$$G_L \sim [\ln(L/\bar{L})]^{1/2} \exp[-L/(\bar{L} \ln^{1/4} L^R)]. \qquad (5.8)$$

Calculating the average size of the molecule in solution using Eqs. (5.7) and (5.8), we obtain a result which agrees with Eq. (5.6).

6. CONCLUSION

We have given a short description of a method which allows us to reduce practically any problem in polymer solution theory to a corresponding magnetic problem, which as a rule has already been solved. In addition to the problems considered in our work, the polymer–magnet analogy may be applied to the description of a mixture of polymers of different composition, and also to solutions of polymers with a non-equilibrium length[4)] distribution[4)] or with a spatially inhomogeneous distribution. Due to lack of space, we could not dwell at length on the description of cyclic polymers, or on mixtures of cyclic and linear polymers.

Finally, we point out an interesting correspondence between the problem of polymer behavior in a limited volume and the problems of magnetism of small particles, and the analogy between surface effects in a solution and in magnets, etc.

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1) Here we introduce the lattice only for simplicity in formulation of the model. We can easily generalize to a continuous model.

2) By the persistent length we mean the distance along a chain, over which the orientation of one link ceases to affect the orientation of another.

3) The result that long rigid moleucles in a fluctuating solution of flexible molecules go over into the globular state has been obtained independently by Grosberg, Erukhimovich, and Shakhnovich.[12] This corresponds to the particular case $c_1 \gg c_2$, as a consequence of which $\tilde{u}_{11}$ and $\tilde{u}_{12}$ are small and are not renormalized; then $u_{11} - \tilde{u}_{12}^2/u_{22}^R < 0$.

4) With the condition, that this distribtuion may be represented as an aggregate of equilibrium distributions. Our method is inapplicable to a monodispersed polymer solution.

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An estimate of the percolation probability in inhomogeneous media

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A universal lower estimate of the probability of percolation is obtained for a random system containing subregions of a conductor and of an insulator. The estimate depends on macroscopic functionals: the effective and mean conductivities of the system, and for anisotropic systems also on the direction of the mean field-intensity vector; that is, on characteristics that can be determined quite simply experimentally.

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We consider a heterogeneous system consisting of subregions occupied by a uniform and isotropic conductor, with conductivity σ_0, and of subregions of zero conductivity (insulators). It is known that with a sufficiently irregular and complicated structure of the medium, there may exist isolated regions of the conductor that do not take part in the transport process. For estimation of the size of such regions, in the theory of percolation[1,2] a quantitative characteristic has been introduced: the percolation probability P, interpreted as the fraction of the conductor volume that takes part in transport. In its essence, the value of P is a char-

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acteristic of the connectivity of the region occupied by the conductor, and in principle it can be determined by purely geometric methods. Kirkpatrick[1] and Shlovskiĭ and Éfros[2] have described examples of calculation of P by procedures of the Monte Carlo type, which carry out a random walk of particles through the volume of the conductor under fixed probability of incidence of a particle on the external boundary of a sufficiently extended region. Besides being very laborious, such a method requires prescription of rather minute characteristics of the internal structure of the random field, which often are unknown in actual situations.

We shall demonstrate the possibility of estimating the percolation probability P from macroscopic characteristics of the transport process that permit direct experimental determination.

Let the local transport equations

$$\mathbf{j}=\sigma\mathbf{h}, \quad \operatorname{div}\mathbf{j}=0, \quad \operatorname{rot}\mathbf{h}=0, \tag{1}$$

be satisfied within the whole region; here $\mathbf{j}$ is the current density, $\mathbf{h}$ is the field intensity, and $\sigma(\mathbf{x})$ is the isotropic random conductivity tensor, which takes the value $\sigma_0 = \text{const}$ with probability p and the value 0 with probability $1-p$.

Let there be prescribed a constant mean field intensity $\mathbf{H}=\langle\mathbf{h}\rangle$, and let the effective conductivity tensor σ_* of an infinite medium be defined by the relations

$$\langle\mathbf{j}\rangle=\mathbf{J}=\sigma_*\mathbf{H}, \quad \operatorname{div}\mathbf{J}=0, \quad \operatorname{rot}\mathbf{H}=0. \tag{2}$$

Here and hereafter, the angular brackets indicate averaging over a volume; in the present case, over the volume of the whole region.

Since in the insulator and in the shielded part of the conductor $\mathbf{j}=0$, we may write

$$Pp\langle\mathbf{j}\rangle_0=\sigma_*\mathbf{H}. \tag{3}$$

In the last formula, the subscript 0 of the averaging sign means that the averaging is carried out only over the connected part of the conductor volume.

Since energy is dissipated only in the connected part of the conductor, the energy-balance equation has the form

$$Pp\langle\mathbf{j}^2\rangle_0=\sigma_0(\mathbf{H}, \sigma_*\mathbf{H}). \tag{4}$$

Equations (3) and (4) are equivalent to the conditions

$$\langle\mathbf{j}\rangle_0^2=(Pp)^{-2}(\sigma_*\mathbf{H}, \sigma_*\mathbf{H}),$$
$$\langle\mathbf{j}^2\rangle_0=(Pp)^{-1}\sigma_0(\mathbf{H}, \sigma_*\mathbf{H}). \tag{5}$$

On taking into account that $\langle\sigma\rangle=\sigma_0 pI$ (here I is the unit tensor) and that the variance of the current in the connected part of the conductor is nonnegative,

$$(D\mathbf{j})_0=\langle\mathbf{j}^2\rangle_0-\langle\mathbf{j}\rangle_0^2\geqslant 0, \tag{6}$$

we get from (5) and (6) an estimate for P:

$$P\geqslant P_*=\frac{(\sigma_*\mathbf{H}, \sigma_*\mathbf{H})}{(\langle\sigma\rangle\mathbf{H}, \sigma_*\mathbf{H})}. \tag{7}$$

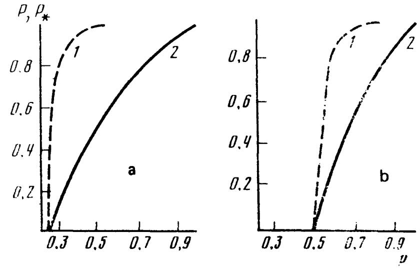

FIG. 1. Percolation probability P (1) and the estimate P_* of it (2) vs. the fraction of conduction links: a) in a simple cubic lattice; b) in a simple square lattice.

If the medium is macroscopically isotropic, we have from (7)

$$P\geqslant\sigma_*/\langle\sigma\rangle. \tag{8}$$

We note that the expression for P_* is universal in the sense that it does not depend on the dimensionality of the field under consideration, but is determined solely by functionals, the effective and mean conductivities, and in the case of anisotropic systems also the direction of the prescribed field intensity H. Since the estimate (7) is realized exactly as an equality in the limiting situation of a layered system, it cannot be improved upon without introduction of additional information about the structure of the medium. It is appropriate to emphasize that the estimate P_* can be found for real systems from an experiment that results in determination of the effective conductivity σ_*.

It is of interest to compare the value of P_* with P as obtained by a mathematical experiment on simple cubic and square lattices.[1] Figure 1 gives the results of such a comparison. In case a, from a regular cubic lattice, $15\times15\times15$, consisting of conducting links, a certain number of them are removed in a random manner. The percolation probability P and the effective conductivity σ_* are calculated as functions of the ratio of the remaining to the original numbers of conductor links. The estimate P_* was obtained by recalculation according to formula (8). Analogous information for the case of a plane square lattice, 50×50, is given in Fig. 1b.

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