+|\Delta_1|^2)^{-1/2}(\omega^2+|\Delta_2|^2)^{-1/2} \tag{44}$$
$$=\begin{cases}\dfrac{7\zeta(3)}{8\pi^2}\dfrac{|\Delta_1||\Delta_2|}{T^2}; & T\gg\Delta_1,\Delta_2,\\ \ln(\Delta/T); & T\ll\Delta.\end{cases}$$

In the case of a degenerate semiconductor, the chemical potential lies much higher than the average position of the bottom of the conduction band, and the fluctuations of the bottom of the band become insignificant. The critical current is then determined by Eq. (41), in which we must put $U=\overline{U}$ (this result agees with that previously obtained for an S–N–S junction[12]). We present also asymptotic formulas for the critical current in the case of strong doping:

$$j_c=B_2|\Delta_1||\Delta_2|\left(\frac{6T}{\pi\tau_{tr}}\right)^{1/2}(\pi^2T^2+|\Delta_1|^2)^{-1/2} \tag{45a}$$
$$\times(\pi^2T^2+|\Delta_2|^2)^{-1/2}\exp\left[-\left(\frac{T}{T_2}\right)^{1/2}\right];\quad T\gg T_2,$$
$$j_c=B_2\left(\frac{6T_2}{\pi\tau_{tr}}\right)^{1/2}\ln\frac{T_2}{T};\quad T\ll T_2\ll\Delta, \tag{45b}$$
$$j_c=B_2\left(\frac{6T_2}{\pi\tau_{tr}}\right)^{1/2}I(\Delta_1,\Delta_2);\quad T_2\gg\Delta, \tag{45c}$$

where the characteristic temperature is $T_2=(\mu-\overline{U})\tau_{tr}/12\pi ma^2$, and $B_2=B(\mu-\overline{U})$.

5. DISCUSSION OF RESULTS

The results show that various mechanisms govern the value of the critical current of S–Sm–S junctions. If the semiconductor is not degenerate, the chemical potential lies much lower than the bottom of the conduction band in the semiconductor, and the weakening of the superconducting current in the junction is due to the tunneling of the pairs through the entire thickness of the semiconductor. In a degenerate semiconductor, the chemical potential lies above the bottom of the conduction band, and the weakening of the superconducting current is due mainly to the loss of coherence of the electrons in the semiconductor layer, where there is no pairing. In the intermediate region, the chemical potential is near the bottom of the band U, and large-scale fluctuations of the potential, whose value at low temperature exceeds the temperature, become significant. The two mechanisms that weaken the superconducting current compete with each other, and flow of the superconducting current turns out to be favored through the rare conducting channels that are formed in the semiconductor layer as a result of the strong fluctuation decrease of the bottom of the band.

Corresponding to the described mechanisms are three forms of the temperature dependence of the critical current of the junction. For a nondegenerate semiconductor, the tunnel formulas (25) and (27) are valid. Formula (25) then describes the temperature dependence of the critical current for junctions with a sufficiently thin semiconducting layer that is uniformly charged (all the electrons go off from the semiconductor into the superconductors), Formula (27) describes the case when the semiconductor-layer thickness exceeds substantially the dimensions of the Schottky barriers, so that the semiconductor is in the main electrically neutral.

For a degenerate semiconductor, the temperature dependence of the critical current is described by formulas (35) in the pure case and (45) in the dirty one. If the semiconductor-layer thickness is large compared with the minimum value of the coherence length in the semiconductor $[a>\xi(T_c)]$, then the critical current increases exponentially, $\sim\exp(-a/\xi)$, when the temperature drops below T_c [formulas (35a) and (45a)]. The coherence length, however, increases with decreasing temperature, and at a certain temperature (T_1 in the pure case and T_2 in the dirty one) becomes comparable with the thickness of the semiconductor layer. At low temperatures $[a<\xi(T)]$ the increase of the critical current is slower, quadratically in the pure case [formula (35b)] and logarithmically in the dirty one [formula (45b)]. For short junctions $[a<\xi(T_c)]$ the temperature dependence of the critical current is determined by the tunneling of the electrons through the Schottky barriers, and formulas (35c) and (35c) are valid.

The temperature dependence of the critical current in the region of intermediate densities is given by (32) and (33) in the pure case and (43) and (44) in the dirty one. It is seen that at not too low temperatures this dependence is exponential but the argument of the exponential is proportional to the temperature raised to some power that depends on the statistics of the fluctuations of the bottom of the conduction band in the semiconductor. At low temperatures, the dependence is the same as in the degenerate case.

For a qualitative explanation of the results, we consider the case of a strongly doped ($N_d a^{\,3} \gg 1$) compensated ($K = N_d/N_a \to 1$) semiconductor. With increasing degree of compensation, the large-scale fluctuations of the potential become stronger (their amplitude and spatial size increase). The electron density then decreases, and the chemical potential drops into the interior of the forbidden band. In the compensation region $K > K_c \sim 1-(N_d a_B{}^3)^{-1/3}$, the chemical potential lies below the average value of the bottom of the band by precisely the value of the typical fluctuation, so that electron drops are produced at places where the bottom of the band is lowered.[10] Long drops (channels) that interconnect the superconducting region have a low probability of being produced. It turns out that the superconducting current flows mainly through deep channels, in which the electron density is much higher than in a typical drop.

The probability $\mathscr{F}(U)$ of formation of such a channel can be easily obtained by assuming that the impurities have a Poisson distribution:

$$\ln\frac{\mathscr{F}(U)}{\mathscr{F}(0)}=\frac{-Z^2}{N_d r_0{}^2 a}$$

where Z is the excess number of impurities in the channel, and r_0 is its radius. The lowering of the bottom of the band in the channel relative to the chemical-potential level U is equal to the energy of the electron in the potential produced by a cluster of impurities: $U = e^2Z/\varkappa a$. The radius of the channel is determined by the screening length: $r_0 \sim a_B{}^{1/2} n^{-1/6}$ at electron densities $n \sim (mU)^{3/2}$. As a result we find that the channel-formation probability is given by Eq. (29), in which $\alpha = 5/2$ and $U_f \sim E_0(N_d a_B{}^4/a)^{2/5}$.

The deeper the channel, the less probable its formation, but at the same time the less the weakening of the superconducting current in it, since the coherence length $\xi \sim v/T \sim U^{1/2}/m^{1/2}T$ increases in this case. The optimal channels have a depth $U \sim U_f(a/\xi_f)^{1/3}$, where $\xi_f = U_f{}^{1/2}/m^{1/2}T$. As a result, at $a>\xi_f$ the critical current of the junction is

$$j_c \sim \exp(-a/\xi) \sim \exp[-(a/\xi_f)^{5/6}],$$

and formula (32) is valid [or (43a) in the dirty case]. At a $<\xi_f$, the temperature dependence of the critical current is determined by the formulas (33) and (43b).

Thus, the fluctuations alter the temperature dependence of the critical current of an S–Sm–S junction in the case of a strongly doped semiconductor at compensations $K>K_c$ and semiconductor layer thicknesses $a > \xi_f$. We note that formulas (32) and (43) for the critical current cease to hold at very high compensation for two reasons. First, the dimension of the Schottky barriers increases with increasing compensation and can exceed the thickness of the semiconductor layer. Second, the excess impurity density in the channel can become comparable with the density $N_d(1-K)$ that determines the thickness of the Schottky barriers, and it is then necessary to take into account the change of their transparency on account of their fluctuations. However, the corresponding compensations

$$K_1 \sim 1-\frac{U_0-\mu}{E_0 N_d a^2 a_B}, \quad K_2 \sim 1-(N_d a_B{}^3)^{-1/2}\left(\frac{T}{E_0}\right)^{1/2}$$

are certainly larger than the compensation K_c at $a>\xi_f$.

Experiment revealed both a tunnel temperature dependence of the critical current of the S–Sm–S junction in the case of a nondegenerate semiconductor, and an exponential dependence in the case of strong doping.[1-3] In experiment, the impurity density $N_d \gtrsim a_B{}^{-3}$ even in the case of strong doping, so that for weak compensation the chemical potential is higher than the average value of the bottom of the conduction band only by a value of the order of the fluctuation scale. The results explain why the temperature dependence absorbed in this case is close to that corresponding to the S–N–S junction: the main contribution to the current is made by the fluctuation channels with strong degeneracy. More accurate measurements of the temperature dependence of the critical current of S–Sm–S junction would reveal the deviation and by the same token yield experimentally the value of α that determines the statistics of the deep fluctuations of the bottom of the conduction band in the semiconductor.

Another convenient possibility of observing the change of the character of the temperature dependence of the critical current is provided by light-sensitive junctions. When these junctions are illuminated, long-lived conduction electrons appear in the semiconductor (the processes of recombination with impurities and with holes at low temperatures are very slow). The statistics of these electrons are determined by the Fermi quasilevel, whose position relative to the bottom of the conduction band depends on the irradiation time. At short times, the electron density is low and the Fermi level lies much lower than the bottom of the conduction band. The temperature dependence of the critical current is then the same as in a tunnel junction. With increasing irradiation, however, the Fermi level rises and may turn out to be in the region of the fluctuations of the bottom of the conduction band. The type of the temperature dependence of the critical current becomes correspondingly fluctuating. A similar transition from a tunnel to a fluctuating temperature dependence of the critical current can be observed in junctions with weakly doped semiconductors when the compensation is increased.

The theory developed makes it possible to determine the critical current of S–Sm–S junctions in which Schottky barriers are produced at the interfaces between the semiconductor and the superconductors. For certain superconductors, ohmic contacts may be produced if electron-enriched potential wells appear at the interfaces in place of the barriers. However, the size of these wells becomes of the same order as the interatomic distances even at very small impurity densities, and these wells have little effect on the passage of the electrons through the interfaces. In the case of a nondegenerate semiconductor, the temperature dependence of the critical current is determined by the tunnel formula (25). In the degenerate case, however, the mutual influence of the superconductors becomes significant and leads to a dependence of the order parameter on the coordinates. The critical current for this case was calculated by Likharev[13] and by Barone and Ovchinnikov.[14]

The authors thank A.A. Abrikosov, A.I. Larkin, and A.L. Éfros for valuable discussions of the results.

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Translated by J. G. Adashko.