less than the width of lower autoionizing states of the He atom and of autoionizing states which appear as a result of excitation of the $4f$ shell. The smallness of the width of these states is associated with multiple oscillations of the wave functions of one-particle states in the main interaction region, which reduces the probability of ionization decay. This is typical of autoionizing states of the valence shells of the heavy atoms.

A good agreement is obtained between the experimental results and those found by numerical (theoretical) calculation of the positions of narrow autoionizing states in the spectrum of the Yb atom near the ionization threshold, which confirms the correctness of the selected experimental method for investigating these states.

The theoretical approach adopted above is equally suitable for the investigation of other many-electron systems, particularly of atoms with three valence electrons.

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Polarization and angular distribution of Cherenkov and transition radiations in the field of a strong electromagnetic wave

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An analysis is made of the influence of an external electromagnetic field on Cherenkov and transition radiations. It is shown that considerable changes appear in the angular and spectral distributions of these radiations. Strong polarization may be acquired and fairly hard photons with energies of several kiloelectronvolts may be emitted.

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1. Quantum theory of transition radiation in the absence of an external electromagnetic field is given in Ref. 1. We shall consider Cherenkov and transition radiations of a longitudinally polarized lepton (electron, muon) moving from medium 1 to medium 2 (Fig. 1) whose permittivities are ε_1 and ε_2. We shall use as independent solutions of the wave equation those which correspond to the case when the principal wave (identified by the thick line in Fig. 1) can be produced, as in Ref. 1, by refraction or reflection of auxiliary waves.

Case a represents a situation in which radiation enters medium 1, whereas case b corresponds to a situation when radiation enters medium 2. The radiating particle moves from medium 1 to medium 2. We shall consider only the situation when we can ignore an electromagnetic (laser) wave reflected from the interface between the two media, i.e., when the reflection coefficient at the laser frequency is low. We shall study transition radiation at frequencies other than the laser frequency.

The matrix element for a transition of a fermion from a state φ_p to a state φ'_p accompanied by the emission of a photon with a 4-momentum $f=(\omega_f, \mathbf{f})$ and a polariza-

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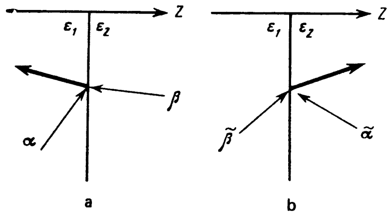

FIG. 1.

tion x is

$$M_{pp'}=-e\int d^4y\,(\bar{\varphi}_{p'}(y)\,[\hat{A}^a(y)+\hat{A}^b(y)]\,\varphi_p(y)), \quad (1)$$

where A^a and A^b are the vector potentials of the emitted electromagnetic field in the cases a and b in Fig. 1. The explicit form of these potentials is given by the following expressions[1]:

$$A_j^a(y)=\sum_{\substack{\varkappa,\omega_f\geq 0\\ l=1,2}}(2\omega_f g^l V)^{-1/2}\Big\{\theta(-z)\Big[\frac{\hat{x}_j^l}{\varepsilon_1^{1/2}}\exp(iyf_{1-})+\frac{\hat{x}_{1j}^l}{\varepsilon_1^{1/2}}\alpha^l\exp(iyf_{1+})\Big]+\theta(z)\frac{\hat{x}_{2j}^l}{\varepsilon_2^{1/2}}\beta^l\exp(iyf_{2-})\Big\} \quad (2)$$

$$A_j^b(y)=\sum_{\substack{\varkappa,\omega_f\geq 0\\ l=1,2}}(2\omega_f \tilde{g}^l V)^{-1/2}\Big\{\theta(-z)\Big[\frac{\hat{x}_j^l}{\varepsilon_2^{1/2}}\exp(iyf_{2+})+\frac{\tilde{\hat{x}}_{1j}^l}{\varepsilon_2^{1/2}}\tilde{\alpha}^l\exp(iyf_{2-})\Big]+\theta(z)\frac{\hat{x}_{2j}^l}{\varepsilon_1^{1/2}}\tilde{\beta}^l\exp(iyf_{1+})\Big\}. \quad (3)$$

The following notation is adopted in Eqs. (2) and (3): $\theta(z)$ is the theta function,

$$yf_{1,2\pm}=\omega_f t-\varkappa\rho\pm\lambda_{1,2}z,\quad \lambda_{1,2}=+(\omega_f^2\varepsilon_{1,2}-\varkappa^2)^{1/2}, \quad (4)$$

$$g^l=1+(\alpha^l)^2+(\beta^l)^2,\quad \tilde{g}^l=1+(\tilde{\alpha}^l)^2+(\tilde{\beta}^l)^2, \quad (5)$$

$\varkappa$ and ρ are the wave vector of the radiation field and the radius vector of the point of observation in the plane of the interface between the media. The index l corresponds to the polarization of the emitted photon; $\mathbf{x}_1=(\mathbf{e}_1+i\mathbf{e}_2)/\sqrt{2}$, $\mathbf{x}_2=(\mathbf{e}_1-i\mathbf{e}_2)/\sqrt{2}$; the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ lie, respectively, in the plane of the three waves and at right-angles to this plane. The amplitudes of the auxiliary waves traveling in a medium[1] are

$$\left.\begin{aligned}&\alpha^1=(\lambda_1\varepsilon_2-\lambda_2\varepsilon_1)/(\lambda_1\varepsilon_2+\lambda_2\varepsilon_1),\quad \alpha^{(2)}=(\lambda_1-\lambda_2)/(\lambda_1+\lambda_2),\\ &\beta^1=(2\lambda_1\varepsilon_2)/(\lambda_1\varepsilon_2+\lambda_2\varepsilon_1),\quad \beta^{(2)}=2\lambda_1(\varepsilon_1\varepsilon_2)^{1/2}/(\lambda_1+\lambda_2).\end{aligned}\right\} \quad (6)$$

The amplitudes of the auxiliary waves traveling in medium 2 (identified by a tilde) are obtained by the index transposition $1 \rightleftharpoons 2$ in the system (6). The plus and minus indices in Eqs. (2)–(4) correspond to the propagation of a wave to the right or left of the interface between the media. The indices 1 and 2 of the wave vector of the emitted photon correspond to the propagation of a wave in medium 1 or medium 2, respectively.

We shall consider a radiating particle in the field of an electromagentic (laser) wave and, therefore, the wave function of the particle $\varphi_p(y)$ is[2]

$$\varphi_p(y)=\Big[1+\frac{e}{2(kp)}(\hat{k}\hat{a}_1\cos\varphi+\hat{k}\hat{a}_2\sin\varphi)\Big]\frac{u(p)}{(2q_0)^{1/2}}\times\exp\Big\{-ie\frac{(a_1p)}{(kp)}\sin\varphi+ie\frac{(a_2p)}{(kp)}\cos\varphi-iqy\Big\}, \quad (7)$$

where

$$q^\mu=p^\mu-e^2\frac{a^2}{2(kp)}k^\mu.$$

It is assumed in Eq. (7) that the external electromagnetic (laser) wave has a vector potential which corresponds to the circular polarization

$$A=a_1\cos\varphi+a_2\sin\varphi,\quad \varphi=ky. \quad (8)$$

Here, $k^\mu=(\omega,\mathbf{k})$ is the wave 4-vector of circularly polarized laser radiation, and the 4-amplitudes a_1 and a_2 are identical in magnitude and mutually orthogonal; the potential A is calibrated by the Lorentz condition, i.e., $(a_1k)=(a_2k)=0$.

Substitution of Eqs. (7), (3), and (2) in Eq.(1) gives

$$M_{p'p}=-\sum_{s=-\infty}^{\infty}\Big(\frac{e}{(2\omega_f g^l V)^{1/2}}M_{pp'}^{1a}+\frac{e}{(2\omega_f\tilde{g}^l V)^{1/2}}M_{pp'}^{1b}\Big)\times(2\pi)^3\delta(\varkappa+\mathbf{q}_\perp'-\mathbf{q}_\perp-s\mathbf{k})\,\delta(s\omega+q_0-q_0'-\omega_f), \quad (9)$$

where

$$M_{pp'}^{1a}=(\varepsilon_1)^{-1/2}Q(\hat{\tilde{x}}^l)\,(\pi\delta(sk_z+q_z-q_z'+\lambda_1)+iP(sk_z+q_z-q_z'+\lambda_1)^{-1})+\alpha_1^l\varepsilon^{-1/2}Q(\hat{\tilde{x}}_1^l)\,(\pi\delta(sk_z+q_z-q_z'-\lambda_1)-iP(sk_z+q_z-q_z'-\lambda_1)^{-1})+\beta^l\varepsilon_2^{-1/2}Q(\hat{x}_2^l)\,(\pi\delta(sk_z+q_z-q_z'+\lambda_2)-iP(sk_z+q_z-q_z'+\lambda_2)^{-1});$$

$$M_{pp'}^{1b}=\varepsilon_2^{-1/2}Q(\hat{x}^l)\,(\pi\delta(sk_z+q_z-q_z'-\lambda_2)-iP(sk_z+q_z-q_z'-\lambda_2)^{-1})+\tilde{\alpha}^l\varepsilon_2^{-1/2}Q(\hat{x}_1^l)\,(\pi\delta(sk_z+q_z-q_z'+\lambda_2)-iP(sk_z+q_z-q_z'+\lambda_2)^{-1})+\tilde{\beta}^l\varepsilon_1^{-1/2}Q(\hat{x}_2^l)(\pi\delta(sk_z+q_z-q_z'-\lambda_1)+iP(sk_z+q_z-q_z'-\lambda_1)^{-1}).$$

Here, $P(x)$ is the principal value of the integral in question, and the function $Q(\hat{x})$ is[2]

$$Q(x)=(4\pi)^{1/2}\bar{u}(p')\Big\{\Big(\hat{x}-\frac{e^2a^2(kx)\hat{k}}{2(kp)(kp')}\Big)B_s+e\Big(\frac{\hat{a}_1\hat{k}\hat{x}}{2(kp')}+\frac{\hat{x}\hat{k}\hat{a}_1}{2(kp)}\Big)B_{1s}+e\Big(\frac{\hat{a}_2\hat{k}\hat{x}}{2(kp')}+\frac{\hat{x}\hat{k}\hat{a}_2}{2(kp)}\Big)B_{2s}\Big\}u(p), \quad (10)$$

where s is an integer ($s=0,\ldots,\pm N$). The functions B_s, B_{1s}, and B_{2s} expressed in terms of Bessel functions J_s and $J_{s\pm1}$ with the argument Δ are given by[2]:

$$\Delta=(b_1^2+b_2^2)^{1/2},\quad b_1=e\Big(\frac{(a_1p)}{(kp)}-\frac{(a_1p')}{(kp')}\Big),\quad b_2=e\Big(\frac{(a_2p)}{(kp)}-\frac{(a_2p')}{(kp')}\Big) \quad (11)$$

We shall first consider Cherenkov radiation in a homogeneous medium ($\varepsilon_1\approx\varepsilon_2\approx\varepsilon$). The probability of emission per unit time is

$$dW_s=(2\pi)^4\delta^4(sk+q-q'-f)\frac{d^3\mathbf{f}\,d^3\mathbf{q}'}{(2\pi)^6 2\omega_f 2q_0 2q_0'\varepsilon}\times{}^1/_2\,\mathrm{Sp}[\hat{Q}(\hat{p}+m)(1-\gamma_5\hat{g})\hat{Q}(\hat{p}'+m)]. \quad (12)$$

Here, $(\hat{p}+m)(1-\gamma_5\hat{g})$ is the projection operator of Michel and Wightman for the initial polarized lepton.

We shall consider a situation in which a change in the 4-momentum of the lepton as a result of photon emission is much less than the initial value since in the case of laser powers currently attainable the probability of emission of photons with the 4-momentum comparable with the initial 4-momentum of the lepton is negligible. Therefore, we can substitute $p\approx p'$ in Eqs. (12) and (10). We note that this cannot be done in Eq. (11). However, it is then found that when the spur in Eq. (12) is calculated, all the terms proportional to the initial lepton polarization vanish. The physical meaning of this result is self-evident: a polarized lepton generates circularly polarized radiation only because of a change in its 4-momentum. If this change is ignored, the initial polarization of the lepton does not affect the characteristics of the emitted radiation. Nevertheless, the radiation is circularly polarized because the laser wave has circular polarization. The substitution of Eq. (10) in Eq. (12) gives, after simple calculations, the following result:

$$dW=\sum_{s=-\infty}^{\infty}\frac{e^2}{4\pi}\delta(s\omega+q_0-q_0'-\omega_f)(C+\delta D)\frac{d\Omega_f f^2\,df}{\omega_f q_0 q_0'}, \tag{13}$$

where

$$C=|B_s|^2\left[\mathbf{p}^2-(\mathbf{pn})^2+\frac{e^2|a|^2}{(kp)}(\mathbf{kp}-(\mathbf{kn})(\mathbf{pn}))\right]+\mathrm{Re}(B_{1s}B_{2s}^*)$$
$$\times\left\{\frac{e^2}{(kp)}\left[\frac{2(\mathbf{a}_1\mathbf{p})(\mathbf{a}_2\mathbf{p})}{kp}(\mathbf{k}^2-(\mathbf{kn})^2)-2(kp)(\mathbf{a}_1\mathbf{n})(\mathbf{a}_2\mathbf{n})-2(\mathbf{pa}_2)(\mathbf{kn})\right.\right.$$
$$\left.\left.\times(\mathbf{a}_1\mathbf{n})-2(\mathbf{pa}_1)(\mathbf{kn})(\mathbf{a}_2\mathbf{n})\right]\right\}+\sum_{\nu=1,2}|B_{\nu s}|^2\frac{e^2}{(kp)^2}[(kp)^2(\mathbf{a}_\nu^2-(\mathbf{a}_\nu\mathbf{n})^2)$$
$$+(\mathbf{a}_\nu\mathbf{p})(\mathbf{k}^2-(\mathbf{kn})^2)-2(\mathbf{a}_\nu\mathbf{p})(\mathbf{kp})(\mathbf{a}_\nu\mathbf{n})(\mathbf{kn})]$$
$$+\mathrm{Re}(B_sB_{\nu s}^*)\left\{\frac{2e}{(kp)}[-(\mathbf{a}_\nu\mathbf{p})(\mathbf{pk})-(\mathbf{pn})(\mathbf{kn})-(kp)[(\mathbf{pa}_\nu)-(\mathbf{pn})(\mathbf{a}_\nu\mathbf{n})]]\right.$$
$$\left.-\frac{e^3|a|^2}{(kp)}\left[(\mathbf{kn})(\mathbf{a}_\nu\mathbf{n})-\frac{(a_\nu p)}{(kp)}(\mathbf{k}^2-(\mathbf{kn})^2)\right]\right\};$$
$$D=\mathrm{Im}(B_{1s}B_{2s}^*)\frac{e^2}{(kp)}\left[2(\mathbf{kp})|\mathbf{a}_1|^2\frac{(\mathbf{kn})}{|\mathbf{k}|}-2|\mathbf{k}|(\mathbf{pa}_2)(\mathbf{a}_2\mathbf{n})\right.$$
$$\left.-2|\mathbf{k}|(\mathbf{pa}_1)(\mathbf{a}_1\mathbf{n})\right]+\sum_{\nu=1,2}\mathrm{Im}(B_sB_{\nu s}^*)\left\{\frac{2e}{(kp)}\{(\mathbf{kp})\mathbf{n}[\mathbf{a}_\nu\times\mathbf{p}]\right.$$
$$\left.+(\mathbf{a}_\nu\mathbf{p})\mathbf{n}[\mathbf{k}\times\mathbf{p}]\}+\frac{e^3|a|^2}{(kp)}[\mathbf{k}\times\mathbf{a}_\nu]\mathbf{n}\right\}.$$

Here, $\mathbf{n}=\mathbf{f}/|\mathbf{f}|$ is the direction of the emitted photon. Since the law of conservation in Eq. (13) admits considerable deviations from the classical distribution of Cherenkov radiation, it is desirable to investigate separately the angular and energy distributions of the radiation and the degree and direction of its circular polarization.

The integral in Eq. (13) can be calculated by investigating the laws of conservation of energy and momentum. The four-dimensional form of the law of conservation is

$$sk+q_0-q_0'-f=0. \tag{14}$$

Simple transformations give the following expression for the energy of an emitted photon:

$$\omega_f=\tfrac{1}{2}(1-n^2(\omega_f))^{-1}[F\pm\{F^2-8s(kp)(1-n^2(\omega_f))\}^{1/2}], \tag{15}$$

where

$$F=2(p_0-n(\omega_f)|p|\cos\chi)+\omega(1-n(\omega_f)\cos\theta)\left(2s-\frac{e^2a^2}{(kp)}\right)$$
$$\cos\theta=\cos\chi\cos\chi_0+\sin\chi\sin\chi_0\cos\varphi,\quad n(\omega_f)\equiv\varepsilon^{1/2};$$

χ is the angle between the vectors $\mathbf{p}$ and $\mathbf{f}$, θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{f}$, χ_0 is the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$, and φ is the azimuthal angle above the direction of $\mathbf{p}$. Similarly, we can obtain the expression for the angle between the initial electron momentum and the emitted photon:

$$\cos\chi=\frac{-B\pm A\cos\varphi(1+A^2\cos^2\varphi-B^2)^{1/2}}{1+A^2\cos^2\varphi}, \tag{16}$$

$$\sin\chi=-\frac{AB\cos\varphi\pm(1+A^2\cos^2\varphi-B^2)^{1/2}}{1+A^2\cos^2\varphi}, \tag{17}$$

$$A=\sin\chi_0\omega n\left(2s-\frac{e^2a^2}{(kp)}\right)\Big/\left[2n|\mathbf{p}|+\omega n\left(2s-\frac{e^2a^2}{(kp)}\right)\cos\chi_0\right], \tag{18}$$

$$B=\left[\omega_f(1-n^2)+\omega\left(\frac{e^2a^2}{(kp)}-2s\right)-2p_0+\frac{2s(kp)}{\omega_f}\right]$$
$$\Big/\left[2n|\mathbf{p}|+\omega n\left(2s-\frac{e^2a^2}{(kp)}\right)\cos\chi_0\right]. \tag{19}$$

The condition for the appearance of radiation is $|\cos\chi|\leqslant 1$. Hence, using Eq. (16), we find that a simple analysis yields the following criteria:

$$\cos^2\varphi\geqslant(|B|-1)/A^2,\quad(1+A^2\cos^2\varphi)\geqslant B^2 \tag{20}$$

or

$$\sigma B/A\cos\varphi>0,\quad|A\cos\varphi|\geqslant|B|. \tag{21}$$

Here, $\sigma=\pm1$ is the sign in front of the square root in Eq. (16). Radiation appears when the conditions of Eq. (20) are satisfied, irrespective of whether the conditions of Eq. (21) are obeyed, and also when the conditions of Eq. (21) are satisfied, irrespective of the conditions of Eq. (20).

We shall consider the simplest cases in order to obtain a physical picture of the phenomena. We shall begin by analyzing Eqs. (18)–(21) in the $s=0$ case, when a lepton does not absorb or emit a single laser photon. Substituting Eqs. (18) and (19) in Eq. (20), we find that in this case a lepton emits radiation only if

$$1+\omega\mu\cos\chi_0/2|\mathbf{p}|>0, \tag{22}$$

$$\left[\frac{c}{nv}+\frac{\xi^2(\beta^2-1)(1-\cos\chi_0)}{\beta-\cos\chi_0}+\frac{\omega_f(n^2-1)}{2|\mathbf{p}|n}\right]\leqslant 1,\quad\beta=\frac{v}{c}, \tag{23}$$
$$\mu=-e^2a^2/(kp)=\xi^2m^2/|\mathbf{p}|\omega[(1+m^2/\mathbf{p}^2)^{1/2}-\cos\chi_0].$$

In the absence of the laser field, Eq. (23) reduces to the classical condition for the possibility of emitting Cherenkov radiation. An external electromagnetic field hinders the emission of radiation by the lepton both because of the second term in Eq. (23) and because the condition (22) may not be satisfied for some angles between the direction of the lepton momentum and the external laser radiation. The possibility of emission associated with the condition (21) appears only for $s\neq 0$ and, therefore, we shall now analyze the situation for $s>0$.

The condition (20) also corresponds to the conditions (22) and (23), where we now have to make the substitutions

$$\mu=\mu+2s, \tag{24}$$
$$p_0=p_0-s(kp)/\omega_f\equiv z_0. \tag{25}$$

It is worth noting that in a wide range of values of ξ^2 and p_0/m we have $\mu\gg1$. In fact, if $m\gg|\mathbf{p}|$, we have $\mu\approx\xi^2m/\omega$, and if $m\ll|\mathbf{p}|$ and $\cos\chi_0=+1$, we have $\mu\approx\xi^2m/\omega|\mathbf{p}|$; if $\cos\chi_0=+1$, we find that $\mu\approx2\xi^2|\mathbf{p}|/\omega$. Therefore, in this range we can ignore the term with s in Eq. (24).

However, it follows from Eq. (25) that the value of z_0 differs considerably from p_0. By way of example, we shall consider the simple case when $A\ll1$. The conditions (21)–(26) yield the following simple expression in the case when $\cos\chi_0=0$:

$$-1\leqslant\frac{c}{vn}\left(1-\frac{s\omega}{\omega_f}\right)\leqslant1. \tag{26}$$

Thus, the emission of Cherenkov radiation may be facilitated or hindered by the emission or absorption of laser photons.

We shall now consider an essentially nonclassical situation of the appearance of Cherenkov radiation, i.e., we shall discuss the case of the conditions given by Eq. (21). Selection of the sign of the root in Eq. (16) can always ensure that the first of the conditions in Eq. (21) is satisfied. Therefore, we have to analyze only the second condition, from which it follows that

$$|\omega_f(1-n^2)/\mu n\omega-2z_0/\mu n\omega-1|\leqslant|\cos\varphi\sin\chi_0|. \tag{27}$$

We note that if $A\gg1$ this condition reduces to the analogous condition applicable to classical radiation in the $A>1$ case.

In the ultrarelativistic range the stimulated Cherenkov radiation represents a set of very narrow spectral lines of width

$$\Delta\omega_f\sim\frac{n\xi^2\omega_f^{\,2}}{|\mathbf{p}|^2 s\omega}m^2. \tag{28}$$

We shall calculate the probability of emission of radiation by integrating Eq. (13) with respect to the angle χ using the δ function. Simple transformations give

$$dW_s=\int\frac{d\varphi\omega_f\,d\omega_f}{q_0q_0'}G \times\frac{s+\mu\omega_f(n\cos\theta-1)/2(p_0-|\mathbf{p}|\cos\chi_0-\omega_f(1-n\cos\theta))-\omega_f/\omega+p_0/\omega}{\omega_f[(\mu/2+s)(\cos\varphi\sin\chi_0\operatorname{ctg}\chi-\cos\chi_0)-|\mathbf{p}|/\omega]}\Bigg|. \tag{29}$$

Here,

$$G=C+\delta D.$$

In Eq. (29) the angle χ may assume two values in accordance with Eqs. (16) and (17). In further calculations it is convenient to use such a coordinate system that the vector $\mathbf{a}_1$ lies in the $\mathbf{k}\cdot\mathbf{p}$ plane and the vector $\mathbf{a}_2$ is directed along the y axis; the z axis is along $\mathbf{p}$. Circular polarization of radiation emitted in a given angle χ with a given frequency ω_f is

$$\mathscr{P}=D/C. \tag{30}$$

We shall now consider the case when $\cos\chi_0=0$, $\varphi=0$ and we shall estimate the polarization in the case of nonrelativistic, relativistic, and ultrarelativistic leptons. For this purpose we shall use Eqs. (14) and (33) and assume in the estimates that for different combinations both B_s and $B_{\lambda s}$ are of the order of 1. If $|\mathbf{p}|\ll m$, then

$$\mathscr{P}\sim\sin\chi+O(\xi\cos\chi). \tag{31}$$

If $|\mathbf{p}|\sim m$, then

$$\mathscr{P}\sim\xi/(\sin\chi+O(\xi)). \tag{32}$$

In the ultrarelativistic case when $|\mathbf{p}|\gg m$, we find that

$$\mathscr{P}\approx\frac{\xi}{\sin\chi}\frac{m}{|\mathbf{p}|}. \tag{33}$$

If $\varphi=\pi/2$, we can easily show that $\mathscr{P}\sim\xi$ for $|\mathbf{p}|\ll m$ and $|\mathbf{p}|\sim m$ and that $\mathscr{P}\sim\xi m\cos\chi/|\mathbf{p}|\sin\chi$ for $|\mathbf{p}|\gg m$. Therefore, it is clear that circular polarization is weak, with the exception of the situation when the radiation is directed parallel or antiparallel to the lepton momentum.

Stimulated Cherenkov radiation of high frequencies (for example, those corresponding to the x-ray range) is of special interest. We shall now estimate the intensity of this radiation. The quantities B_s, B_{1s}, and B_{2s} occurring in the expression for the radiation intensity are expressed in terms of Bessel functions with the argument Δ [see Eq. (11)]. It is clear from Eq. (26) that if $s\sim1$ and $1-v^2/c^2\ll1$, because

$$\frac{s\omega}{1+\beta n}\leqslant\omega_f\leqslant\frac{s\omega}{1-\beta n},$$

and the refractive index is $n\gtrsim1$, the maximum energy of the emitted photon may be much greater than $s\omega$. In this case the field of x rays will be high. In the other cases we have $\omega_f\sim s\omega$ and, therefore, the high energy of the emitted photons corresponds to a large index s of the Bessel function. If $s-\Delta\gg\Delta^{1/3}$, then the estimates of Ref. 4 show that the yield of hard (x-ray) radiation is exponentially small: $\sim e^{-s}$. However, if $s-\Delta\ll\Delta^{1/3}$, the yield of this radiation is higher since the Bessel function is of the order of unity. If $n\gtrsim1$, the maximum value of the ratio Δ/s is

$$\gamma=\max\frac{\Delta}{s}=\frac{\xi}{(1+\xi^2)^{1/2}}. \tag{34}$$

Thus, hard radiation is obtained if

$$(1-\xi(1+\xi^2)^{-1/2})\ll s^{-2/3}. \tag{35}$$

This imposes restrictions on the maximum possible value of s, i.e., on the wavelength of the stimulated Cherenkov radiation. For example, for $\xi\sim1$, we have $s\ll5$. It should be stressed that since the condition (35) does not require a high lepton energy, the resultant radiation has a strong circular polarization.

Finally, we shall estimate the probability of emission of Cherenkov radiation. For example, in the case of an ultrarelativistic electron, we find from Eq. (29) that

$$J_s^2\Delta\omega\sim nK^2m^2\xi s\omega/|\mathbf{p}|^2, \tag{36}$$

where K is a numerical coefficient which appears because of the asymptotic nature of the Bessel function with a large argument when the condition (35) is satisfied:

$$K^2=(3^{8/3}2^{4/3}\Gamma^2(3/2)s^{2/3})^{-1}.$$

Then, the radiation intensity is of the order of

$$I\approx\frac{1}{137}\frac{\hbar\omega_f n\xi s^{1/3}\omega m}{10^2|\mathbf{p}|}\sin^2\chi. \tag{37}$$

If $\xi\sim1$, the emission of radiation corresponds to $s\approx3000$ and $\sin^2\chi\sim0.1$. If we assume that $\hbar\omega\sim2$ eV and take $|\mathbf{p}|/m\sim10$, the number of emitted photons of energy 5 keV is $I/\hbar\omega_f\sim10^9\ \mathrm{sec}^{-1}$, which is comparable with the number of photons in the emission of the nonstimulated Cherenkov radiation. By way of illustration, Figs. 2 and 3 show the results of machine calculations of the angular and spectral dependences of Cherenkov radiation.

We shall now consider transition radiation. We shall deal with case a in Fig. 1, which corresponds to the emission of a photon in the first medium from which the electron is arriving. In the matrix elements [Eq. (10)]

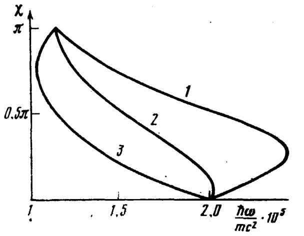

FIG. 2. Dependence of the angle of emission on the radiation frequency calculated for different azimuthal angles: 1) $\varphi=0$; 2) $\varphi=\pi/2$; 3) $\varphi=\pi$; $\xi^2=0.001$, $\beta=10^{-2}$.

the terms with the δ functions correspond to Cherenkov radiation which does not interfere with transition radiation if measurements are carried out far from the emission region, because the waves travel in different directions. Therefore, only the principal values of the integrals remain in Eq. (10). The matrix element of transition radiation is

$$M_{pp'}=-\frac{e}{(2\omega_f g^l V)^{1/2}}\tilde{M}^{1a}_{pp'}(2\pi)^3\delta(\boldsymbol{\varkappa}+\mathbf{q}_\perp-\mathbf{q}_\perp'-s\mathbf{k}_\perp)\delta(s\omega+q_0-q_0'-\omega_f),$$
$$\tilde{M}^{1a}_{pp'}=\frac{i}{\varepsilon_1^{1/2}}\hat{Q}(\hat{x}^l)\,\mathrm{P}\left(\frac{1}{sk_z+q_z-q_z'+\lambda_1}+\frac{\alpha^l}{sk_z+q_z-q_z'-\lambda_1}-\frac{\beta^l(\varepsilon/\varepsilon_2)^{1/2}}{sk_z+q_z-q_z'+\lambda_2}\right). \quad (38)$$

Equation (38) is derived bearing in mind that all three waves in Fig. 1 have the same polarization. The notation is the same as in Eq. (9).

Following Ref. 1, we use the expression

$$W_s=\iint e^2|\tilde{M}^{1a}_{pp'}|^2(2\pi)^3\delta(\boldsymbol{\varkappa}+\mathbf{q}_\perp-\mathbf{q}_\perp'-s\mathbf{k})\times\delta(s\omega+q_0-q_0'-\omega_f)\frac{d\mathbf{q}'\,d\mathbf{f}}{\beta g^l 2q_0 2q_0' 2\omega_f}$$

to obtain the following formula for the probability of emission of transition radiation per unit time:

$$W=\int d\mathbf{f}\frac{e^2|\tilde{M}^{1a}_{pp'}|^2}{(4\pi)^3 q_0 q_0'\omega_f\beta g^l}\left|\frac{dq_0'}{dq_z'}\right|^{-1}. \quad (39)$$

Here, $\beta=v/c$. It follows from Eq. (7) that

$$\left|\frac{dq_0'}{dq_z'}\right|=\left|1+\frac{p_z-p_0'}{p_0'+e^2a^2\omega(\omega p_z'-k_z p_0')/2(kp')^2}\right|. \quad (40)$$

In Eqs. (39) and (40) we have already used the laws of conservation from which it follows

$$\left.\begin{aligned}&s\omega+q_0-\omega_f=(m^2+(p_\perp')^2+(p_z')^2)^{1/2}-\frac{e^2a^2\omega}{2(\omega(m^2+(p_\perp')^2+(p_z')^2)^{1/2}-\mathbf{k}_\perp\mathbf{p}_\perp'-k_z p_z')},\\&\mathbf{q}_\perp+s\mathbf{k}_\perp-\boldsymbol{\varkappa}=\mathbf{p}_\perp-\frac{e^2a^2\mathbf{k}_\perp}{2[\omega(m^2+(p_\perp')^2+(p_z')^2)^{1/2}-\mathbf{k}_\perp\mathbf{p}_\perp'-k_z p_z')}\end{aligned}\right\} \quad (41)$$

The system (41) should be used to find $\mathbf{p}_\perp'$ and p_z'. It is convenient to introduce the variables $y=p_z'$ and u, which are defined by

$$\left.\begin{aligned}&\mathbf{p}_\perp'=\mathbf{b}+u\mathbf{E},\\&\mathbf{E}=\mathbf{k}_\perp e^2a^2/2,\ \mathbf{b}=\mathbf{q}_\perp+s\mathbf{k}_\perp-\boldsymbol{\varkappa}.\end{aligned}\right\} \quad (42)$$

In terms of new variables, we find that the three equations of the sytem (41) reduce to two equations:

$$\left.\begin{aligned}&a=(m^2+y^2+(\mathbf{b}+u\mathbf{E})^2)^{1/2}-Du,\\&\omega(a+Du)-\mathbf{k}_\perp\mathbf{b}-u\mathbf{E}\mathbf{k}_\perp-k_z y=u^{-1},\\&a=s\omega+q_0-\omega_f,\ D=e^2a^2\omega/2.\end{aligned}\right\} \quad (43)$$

The solution of the system (43) reduces to the solution of a general quartic algebraic equation for u. It is normally best to solve this equation on a computer. Therefore, to obtain a physical picture, we shall consider the simplest geometries which can be treated analytically.

We shall consider a situation in which a laser wave is directed along the interface between the two media ($k_z=0$). In this case the system simplifies drastically and its solution gives

$$\left.\begin{aligned}&p_z'=[p_z^2+(\omega_f^2-\varkappa^2)-2(\omega_f p_0-\boldsymbol{\varkappa}\mathbf{p}_\perp)+2s[\omega(p_0-\omega_f)-\mathbf{k}_\perp(\mathbf{p}_\perp-\boldsymbol{\varkappa})]+\mu(\mathbf{k}_\perp\boldsymbol{\varkappa}-\omega\omega_f)]^{1/2},\\&\mathbf{p}_\perp'=\mathbf{p}_\perp+s\mathbf{k}_\perp-\boldsymbol{\varkappa}-\frac{\mu\mathbf{k}_\perp}{2}\frac{(\omega\omega_f-\mathbf{k}_\perp\boldsymbol{\varkappa})}{\omega(p_0-\omega_f)-\mathbf{k}_\perp(\mathbf{p}_\perp-\boldsymbol{\varkappa})}\end{aligned}\right\} \quad (44)$$

It follows from Eq. (44), which reduces to the formula of Ref. 1 in the absence of a laser field, that a nonrelativistic lepton may generate hard transition radiation if $s\neq 0$.

Circular polarization of the transition radiation is given by the same formulas but now the angles χ and φ are arbitrary, whereas in the case of Cherenkov radiation they are given by Eqs. (17) and (16). It is interesting to note that circular polarization is strong for precisely those values of the initial lepton momentum for which the very probability of emission of transition radiation is high; we shall now consider this probability.

We shall begin with a nonrelativistic lepton. We shall confine our attention to two situations: 1) $k_z=0$; 2) $\mathbf{k}_\perp=0$. We shall consider the case of a weakly relativistic or a nonrelativistic lepton, since the probabilities of the emission of transition radiation are high precisely in those two cases; in the case of an ultrarelativistic lepton the relative changes in the momentum and energy in the field of a laser wave are small, whereas in the absence of this field the probability of emission of transition radiation decreases rapidly on increase in the energy of the radiating particle.

We shall obtain estimates for the case of perpendicular direction of the initial electron momentum ($k_z=0$). In this case the quantity Δ [see Eq. (11)] is given by

$$\Delta^2=(\Delta_{II}-\Delta_I)^2+\frac{\xi^2m^2\varkappa_y^2}{\omega^2p_0^2}, \quad (45)$$

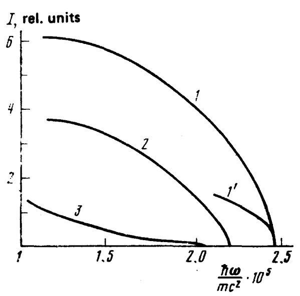

FIG. 3. Dependences of the total radiation intensity on the frequency of the radiation calculated for different azimuthal angles: 1) $\varphi=0$; 2) $\varphi=\pi/2$; 3) $\varphi=\pi$ (1′—corresponds to $\varphi=0$ representing the intensity of the stimulated radiation); $\xi^2=10^{-3}$, $\beta=10^{-2}$.

where

$$\Delta_{\text{I}}=\frac{\xi m}{\omega^2 p_0^2}(\omega|\mathbf{p}|(\omega_f-\varkappa_x)),$$

$$\Delta_{\text{II}}=\frac{\xi m}{\omega^2 p_0^2}\left(\omega\frac{p_0^2}{|\mathbf{p}|}\left(s\omega-\omega_f-\frac{\xi^2m^2}{2p_0^2}(\omega_f-\varkappa_x)\right)\right).$$

If the number of the laser field photons does not change ($s=0$) or if it changes little ($s\sim 1$), the probability of emission of transition radiation is high only if $\Delta^2\lesssim 1$. This condition can be ensured, in particular, when the second term in Eq. (45) is small, i.e., when radiation travels in the $\mathbf{k}\cdot\mathbf{p}$ plane. The smallness of the first term can be ensured by the smallness of Δ_{I} or Δ_{II} or by ensuring that these quantities compensate each other exactly. We shall consider only the case when both of them are small:

$$\varkappa_x\approx\omega_f,\quad s\omega\approx\omega_f.$$

Thus, we can expect needle-shaped distribution of the radiation parallel or antiparallel to the laser wave.

In the case of emission of radiation along any angle, we have to satisfy the condition $\Delta\approx s$ and it then follows from Eq. (45) ($\varkappa_y\neq 0$) that

$$\left.\begin{aligned}\omega_f=s\omega\left[R\pm\frac{p_0}{\xi m}\left(\frac{\mathbf{p}^2}{p_0^2}R^2+\left(\frac{\varkappa_y}{\omega_f}\right)^2\left(1-\frac{\xi^2m^2}{|\mathbf{p}|^2}\right)\right)^{1/2}\right]\Big/\left[\left(\frac{\varkappa_y}{\omega_f}\right)^2\right.\\ \left.+\left(\frac{\mathbf{p}}{p_0}\right)^2R^2\right],\\ R=\frac{p_0^2}{\mathbf{p}^2}+\left(1+\frac{\xi^2m^2}{2\mathbf{p}^2}\right)\left(1-\frac{\varkappa_x}{\omega_f}\right).\end{aligned}\right\}\qquad(46)$$

We can thus see that at high values of s the emitted radiation is always very hard and that a discrete spectrum with a step of the order of the laser frequency is emitted along each angle.

We shall now consider the situation when $\mathbf{k}_\perp=0$. We shall confine ourselves to the case $\mathbf{p}_\perp=0$. Applying the formulas of Eq. (11), we obtain

$$\left.\begin{aligned}\Delta^2=\xi^2\varkappa^2m^2u^2,\\ u^{-1}=\omega\left\{p_0+s\omega-\omega_f+\frac{\xi^2m^2}{2(p_0-p_z)}\right.\\ \left.\pm\left[\left(p_0+s\omega-\omega_f+\frac{\xi^2m^2}{2(p_0-p_z)}\right)^2-\varkappa^2+m(1+\xi^2)\right]^{1/2}\right\}.\end{aligned}\right\}\qquad(47)$$

For low values of s the probability of emission of transition radiation is high only if $\Delta<1$. In the weakly relativistic case it follows from Eq. (46) that the frequency of the transition radiation depends strongly on the angle of emission relative to the normal to the surface:

$$\omega_f\sim\omega/\xi|\sin\chi|.\qquad(48)$$

In the case of high values of $s>1$ and for $\Delta\approx s$, we find that ω_f is given by

$$\omega_f^2\left[\delta^2+\frac{2|s|\omega\delta}{m}+\frac{\omega^2}{m^2}\frac{s^2}{\xi^2}\delta^2\right]-2\omega_f\omega\frac{\delta|s|}{\xi}\left(\frac{p_0}{m}+\frac{s\omega}{m}+\frac{\xi^2m}{2(p_0-p_z)}\right)+\frac{1+\xi^2}{\xi}s^2\omega^2=0,\qquad(49)$$

where

$$\delta=\varkappa/\omega_f.$$

We shall note first that transition radiation does not appear always. If the discriminant of Eq. (49) is negative, this radiation is not emitted. For example, in the case of a nonrelativistic electron and $\xi^2<1$, we find that ($s\omega\sim\xi m$)

$$\omega_f\sim\frac{\xi m}{2|s|\omega[1+(\xi m/\omega s)^2]}[p_0^{1/2}\pm(2s\omega)^{1/2}]^2\qquad(50)$$

and the corresponding condition applicable to the angle is

$$|\sin\chi|<\frac{2\xi m}{\omega|s|\pm 2\xi^2m},\qquad(51)$$

where the plus sign should be taken for $s<0$ and the minus sign for $s>0$. It is clear from Eq. (50) that the maximum frequency corresponds to $s\sim\xi m/\omega$ and we then have $\omega_f\sim m/2$. This high frequency occurs only with the aid of the external laser field.

We shall now consider the probability of emission of transition radiation. Using Eq. (41), we can show that the probability is in almost all cases close to the classical value and only when the emission frequency is given by Eq. (50), can we expect strong amplification

$$dW\sim\frac{1}{137}\frac{1}{(4\pi)^2}\frac{m^2}{\omega^2}s^{8/3},\qquad(52)$$

from which it follows that if $\omega\sim 1$ eV, $\xi^2\sim 10^{-5}$, $\omega_f\sim 1$ keV, and $s\sim 10^4$ the emission probability is $W\sim 10^{-2}$, which is two orders of magnitude higher than the usual classical value for relativistic electrons. [It should be noted that Eq. (52) is obtained for a weakly relativistic electron, but in an external electromagnetic field.]

We can thus see that an external electromagnetic field alters greatly the characteristics of Cherenkov and transition radiations. In the case of Cherenkov radiation there is a change in the angular distribution and an azimuthal asymmetry of the radiation appears even in a homogeneous medium. In addition to the branch which reduces to classical Cherenkov radiation in the absence of an external field, there is a new branch in the presence of this field and it originates from the Compton scattering. The radiation corresponding to both branches is strongly polarized in the case of nonrelativistic leptons. The degree of polarization depends on the angle of emission. The presence of an external electromagnetic field makes it possible to expect the emission of hard (~10 keV) photons at a fairly high rate of $\sim 10^9$ sec^{-1}.

There is a considerable change also in the characteristics of transition radiation in a laser field. Particularly interesting is the appearance of a highly directional fairly hard radiation under multiphoton absorption conditions ($s\gg 1$) with a probability of the order of a percent.

We shall conclude by noting that we have considered a medium transparent to laser radiation and all the effects associated with the influence of the dielectric properties of the medium are assumed to be small. It should be noted that the intensity of the electric field of the laser needed to observe these effects is fairly high: $E\sim 10^6$ V/cm ($\xi^2\sim 10^{-6}$).

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Effect of thermal conditions on the generation threshold of Mössbauer γ radiation in a system of excited nuclei

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We consider the influence of thermal expansion and of the relaxation processes in the establishment of thermodynamic equilibrium on the attainment of the threshold condition for γ amplification in a system of excited short-lived Mössbauer nuclei. It is shown that at an initial crystal temperature $T \gtrsim 10^{-2}$ K the threshold condition is not attainable because of the inhomogeneous Doppler line broadening due to the γ-laser working-medium linear thermal expansion by the heat released in the pumping and by the absorption of the energy of conversion electrons and of the resonant γ quanta. At infralow temperatures $T < 10^{-3}$–10^{-5} K the phonon-electron and phonon-phonon relaxation times can exceed the lifetime of the excited states of the nuclei. As a result, the activation and emission of the nuclei will proceed more rapidly than the heating and linear thermal expansion of the sample, so that the attainment of the threshold condition for γ amplification in a system of polarized Mössbauer nuclei is feasible in principle.

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An exhaustive analysis of various γ-laser models makes it possible to specify concretely the lasing conditions, as well as the parameters of the working isotope and of the pump. This stimulates in final analysis experimental research in this field. In this article we estimate the temperature regime of the operating medium of a γ laser using short-lived Mössbauer isotopes ($\tau \sim 10^{-6} - 10^{-8}$ sec) in the pumping and emission processes, and demonstrate that the rate of change of the temperature with time, $T_t = \partial T/\partial t$ influences substantially the possibility of realizing the laser process.

The dependence of the resonant gain on the temperature was considered earlier only from the point of view of the "static" action of the heating, i.e., of the inhomogeneous and homogeneous temperature broadenings of the spectral lines[1] and of the decrease of the probability f of the Mössbauer effect with increasing temperature.[2] However, the sample temperature varies with time because of the heating of the laser working medium by the pumping as well as by absorption of the energy of the conversion-electron and of the resonant γ quanta upon decay of the excited nuclei. The linear thermal expansion causes a relative motion of the nuclei with velocity

$$v = \gamma l T_t,$$

where $\gamma(T)$ is the thermal-expansion coefficient and l is the distance between nuclei. The corresponding Doppler shift of the frequency leads to an inhomogeneous broadening of the emission line and to a decrease of the gain. Since noticeable lasing is possible only if the active medium is extensive enough, the gain length must be larger than or of the order of the length l_{ph} of the photoabsorption of the resonant γ quanta. If the threshold value of the line width is denoted by Γ^*, then at $l = l_{ph}$ the critical rate of change of the temperature is determined by the condition

$$T_t^* = \Gamma^* c / \nu \gamma(T) l_{ph}, \qquad (1)$$

where c is the speed of light and ν is the frequency of the resonant transition.

In the γ-laser models of Refs. 1–3, appreciable cooling of the working medium is proposed, but since the relaxation processes in a solid slow down with decreasing temperature, and the characteristic lasing time is of the order of the lifetime of the short-lived transitions, we must, before proceeding to estimate the influence of thermal deformations on the possibility of lasing, answer the following question: can heating and thermal expansion of the crystal take place within a time of the order of τ?

In the case of sufficiently pure metals, the principal mechanism whereby the electron-gas energy is transferred to the lattice is the electron-phonon interaction. Heating of the sample by absorption of the energy of conversion electrons and of x-ray and γ quanta is effected entirely via this mechanism. The time of establishment of thermodynamic equilibrium in the system made up of the electron gas and the lattice is characterized by the phonon-electron relaxation time τ_{pe}.[4,5] Since the relaxation of the phonon distribution function to an equilibrium form is determined also by the phonon-phonon relaxation time τ_{pp}, it is obvious that these two parameters are the ones that characterize the time of establishment of an equilibrium crystal temperature.

In the quasiharmonic approximation, the process of

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