Theory of propagation of an electromagnetic or sound wave in cholesteric liquid crystals

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It is shown that electromagnetic or sound waves propagating in a layer of a cholesteric liquid crystal, transverse to the cholesteric axis, lead to the appearance of additional waves. At wave frequencies below some certain value these waves are surface waves. In a conducting cholesteric liquid crystal, the additional wave can, upon attentuation, experience multiple oscillations. The amplitudes of the waves are determined. The sound wave in an external field can be unstable. It is shown that the sound wave can generate an electromagnetic field. The formation of an acousto-electric field and accompanying acousto-magnetic field is considered. It is shown that in a conducting cholesteric liquid crystal the Alfvén wave also has an additional branch.

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1. STATEMENT OF THE PROBLEM AND BASIC RESULTS

In the study of propagation of electromagnetic waves (EMW) in a cholesteric liquid crystal (CLC) is usually limited to the case of a wave directed along the cholesteric axis (CA) of a nonconducting sample. It turns out that the incidence of an EMW or a sound wave on a CLC layer in a direction transverse to the CA, and also the propagation of EMW in conducting CLC, leads to a number of interesting features. In a dielectric CLC, the case is of interest in which the magnetic field of the EMW is parallel to the CA. (An analysis shows that in the EMW magnetic field transverse to the CA the results are not of interest.) In this case, in addition to the ordinary EMW, which appear in a CLC with a permittivity

$$\varepsilon_{in}=\varepsilon\delta_{in}+\varepsilon\delta\beta_{in},\quad \delta\ll1,$$
$$\beta_{xx}=-\beta_{yy}=\cos(2qz),\quad \beta_{xy}=\sin(2qz),\quad \beta_{iz}=0, \tag{1}$$

additional EMW appear at $\delta\neq0$ (q is the inverse of the pitch of the cholesteric helix directed along the z axis). These additional waves do not at all possess the usual features. If the incident wave of frequency ω propagated along the y axis and had only the component of the electric field E_x, then the additional wave has the components $E_{x,y,z}$, the amplitudes of which are of order

$$E_0\omega\omega_{max\,\varepsilon}^{-1},\quad E_0\omega^2\omega_{max\,\varepsilon}^{-2},$$

where E_0 is the amplitude of the ordinary EMW in the layer, while $\omega_{max\,\varepsilon}$ is determined by the condition

$$\omega_{max\,\varepsilon}=2cq(\delta\varepsilon)^{-1/2} \tag{2}$$

(c is the speed of light). The amplitudes of the additional EMW can be greater than δE_0. The dependence of these additional waves on the coordinates is different in different frequency ranges. Our consideration, as is shown below, is limited to the condition $\omega<\omega_{max\,\varepsilon}$. If

$$\omega>\omega_0=2cq, \tag{3}$$

then the additional EMW in the layer

$$\sim\begin{matrix}\cos\\ \sin\end{matrix}(2qz)\begin{matrix}\cos\\ \sin\end{matrix}(\pm\varkappa y-\omega t), \tag{4}$$

and outside the layer (transmitted and reflected)

$$\sim\begin{matrix}\cos\\ \sin\end{matrix}(2qz)\begin{matrix}\cos\\ \sin\end{matrix}(\pm py-\omega t), \tag{5}$$

$$p=2q(\omega^2\omega_0^{-2}-1)^{1/2},\quad \varkappa=2q(\varepsilon\omega^2\omega_0^{-2}-1)^{1/2}, \tag{6}$$

i.e., inside and outside the layer, EMW appear traveling at angles

$$\operatorname{arctg}(\omega^2\omega_0^{-2}-1)^{1/2},\quad \operatorname{arctg}(\varepsilon\omega^2\omega_0^{-2}-1)^{1/2}$$

to the y axis and modulated along the CA.

In the frequency range $\omega_0>\omega>\omega_1=\omega_0\epsilon^{-1/2}$ the waves in the layer are described by (4) and (6), and outside the layer we have, in place of (5),

$$\sim\begin{matrix}\cos\\ \sin\end{matrix}(2qz)\begin{matrix}\cos\\ \sin\end{matrix}(\omega t)\exp(\pm\Gamma y),\quad \Gamma^2=-p^2 \tag{7}$$

(the plus for the reflected wave, the minus for the tramsitted wave: the layer occupies the region $0\leqslant y\leqslant d$), i.e., outside the layer, surface EMW are formed. Finally, at $\omega<\omega_1$, the waves outside the layer are described by (7), and inside the layer are formed surface waves

$$\sim\begin{matrix}\cos\\ \sin\end{matrix}(2qz)\begin{matrix}\cos\\ \sin\end{matrix}(\omega t)\exp(\pm fy),\quad f^2=-\varkappa^2 \tag{8}$$

(the plus and minus are for waves at the boundaries $y=0$ and d, respectively). In the conducting CLC layer, in addition to the ordinary wave with amplitude of magnetic field H_0, which is attenuated $\sim\exp[(i-1)(2\pi\sigma\omega)^{1/2}c^{-1}y]$, (where σ is the isotropic part of the conductivity and σ_{ik} has a form similar to (1)), additional waves are formed, traveling almost along the CA with amplitudes $\delta H_0\omega\omega_{max\,\sigma}^{-1}$ or $\delta H_0\omega^{1/2}\omega_{max\,\sigma}^{-1/2}$, and in this case the consideration is valid at frequencies lower than

$$\omega_{max\,\sigma}=c^2q^2\sigma^{-1}. \tag{9}$$

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In the case of a wave traveling parallel to the CA, in addition to the ordinary wave, which is damped within the skin depth, there is an additional wave, which undergoes multiple oscillations as it is damped. The number of these oscillations increase with increase in the quantity $\omega_{\max\sigma}\omega^{-1}$.

Additional waves that are not elecromagnetic but acoustic, appear in the passage of a sound wave transverse to the CA if an external constant electric field is applied to the layer, parallel to the direction of propagation of the sound. In this case, the surface waves appear at sound frequencies satisfying the condition

$$\omega<\omega_s=2qs \tag{10}$$

(s is the speed of sound). The amplitude of the additional waves increases with increase in the frequency.

We note at once that the method for finding additional waves is different for passage of the wave along and transverse to the CA. In the first case, following Ref. 1, we can find the dispersion equation, from which we can determine both the ordinary and the additional waves. The situation is more complicated in the propagation of waves transverse to the CA, when the set of Maxwell's equations or the equations of hydrodynamics in an external electric field is solved by iteration in δ. In this case, the dispersion equation is found for the fundamental waves, while the additional waves appear as "induced" solutions of the set of equations in which the fundamental waves play the role of "external" fields.

In a conducting CLC, the sound wave is unstable if the external electric field is so large that the drift velocity $v_d\approx 2s$ (and not $v_d=s$ as in the case of instability of sound in piezosemiconductors). We note that the excitation of acoustic oscillations by an electric field is difficult in a liquid crystal and exceptional in a CLC.

In the propagation of sound waves in a layer, alternating solenodial fields appear, in which the amplitude of the magnetic field of the wave is greater than the amplitude of the electric, while their velocity is of the order $s\ll c$, i.e., these waves are similar to thermomagnetic waves.[3]

A potential difference, constant in time, and transverse to the CA appears, in second order in the small ratio of the density oscillations to the unperturbed density, for the case of sound waves propagating at an angle to the CA and to the external electric field. In contrast with ordinary media, this acoustic-electric field has a solenoidal increment which leads to the appearance of a stationary magnetic field–the acousto-magnetic field.

Before proceeding to the calculation, we make a few reservations. At frequencies $\omega>\omega_{\max}$ or $\omega>\omega_{\max\sigma}$, the iteration treatment is inapplicable. However, as is seen from (9), $\omega_{\max\sigma}$ is very large and the frequencies which do not satisfy (9) require account of temporal dispersion, which is beyond the scope of our present work.

At $\omega>\omega_{\max\varepsilon}$, it is necessary to take into account the damping of the EMW, which is very considerable at these frequencies. The ratio of the imaginary and real frequencies of the permittivity becomes greater than the ratio of the anisotropic and isotropic parts of the permittivity. (Therefore, in particular, waves with frequencies greater than $\omega_{\max\varepsilon}$ are stable. With neglect of damping, parametric[4] excitation of the EMW appears). In carrying out the estimates, two cases are possible. First, a pure CLC, in which $q\approx 10^5$ cm^{-1}, $\delta=0.1-0.1$. Second, solutions of CLC and a nematic liquid crystal, in which the quantities q and δ decrease with decrease in the concentration of the CLC.[1] Therefore, in solutions of CLC and a nematic liquid crystal, the characteristics frequencies decrease while the amplitude of the additional waves increase. We note that as the concentration of CLC approaches zero, not only do $\omega_{\max\varepsilon}$ and $\omega_{\max\sigma}$ approach zero, but account of damping also becomes necessary.

2. DIELECTRIC CHOLESTERIC LIQUID CRYSTAL. PROPAGATION OF ELECTROMAGNETIC WAVES TRANSVERSE TO THE CHOLESTERIC AXIS (CA)

Let the field of the incident EMW be directed along the x axis. Then, neglecting δ, we find that a wave also develops in the layer of CLC. The wave field is directed along the x axis, and is given by (the index "ord" denotes the ordinary wave)

$$E^{\mathrm{ord}}=E_0\cos(ky\mp\omega t). \tag{11}$$

We now take into account the terms proportional to δ in (1). Here we must keep it in mind that the y and z components of the electric field inside and outside the layer and the increment to the x component are proportional to δ. Neglecting the difference in the magnetic susceptibility from unity and keeping terms that are linear in δ, we obtain the following equations for the additional waves $E_{x,y,z}$:

$$\begin{gathered}(\Delta-\varepsilon\partial^2/\partial t^2)E_{x,y}=-\delta k/N_{x,y}E_0\cos(2qz)\cos(ky\mp\omega t),\\ N_x=k,\quad N_y=2q,\quad \Delta=\partial^2/\partial y^2+\partial^2/\partial z^2,\\ \partial E_y/\partial y+\partial E_z/\partial z=\delta kE_0\sin(2qz)\sin(ky\mp\omega t).\end{gathered} \tag{12}$$

This system must be solved under the usual boundary conditions, in which the induction-vector component normal to the surface is equal to

$$\varepsilon E_y+\delta\varepsilon E_0\sin(2qz)\cos(ky\mp\omega t),$$

where E_y is determined from (12).

The solution of (12) is different in the different frequency ranges indicated in Sec. 1. The field compnents $E_{x,y,z}$ inside and outside the layer (the reflected and transmitted waves) and proportional to the expressions (4), (5), (7), (8) and to $\cos(ky\mp\omega t)$.

For example, for the x component at $\omega>\omega_0\varepsilon^{1/2}$ inside the layer,

$$\begin{gathered}E_x^{\mathrm{in}}=\delta k^2(2q)^{-2}E_0\cos(2qz)\cos(ky\mp\omega t)\\ +[E_{1,3}\cos(\varkappa y\mp\omega t)+E_{2,4}\sin(\varkappa y\mp\omega t)]\cos(2qz),\end{gathered} \tag{13}$$

and outside the layer,

$$E_x^{\mathrm{out}}=[E_{1,3}^{\mathrm{ref/tr}}\cos(py\mp\omega t)+E_{2,4}^{\mathrm{ref/tr}}\sin(py\mp\omega t)]\cos(2qz), \tag{14}$$

where $\varkappa$ and p are given in (6).

Substituting $E_{x,y,z}$ in the form (13) and (14) (or with replacement of expressions of the type (4) and (5) by expressions of the type (7) or (8), as indicated in Sec. 1) in the boundary conditions, we find at $\varkappa, p \neq 0$:

$$E_{zi}^{in},\ E_{zi}^{out} \approx \delta k(2q)^{-1}E_0, \qquad E_{xi}^{in},\ E_{xi}^{out} \approx \delta k^2(2q)^{-2}E_0,$$
$$E_{yi}^{in} \approx 2q\varkappa^{-1}E_{zi}, \qquad E_{yi}^{out} \approx 2qp^{-1}E_{zi}.$$

As $\varkappa \to 0$, $E_z^{in} \to 0$; as $p \to 0$, $E_z^{out} \to 0$.

The largest of the additional waves is E_x (at $\omega > \omega_0$). With decrease in the frequency, the amplitudes of the additional waves fall off, while as $\omega \to 0$, the additional waves disappear.

Estimates show that in pure CLC, $\omega_{\max\varepsilon} \approx 10^{16} s^{-1}$ and $\omega_0 \approx 10^{15} s^{-1}$.

3. CONDUCTING CLC. PROPAGATION OF EMW

If an EMW is propagated parallel to the CA, then we obtain by the standard method[1] the following equation for $H_\pm = H_x \pm iH_y$:

$$\left(\frac{\partial}{\partial t} - \frac{c^2}{4\pi\sigma}\frac{\partial^2}{\partial z^2}\right)H_\pm = \frac{-\delta c^2}{4\pi\sigma}\frac{\partial}{\partial z}\left[\exp(\pm 2iqz)\frac{\partial H_\mp}{\partial z}\right] \tag{15}$$

for frequencies that satisfy the condition $\omega \ll \sigma$ [in most cases, this condition is stringent than the condition (9)].

Similar to Ref. 1, we set $H_\pm = H_\pm^0 \exp[i(1 \pm q)z - i\omega t]$. We then find for the frequencies satisfying (9),

$$l = \mp q + (1+i)\tilde{k} + \frac{i-1}{2q}\delta^2\tilde{k}^2, \quad \tilde{k} = (2\pi\sigma\omega)^{1/2}c^{-1}. \tag{16}$$

Substituting (16) in (15), we obtain

$$H_\mp^0 = -i\delta\tilde{k}^2(2q)^{-2}H_\pm^0 \ll H_\pm^0 \tag{17}$$

[for the upper and lower signs in (16)].

Therefore, the components $H_{x,y}$ of the field in CLC consist of two parts: an ordinary part proportional to $\exp[(i-1)\tilde{k}z)]$ and damped over a wavelength, and the part which exists in the case $\delta \neq 0$ and which is proportional to $\exp[(-\tilde{k} \pm 2iq)z]$. Since $\tilde{k} \ll 2q$ upon satisfaction of the condition (9), then this part undergoes $\omega_{\max\sigma}^{1/2}\omega^{-1/2} \gg 1$ oscillations over the damping distance. In the solution, at low concentration of CLC, q, $\delta \to 0$, while the ratio of the amplitude of the additional and fundamental waves increases in inverse proportion to the CLC concentration. However, the quantity $\omega_{\max\sigma}$ here decreases in proportion to the square of the concentration. In the case of frequencies $\omega > \omega_{\max\sigma}$ there are no singularities in the passage of the waves.

If the EMW is incident on a CLC layer in the y direction, then the z component of the ordinary EMW in the layer has the form $H_0 \exp[(i-1)\tilde{k}y - i\omega t]$. Under the condition (9), and at $\omega \ll \sigma$, iteration makes it possible to find the additional waves

$$H_x = -\delta\tilde{k}(2q)^{-1}H_0\psi + \text{c.c.} \quad H_y = -i\delta\tilde{k}(2q)^{-1}H_0\psi + \text{c.c.},$$
$$H_z = -i\delta\tilde{k}^2(2q)^{-2}H_0\psi + \text{c.c.}, \quad \psi = \exp[\tilde{k}y(i-1) - i\omega t \pm 2iqz]. \tag{18}$$

Since $\tilde{k} \ll q$, then these waves propagate almost along the CA, attenuating in the y direction, i.e., surface EMW are generated in the conducting layer.

The reflected EMW are surface waves at $\omega < \omega_0$. The condition $\omega < \sigma$ is usually more stringent than $\omega < \omega_0$.

The analysis of the propagation of the wave transverse to CA is valid so long as $\omega < \omega_{\max\sigma}\delta^{-1/2}$. Therefore, as also in the case of nonconducting CLC, in which the amplitudes of the additional and fundamental waves become comparable at $\omega \to \omega_{\max\varepsilon}$, the amplitudes of the additional waves in conducting CLC tend toward the amplitude of the fundamental wave in the case $\omega \to \omega_{\max\sigma}\delta^{-1/2}$.

It is necessary here that temporal and spatial dispersions do not arise. The presence of an oscillating part of the magnetic field can, in the presence of an external magnetic field H_{ext}, lead to oscillations of the velocity of the medium. Actually, the force acting on the medium is equal to $(\mathrm{curl}\, \mathbf{H} \times \mathbf{H}_{ext})/4\pi$ and the equation of motion of the medium can be written in the form

$$\left(\frac{\partial}{\partial t} - \nu\Delta\right)\mathbf{v} = (4\pi\rho)^{-1}[\mathrm{rot}\,\mathbf{H} \times \mathbf{H}_{ext}].$$

The alternating field H is attenuated over the skin depth $\tilde{k}^{-1}$. We consider the case of a CLC layer thickness d satisfying the condition $\tilde{k}d \gg 1$, i.e., $d \gg c(\sigma\omega)^{-1/2}$ (at $\omega \ll \sigma$). Of greatest interest is the case $\omega \gg \nu q^2$ at $\sigma \gg \nu q^2$. The latter condition is satisfied upon introduction into the CLC of donor impurities with concentrations greater than 10^{17} cm^{-3}. In this case, the ratio of the oscillation part of the velocity to the nonoscillating part is of the order of $\delta\tilde{k}(2q)^{-1}$, i.e., it is $2q\tilde{k}^{-1}$ times larger than the analogous ratio for magnetic fields.

The case of the incidence of a wave transverse to the CA in an external magnetic field H_{ext} can be considered in similar fashion if the thickness of the sample is less than the thickness of the skin depth; if however $\tilde{k}d < 1$, then the consideration applies to the surface layer. In this case, motions of the medium arise in all three directions, while at $\tilde{k} > q$ the additional components of the velocity are smaller by a factor of $\delta\tilde{k}q^{-1}$ than the fundamental, and at $\tilde{k} < q$ the additional waves of motion of the medium are δ times smaller than the fundamental.

4. PROPAGATION OF SOUND WAVES IN A DIELECTRIC CHOLESTERIC LIQUID CRYSTAL

Let a constant external field be applied to a layer of CLC, and let a weakly damped sound wave propagate along it; the velocty in the wave is described in the following way:

$$v_x = v_0 \cos(k_s x - \omega t)\exp(-\gamma x),$$
$$\gamma = \nu\omega^2(2s^3)^{-1}, \tag{19}$$

(ν is the viscosity, the anistropy of which can be neglected). The linearized set of equations that describe small deviations of the density ρ' from the equilibrium value ρ, of the field $\nabla\Phi$, and of the velocity have the form[5]

$$\frac{\partial\rho'}{\partial t} + \rho\frac{\partial v_i}{\partial x_i} = 0,$$
$$\left(\frac{\partial}{\partial t} - \nu\Delta\right)v_i + s^2\frac{\partial}{\partial x_i}\frac{\rho'}{\rho} = -\frac{E_nE_m}{8\pi\rho}\frac{\partial}{\partial x_i}\varepsilon_{nm} + \frac{1}{8\pi}\frac{\partial}{\partial x_i}\left(E_nE_m\frac{\partial\varepsilon_{nm}}{\partial\rho}\right)$$
$$\frac{\partial}{\partial x_i}\varepsilon_{in}E_n = 0. \tag{20}$$

It is not difficult to see that even at $k_s = 2q$ there is no parameteric excitation. Actually, the maximum value of the external electric field is limited by the condition[1]

$$\delta\varepsilon E_0^2 < Kq^2$$

(K is the coefficient in the expansion of the free energy density in terms of the spatial derivative of the director). For the condition of parameteric excitation, following Ref. 4, we find

$$\delta\varepsilon \frac{\partial \ln \varepsilon}{\partial \ln \rho}(8\pi\rho\nu qs)^{-1}E_0^2 > 1.$$

The last two inequalities are incompatible.

The solution of (20) in the cases of (1) and (19) can be found by using iteration in δ. At $\omega > \omega_s$

$$\begin{gathered} v_{x,z} = \delta\varepsilon v_0 \omega (8\pi\rho s^2\omega_s)^{-1} E_{0x}^2 F_{x,z}, \\ F_x = \omega\omega_s^{-1} \frac{\partial \ln \varepsilon}{\partial \ln \rho}[\cos(k_s x - \omega t) + A_x \cos(\varkappa_s x - \omega t)]\cos(2qz), \\ F_z = \left(\frac{\partial \ln \varepsilon}{\partial \ln \rho} - 1\right)[\sin(k_s x - \omega t) + \sin(\varkappa_s x - \omega t)]\sin(2qz), \\ \varkappa_s = 2q(\omega^2\omega_s^{-2} - 1)^{1/2}. \end{gathered} \tag{21}$$

The explicit form of the constants $A_{x,z}$ depends on the form of the boundary conditions at $x = 0$ and d. At $\varkappa_p \neq 0$, the quantity A is of the order unity. At $\omega < \omega_s$ we must replace $\cos(\varkappa_s x - \omega t)$ and $\sin(\varkappa x = \omega t)$ in (21) by $\cos(\omega t)\exp(-p_s x)$ and $\sin(\omega t)\exp(-p_s x)$, where $p_s^2 = -\varkappa_s^2$. Thus, just as in the propagation of EMW, additional waves develop which, at $\omega < \omega_s$, are surface waves. Additional surface and nonsurface reflected and transmitted waves appear in similar fashion. In the additional waves, there is an oscillating electric field $|\nabla\Phi|$ of order

$$\approx \varkappa_s v \frac{\partial \ln \varepsilon}{\partial \ln \rho}(k_s s)^{-1} E_0, \quad \approx p_s v \frac{\partial \ln \varepsilon}{\partial \ln \rho}(k_s s)^{-1} E_0.$$

At $\omega \gg \omega_s$,

$$|\nabla\Phi| \approx \omega v \frac{\partial \ln \varepsilon}{\partial \ln \rho}(\omega_s s)^{-1} E_0,$$

i.e., the additional wave is not acoustic but electroacoustic. Iteration is allowable so long as

$$\omega < \omega_s \left[8\pi\rho s^2 \left(\delta\varepsilon E_0^2 \frac{\partial \ln \varepsilon}{\partial \ln \rho}\right)^{-1}\right]^{1/2}.$$

5. INSTABILITY OF SOUND WAVES IN A CONDUCTING CHOLESTERIC LIQUID CRYSTAL IN AN EXTERNAL ELECTRIC FIELD

Let a sound wave be propagated in a CLC with conductivity of type (1), parallel to the electric field and transverse to the CA, while the CLC layer is located between two ordinary semiconductors. In this case the deviation of the density and of the potential φ leads to a deviation of the concentration n' of the carriers from the stationary value n_0. The system of equations which describes the problem in the linear approximation, in addition to the first two equations of (20), also contains

$$\begin{gathered} \varepsilon^{-1}\frac{\partial}{\partial x_i}\left(\varepsilon_{in}\frac{\partial \Phi}{\partial x_n}\right) - (1+\delta\beta_{xx})\frac{\partial \ln \varepsilon}{\partial \ln \rho} E_{0n}\frac{\partial}{\partial x_n}\frac{\rho'}{\rho} = \frac{4\pi e}{\varepsilon}n', \\ \left(\frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x}\right)n' - \mu n_0\left(\Delta + \delta\beta_{xx}\frac{\partial^2}{\partial x^2}\right)\varphi = 0 \end{gathered} \tag{22}$$

(μ is the mobility, v_d the drift velocity of the carriers). Account of the terms proportional to δ will be carried out by the method proposed in Ref. 4 for solution of the problem of parameteric resonance We set

$$\begin{gathered} \varphi', \rho', n', v \sim [A_\pm \cos(kx \pm qz - \omega t) \\ + B_\pm \sin(kx \pm qz - \omega t)]\exp(\lambda t), \end{gathered} \tag{23}$$

where the constants A, B are different for φ', ρ', n', v, while $\lambda \ll \omega$, so that the terms proportional to λ^2 can be neglected. We substitute (23) in the first two equations of (2) and also in (22). For $k > q$, $\sigma < qs$, $\omega = ks$, we find that the quantity λ is maximal at

$$v_d = 2s[1 - 2\pi\sigma(\varepsilon qs)^{-1}] \approx 2s. \tag{24}$$

Here $\lambda > 0$ if

$$\delta s^2 \frac{\partial \ln \varepsilon}{\partial \ln \rho} > 32\pi\nu\sigma, \tag{25}$$

which is easily achieved The requirement (24) is consistent with the condition $\delta\varepsilon E_0^2 < Kq^2$ (in which the pitch of the helix does not depend on the field[1]), since $K\mu^2q^2 > \delta\varepsilon s^2$ (in solutions with low CLC concentrations, where q and δ are proportional to this concentration, the latter inequality can not be satisfied). The results can be written down as the condition for the carrier density:

$$n_0 < qs^2(e\mu)^{-1}\min[1, \delta\varepsilon s(32\pi\nu q)^{-1}] \approx qs(e\mu)^{-1}$$

(it is taken into account that $\partial \ln \varepsilon / \partial \ln p \approx 1$). We note that the condition obtained for the carrier density is stricter than the condition for the concentration that follows from the requirement of the absence of heating of the sample over the time of several periods of the sound oscillations in the field, which is required for satisfaction of (24). The excited oscillations propagate at an angle to the CA and to the applied field. In them, the quantities $|\rho'\rho^{-1}|$, $|vs^{-1}|$, $|E'E_0^{-1}|$ are of the same order, while

$$|n'n_0^{-1}| \approx |\varepsilon q E_0(4\pi e n_0)^{-1}\|E'E_0^{-1}| < |E'E_0^{-1}|.$$

At $k < q$, there are no excitations. On the interface of the CLC with the semiconductor, the boundary conditions lead to the appearance in the semiconductor of perturbations of the potential of the concentratuon and of the lattice vibrations, which are damped upon penetration into the semiconductor.

6. EXCITATION OF A SOLENOIDAL FIELD OF THE SOUND WAVE

We shall show that sound wave in CLC can cause the appearance of a solenoidal EM field propagating with velocity $s \ll c$. Such a field is similar to the thermomagnetic field which arises in ordinary conducting media in the presence of a temperature gradient.[3] Actually in dielectric CLC, as is seen from the last equation of (20), in the passage of a sound wave of the type (19) parallel to the external electric field and transverse to the CA, we obtain the following by integrating over δ and setting $E = E_1 + \delta E_2$,

$$\frac{\partial E_{2i}}{\partial x_i} = k_s v_0 s^{-1}\frac{\partial \ln \varepsilon}{\partial \ln \rho}E_0\cos(2qz)\cos(k_s x - \omega t). \tag{26}$$

The solution of (26) depends on the boundary conditions. Let $E_2 = 0$ at $x = 0$ for all z; then

$$E_2 = E_{2x} = v_0 s^{-1}\frac{\partial \ln \varepsilon}{\partial \ln \rho}E_0\cos(2qz)[\sin(k_s x - \omega t) - \sin\omega t].$$

If now $E_2 = 0$ at $z = 0$ and $qL = 2\pi m$, where $m \gg 1$ is an integer, then

$$E_z = E_{2z} = k_s v_0 (2qs)^{-1} E_0 \sin(2qz) \cos(k_s x - \omega t).$$

It is seen from these solutions that an alternating magnetic field is produced.

$$H_y = \delta c v_0 s^{-2} \frac{\partial \ln \varepsilon}{\partial \ln \rho} E_0 F \sin(2qz) \sin(k_s x - \omega t); \quad F = k_s (2q)^{-1}, \quad E_z = E_{2x}, \quad F = 2qk_s^{-1}, \quad E_z = E_{2z}. \tag{27}$$

The magnitude of this magnetic field can be greater than the alternating electric field if

$$\delta c s^{-1} \frac{\partial \ln \varepsilon}{\partial \ln \rho} F > 1.$$

Thus, waves of solenoidal fields are generated, proportional to $\cos(k_s x \pm 2qz - \omega t)$. The amplitude of the waves radiated outside the CLC will be smaller than (27) by a factor of sc^{-1}, while the ratio of the flux densities of the radiated EMW to the sound wave flux density is

$$\delta^2 c (\partial \ln \varepsilon / \partial \ln \rho)^2 E_0^2 (4\pi \rho s^3)^{-1} \ll 1.$$

Similar solenoidal waves arise because there exist additional magnetic waves in the CLC. Their amplitude is smaller than (27) but in the case $\omega < \omega_s$ they are of interest as surface waves. In the excitation of sound waves in conducting CLC, the solenoidal fields also grow, leading to radiation from the CLC layer.

In conducting CLC in the presence of an external magnetic field H_{ext}, the presence of the solenoidal field can lead to instability of the sound wave.

Actually, an additional force appears in the CLC, acting on the medium and equal to $(\mathrm{curl}\, \mathbf{H} \times \mathbf{H}_{ext})/4\pi$. Taking this force into account in (20) and proceeding as in Sec. 5, we find that the sound waves are unstable if

$$\delta c (2\pi \nu \rho q s^2)^{-1} \frac{\partial \ln \varepsilon}{\partial \ln \rho} E_0 H_{ext} > 1.$$

This condition is compatible with the condition of independence of q of the field

$$Kq^2 > \delta \varepsilon E_0^2, \quad \delta \chi H_{ext}^2$$

(χ is the isotropic part of the magnetic susceptibility) since

$$c \frac{\partial \ln \varepsilon}{\partial \ln \rho} Kq > \rho s^2 \chi^{1/2}.$$

In the developing waves, the amplitude of the alternating magnetic field is greater than the amplitude of the alternating electric field by a factor sx^{-1}. The ratio

$$|\mu H' s (cv)^{-1}| \approx v_d s^{-1} < 1.$$

7. ACOUSTO-ELECTRIC AND ACOUSTO-MAGNETIC EFFECTS

Let a sound wave be propagated at an angle to the CA, along which is applied a constant electric field E_0

$$\rho' = \rho_1 \cos(k_x x + k_z z - \omega t) \exp(-\gamma_x x - \gamma_z z).$$

We substitute this expression in Poisson's equation and seek the potential with accuracy to within ρ_1^2. We average the resultant solution over the period of the sound wave, after which we determine $U_x = \varphi(L) - \varphi(0)$ the potential difference in a direction transverse to the external field

$$U_x = E_0 L I \gamma_z k_z (\rho s^3)^{-1} (k_x^2 + k_z^2)^{-1} \left(\frac{\partial \ln \varepsilon}{\partial \ln \rho} \right)^2 [1 + 2\delta \cos(2qz)],$$

(L is the dimension in the x direction, I is the flux density of the acoustic energy). U_x is maximal upon incidence of the sound wave at an angle $\pi/4$ to the CA. It is essential that U_x contain a solenoidal contribution. Substituting the electric field in the Maxwell equation at $\sigma \neq 0$, we find that magnetic field structures.

$$H_y = -4\pi \delta \sigma \gamma_z k_x E_0 I \left(\frac{\partial \ln \varepsilon}{\partial \ln \rho} \right)^2 (cq\rho s^3)^{-1} (k_x^2 + k_z^2)^{-1} \cos(2qz) \exp(-\gamma_x x - \gamma_z z)$$

are formed in the conducting CLC. (We note that $\partial_z \to 0$ as $k_z \to 0$.) The most acceptable value of E_0 is limited by the condition of the independence of q of E_x, which gives

$$U_x < k \rho s^3 q L (\gamma I)^{-1} K^{1/2} \delta^{-1/2}.$$

The developing constant electric field leads to the appearance of all the phenomena already treated above in the absence of E_x.

8. ALFVÉN WAVES IN A CHOLESTERIC LIQUID CRYSTAL

Consideration of Alfvén waves has meaning only in the case of a magnetic field parallel to the CA, since the frequency of the weakly attenuating waves must satisfy the condition

$$\omega < \sigma H_0^2 (\rho c^2)^{-1}.$$

In a magnetic field transverse to the CA, this condition has the form

$$\omega < K \sigma q^2 (\delta \chi \rho c^2)^{-1}.$$

The right side of the latter inequality does not exceed $10^{-1}\ \mathrm{s}^{-1}$. We first consider $H_0 = H_{0z}$ in an incompressible liquid. We set

$$v_x \pm i v_y = v_\pm, \quad v_\pm, \ H_\pm \sim \exp[-i\omega t + i(l \pm q) z].$$

From the equations of magnetohydrodynamics, we find

$$(l \pm q)^2 = k_A^2 [1 + \delta k_A c (32\pi \nu q^2)^{-1}], \quad k_A = \omega v_A^{-1} = \omega H_0^{-1} (4\pi\rho)^{1/2}.$$

In contrast to the case $\delta = 0$ there exist two Alfvén waves in CLC: the ordinary, which is proportional to $\exp(ik_A z)$, and the additional wave, which is proportional to $\exp[i(k_A \pm 2q)z]$. In the additional wave, the amplitudes of the magnetic field and of the velocity change over a distance equal to the pitch of the cholesteric helix. This additional wave is modulated by a wave with a period k_z. The ratio of the amplitudes of the additional and the fundamental waves is of order

$$\delta \omega_A^3 (4 \nu v_A^2 q^4)^{-1}.$$

We note that this ratio $\sim \delta q^{-4}$ and increases in CLC solutions with decrease in the CLC concentration. Our consideration loses meaning at q of the order $\sigma v_A c^{-2}$, i.e., as $q \to k_A$. The maximum frequency is $\omega_{A\max} \approx 10^2\ \mathrm{s}^{-1}$. In a magnetic field not parallel to the CA, still another reason for the existence of the additional Alfvén wave is possible. Actually, in the equation of motion of the medium, there is a striction force directed along the CA and equal to

$$\approx\delta\chi H_{0x}H_x'q\sin(2qz).$$

This force creates the additional Alfvén wave even at

$$(l\pm q)^2=k_A^2.$$

The ratio of the amplitudes of the additional and fundamental waves (for magnetic fields) is of the order of

$$\delta\sigma\chi H_{0x}H_{0z}(\nu\rho qk_Ac^2)^{-1},$$

and for the velocities in a direction transverse to the CA,

$$\delta\chi H_{0x}H_{0z}k_A(\nu\rho\omega q)^{-1}.$$

The first of these ratios is much smaller than the second, i.e., a wave of hydrodynamic motions of the medium is generated. In a compressible medium, magneto-acoustic waves develop, which also consist of an ordinary and an additional wave, while the ratio of the amplitudes of the additional magnetic field to the fundamental is smaller than in an incompressible medium.

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