Influence of quantum gravitational processes on the evolution of the isotropic Universe

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Exact solutions are obtained and analyzed for the flat isotropic model of the Universe in the framework of Einstein's theory of gravitation with energy-momentum tensor of an ultrarelativistic medium that takes into account quantum gravitational effects of spontaneous particle production, bulk viscosity, and interaction of the particles with the gravitational field. It is shown that these solutions do not contain mathematical singularities of the physical parameters.

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INTRODUCTION

The cosmological evolution of the isotropic Universe in the framework of gravitational theory with vacuum quantum effects has been discussed by Gurovich and Starobinskii.[1] In our paper, one of the vacuum effects (vacuum polarization) is not considered, i.e., the geometrical part of the gravitational equations is assumed to be as in Einstein's theory. The model investigated here is of interest in enabling one to obtain exact analytic solutions that take into account other quantum and dissipative processes, namely, spontaneous particle production, interaction of the particles with the self-consistent field, and bulk viscosity. These processes change the form of the energy-momentum tensor compared with the standard hydrodynamic expression; they lead to a growth of the conformal temperature and entropy of the medium.

The main contribution to quantum phenomena in an isotropic gravitational field is made by nonconformal particles.[1)] It is shown in Ref. 2 that longitudinally polarized vector particles are of this kind. Interest in them is justified from the point of view of modern theories of gauge fields with spontaneously broken internal symmetries. In addition, it is possible that scalar (in gauge theories, Higgs) particles are also nonconformal.

The energy-momentum tensor and equation for the entropy increase for vector particles were obtained in Ref. 2. For scalar nonconformal particles, the calculations lead to a similar result differing from that given in Ref. 2 by numerical coefficients. This circumstance makes it possible, using the method of Ref. 2, to formulate a system of gravitational equations that take into account local quantum gravitational effects in an ultrarelativistic medium of real fundamental particles in a state of local thermodynamic equilibrium:

$$R_i^k-\delta_i^k R/2=\varkappa T_i^k,$$

$$T_i^k={}^1/_4\sum_s b_s N_s T^4(4u_iu^k-\delta_i)+{}^1/_3\sum_s c_sN_s[{}^1/_6(4u_iu^k-\delta_i^k)d_m^l d_l^m$$

$$+(d_i^k u^l{}_{;l}+3u^l d_i^k{}_{;l})]T^2+\xi_{tot}d_i^k\left[1+\left(d_l^m d_m^l\sum_s c_sN_s\right)\Big/\left(T^2\sum_s b_sN_s\right)\right],$$

$$(\sigma u^l)_{;l}=\sum_s b_sN_s(T^3u^l)_{;l}=\xi_{tot}d_m^l d_l^m/3T,$$

where[2)]

$$\xi_{tot}=\frac{1}{576\pi}\sum_s \beta_sN_s\frac{b_s}{\gamma_s}R^2\frac{3T}{d_q{}^p d_p{}^q}+\xi T^3,$$

T is the physical temperature of the medium, σ is the entropy density of the system, u^l is the 4-velocity,

$$d_m^l=(\delta_m^l-u^lu_m)u^p{}_{;p}$$

is the volume strain tensor of the velocity field,

$$b_s=\begin{cases}\dfrac{2\pi^2}{45}g_s, & \text{boson}\\ \dfrac{2\pi^2}{45}\cdot\dfrac{7}{8}g_s, & \text{fermions}\end{cases},\qquad \gamma_s=\begin{cases}0.122\,g_s, & \text{boson}\\ 0.122\cdot\dfrac{3}{4}g_s, & \text{fermions}\end{cases},$$

g_s is the number of independent polarizations of a particle of spin s, N_s is the number of species of particles of spin s (the particles and antiparticles are counted separately), and ξ is the dimensionless numerical coefficient of bulk viscosity of the matter. For the considered types of fundamental fields, the numerical coefficients β_s (spontaneous production from vacuum of real particles) and c_s (interaction of real particles with the self-consistent gravitational field) have the values

$$\beta_0=1,\quad c_0=1/108,$$

$$\beta_1^{(0)}=0,\quad c_1^{(0)}=0\ (m_1=0),\quad \beta_1=1,\quad c_1=1/108\ (m_1\neq 0),$$

$$\beta_{1/2}=0,\quad c_{1/2}=0.$$

1. PROPERTIES OF THE SYSTEM OF GRAVITATIONAL EQUATIONS IN A HOMOGENEOUS ISOTROPIC SPACE. SPECIAL (STEADY-STATE) SOLUTION

In the Friedmann metric

$$ds^2=a^2(\tau)(d\tau^2-dx^2-dy^2-dz^2),\quad a(\tau)d\tau=dt\quad (c=\hbar=1)$$

the system of gravitational equations formulated in the Introduction takes the form[3)]

$$l_g^{-2}\dot a^2=k_1\Theta^4+k_2\Theta^2\dot a_-{}^2a^{-2},$$

$$S\Big/\left(3\sum_s b_sN_s\right)=\Theta^2\dot\Theta=\alpha_1\ddot a^2a^{-2}+\alpha_2\Theta^2\dot a^2a^{-2},$$

$$k_1=\frac{1}{4}\sum_s b_sN_s,\quad k_2=\frac{3}{2}\sum_s c_sN_s=\frac{N_1+N_0}{72},\tag{1.1}$$

$$\alpha_1=\pi(N_1+N_0)/1080\cdot 0.122\sum_s b_sN_s,\quad \alpha_2=3\xi\Big/\sum_s b_sN_s,$$

where Θ is the conformal temperature of the medium and is related to the physical temperature T by $\Theta=Ta$, S is the entropy of the system, $S=\sigma a^3$, and $l_g=\varkappa^{1/2}=10^{-33}$ cm ($c=\hbar=1$).

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The values of the dimensionless numerical coefficients k_1, k_2, α_1, α_2 must in principle be established by the theory of elementary particles. However, the present state of this theory does not permit a definite choice of the model, so that, in the present paper, we shall investigate the system (1.1) analytically for arbitrary values of the coefficients $k_1, k_2, \alpha_1, \alpha_2 \geq 0$.

The absence in explicit form in the system (1.1) of the independent variable τ makes it possible to lower its order once. After the substitution

$$\dot{a}=z, \quad \ddot{a}=\frac{1}{2}\frac{dz}{da}=\frac{1}{2}z', \tag{1.2}$$

we obtain

$$\begin{aligned} l_g^{-2}z&=k_1\Theta^4+k_2\Theta^2 za^{-2}, \\ z^{1/2}\Theta^2\Theta'&={}^1/_4\alpha_1(z')^2a^{-2}+\alpha_2\Theta^2 za^{-2}. \end{aligned} \tag{1.3}$$

The most important property of the system (1.3) is its invariance under the similarity transformation group

$$z\to z\exp\sigma, \quad \Theta\to\Theta\exp(\sigma/4), \quad a\to a\exp(\sigma/4).$$

This permits the change of variables

$$z=Pa^4, \quad \Theta=Ta, \quad a=a_0\exp\chi \quad (\chi\in(-\infty,\infty)), \tag{1.4}$$

which reduces the system (1.3) to the form

$$l_g^{-2}P=k_1T^4+k_2T^2P, \tag{1.5}$$

$$P^{1/2}T^2(dT/d\chi+T)=\alpha_1[4P^2+2PdP/d\chi+(dP/d\chi)^2/4]+\alpha_2T^2P, \tag{1.6}$$

which does not contain the independent variable χ and separates in explicit form the special (steady-state) solution of the system

$$P=\text{const}, \quad T=\text{const}. \tag{1.7}$$

We show that for the expanding Universe ($\dot{a}>0$) the solution (1.7) is physical. We note first that in accordance with (1.2) and (1.4) the quantities T and P are observable; the first of them is the real temperature of the medium and the second is the combination $(R_0^0-\frac{1}{2}R)/3$ of the components of the space-time curvature tensor. We now consider the system (1.5) and (1.6), assuming constancy of its physical parameters. We obtain

$$l_g^{-2}P_c=k_1T_c^4+k_2T_c^2P_c, \quad T_c^3=4\alpha_1P_c^{3/2}+\alpha_2T_c^2P_c^{1/2}. \tag{1.8}$$

From the first equation of the system (1.8) we obtain

$$P_c=k_1T^4/(l_g^{-2}-k_2T_c^2), \quad P_c^{1/2}\equiv\dot{a}/a^2=\pm k_1^{1/2}T_c^2(l_g^{-2}-k_2T_c^2)^{-1/2}. \tag{1.9}$$

The symbols $\pm$ in the expression for the Hubble parameter $P_c^{1/2}$ correspond to the expansion (+) and contraction (–) stages. From obvious considerations—positivity of P_c and T_c and reality of $P_c^{1/2}$—there follows a restriction on the physical temperature:

$$T_c\leq l_g^{-1}k_2^{-1/2}. \tag{1.10}$$

The second equation of the system (1.8) after elimination of P_c by means of (1.9) takes the form

$$4\alpha_1k_1^{3/2}[T_c(l_g^{-2}-k_2T_c^2)^{-1/2}]^3+\alpha_2k_1^{1/2}T_c(l_g^{-2}-k_2T_c^2)^{-1/2}=\pm1.$$

It has two complex solutions, which are of no interest for us, and one real solution

$$\begin{aligned} T_c&=\pm l_g^{-1}u_c(1+k_2u_c^2)^{-1/2}, \\ u_c&={}^1/_2k_1^{-1/2}\alpha_1^{-1/3}\{[1+(1+\alpha_2^3/27\alpha_1)^{1/2}]^{1/3}+[1-(1+\alpha_2^3/27\alpha_1)^{1/2}]^{1/3}\}. \end{aligned} \tag{1.11}$$

It can be seen from Eqs. (1.11) that a stationary solution is impossible during the contraction stage ($T_c<0, P_c<0$). However, during the expansion stage it is physical, since besides the condition of positivity of T_c strict fulfillment of the inequality (1.10) holds, and this ensures that P_c and $P_c^{1/2}$ are positive and finite for all values of the numerical coefficients of the original system (1.1).[4)]

Complete integration of the problem gives

$$P_c=l_g^{-2}k_1u_c^4/(1+k_2u_c^2), \quad a=a_0\exp[l_g^{-1}k_1^{1/2}u_c^2(1+k_2u_c^2)^{-1/2}(t-t_0)]. \tag{1.12}$$

In (1.12), t is the physical time $[t\in(-\infty,\infty)]$. The entropy of the system also increases exponentially with the time:

$$\begin{aligned} S&=S_0\exp[3l_g^{-1}k_1^{1/2}u_c^2(1+k_2u_c^2)^{-1/2}(t-t_0)], \\ S_0&=\sum_s b_sN_s(T_ca_0)^3. \end{aligned} \tag{1.13}$$

Turning to the interpretation of the solutions (1.11)–(1.13), we note first that the stationary local physical parameters T_c and P_c are, apart from dimensionless factors of the order of unity, the Planck quantities, and the expansion (1.12) is in fact in accordance with Hoyle's[5)] law.[4] In this case, we see that the quantum gravitational process of particle production at Planck curvatures takes on, in a certain sense, the part of Hoyle's hypothetical C field.

The obtained solution is at the limit of classical notions of space-time as a continuous manifold. Nevertheless, its existence can be expected from simple physical considerations. This is because the solution is realized when there is exact compensation of two competing processes, namely, the cooling of the system by the cosmological expansion is compensated by its heating due to production of particles by the gravitational field. The absence of such a solution during the contraction stage is also obvious for similar reasons, since in such a case both contraction and particle production heat the system, so that its local physical parameters cannot be constant.

As can be seen from (1.12) and (1.13), the specification of the initial conditions for the Hoyle universe reduces to fixing the instant of time t_0 corresponding to the entropy S_0. From the point of view of the use of the obtained results for interpreting the data of experimental cosmology, we note that the hypothesis of Hoyle evolution of the Universe at the Planck parameters makes it possible to relate the currently observed entropy of the Universe to the duration of the Hoyle stage in the early phase of cosmological expansion.[6)]

2. GENERAL SOLUTION OF THE SYSTEM OF GRAVITATIONAL EQUATIONS. ABSENCE OF A SINGULARITY OF THE PHYSICAL PARAMETERS

The system of equations (1.5) and (1.6) has a general solution in the form of quadratures. Indeed, it follows from Eq. (1.5) that

$$\begin{aligned} P&=k_1T^4/(l_g^{-2}-k_2T^2), \quad P^{1/2}\equiv\dot{a}/a^2=\eta k_1^{1/2}T^2(l_g^{-2}-k_2T^2)^{-1/2}, \\ T&\leq k_2^{-1/2}l_g^{-1}, \end{aligned} \tag{2.1}$$

where $\eta=1$ for the expansion stage ($\dot{a}>0$) and -1 for the contraction stage ($\dot{a}<0$). Substitution of (2.1) and (1.6) leads to an equation for T:

$$\left(2+\frac{k_1T^2}{l_g^{-2}-k_2T^2}\right)^2\left(\frac{dT}{d\chi}\right)^2+\left[4T\left(2+\frac{k_1T^2}{l_g^{-2}-k_2T^2}\right)-\eta\frac{(l_g^{-2}-k_2T^2)^{3/2}}{\alpha_1k_1^{3/2}T^2}\right]\frac{dT}{d\chi}$$
$$+4T^2-\eta\frac{(l_g^{-2}-k_2T^2)^{3/2}}{\alpha_1k_1^{3/2}T}+\frac{\alpha_2}{\alpha_1k_1}(l_g^{-2}-k_2T^2)=0. \tag{2.2}$$

Equation (2.2) can be readily integrated by quadrature, though to simplify the complicated integrands it is convenient to replace the physical T by a dimensionless quantity u uniquely related to it:

$$u=T/(l_g^{-2}-k_2T^2)^{1/2},\quad u\in(0,\infty]. \tag{2.3}$$

Going over in (2.2) from T to u and solving the obtained equation for $du/d\chi$, we find that

$$a=a_0\exp\int_{u_0}^{u}du\frac{2\eta\alpha_1k_1^{1/2}u^2(2+k_2u^2)^2}{(1+k_2u^2)F(u,\eta,\xi)}, \tag{2.4}$$

$$F(u,\eta,\xi)=1-4\alpha_1\eta k_1^{1/2}u^3(2+k_2u^2)$$
$$+\xi[1+4\alpha_1\eta k_1^{1/2}k_2u^5(2+k_2u^2)-4\alpha_1\alpha_2k_1^2u^4(2+k_2u^2)^2]^{1/2}. \tag{2.5}$$

Here, $\xi=\pm1$ corresponds to the two signs in front of the radical in the solution of the quadratic equation (2.2) for $dT/d\chi$.

Equation (2.4) gives the dependence $\varphi[u(T)]=a$. We now form an equation that should determine the dependence of u (i.e., T) on the time. Since

$$d\chi=\frac{da}{a}=\frac{\dot{a}}{a^2}a\,d\tau=P^{1/2}dt=\eta k_1^{1/2}l_g^{-1}u^2(1+k_2u^2)^{-1/2}dt$$

(the time t is physical!), it follows in accordance with (2.4) that the required dependence $\psi[u(T)]=t$ has the form

$$\eta(t-t_0)l_g^{-1}=\int_{u_0}^{u}du\frac{2\eta\alpha_1k_1(2+k_2u^2)^2}{(1+k_2u^2)^{1/2}F(u,\eta,\xi)}. \tag{2.6}$$

The two constants in the cosmological solution (2.4), (2.6) are fixed by specifying the initial conditions

$$T|_{t=t_0}=T_0(u|_{t=t_0}=u_0),\quad a|_{t=t_0}=a_0.$$

Equations (2.4) and (2.6) determine the general solution of the system of gravitational equations (1.1) in parametric form in quadratures of Abelian type, the parameter in the obtained solution being the physical temperature T of the system[7] (the parameter u is uniquely related to T). The region of admissible values of the local physical parameters for the solution (2.4)–(2.6) follows from obvious considerations (positivity of P, T, and a and reality of $P^{1/2}$):

$$T\in(0,k_2^{-1/2}l_g^{-1}]\quad(u\in(0,\infty]),$$
$$P_8(u)=1+4\alpha_1\eta k_1^{1/2}k_2u^5(2+k_2u^2)-4\alpha_1\alpha_2k_1^2u^4(2+k_2u^2)^2\geq0. \tag{2.7}$$

The solution (2.4)–(2.6) contains for each of the stages (expansion, $\eta=1$, or contraction, $\eta=-1$) two branches ($\xi=\pm1$), one of the branches ($\xi=-1$) having the asymptotic behavior of the Friedmann solution as $t\to\pm\infty$, while the other ($\xi=1$) lies entirely in the quantum domain. Therefore, in what follows we shall take $\xi=-1$ in (2.4)–(2.6), assuming that the treatment of the second branch in the framework of Eqs. (1.1) is incorrect.

The most important property of the solution (2.4)–(2.6) ($\xi=-1$) is the absence of singularities of the physical parameters of the system. Indeed, in accordance with (2.7) we have $P_8(\infty)<0$, i.e., u is bounded: $u\in(0,u_{\max}]$ ($T\in(0,T_{\max}<(k_2l_g^2)^{-1/2}]$; see (2.3)). It can be seen from the structure of the polynomial $P_8(u)$ that the equation $P_8(u)=0$ has only one positive root, which is uniquely associated with $u_{\max}$. The boundedness of u also entails boundedness of the combination

$$(R_0^0-R/2)_{max}=3P_{max}=3k_1l_g^{-2}u^4_{max}/(1+k_2u^2_{max}). \tag{2.8}$$

In addition, the maximal value of the curvature scalar,

$$R_{max}=3\left(\frac{dP}{d\chi}\right)_{max}+12P_{max}=3\left(\frac{dP}{du}\right)_{max}\left(\frac{du}{d\chi}\right)_{max}+12P_{max}, \tag{2.9}$$

is also bounded, since the fact that $du/d\chi=a\,du/da$ is bounded follows directly from (2.4) ($\xi=-1$). The relations (2.8) and (2.9) prove that there are no singularities of the components of the Ricci tensor: $|R_i^k|_{\max}<\infty$.

Note that the fact that the system of equations (1.1) has a finite domain of admissible values of the physical parameters is not related to the effect of interaction of the particles with an external field, since for $k_2=0$ all of the above conclusions remain valid. The finiteness of the physical parameters is generated by the nonlinearity in the curvature of the employed gravitational equations as the condition of conservation of reality of the metric and is, thus, a combination of the spontaneous creation and viscosity effects.

3. FRIEDMANN SOLUTIONS DISTORTED BY QUANTUM EFFECTS

Because of the effects of the spontaneous particle production and bulk viscosity, the evolution of the Universe is not symmetric under the operation of time reversal. This asymmetry is manifested in not only the cosmological solutions but also in the different domains of the admissible values of the physical quantities, which are not carried into each other under the operation of time reversal.

The range of admissible values of T, a, and u can be found from the solution (2.4)–(2.6) itself with allowance for the restriction (2.7). For the contraction region, we have

$$t\in(-\infty,0],\quad a\in(\infty,a_{min}^{(-)}>0],\quad u\in(0,u_{max}^{(-)}<\infty],\quad P\in(0,P_{max}^{(-)}<\infty]. \tag{3.1}$$

In this case, it is natural to specify the initial conditions

$$a|_{t\to-\infty}\to\infty,\quad u|_{t\to-\infty}\to Tl_g|_{t\to-\infty}\to\frac{\Theta_{(-)}l_g}{a}\bigg|_{t\to-\infty}\to0. \tag{3.2}$$

Thus, the family of solutions for the contraction stage can be parametrized by the values of the initial entropy of the Universe:

$$S_{(-)}=\sum_s b_sN_s\Theta^3_{(-)}.$$

In contrast to the contraction stage, in which the cosmological solution does not depend qualitatively on the relationship between the numerical coefficients of the original system of equations (1.1), the early evolution of the Universe described by the solution (2.4)–(2.6) ($\xi=-1$) in the expansion stage $\eta=1$ has a different nature depending on the relationship between α_1 (spontaneous particle production) and α_2 (bulk viscosity). Indeed, in the expansion stage the function $F(u, 1, -1)$ can, besides the zero at the point $u=0$ (corresponding to the transition to the Friedmann solution), also have a zero at the point satisfying the equation

$$4\alpha_1 k_1^{1/2} u^3 + \alpha_2 k_1^{1/2} u = 1,$$

i.e., for $u=u_c$ and $T=T_c$ (see Sec. 1). However, the existence of this zero depends on the relationship between the numerical coefficients of the system (1.1). Namely, if

$$\alpha_1 > \alpha_2^3 \frac{(1+k_2 u_c^2/2)^2}{(1+k_2 u_c^2)^3}, \tag{3.3}$$

the the function $F(u, 1, -1)$ will not have a zero for finite u. Assuming that the condition (3.3) is satisfied (spontaneous production predominates over viscosity), and using the inequality (2.7), we obtain the following domain of admissible values:

$$t\in[0,\infty),\quad a\in[a_{min}^{(+)},\infty),\quad u\in[u_{max}^{(+)},0),\quad T\in[T_{max}^{(+)},0),\quad P\in[P_{max}^{(+)},0),$$
$$a_{min}^{(+)}\neq 0,\quad u_{max}^{(-)}\leqslant u_{max}^{(+)}<\infty,\quad P_{max}^{(-)}\leqslant P_{max}^{(+)}<\infty,\quad u_{max}^{(+)}>u_c. \tag{3.4}$$

In principle, the initial conditions for an expanding Universe with spontaneous production that predominates over viscosity can be specified for arbitrary t (for example, for $t=0$) in the interval (3.4). However, it is convenient to parametrize the family of solutions by the values of $\Theta_{(+)}=\Theta|_{t\to\infty}$, which is related to the possibility of finding this parameter from data on the microwave background ($\Theta_{(+)}^{exp}\sim 10^{27}$). Thus, suppose

$$a|_{t\to\infty}\to\infty,\quad u|_{t\to\infty}\to T l_g|_{t\to\infty}\to \frac{\Theta_{(+)} l_g}{a}\Big|_{t\to\infty}\to 0. \tag{3.5}$$

Then the cosmological solution for the stages of expansion ($\eta=1$) and contraction ($\eta=-1$) can be represented in a unified form by means of the quadratures

$$a=\frac{\Theta_{(\eta)} l_g}{u}\exp\left[\int_0^u du\left(\frac{1}{u}+\frac{2\eta\alpha_1 k_1^{1/2}(2+k_2u^2)^2u^2}{(1+k_2u^2)F(u,\eta,-1)}\right)\right],$$
$$\eta k_1^{1/2} t l_g^{-1}=\frac{1}{2}\left(\frac{1}{u^2}-\frac{1}{u_{max}^{(\eta)2}}\right)+\eta\alpha_2 k_1^{1/2}\left(\frac{1}{u}-\frac{1}{u_{max}^{(\eta)}}\right)-\left(\alpha_2^2 k_1-\frac{k_2}{2}\right)\ln\frac{u}{u_{max}^{(\eta)}}$$
$$+\int_{u_{max}^{(\eta)}}^{u} du\left[\frac{1}{u^3}+\eta\frac{\alpha_2 k_1^{1/2}}{u^2}+\frac{\alpha_2^2 k_1-k_2/2}{u}+\frac{2\eta\alpha_1 k_1^{3/2}(2+k_2u^2)^2}{(1+k_2u^2)^{1/2}F(u,\eta,-1)}\right]. \tag{3.6}$$

The integrands in (3.6) do not contain singularities for any u in the domain of admissible values (3.2) and (3.5). We also write down the expression for the change in the entropy during the entire admissible time of evolution of the Universe:

$$\Delta S=\sum_s b_s N_s \Theta_{(\eta)}^3\left\{\eta-\frac{\eta}{(1+k_2u_{max}^{(\eta)2})^{3/2}}\exp\left[3\int_0^{u_{max}^{(\eta)}} du\left(\frac{1}{u}+\frac{2\eta\alpha_1k_1^{1/2}(2+k_2u^2)^2u^2}{(1+k_2u^2)F(u,\eta,-1)}\right)\right]\right\}.$$

It is readily seen that at large $|t|$ the solution (3.6) approaches the Friedmann solution asymptotically. The degree of distortion of the "ideal" Friedmann solution is determined by the values of the integrals in (3.6) and does not depend on the initial conditions. The universal functions $a/(\Theta_{(\eta)} l_g)$ and $k_1^{1/2}|t|/l_g$, obtained by numerical integration, are shown in Figs. 1(a) and 1(b).

Now suppose that the inequality opposite to (3.3) is

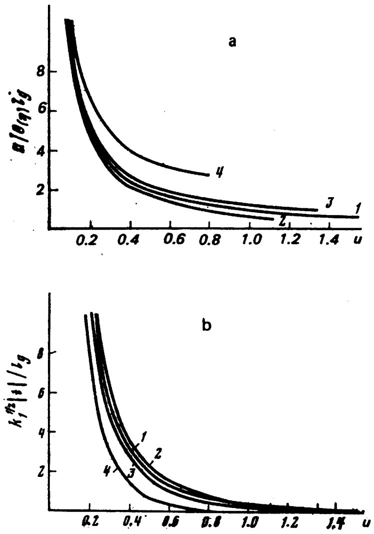

FIG. 1. 1) $\dot a>0$, $\alpha_2=0.033$; 2) $\dot a>0$, $\alpha_2=0.100$; 3) $\dot a<0$, $\alpha_2=0.033$; 4) $\dot a<0$, $\alpha_2=0.350$.

satisfied:

$$\alpha_1\leqslant\alpha_2^3\frac{(1+k_2u_c^2/2)^2}{(1+k_2u_c^2)^3}, \tag{3.7}$$

i.e., bulk viscosity predominates over spontaneous production. In this case, besides the zero at the point $u=0$ ($T=0$), the function $F(u, 1, -1)$ will have a zero at the point $u=u_c$, which leads to the following domain of admissible values[8]:

$$t\in(-\infty,\infty),\quad a\in(0,\infty),\quad u\in(u_c,0),\quad T\in(T_c,0),\quad P\in(P_c,0). \tag{3.8}$$

The values of u_c, T_c, and P_c are given in Sec. 1. It is here also convenient to parametrize the family of solutions by means of the value of $\Theta_{(+)}$ [see (3.5)].[9] We write down the corresponding quadratures:

$$a=\frac{\Theta_{(+)}l_g}{u}\left(\frac{u_c-u}{u_c}\right)^{\beta}\exp\left\{\int_0^u du\left[\frac{2\alpha_1k_1^{1/2}u^2(2+k_2u^2)^2}{(1+k_2u^2)F(u,1,-1)}+\frac{1}{u}+\frac{\beta}{u_c-u}\right]\right\},$$
$$k_1^{1/2}(t-t_0)l_g^{-1}=\frac{1}{2}\left(\frac{1}{u^2}-\frac{1}{u_c^2}\right)+\alpha_2k_1^{1/2}\left(\frac{1}{u}-\frac{1}{u_c}\right)-\left(\alpha_2^2k_1-\frac{k_2}{2}\right)\ln\frac{u}{u_c}$$
$$+\beta\frac{(1+k_2u_c^2)^{1/2}}{u_c^2}\ln\frac{u_c-u}{u_c}+\int_{u_c}^{u}du\left[\frac{2\alpha_1k_1^{3/2}(2+k_2u^2)^2}{(1+k_2u^2)^{1/2}F(u,1,-1)}+\frac{1}{u^3}\right.$$
$$\left.+\frac{\alpha_2k_1^{1/2}}{u^2}+\frac{\alpha_2^2k_1-k_2/2}{u}+\frac{\beta(1+k_2u_c^2)^{1/2}}{u_c^2(u_c-u)}\right], \tag{3.9}$$
$$\beta=\frac{1-4\alpha_1k_1^{1/2}u_c^3(2+k_2u_c^2)}{(1+k_2u_c^2)(3-2\alpha_2k_1^{1/2}u_c)}.$$

The integrals in (3.9) do not have singularities. The quantity t_0 corresponds to a deformation shift of the point $t=0$. Because of the infinite time interval [see (3.8)], such a shift has no meaning, and we can set $t_0=0$.

The solution (3.9) belongs to the type first considered by Murphy[6] [Murphy's analytic solution is obtained from (3.9) in the limit $\alpha_1, k_2\to 0$]. The initial value of the entropy in this solution is zero, and $S\to\Sigma_s b_s N_s \Theta_{(+)}^3$ asymptotically as $t\to\infty$. A distinctive feature of the cosmological expansion when viscosity plays the dominant part is that the evolution commences from a state in which space is compressed into a point and at the same time has a finite physical curvature, while the matter has a finite physical temperature.

In accordance with (3.7), $0<\beta\leqslant 1$, and the value $\beta=1$ is attained only in the limit $k_2\to 0$ (Murphy's "ideal" solution). In the general case $\beta<1$, and therefore for $|u-u_c|\ll u_c$ (the start of expansion) there is a qualitative change in Murphy's solutions, namely, the early evolution when viscosity is dominant takes place in accordance with Hoyle's law [the law (1.12) holds approximately]—the parameter a increases and the physical temperature remains virtually constant. There is effective accumulation of entropy. For $u\ll u_c$, the expansion law departs from the Hoyle law, and the transition to the Friedmann solution begins. The functions $a/(\Theta_{(+)}l_g)$ and $k_1^{1/2}t/l_g$, obtained as a result of numerical integration, are shown in Figs. 2(a) and 2(b).

In the numerical calculation of the cosmological solutions (3.6) and (3.9), it is necessary to take some particular model of the elementary-particle physics. To be specific, we chose the $U(1)\times SU_L(2)$ and $U_c(3)$ model (QHD) (see Ref. 7), in the framework of which the matter of the hot Universe at temperatures $T>V$ (V is the threshold for the production of free quarks) must consist of color quarks ($s=\frac{1}{2}$), eight species of massless vector gluons ($s=1, m_1=0$), leptons ($s=\frac{1}{2}, m_{1/2}\neq 0$), the intermediate bosons $W^\pm, Z^0$ ($s=1$, $m_1\neq 0$), one scalar Higgs particle ($s=0$, $m_0\neq 0$), and also photons. Restricting ourselves to three multiplets of fundamental fermions, which includes the u, d, s, c, b, t quarks and their corresponding leptons, we obtain for the numerical coefficients of the system of gravitational equations (1.1)

$$k_1=16.778,\quad k_2=0.0556,\quad \alpha_1=0.0014. \tag{3.10}$$

The absence of information about the interaction laws of the elementary particles makes it impossible to calculate α_2 (the coefficient of bulk viscosity). For the coefficients (3.10), the value of $\alpha_2^{\rm cr}$ [the value of the coefficient α_2 at which the inequality (3.7) becomes an equality] was found to be 0.1166. Therefore, in the numerical integration the coefficient α_2 played the part of a parameter, and values of α_2 both greater and smaller than $\alpha_2^{\rm cr}$ were chosen.

CONCLUSIONS

The results of the present paper show that quantum gravitational processes in a system of real particles at large space-time curvatures significantly change the evolution of the homogeneous isotropic cosmological model. The change is most strongly manifested in the fact that the employed system of gravitational equations has an effective range of admissible values of the physical quantities bounded at large curvatures and high temperatures. This has the consequence that the physical parameters of the system do not develop mathematical singularities during the evolution process.

The existence of a nonsingular region of admissible values of the physical quantities appears as the condition for the Riemannian geometry to remain real. It seems to us entirely natural that other possible systems of equations containing not only linear terms but also terms with a nonlinear dependence on the curvature should have similar properties.

The effects of the particle production and bulk viscosity also lead to an asymmetry between the expansion and contraction stages. The asymmetry is manifested in differences in the ranges of admissible values and also in the rates of evolution. For expansion, for example, the energy relations have the form[10] $-T_0^0\leqslant\frac{1}{3}T_\alpha^\alpha\leqslant\frac{1}{3}T_0^0$, whereas for contraction $\frac{1}{3}T_0^0\leqslant\frac{1}{3}T_\alpha^\alpha\leqslant\frac{1}{3}(T_\alpha^\alpha)_{\max}$.

The inclusion among the considered effects (besides viscosity) of spontaneous particle production also changes our picture of the evolution of a gravitating system in the expansion stage. First, it is found that, irrespective of the values of the numerical coefficients of the original system of equations, the early stage of the expansion can proceed in accordance with the law (1.12) with stationary physical parameters (steady-state solution). Second, there is a change in the Murphy-type cosmological solutions, which in the early stage of evolution acquire the Hoyle asymptotic behavior, solutions of such type with Friedmann asymptotic behavior in the region of small curvature being possible only in a system with strong viscous effects, so that in the initial stage of evolution the total heating of the system by the spontaneous production processes and the viscosity almost compensates the cooling of the system by expansion. Third, the Murphy-type solutions for an expanding system cease to be unavoidable; for if spontaneous production predominates over viscosity [see (3.3)], it is possible to have solutions without the exotic singularities inherent in the Murphy solutions. Inclusion of the effect of the interaction of the particles with an external gravitational field ($k_2\neq 0$) raises the viscosity barrier for the existence of Murphy-type solutions [instead of the condition $\alpha_2^3\geqslant\alpha_1$, the condition (3.7) must hold]. From the practical point of view, the existence of Murphy-type solutions requires a choice of the model of the elementary-particle physics for which the coefficient of bulk viscosity α_2 exceeds the coefficient of spontaneous production α_1 by 1.5 orders of magnitude.

In our opinion, evolution with spontaneous particle production playing the dominant role is more probable. In support of this conclusion we may note that the bulk viscosity effect, which arises entirely because the various internal degress of freedom of the particles are not all on the same footing dynamically, must evidently decrease with increasing energy and restortation of the interaction symmetries.

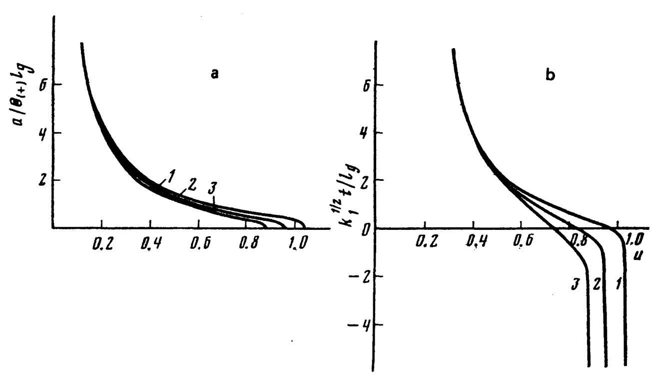

FIG. 2. 1) $\alpha_2=0.133$; 2) $\alpha_2=0.167$; 3) $\alpha_2=0.200$.

Some of the problems of evolution of the Universe at extremal curvatures and energy densities remain unresolved in the framework of the model theory employed in the present paper. Indeed, the cosmological solutions obtained above do not contain unphysical infinities but, nevertheless, they cannot be analytically continued into the region $t < t_0$. Therefore, there is here, as in the classical theory, a big-bang effect. The quantum effects merely set upper bounds for the parameters of the "nascent Universe," and the origin of the "bang" remains open. One can of course assume that it is the result of dynamical instability of the regime of stationary expansion,[5] but then the problem simply reduces to the origin of this regime. It should be emphasized that quantum gravitational theory is as yet insufficiently developed for any definite conclusions to be drawn about the extremal stage in the evolution of the Universe. It was already noted in Refs. 1, 2, and 5 that a system of equations which takes into account only local effects is no more then a rough approximation to a consistent theory. However, we can already say that quantum gravitational phenomena can have a profound influence on the formation of the macroscopic properties of the Universe.

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1) By nonconformal particles we mean particles whose conformal invariance remains broken on the transition to the ultrarelativistic limit.

2) In the convariant expressions given above, we omit the terms that vanish in conformally flat spaces (such as, for example the second invariant of the Weyl tensor and the shear deformation tensor of the velocity field). Therefore, in an investigation of quantum gravitational processes in a system of real particles in models with lower symmetry is it necessary to make a corresponding modification of the system of gravitational equations (see Ref. 3).

3) In the considered type of space, the independent equations are the $\binom{0}{0}$ component of the gravitational equations and the equation of entropy increase. In (1.1) and below, the dot denotes the derivative with respect to the cosmological time τ.

4) Note that the solution (1.11)–(1.13) is also an exact special solution for the system of equations containing not only the Einstein gravitational term but also the radiative corrections quadratic in the curvature (see Ref. 2).

5) In theories that take into account quantum gravitational phenomena solutions with stationary physical parameters were discussed earlier in Refs. 1 and 5. In Ref. 1, the steady-state regime arises because of viscosity of the vacuum; in Ref. 5. a nondissipative version of this stage of the evolution is proposed.

6) For the system of gravitational equations containing the invariants quadratic in the curvature,[2] the solution with stationary physical parameters is unstable. At large a (large physical times t), the instability has an exponential nature in the time t. For a discussion of the instability of the steady-state solution, see also Ref. 1.

7) As physical parameter of the solution (2.4)–(2.6), one can also use the combination $R_0^0 - R/2$ of the components of the curvature tensor [See Eqs. (2.1) and (2.3) and Sec. 1].

8) If the inequality (3.7) is satisfied, the point $u = u_c(T = T_c)$ is a bifurcation point of the cosmological solution (2.4)–(2.6) ($\eta = 1$, $\xi = -1$). One of the branches [see (3.9) below] describes a system cooled (from T_c to $T = 0$) by expansion; the other, which lies entirely in the quantum domain, describes a system which is heated (from T_c to $T_x^{(+)} > T_c$). Murphy's "ideal" solution[6] has a similar bifurcation. For the reasons given in Sec. 2, we do not consider the second branch ($\eta = 1$, $\xi = -1$) of the solution (2.4)–(2.6).

9) It is impossible to specify initial conditions at the point $a = 0$ ($T = T_c$), corresponding to the start of evolution, because of its singular nature.

10) The relation $T_\alpha^\alpha/3 = -T_0^0$ has a solution with stationary physical parameters and a Murphy-type solution at the singular point $a = 0$.

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