The motion of a Josephson vortex in the field of a random potential

M. B. Mineev, M. V. Feĭgel'man, and V. V. Shmidt

Institute of Solid State Physics, Academy of Sciences of the USSR
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We consider the motion and pinning of a Josephson vortex in a field produced by random inhomogeneities in a long junction. We find the distribution function of the force of vortex pinning on the inhomogeneities. We construct the current-voltage characteristic (CVC) of the junction. For inhomogeneities which are weak compared to the ohmic losses the CVC has a single hysteresis, in the opposite case it has two.

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1. STATEMENT OF THE PROBLEM

The aim of the present paper is a study of the behavior of an isolated Josephson vortex (soliton) under conditions of random inhomogeneities in the junction which include, in particular, random microcontacts of the junction sides. Here the junction is assumed to be one-dimensional and long, i.e., $L \gg \lambda_J$, where L is the length of the junction and λ_J the Josephson penetration depth.

It is well known[1] that the equation for the phase difference between the sides of an ideal (lossless) Josephson junction has the form of the sine-Gordon equation:

$$\frac{\partial^2\varphi}{\partial x^2}-\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2}=\frac{\sin\varphi}{\lambda_J^2}. \qquad (1)$$

Here c_0 is the velocity of propagation (Swihart velocity) of an electromagnetic wave in the junction.

A solitary vortex in an ideal long junction is described by the one-soliton solution

$$\varphi_0=4\,\mathrm{arctg}\left(\exp\frac{x-vt}{\lambda_J(1-\beta^2)^{1/2}}\right). \qquad (2)$$

Here v is the velocity of the vortex and $\beta=v/c_0$. In reality such a solitary vortex is obtained from a rarefied chain of vortices in the limit of a low vortex concentration. It can also occur in the form of a "shuttling vortex"[2] which performs a finite motion, periodically being reflected from the edges of the junction and reversing in that process the direction of the magnetic flux quantum contained in the vortex. In both cases the difference in the potentials between the sides is equal to the velocity of the vortex to within a constant factor:

$$V=\Phi_0 nv/Lc, \qquad (3)$$

where V is the potential difference, $\Phi_0=\pi\hbar c/e=2\times10^{-7}$ G·cm^2 is the magnetic flux quantum, v the vortex velocity, n the number of vortices in the junction (for a shuttling vortex $n=1$) and $c=3\times10^{10}$ cm/s is the velocity of light *in vacuo*.

The presence of random contact inhomogeneities reduces in the equations to the fact that λ_J together with the factor in front of $\sin\varphi$ in (1) becomes a random function of the coordinate:

$$\lambda_J^{-2}(x)=\langle\lambda_J\rangle^{-2}[1+f(x)], \qquad (4)$$

where $\langle\lambda_J\rangle$ is an average of λ_J along the junction, while $f(x)$ is the random deviation from the average; thus

$$\langle f(x)\rangle=0. \qquad (5)$$

We can choose the correlator of the random quantity f is the form

$$K(x-x')=\langle f(x)f(x')\rangle=\frac{\alpha}{2l}\exp\left(-\frac{|x-x'|}{l}\right), \qquad (6)$$

where the correlation radius l of the random potential is assumed to be much smaller than the size of the vortex λ_J,[1)] while α is the so-called Gaussian correlation parameter. As $\lambda\to0$ we have from (6)

$$K(x-x')\to\alpha\delta(x-x').$$

The parameter α can be expressed in terms of the mean squared fluctuation of the critical current density of the Josephson junction:

$$\alpha=2l\langle(\delta J_c)^2\rangle/J_c^2.$$

We shall in what follows use as system of units

$$\langle\lambda_J\rangle=c_0=1.$$

Bearing (4) in mind and also the presence of ohmic losses which are unavoidable in real junctions we can write the equation

$$\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial t^2}-\eta\frac{\partial}{\partial t}\right)\varphi=(1+f)\sin\varphi+J_0, \qquad (7)$$

where η is the coefficient of the viscosity in the junction and is equal to

$$\eta=\Phi_0c_0/2\pi cJ_c\rho d\lambda_J,$$

where ρ is the ohmic resistivity of the Josephson junction, d the thickness of the dielectric layer, and J_c the critical current density.

The uniformly distributed current J_0 introduced in (7) compensates for the ohmic energy losses of the vortex and thus guarantees its uniform motion (when $f=0$). In Refs. 2 to 4 it was shown that the velocity β of a uniformly moving vortex is connected with J_0 as follows:

$$J_0=4\eta\beta/\pi(1-\beta^2)^{1/2}. \qquad (8)$$

Eq. (8) is rigorously applicable when $J_0\ll1$, is qualitatively valid when $J_0\lesssim1$, and when $J_0\gtrsim1$, as was shown by Todorov,[5] it goes over as $\beta\to1$ into Aslamazov and Larkin's formula[6]

$$J_0=[J_c+(\eta V)^2]^{1/2}.$$

This can be understood from the following considerations. When $J_0<1$ the voltage in the junction is connected with the motion of a single vortex, i.e., of a local region when φ changes by 2π, while when $J_0>1$ the

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voltage in the contact is caused by the Josephson generation, at which φ changes with time practically uniformly along the whole of the junction.

Equation (8), when (3) is taken into account, is the current-voltage characteristic (CVC) of the junction described here when there are no random inhomogeneities.

We shall examine what changes in the CVC result from the presence of the random quantity f in (7). When $f \neq 0$ the vortex is slowed down through friction against the random inhomogeneities, and for a uniform motion of the vortex it is necessary to introduce a current J_1 additional to J_0 in the junction. We must thus add a term J_1 to the right-hand side of (7).

2. VORTEX MOTION IN THE FIELD OF A RANDOM POTENTIAL

To isolate in a pure form the role of the random inhomogeneities in the junction we neglect temporarily the ohmic losses in the junction ($\eta = 0$) and the current J_0 which compensates for these losses. The initial equation then takes the form

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial t^2} = [1 + f(x)]\sin\varphi + J_1. \tag{9}$$

Let $|f| \ll 1$. We look for a solution of the equation in the form

$$\varphi = \varphi_0 + \varphi_1 + \varphi_2,$$

where φ_0 is given by Eq. (2), and $|\varphi_2| \ll |\varphi_1| \ll 1$. Equation (9) linearized in φ_1 will then, in the coordinates ξ and τ, which move with the vortex, take the form[2)]

$$L_\tau\varphi_1(\xi,\tau) \equiv \left[\frac{\partial^2}{\partial\xi^2} - \frac{\partial^2}{\partial\tau^2} - \cos\varphi_0(\xi)\right]\varphi_1(\xi,\tau) = f\left(\frac{\xi+\beta\tau}{\gamma}\right)\sin\varphi_0(\xi),$$
$$\xi = (x-\beta t)/\gamma, \quad \tau = (t-\beta x)/\gamma, \quad \gamma = (1-\beta^2)^{1/2}.$$

Hence we find

$$\varphi_1(\xi,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-i\omega\tau - \frac{i\omega\xi'}{\beta}\right) f_{-\omega\gamma/\beta}\sin\varphi_0(\xi')\,G_\omega(\xi,\xi')\frac{\gamma\,d\xi'\,d\omega}{2\pi\beta},$$

where $f_{-\omega\gamma/\beta}$ is the Fourier transform of $f(x)$ with wave vector $-\omega\gamma/\beta$, while $G_\omega(\xi,\xi')$ is the Fourier transform of the Green operator L_τ with respect to time and is known from Ref. 7:

$$G_\omega(\xi,\xi') = -\frac{\exp(-q_0|\xi-\xi'|)}{2q_0(1-q_0^2)}(\operatorname{th}\xi \pm q_0)(\operatorname{th}\xi' \mp q_0), \tag{10}$$
$$q_0 = \begin{cases} -i(\omega^2-1)^{1/2} & \text{if} \quad \omega > 1 \\ i(\omega^2-1)^{1/2} & \text{if} \quad \omega < -1. \\ (1-\omega^2)^{1/2} & \text{if} \quad |\omega| \leq 1 \end{cases}$$

The upper sign corresponds to the case $\xi > \xi'$ and the lower one to the case $\xi < \xi'$.

To second order in f we have from (9)

$$L_\tau\varphi_2(\xi,\tau) = f\left(\frac{\xi+\beta\tau}{\gamma}\right)\cos\varphi_0(\xi)\,\varphi_1(\xi,\tau) - \frac{\sin\varphi_0(\xi)}{2}\varphi_1^2(\xi,\tau) + J_1. \tag{11}$$

We change in (11) to the frequency representation and perform the averaging $\langle f_k f_{k'}\rangle$ under the integral signs, using the condition $l \ll 1$. We get

$$L_\omega\varphi_2(\xi,\omega) = 2\pi\delta(\omega)\frac{\gamma}{\beta}\int_{-\infty}^{\infty} K_{-\omega'\gamma/\beta}$$
$$\times\left\{\int_{-\infty}^{\infty} G_\omega(\xi,\xi')\exp\left(-\frac{i\omega'\xi'}{\beta}\right)\sin\varphi_0(\xi')\,d\xi'\exp\left(\frac{i\omega'\xi}{\beta}\right)\cos\varphi_0(\xi)\right.$$
$$\left.-\frac{\sin\varphi_0(\xi)}{2}\left|\int_{-\infty}^{\infty} G_{\omega'}(\xi,\xi')\exp\left(-\frac{i\omega'\xi'}{\beta}\right)\sin\varphi_0(\xi')\,d\xi'\right|^2\right\} + 2\pi\delta(\omega)J_1. \tag{12}$$

We used the fact that $\langle f_k f_{k'}\rangle = 2\pi\delta(k+k')K_k$ where

$$K_q = \int_{-\infty}^{\infty} K(x)e^{-iqx}\,dx = \frac{\alpha}{1+l^2q^2}.$$

Requiring that φ_2 be finite, which corresponds to a uniform motion of the soliton, we get the following expression for J_1:

$$J_1 = \frac{\gamma}{\pi\beta}\int_{-\infty}^{\infty}\frac{\sin\varphi_0(\xi)}{2\operatorname{ch}\xi}\int_{-\infty}^{\infty} K_{-\omega'\gamma/\beta}\left|\int_{-\infty}^{\infty} G_{\omega'}(\xi,\xi')\exp\left(-\frac{i\omega'\xi'}{\beta}\right)\sin\varphi_0(\xi')\,d\xi'\right|^2$$
$$\times\frac{d\omega'}{2\pi}d\xi - \frac{\gamma}{\pi\beta}\int_{-\infty}^{\infty}\frac{\cos\varphi_0(\xi)}{\operatorname{ch}\xi}\int_{-\infty}^{\infty} K_{-\omega'\gamma/\beta}\int_{-\infty}^{\infty} G_{\omega'}(\xi,\xi')$$
$$\times\exp\left(-\frac{i\omega'\xi'}{\beta}\right)\sin\varphi_0(\xi')\exp\left(\frac{i\omega\xi}{\beta}\right)\frac{d\omega'}{2\pi}d\xi. \tag{13}$$

After cumbersome calculations the integral J_1 takes the form

$$J_1 = \frac{\gamma^5}{8\beta^5}\int_1^{\infty} d\omega\,K_{-\omega\gamma/\beta}\left[(b_+^2 - b_-^2)\frac{\omega^3}{q\beta} - (b_+^2 - b_-^2)\omega^2\right], \tag{14}$$

where

$$b_\pm = \frac{1}{\operatorname{ch}(\pi q_\pm/2)}, \quad q_\pm = q \pm \frac{\omega}{\beta}, \quad q = (\omega^2-1)^{1/2}.$$

We emphasize that the change from (13) to (14) was made without any approximations.

We give in Fig. 1 the result of a numerical integration in (14) for different values of the correlation length l. It is clear from these curves that J_1 reaches a maximum for some β^* and that the $J_1(\beta)$ curves are practically insensitive to a change in l at $l < 0.1$.

When $\beta \ll 1$ we can estimate expression (14) using the steepest descent method:

$$J_1 = \alpha e^{-\pi/\beta}/2^{1/2}\beta^{11/2}. \tag{15}$$

We shall now discuss the results.

From the very start, the problem was to find a cur-

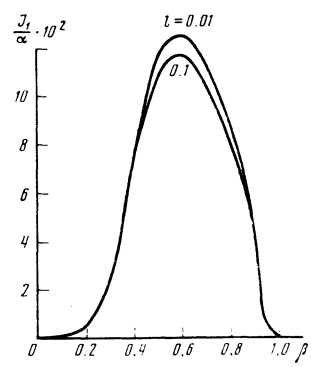

FIG. 1. Current compensating for radiative losses vs. the potential difference on the junction for different values of l.

rent J_1 that would guarantee the motion of a vortex in a uniform junction with a constant velocity β. It is well known[7] that when a vortex moves along a junction with a periodically varying critical current it emits electromagnetic waves which have a plasma dispersion law $\omega^2 = k^2 + 1$, which is easily obtained from (1) when $\varphi \ll 1$. This means that the emission of such a vortex starts from a threshold frequency $\omega = 1$ in the coordinate system of the vortex. The vortex emits therefore starting from some threshold velocity. It is clear that if the vortex moves along a junction with randomly distributed inhomogeneities it must emit not a monochromatic wave but noise. Correspondingly, it has a velocity distribution of the emission thresholds. As a result the intensity of the radiation of such a vortex will increase with increasing vortex velocity, since ever newer frequencies will take part in the emission.

On the other hand, it was shown in Ref. 7 that the emission of a vortex moving along a junction with periodic inhomogeneities tends to zero as $\beta \to 1$. It is thus clear that also when the vortex moves along a junction with random inhomogeneities the emission by the vortex will be damped as $\beta \to 1$. It is clear from (14) that the emission occurs at all frequencies $\omega \geq 1$ and the mode with frequency ω had a radiation amplitude proportional to $f_{-\omega\gamma/\beta}$.

It now remains to connect the vortex radiation with the current J_1. This can easily be understood. Indeed, to sustain the vortex velocity at a constant value it is necessary to compensate for the radiative energy losses of the vortex at the expense of the energy of the current source. Therefore J_1 increases with β for small values of β and $J_1 \to 0$ as $\beta \to 1$.

3. PINNING OF A VORTEX BY RANDOM INHOMOGENEITIES

We consider an unperturbed vortex at rest and localized in the vicinity of a point x_0; it is thus described by the formula

$$\varphi(x-x_0)=4\,\mathrm{arctg}\exp(x-x_0). \tag{16}$$

The equation with a random f, which in the case of a vortex at rest has the form

$$\frac{\partial^2\varphi}{\partial x^2}=(1+f)\sin\varphi,$$

corresponds to the Hamiltonian

$$H=\int_{-\infty}^{\infty}\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2+1-(1+f)\cos\varphi\right]dx,$$

which for $f \ll 1$ [with allowance for (16)] is equal to

$$H=8-\int_{-\infty}^{\infty}f(x)\cos\varphi_0(x-x_0)\,dx. \tag{17}$$

The force acting upon the vortex in this case is equal to

$$F=-\frac{\partial H}{\partial x_0}=-4\int_{-\infty}^{\infty}\frac{f(x)\,\mathrm{sh}(x-x_0)}{\mathrm{ch}^3(x-x_0)}dx. \tag{18}$$

We find the distribution function of the quantity F corresponding to different realizations of $f(x)$. To do this we shall consider F as a function of y, the upper limit of the integral:

$$F(y)=-4\int_{-\infty}^{y}f(x)\frac{\mathrm{sh}(x-x_0)}{\mathrm{ch}^3(x-x_0)}dx.$$

We write further

$$F(y+\Delta y)=F(y)-4\int_{y}^{y+\Delta y}f(x)\frac{\mathrm{sh}(x-x_0)}{\mathrm{ch}^3(x-x_0)}dx.$$

The distribution function $W(F,y)$ of the force F produced by all inhomogeneities with coordinates less than y and $W(F,y+\Delta y)$ are obviously connected, as follows from the last equation, by the relation:

$$W(F,y+\Delta y)=\left\langle W\left(F+4\int_{y}^{y+\Delta y}f(x)\frac{\mathrm{sh}(x-x_0)}{\mathrm{ch}^3(x-x_0)}dx,y\right)\right\rangle. \tag{19}$$

The averaging in the right-hand side of (19) is done over the random force f in the interval $y < x < y+\Delta y$. We choose $l \ll \Delta y \ll 1$ and, expanding the right-hand side of (19) in a series in Δy, we get after averaging and using (6)

$$\frac{\partial W(F,y)}{\partial y}=8\alpha\frac{\mathrm{sh}^2(y-x_0)}{\mathrm{ch}^6(y-x_0)}\frac{\partial^2 W(F,y)}{\partial F^2}.$$

The solution of this equation has the form

$$W(F,y)=[2\pi\Phi(y)]^{-1/2}\exp(-F^2/2\Phi(y)),$$

$$\Phi(y)=16\alpha\int_{-\infty}^{y}\frac{\mathrm{sh}^2(x-x_0)}{\mathrm{ch}^6(x-x_0)}dx.$$

For the quantity $W(F)\equiv W(F,\infty)$ in which we are interested we have

$$W(F)=(15/64\pi\alpha)^{1/2}\exp(-15F^2/64\alpha), \tag{20}$$

i.e., the force acting from the random inhomogeneities on the vortex has a Gaussian distribution with a mean square deviation of the order $\alpha^{1/2}$. One shows similarly that in a random potential the vortex energy U which is added to the energy $E_0 = 8$ of the unperturbed vortex [see (17)] also has a Gaussian distribution:

$$W(U)dU=(3/8\pi)^{1/2}\exp(-3U^2/8\alpha)\,dU. \tag{21}$$

It follows from (17) and (18) that the characteristic scale of the changes in the field of the random potential (not to be confused with the correlation radius l!) in which the soliton moves is of the order of unity or, in dimensional notation of order λ_j.

It is clear from (20) that the characteristic pinning force which constrains the vortex is of order $\alpha^{1/2}$ and fluctuates around that value according to (20).

4. CVC OF CONTACTS WITH RANDOM INHOMOGENEITIES

We discuss the form of the CVC of a long junction with random inhomogeneities. The dependence of the total external current on the average voltage across the junction can be obtained by simply adding the current J_0 (8), which compensates for the ohmic losses in the junction, and J_1 [see (15)], which compensates for the radiative losses of the vortex at random defects. Thus,

$$J(\beta)=J_0(\beta)+J_1(\beta), \tag{22}$$

where according to (3) $\beta = LcV/\Phi_0 c_0 = V/V_0$.

The shape of this CVC is shown in Fig. 2. Whether or

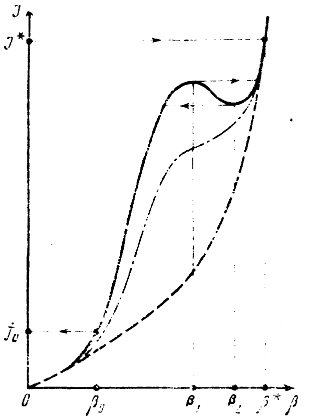

FIG. 2. CVC of a junction with random inhomogeneities; the continuous curve corresponds to $\eta \ll \alpha$, the dashed curve to $\alpha = 0$, and the dash-dot curve to $\eta \approx \alpha$.

not $J(\beta)$ is monotonic depends on the relation between the damping η and the force α of the random defects. In particular, if $\eta \ll \alpha$ (in dimensional units $\eta\lambda_J \ll \alpha$) the CVC will be non-monotonic (Fig. 2, continuous curve). When $\eta \gg \alpha$ the CVC is monotonic and differs little from the case $\alpha = 0$ shown by the dashed curve in Fig. 2. When $\eta \approx \alpha$ a characteristic plateau is formed on the CVC (Fig. 2, dash-dot curve). As the value of $J_1(\beta)$ is exponentially small when $\beta \ll 1$ and the current $J_1(\beta) \to 0$ as $\beta \to 1$, it is clear that in those extreme regions the shape of the CVC is determined by the current J_0 that compensates for the Ohmic losses.

The behavior of the vortex in the field of a random potential seems to us to be as follows.

The vortex is at rest until, in accordance with (20), the current supplied in the junction reaches a value J^* of the order of $\alpha^{1/2}$, as a result of which the vortex breaks loose and starts to move. The vortex will accelerate until it reaches a velocity β^* at which the Lorentz force produced by the external current and accelerating the vortex is not compensated by the ohmic friction force which, although assumed to be small, becomes important, according to (8), when the velocity has increased sufficiently. Since the maximum current connected with the emission $J_1(\beta) < \alpha \ll 1$ (see Fig. 1) and the rupture current is $J^* \sim \alpha^{1/2} \gg \alpha$ [see (20)], a stationary regime of vortex motion cannot be reached via radiative friction alone, but is established because of the ohmic friction given by Eq. (8).

The process described here has a mechanical analog. The vortex is at rest, pinned by the junction inhomogeneities, until the external current reaches the magnitude J^*. At that moment the static friction force will be overcome by the Lorentz force applied to the vortex, which will be accelerated until the Lorentz force is balanced by the gliding friction force.

On the section between β_1 and β_2 in Fig. 2 we have $\partial J/\partial V < 0$, so that the CVC on that section shows hysteresis as is shown in the same figure. We estimate now the order of the velocity β_0 for which Eq. (15) is no longer applicable. It follows from the above that a vortex moving in a random field spends energy only on radiation (we have thus an active resistance to the current) while the effective field of the random forces which, as we have mentioned, changes over distances of the order λ_J, plays the role of a reactance in the sense that the soliton velocity fluctuates, but does not decrease on the average during the motion along such a relief. The minimum velocity with which the vortex can still "roll over the hump" of the effective field of the random forces can easily be estimated by equating the kinetic energy of the soliton [the mass of which is equal to 8 according to (17)] to the height U of the energy barrier produced by the random potential:

$$4\beta_0^2 \approx U. \tag{23}$$

It is clear from (21) and (23) that the quantity β_0 has a broad distribution, the asymptotic behavior of which for $\beta_0 \gg \langle\beta_0\rangle$ can be obtained by substituting (23) in the form of an equality into (21):

$$P(\beta_0)\,d\beta_0 = A\beta_0 \exp(-24\beta_0^4/\alpha)\,d\beta_0, \quad A \sim 1. \tag{24}$$

We now determine the average stopping power j_0. If at $\beta \sim \beta_0$ the ohmic-loss current (8) dominates in (22), the value of j_0 is as to order of magnitude determined by substituting into (8) $\beta = \beta_0 \sim \langle U^2\rangle^{1/2} \sim \alpha^{1/2}$:

$$j_0 \sim \eta\alpha^{1/2}. \tag{25}$$

In the case when the radiative losses are more important, j_0 is determined by averaging $J_1(\beta)$ from (15) with the distribution function (24):

$$j_0 = \int \frac{A\alpha}{2^{1/2}} \beta_0^{-9/2} \exp\left(-\frac{\pi}{\beta_0} - \frac{24\beta_0^4}{\alpha}\right) d\beta_0$$
$$= \tilde{A}\alpha^{2/5} \exp\left[-\frac{5\pi}{4}\left(\frac{\pi\alpha}{96}\right)^{-1/5}\right], \quad \tilde{A} \sim 1. \tag{26}$$

When evaluating the integral in (26) it turns out that $\beta_0 \gg \alpha^{1/2}$ are the important values. This justifies the use of the asymptotic formula (24).

We now discuss the conditions for observing the predicted effects. Firstly, the ohmic losses in the junction must be sufficiently small, viz., the condition

$$\eta \ll \alpha/\lambda_J \tag{27}$$

must be satisfied. A characteristic value is $\eta \approx (1 \text{ to } 3) \times 10^{-3}$. The quantity α is determined by the mean square fluctuations in the critical current density

$$\frac{\alpha}{\lambda_J} = \frac{2l}{\lambda_J} \frac{\langle(\delta J_c)^2\rangle}{J_c^2}$$

(l is the characteristic size of the inhomogeneities), and it depends strongly on the way the junction is prepared. Since $\eta \ll 1$, the condition (27) can be satisfied when $\alpha/\lambda_J \ll 1$, i.e., in the region where the theory developed in this paper is applicable. Secondly, the length L of the junction must be much larger than the characteristic size of a vortex λ_J.

Recently Ref. 8 has appeared in which the CVC of a long ($L \approx 20\,\lambda_J$) Josephson junction with a strongly pronounced plateau was observed. Such a form of CVC is described by our theory when $\alpha \sim \eta$ (Fig. 2, dash-dot curve). In conclusion we note that the CVC obtained by us contains a "large" hysteresis between the points β_0 and β^* and in the non-monotonic case ($\eta \ll \alpha$, continuous curve in Fig. 2) a "small" hysteresis between the points

β_1 and β_2.

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[1)] The characteristic value of l is of the order of the size of the granules of the superconducting films which make up the junctions, i.e., $l \lesssim 10^{-4}$ cm. A characteristic value is $\lambda_J \sim 10^{-2}$ cm. Thus $l \ll \lambda_J$.

[2)] Here we neglect the current J_1 which, as will become clear in what follows, is of second order of smallness in f.

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