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Effect of electron-electron collisions on the energy distribution of excess quasiparticles in superconductors

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A solution is obtained of the kinetic equation for the distribution function of the excess quasiparticles produced by a broad source, with account taken of the electron-electron collisions. It is shown that these collisions do not alter qualitatively the form of the quasiparticle distribution function if the electron-electron interaction constant is not zero.

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INTRODUCTION

It can be regarded as established that many properties of nonequilibrium superconductors with an excess of quasiparticles are determined by the form of the quasiparticle distribution function (QDF), particularly by the character of the dependence of the order parameter Δ on the temperature and on the strength β of the quasiparticle source, by the sign of the current that describes the linear response to the magnetic field, by the stability of the states to inhomogeneous perturbations, and by others. The foregoing effects are exceedingly sensitive to the energy distribution of the nonequilibrium quasiparticles produced by the source (optical pumping or tunnel injection). In fact even small deviations of the QDF from the equilibrium distribution

$$n_T=(e^{\varepsilon/T}+1)^{-1}$$

can radically alter the picture and lead to ambiguous dependences of Δ on T and β, to the appearance of instabilities of different types, and to reversal of the sign of the current in the magnetic field. For a correct description it is therefore necessary to find the QDF from the corresponding kinetic equations obtained by Eliashberg.[1]

For a broad source, such solutions were obtained earlier[2] near the phase transition region, where $\Delta/\Delta_0 \ll 1$(Δ_0 is the order parameter in the absence of pumping), and in the entire interval of Δ/Δ_0.[3] It was shown that $n(\xi)$ is a monotonically decreasing function ($\xi=p^2/2m-E_F$, E_F is the Fermi energy), does not exceed $\frac{1}{2}$, and is localized in an energy interval $\xi\sim\Delta_0$. If $n(\xi)$ is compared with n_T, then a characteristic property is the "superheat" of the energy distribution of the nonequilibrium quasiparticles,[1)] i.e., the decrease of the number of quasiparticles near small ξ, particularly at $\xi=0$:

$$n(\xi=0)<n_T(\xi=0). \tag{1}$$

This decrease is due to the increase of the recombination rate of the quasiparticles at small ξ because of the coherent factors in the kinetic equation, which take into account the coherent character of the interaction of the quasiparticles with the phonons.

Coherent factors play a major role in the considered phenomena, so that it is convenient to separate their contribution to the QDF by changing over to the form[2]

$$n(\xi)=n_0+n_1,\quad \varepsilon=(\xi^2+\Delta^2)^{1/2}, \tag{2}$$

where n_0 satisfies the kinetic equation with zero coherent factors and n_1 is the correction necessitated by the coherent factors. It has been shown[3,6] that the function n_0 reaches a maximum equal to $\frac{1}{2}$ at $\xi=0$:

$$n_0(0)=1/2, \tag{3}$$

after which it decreases monotonically. In the interval $0<\xi<\bar{\xi}$, where $\bar{\xi}/\Delta_0\sim4$, the function n_0 is close to the thermal one, after which it falls off more slowly than n_T.

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The coherent correction n_1, which differs from zero only in the nonequilibrium state, can be represented in the form[6]

$$n_1=\frac{\psi_{ef}}{2a_2},\quad \psi_{ef}=-\frac{\Delta^2 a_1}{\varepsilon\pi}\varphi_{ef},\quad \varphi_{ef}=\pi\int_0^\infty dx\, x\varphi_{ef}(x), \tag{4}$$

$$a_i=\int_0^\infty \xi^i n_0(\xi)\,d\xi,\quad \varphi_{ef}(x)=n_0(x)-N_x(1-2n_0(x)),\quad x=\xi/a_1^{1/2}, \tag{5}$$

where N_x is the distribution function of the nonequilibrium phonons.

The function $\varphi_{ef}(x)$ describes the balance of the processes of recombination and production of quasiparticles in interactions with phonons. At equilibrium (when $n_0=n_T$), φ_{ef} and with it n_1 vanish. For sufficiently thin films, when the phonons go off to the substrate without being reabsorbed, $N_x=0$ and φ_{ef} reach their maximum values.

It must be emphasized that the coherent correction n_1 determines the character of the phase transition. In fact, substituting (2) in the equation for Δ (Ref. 1) we obtain at $T=0$

$$\frac{\beta-\beta_c}{\beta_c}=2\zeta\int_0^{\omega_D}\frac{n_1}{\varepsilon}d\xi,\quad \zeta\sim 1. \tag{6}$$

Analogously, for N_S^0 (Ref. 7) (the number of superconducting electrons)

$$N_S=1+2\int_0^\infty\frac{dn}{d\varepsilon}d\xi,$$

which is proportional to the current in the magnetic field and which determines also the stability of the system to inhomogeneous perturbations,[8] we have

$$N_S=2\int_0^\infty\frac{dn_1}{d\varepsilon}d\xi. \tag{7}$$

The foregoing results, particularly (1)–(4), were obtained under the assumption that the QDF is the result of only the interaction between the quasiparticles and the lattice, while the influence of the electron-electron collisions can be neglected. The last assumption is usually associated with the smallness of the electron-electron interaction constant compared with that of the electron-phonon interaction (we shall designate their ratio by C). It must be noted, however, that there are superconductors with $C>1$; in addition, even at small C the corrections to n on account of the electron-electron interaction can generally speaking play a significant role in view of the unusual sensitivity of the properties of a nonequilibrium superconductor to the form of the QDF.

In this paper we obtain a solution for n with account taken of the electron-electron collisions, and show that the behavior of $n(\xi)$ at small ξ (which plays the principal role in the considered phenomena) remains qualitatively unchanged at nonzero values of the parameter C.

QUASIPARTICLE DISTRIBUTION FUNCTION WITH ALLOWANCE FOR ELECTRON ELECTRON COLLISONS

The quasiparticle distribution function satisfies the following kinetic equation[1]:

$$(1-n(\xi))S^+-n(\xi)S^-=Q(\xi),$$

$$S^\pm=S_{ef}^\pm+CS_{ee}^\pm,\quad S=S^++S^-,\quad \beta=\int_0^\infty Q(\xi)\,d\xi/\Delta_0, \tag{8}$$

where Q is the broad source of quasiparticles, $S_{ef}^\pm$ are the collision integrals of the quasiparticles with the phonons, $S_{ee}^\pm$ are the collision integrals of the particles with one another on account of the electron-electron interaction and are given by

$$S_{ee}^+(n,\Delta,\xi)=\frac{1}{\Delta_0^2}\iiint_0^\infty d\xi_1\,d\xi_2\,d\xi_3\,\{M_1n_1n_2n_3\delta(\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3)$$
$$+3M_2n_1n_2(1-n_3)\delta(\varepsilon-\varepsilon_1-\varepsilon_2+\varepsilon_3)+3M_3n_1(1-n_2)(1-n_3)\delta(\varepsilon-\varepsilon_1+\varepsilon_2+\varepsilon_3)\}, \tag{9}$$

$$S_{ee}^-(n,\Delta,\xi)=\frac{1}{\Delta_0^2}\iiint_0^\infty d\xi_1\,d\xi_2\,d\xi_3\{M_1(1-n_1)(1-n_2)(1-n_3)\delta(\varepsilon-\varepsilon_1-\varepsilon_2-\varepsilon_3)$$
$$+3M_2(1-n_1)(1-n_2)n_3\delta(\varepsilon-\varepsilon_1-\varepsilon_2+\varepsilon_3)+3M_3(1-n_1)n_2n_3\delta(\varepsilon-\varepsilon_1+\varepsilon_2+\varepsilon_3)\}, \tag{10}$$

$$M_1=a\left(1-\frac{\Delta^4}{\varepsilon\varepsilon_1\varepsilon_2\varepsilon_3}\right)-\frac{b\Delta^2}{3\varepsilon}\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}+\frac{1}{\varepsilon_3}-\frac{\varepsilon^2}{\varepsilon_1\varepsilon_2\varepsilon_3}\right),$$
$$M_2=M_1(-\varepsilon_3),\quad M_3=M_1(-\varepsilon_2,-\varepsilon_3),\quad \varepsilon_i\equiv\varepsilon(\xi_i),\quad n_i\equiv n(\xi_i),$$
$$C=\omega_D^2 b/\pi\lambda E_F a_1^{1/2}, \tag{11}$$

where a and b are quantities of the order of unity and depend on the matrix element of the electron-electron interaction; the combinations made up of ε_i and Δ in M_i stem from the coherence of the quasiparticle interaction with one another, λ is the electron-phonon interaction constant, and ω_D is the Debye frequency.

Following our proceeding paper,[2] we seek a solution of (8) in the form (2). For the function n_0 we obtain the equation

$$(1-n_0(\xi))S^+(n_0,0,\xi)-n_0(\xi)S^-(n_0,0,\xi)=0. \tag{11'}$$

We obtain first n_0 for $\xi\to 0$. Taking into account the equality

$$S_{ef}^+(n_0,0,0)=S_{ef}^-(n_0,0,0)$$

(see Ref. 2) we obtain from (11′) the difference

$$2n_0-1=C[S_{ee}^+(n_0,0,0)-S_{ee}^-(n_0,0,0)]/S(n_0,0,0). \tag{12}$$

The interchange of variables $\xi_1\rightleftarrows\xi_3$ in S_{ee}^- leads to the equality $S_{ee}^+=S_{ee}^-$, with the aid of which we obtain from (12)

$$n_0(0)=1/2, \tag{13}$$

i.e., the property (3) remains unchanged for arbitrary C.

It must be noted that were we to have $S_{ee}^-\neq S_{ee}^+$, then even at small C the correction to n_0 could be of importance, say, for the sign of N_S. It can also be shown that $n_0\to 0$, $\xi\to\infty$, and $n_0(\xi)=n_0(\xi/a_1^{1/2})$, therefore the relations (6) and (7) remain in force for all C.

The corresponding equation for the function n_1 can be represented in the form

$$n_1(\xi)S(n_0,0,\xi)+\hat{S}\{n_1\}=\psi_{ef}+C\psi_{ee}, \tag{14}$$

where $\hat{S}\{n_1\}$ is an integral operator that leads to corrections of order Δ^2, while ψ_{ef} and ψ_{ee} are functions that arise because of the coherent factors in $S_{ef}^\pm$ and $S_{ee}^\pm$; ψ_{ef}

is given by (4), and ψ_{ee} is given by the expression (given here for small ξ)

$$\psi_{ee}=\frac{\Delta^2 a_1}{\varepsilon\pi}\varphi_{ee},\quad \varphi_{ee}=\int\int_0^\infty dx_1\,dx_2\frac{x_1^2+x_2^2+x_1x_2}{(x_1+x_2)x_1x_2}\varphi_{ee}(x_1x_2), \tag{15}$$

in which we put $n_0(0)=\frac{1}{2}$,

$$\varphi_{ee}(x_1,x_2)=n(x_1+x_2)(1-n_1)(1-n_2)-n_1n_2(1-n(x_1+x_2)). \tag{16}$$

The function φ_{ee} is proportional to the difference between the rates of the recombination and production of the quasiparticles on account of the electron-electron collisions. It is easy to verify that at equilibrium, when $n_0=n_T$, both φ_{ee} and φ_{ef} vanish. There is however a principal difference between φ_{ef} and φ_{ee}. Its gist is that in thin films, where the number of phonons is small, quasiparticle production by phonons is suppressed, and φ_{ef} reaches values on the order of unity, whereas φ_{ee} always contains a term that describes quasiparticle production, and φ_{ee} is small. Therefore, φ_{ee} is apparently always smaller than φ_{ef}. This circumstance is additionally added by the fact that the functions n_0 are close to n_T in a certain interval $0<\xi<\bar{\xi}$ (see above, as well as Ref. 6). Therefore a contribution to φ_{ee} is made only by large ξ, at which the functions are small.

We now determine n_1. In the approximation linear in Δ we obtain from (14)

$$n_1=\psi/2\tilde{a}_2,\quad \psi=\psi_{ef}+C\psi_{ee},\quad \tilde{a}_2=a_2+CS_{ee}^+(n_0,0,0). \tag{17}$$

At $C=0$, Eq. (17) yields (4), and from (6) and (7) with the aid of (4) we get

$$\left.\frac{d\Delta}{d\beta}\right|_{\Delta\to 0}=\frac{a_2}{\pi a_1\beta_c}>0,\quad N_s=\frac{\Delta\pi a_1}{2a_2}>0. \tag{18}$$

This corresponds to an ambiguous dependence of Δ on β, to a diamagnetic response ($N_S>0$), and to stability against inhomogeneous perturbations ($N_S>0$).

At $C\neq 0$ the sign of the coherent correction is determined by the contributions from the coherent factors due to the electron-phonon and electron-electron interactions

$$\psi=-\frac{\Delta^2 a_1}{\varepsilon\pi}(\varphi_{ef}-C\varphi_{ee}), \tag{19}$$

where the electron-electron contribution is of opposite sign.

At small values of C ($C<1$) the main contribution is made by φ_{ef}, so that we arrive at the previous results with a certain renormalization. If C is large, the situation is more complicated, since this poses the complicated problem of finding n_0 at large C and of calculating φ_{ef} and φ_{ee}. Although estimates based on the arguments advanced above favor predominance of the phonon contribution to the function ψ at all C, one cannot rule out the possibility that ψ can reverse sign at large C. The final answer can be obtained through actual calculations.

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[1)] If the QDF $n(\xi)$ is approximated by a function of the type $n_F=(\exp[(e-\mu^*)/T^*]+1)^{-1}$ and the parameters μ^* and T^* are suitable chosen, then, as shown in Ref. 3 and confirmed in Ref. 5, μ^* is negative and T^* exceeds the critical temperature T_c. In this case $\Delta\neq 0$, and this is why the quasiparticle distribution was called superheated. When the critical strength $\beta=\beta_c$ is reached, so that $\Delta\neq 0$, then μ^* vanishes and T^* conicides with T_c.

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