Stability of microwave-irradiated nonequilibrium superconductors

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The stability of microwave-irradiated nonequilibrium superconductors is investigated. It is shown that even at rather low radiation intensities the superconductor becomes unstable to fluctuations of the order-parameter modulus. The connection between this instabilitity and the enhancement of the superconductivity by microwave radiation is discussed.

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1. INTRODUCTION

The enhancement of superconductivity by microwave radiation has been studied both experimentally[1,2] and theoretically.[3,4] The experimentally observed increase in the critical temperature T_c under the action of the irradiation is significantly less than that predicted theoretically. Furthermore, in the experiment[5] sharp jumps are observed in the functions $R(I)$ and $R(T)_{I=\text{const}}$ (I is the radiation intensity, R is the resistance of the sample), are apparently due to a charge in the conditions under which the steady-state quasiparticle distribution functions are established, and are not described by the theory. On the other hand, the stability of superconductor nonequilibrium states that set in upon microwave irradiation have not been completely studied.

The goal of this work is to show that even at relatively low microwave radiation intensities, a homogeneous superconductor state becomes unstable to infinitesimally small fluctuations of the order-parameter modulus.

We will investigate fluctuations with frequencies ω and wave vectors q satisfying the inequalities $\hbar\omega \ll \tilde{\varepsilon}$ and $qV_F \ll \tilde{\varepsilon}$. Here $\tilde{\varepsilon}$ is the characteristic energy of the quasiparticles, which will be determined below. In this case, for the description of the superconductor state we may make use of the kinetic equation and the usual self-consistency equation for the order-parameter modulus (we do not consider fluctuations of the phase of the order parameter in this paper). Linearizing these equations relative to fluctuations of the energy gap Δ' and of the quasiparticle distribution function n'_ε, we can obtain at $\tau_\varepsilon^{-1} \ll \omega \ll \tau_{\text{im}}^{-1}$ (τ_{im} and τ_ε are the quasiparticle momentum and energy relaxation times) the dispersion relation connecting ω and q (Refs. 6–8):

$$\frac{N_s}{N} = -2\int_\Delta^\infty d\varepsilon \frac{\Delta^2}{\varepsilon^2} \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \frac{\partial n_\varepsilon}{\partial \varepsilon} \frac{i\omega}{-i\omega + Dq^2(\varepsilon^2-\Delta^2)^{1/2}/\varepsilon}, \qquad (1)$$

$$\frac{N_s}{N} = 1 + 2\int_\Delta^\infty \frac{\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}} \frac{\partial n_\varepsilon}{\partial \varepsilon} d\varepsilon. \qquad (1a)$$

Here ε is the quasiparticle energy, n_ε is the steady-state quasiparticle distribution function in the microwave-pumped superconductor, Δ is the half-width of the energy gap, D is the diffusion coefficient for electrons in the normal metal, N is the electron density in the metal, and N_s is the superfluid density.

If $\operatorname{Im}\omega < 0$, the system is stable; if $\operatorname{Im}\omega > 0$, the system is unstable. From Eq. (1) it is easy to find[6-8] that at $N_s < 0$ the system is unstable to the fluctuations under consideration.[1)]

It is seen that the quasiparticle distribution function itself does not enter in the dispersion equation and in the stability criteria, but rather its derivative with respect to energy. Thus, the detailed structure of the quasiparticle distribution function proves to be very essential for investigation of the stability.

2. CALCULATION OF THE STEADY-STATE QUASIPARTICLE DISTRIBUTION FUNCTION IN A MICROWAVE-IRRADIATED SUPERCONDUCTOR

The quasiparticle distribution function arising in superconductors subjected to the action of microwave radiation of frequency Ω has been studied theoretically in Ref. 4. However, we need a more accurate solution than the one in Ref. 4. As will be evident from what follows, for us the low energy $\varepsilon - \Delta \ll kT$ region will be essential.

It is known that in this energy region the phonon collision operator can be written accurate to $((\varepsilon-\Delta)/kT_c)^3 \ll 1$ in the relaxation-time approximation (we consider elastic scattering of quasiparticles by impurities to be the fastest process; therefore the quasiparticle distribution function does not depend on the momentum direction). Then the kinetic equation for n_ε has the form[4]

$$A\left\{\frac{\varepsilon(\varepsilon-\hbar\Omega)+\Delta^2}{\varepsilon((\varepsilon-\hbar\Omega)^2-\Delta^2)^{1/2}}(n_{\varepsilon-\hbar\Omega}-n_\varepsilon)\theta(\varepsilon-\Delta-\hbar\Omega) - \frac{\varepsilon(\varepsilon+\hbar\Omega)+\Delta^2}{\varepsilon((\varepsilon+\hbar\Omega)^2-\Delta^2)^{1/2}}(n_\varepsilon-n_{\varepsilon+\hbar\Omega})\theta(\varepsilon+\hbar\Omega-\Delta)\right\} = \frac{n_\varepsilon-n_\varepsilon^0}{\tau_\varepsilon}. \qquad (2)$$

Here, $A = 8\pi e^2 ID\alpha/c(\hbar\Omega)^2 d$ is a parameter proportional to the radiation intensity; $n_\varepsilon^0 = (e^{\varepsilon/kT}+1)^{-1}$ is the equilibrium quasiparticle distribution function; T is the thermostat temperature; α and d are the skin depth and the sample thickness.

For simplicity, we consider the case $\hbar\Omega < 2\Delta$; but we can show that the results obtained below also carry over to the case $\hbar\Omega > 2\Delta$. Furthermore, we regard the phonon subsystem as in equilibrium, i.e., we assume that nonequilibrium phonons are not reabsorbed before they

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leave the sample. Taking into account reabsorption of phonons also does not result in a change in the results, primarily because at $\Delta \ll kT$ nonequilibrium phonons relax mainly on high-energy quasiparticles with $\varepsilon \approx kT$.

We will search for a solution of Eq. (2) for relatively low radiation intensities, when $\Gamma = A\tau_\varepsilon(2\Delta/\hbar\Omega)^{1/2} \ll 1$, and show that even in this case the solution is unstable.

By virtue of $\Gamma \ll 1$, Eq. (1) may be solved by perturbation theory[4] and we can substitute in the left-hand side of Eq. (2) the equilibrium quasiparticle distribution function n_ε^0. The energy regions $\varepsilon \to \Delta$ and $\varepsilon \to \Delta + \hbar\Omega$ are exceptions. Due to singularities in the density of states, the arrival of quasiparticles from the region $\varepsilon = \Delta$ into the region $\varepsilon \approx \Delta + \hbar\Omega$ as a result of photon absorption tends toward infinity, and therefore the accurate solution of Eq. (2) has the property $n(\varepsilon) = n(\varepsilon + \hbar\Omega)$ as $\varepsilon \to \Delta$. As for the departure of the quasiparticles from the region $\varepsilon \approx \Delta + \hbar\Omega$ to the region $\varepsilon \approx \Delta + 2\hbar\Omega$ as a result of photon absorption, it can be considered by perturbation theory just as in Ref. 4.

To obtain a solution, we must divide the entire energy region into two parts, $(\Delta, \Delta + \hbar\Omega)$ and $(\Delta + \hbar\Omega, \infty)$ and neglect by virtue of $\Gamma \ll 1$ the second term in Eq. (2) in the second region. The equation can then be solved exactly and

$$n_z = n_z^0 + \frac{\Gamma z^{1/2}}{\Gamma z^{1/2} + (z+1)^{1/2}(\Gamma + z^{1/2})}(n_{z+1}^0 - n_z^0), \quad z = \frac{\varepsilon - \Delta}{\hbar\Omega} < 1, \tag{3}$$

$$n_z = n_z^0 + \frac{\Gamma z^{1/2}(n_{z-1}^0 - n_z^0)}{\Gamma(z-1)^{1/2} + z^{1/2}(\Gamma + (z-1)^{1/2})}, \quad z = \frac{\varepsilon - \Delta}{\hbar\Omega} > 1. \tag{3a}$$

The obtained solution (3) is shown in Fig. 1. For $z \gg \Gamma^2$, it agrees with the solution obtained by perturbation theory in Ref. 4 (and shown in Fig. 1 by the dashed line).

The integrated effects of an increase in T_c, investigated in Ref. 4, are not sensitive to the details of the behavior of n_ε in the small region $z < \Gamma^2$. However, this region is quite essential for determination of the stability. In fact, from Eq. (3) it is evident that at $z \ll \Gamma^2$ we have $n_z = n_z^0 - \hbar\Omega z^{1/2}/4kT$ (it is important that in this region n_z does not depend on the irradiation intensity). From Eq. (3) it is seen, first, that $N_s < 0$ and second, that the integral in (1a) diverges logarithmically as a result of the square-root singularity of the distribution function as $z \to 0$. This singularity occurs as a

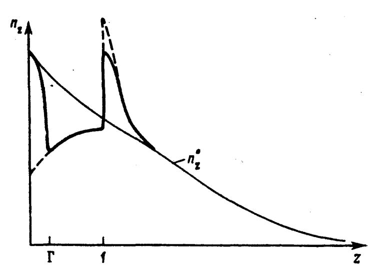

FIG. 1. Steady-state quasiparticle distribution function following microwave irradiation. The thin solid line is the equilibrium distribution. The thick solid line is the distribution function (3). The dashed line is the distribution function obtained in Ref. 4.

result of the square-root singularity in the left-hand side of the kinetic equation (2) at $\varepsilon = \Delta + \hbar\Omega$ and is not "smeared" by any relaxation processes that do not have the indicated singularity. In particular, by going outside the framework of the "relaxation time approximation" in the phonon collision integral (2) we do not change the indicated result. Thus, we arrive at the conclusion that within the scope of the model considered above the parameter N_s is negative and is infinite at any pump intensity and, consequently, we must look for the reason why the singularity indicated above is smeared out at $z = 0$. There are at least two such reasons. First, the microwave radiation source has a finite line width, and allowance for this fact "smears out" the singularity in Eq. (2). Secondly and more importantly, in real superconductors the singularity (averaged over all angles) in the density of states at $\varepsilon = \Delta$ is smeared over an energy of the order of γ. This smearing is determined by the finite lifetime of the quasiparticles (corresponding to $\gamma_1 = \tau_\varepsilon^{-1} \approx (kT_c)^3/\Theta_D^2$, Θ_D is the Debye energy) and by the anisotropy of the energy gap of the superconductor (corresponding to $\gamma_2 = \Delta[1 + (\Delta\tau_{\text{im}})^2]^{-2}$ (Ref. 8)).

Consideration of the enhancement with the aid of the kinetic equation (2) is valid if $\hbar\Omega \gg \gamma$. If $\Gamma^2 > \gamma/\hbar\Omega$, then

$$\frac{N_s}{N} = -\frac{(\hbar\Omega)^{1/2}\Delta^{1/2}}{2^{5/2}kT}\ln\frac{\Gamma^2\hbar\Omega}{\gamma} + \frac{2}{9}\left(\frac{\Delta}{kT}\right)^2. \tag{4}$$

The second term in Eq. (4) comes from the equilibrium term in Eq. (3). Thus, the instability arises if

$$\frac{9(\hbar\Omega)^{1/2}kT}{2^{5/2}\Delta^{3/2}}\ln\frac{\Gamma^2\hbar\Omega}{\gamma} > 1.$$

A rough estimate shows that if $2(\hbar\Omega)^{1/2}kT\Delta^{-3/2} > 1$, then at $\Gamma^2 > \Gamma_c^2 = \gamma/\hbar\Omega$, i.e., at

$$I > I_c = \left(\frac{\gamma}{\hbar\Omega}\right)^{1/2}\left(\frac{\hbar\Omega}{2\Delta}\right)^{1/2}\tau_\varepsilon^{-1}\frac{c(\hbar\Omega)^2}{8\pi e^2}\frac{d}{\alpha D} \tag{5}$$

the system becomes unstable to infinitesimal fluctuations of the order-parameter modulus. The critical radiation intensity I_c is too high in this estimate. However, an accurate calculation of I_c requires that we consider the energy region $\varepsilon - \Delta \approx \gamma$, i.e., that we go outside the framework of the kinetic equation, which presents certain difficulties. From what has been said above it is clear that the essential energies in the problem are $\gamma < \bar{\varepsilon} - \Delta < \Gamma^2\hbar\Omega$. It is significant that ω and q enter in (1) only in the ratio ω/Dq^2 and therefore the inequalities indicated above for ω and q can always be satisfied.

The value of γ is usually rather low under the experimental conditions in thin films (i.e., at low τ_{im}). For Al, for example, $\gamma_1 \approx 10^{-7}$ s^{-1} at $\tau_{\text{im}} \approx 10^{-13}$ s^{-1}, $\Delta \approx 10^{-11}$ s^{-1}, and $\gamma_2 \approx 10^{-7}$ s^{-1}. This is the reason for the relatively low value of the critical radiation intensity. At $\Gamma < \Gamma_c$, the system is stable and the change in the critical temperature can be found from the equations derived in Ref. 4.

Generally speaking, the above discussion does not mean that at $\Gamma > \Gamma_c$ stimulation of superconductivity is impossible. Only homogeneous solutions are impossible. In particular, the possibility is not excluded that the onset of instability can lead to a state in which the spatial fluctuations of the gap become of the order of Δ'

$\sim\Gamma^2\hbar\Omega$, after which the instability is stabilized by the smearing of the coordinate-averaged state density. However, the ultimate resolution of this question requires the solution of a complex system of spatially inhomogeneous nonlinear equations.

The instability under consideration is connected with the singularity of the kinetic equation at the source and therefore arises not only at $T\approx T_c$ but also at $T<T_c$, $\hbar\Omega\ll kT$. We cannot exclude the possibility that discontinuities in the function $R(I)$ which were observed in Ref. 5, are connected with the instability described above.

In conclusion, we note that the question of the stability of the distribution functions obtained by perturbation theory was first posed in Ref. 10. The case $\omega\tau_\varepsilon\ll 1$ was studied (i.e., the case opposite to the one considered in this paper). However, due to errors in the calculations in Ref. 10 (omission of one of the terms in the kinetic equation), the question of the stability of the system in the indicated frequency region remains open.

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