# Radiowave propagation in a metal plate in a perpendicular magnetic field

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A general method is developed for the investigation of an RF field in a metal plate from whose surface electrons are diffusely reflected. It is shown that the field in the plate, whose thickness greatly exceeds the extremal displacement of the electrons during one cyclotron period, is expressed in terms of the field in the semi-infinite metal. General expressions are obtained for the distribution of the field and of the impedance of the plate in the case of antisymmetrical excitation. These expressions yield the dependence of the slowly varying part of the impedance, of the Doppler-oscillation amplitude, and of the Gantmakher-Kaner oscillation amplitude on the constant magnetic field for both strong and weak fields.

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Interest in how the character of electron reflection from a surface influences the kinetic properties of a metal has greatly increased of late. In particular, many reports have been published of investigations of the penetration of a radio-frequency field through a metallic plate placed in a perpendicular magnetic field. The reason for this interest is that significant differences were observed between specular and diffuse reflection under conditions of Doppler-shifted cyclotron resonance (DSCR). Whereas for specular reflection the problem of propagation of an electromagnetic field in a plate is relatively easy to solve, in the case of diffuse reflection the problem is much more complicated. Many approximate methods of solving this problem in the case of diffuse reflection have been developed in different papers, with contradictory results. This gave rise to lively discussions. The authors of Refs. 1–5 assume that the oscillations of the plate impedance are due to penetration of dopplerons and of Gantmakher-Kaner "waves" present in the infinite metal. On the other hand, the authors of Refs. 6 and 7 state that the major role is played by the "surface-current oscillations" (SCO) connected with the Sondheimer effect and with the presence of surface-conductivity branch points "which are not related to the penetration of an electromagnetic field different from the skin wave into the interior of the metal."[7]

It is shown in the present paper that the RF field in a metallic plate whose thickness exceeds greatly the electron displacement during one cyclotron period can be expressed in elementary fashion in terms of the field in the semi-infinite metal and in terms of its impedance. This problem is solved in general form in §2. It is preceded by §1, devoted to the study of the field in a semi-infinite-metal. Although this problem poses no difficulty in principle and can be solved by the Wiener-Hopf method, we have included §1 for the following reasons. First, it contains the simplest and shortest method of finding the field distribution. Second, it is used to introduce the basic notation and to derive the relations needed in the succeeding section. The general formulas of the theory are used in §3 to derive simple expressions for a number of limiting cases; these expressions describe the most frequently encountered experimental situations. It is also demonstrated that the results of the preceding studies,[1–5] whose validity is more restricted, are particular cases of the formulas of §3. The discrepancies between the different papers are discussed in §4.

## §1. FIELD IN A SEMI-INFINITE METAL

1. In the case of diffuse reflection of electrons from a surface, the field distribution $E(\zeta)$ in a semi-infinite metal, is determined in circular polarizations by the integro-differential equation[8–10]

$$\frac{d^2e(\zeta)}{d\zeta^2}+\int_0^\infty K(\zeta-\zeta')e_0(\zeta')d\zeta'=0 \tag{1}$$

with the boundary conditions

$$e_0(0)=1,\quad e_0(\infty)=0. \tag{1a}$$

Here $e_0(\zeta)=E(\zeta)/E(0)$; $\zeta=2\pi z/u$; $z$ is the coordinate measured along the inward normal to the surface; $u$ is the maximum electron displacement during the cyclotron period,

$$K(\zeta)=\xi s_\pm(\zeta),\quad s_\pm(\zeta)=\mp iH\sigma_\pm(\zeta)/nec,\quad \xi=\omega neu^2/\pi cH, \tag{2}$$

$\sigma_\pm(\zeta)$ is the nonlocal conductivity in the infinite metal for the corresponding circular polarization: $H$ is the constant magnetic field perpendicular to the surface; $\omega$ is the frequency of the electromagnetic wave incident on the metal; $n$ is the density of the conduction electrons. To simplify the notation, we omit here and elsewhere the ± subscripts of the functions $E$, $e_0$, and $K$.

Representing the function $e_0(\zeta)$ in terms of its Fourier transform

$$e_0(\zeta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e_0(q)e^{iq\zeta}dq, \tag{3}$$

and taking the Fourier transform of Eq. (1), we solve the latter by the known Wiener-Hopf method

$$e_0(q)=-\frac{i}{q-i\eta}\tau_2(q), \tag{4a}$$

$$\tau_2(q)=\frac{q^2+\eta^2}{D(q)}\tau_1(q), \tag{4b}$$

$$D(q)=q^2-K(q). \tag{5}$$

$D(q)$ is the left-hand side of the dispersion equation

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$$D(q)=0, \tag{6}$$

that corresponds to Eq. (1), and $K(q)$ is the Fourier transform of $K(\zeta)$ [and is even because $K(\zeta)$ is even]. The functions $\tau_1$ and $\tau_2$ are given by

$$\tau_{1,2}=\exp\left[\frac{1}{2\pi i}\int_{-\infty\mp i\varepsilon}^{\infty\mp i\varepsilon}\frac{dz}{z-q}\ln\frac{D(z)}{z^2+\eta^2}\right], \tag{7}$$

where $|\mathrm{Im}\, q|<\varepsilon<\eta$. We chose in (7) a band of width $2\varepsilon$ containing not a single root of the dispersion equation.

2. We consider a frequency region and magnetic-field region satisfying the inequalities $\omega\ll\nu\ll\omega_c$, where $\omega_c$ is the electron cyclotron frequency and $\nu$ is the frequency of the collisions of the electrons with phonons and impurities. We consider a metal model in which the displacement of the electrons as a function of the longitudinal momentum has one extremum (maximum). If other groups of carriers are present, we assume their displacements to be small compared with the maximum electron displacement $u$ and describe their contribution to the conductivity in the local approximation. In this case the function $K(q)$ has two branch points, and the cuts from them are best drawn as shown in Fig. 1. We put $\gamma=\nu/\omega_c$.

In the upper half-plane of the first sheet, the dispersion equation (5) has in the considered model two roots for minus polarization (we denote them by $q_1$ and $q_2$), and one root $q_1$ for plus polarization. The entire analysis that follows will be for minus polarization, and the corresponding formulas for plus polarization are obtainable in similar fashion.

The functions $\tau_1(q)$ and $\tau_2(q)$ can be continued analytically to the entire $q$ plane. It follows then from (7) and from (4b) that $\tau_1(q)$ is a regular function having no zeros in the upper half-plane, while in the lower half-plane it has zeros at the points $-q_1$ and $-q_2$, a pole at the point $-i\eta$, and a cut from the branch point $q=1-i\gamma$. The function $\tau_2(q)$ is regular and has no zeros in the lower half plane, while in the upper one it has poles at the points $q_1$ and $q_2$, a zero at the point $i\eta$, and a cut from the branch point $q=-1+i\gamma$. We note that from (4) and (7) follows the relation

$$e_0(q)e_0(-q)D(q)=1, \tag{8}$$

where $e_0 q$ is regular in the lower half-plane, and $e_0(-q)$ in the upper. We note also that since $\tau_1$ and $\tau_2$ tend to unity as $q\to\infty$, $e_0(q)$ decreases like $-i/q$ at large $q$.

We deform the contour of integration with respect to $q$ in (3) in the upper half plane. The field $e_0(\zeta)$ is then

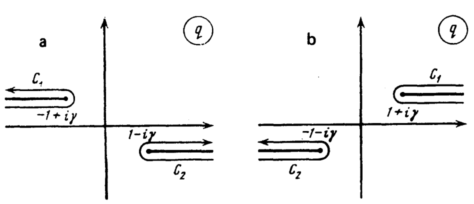


FIG. 1. Positions of branch points of the function $K(q)$ and of the cuts (thick lines) in the complex $q$ plane for minus (a) and plus (b) polarization. The thin lines show the integration contours $C_1$ and $C_2$.

a sum of the contributions from the poles at the points $q_1$ and $q_2$ and from the integral along the edges of the cut:

$$e_0(\zeta)=a_0e^{iq_1\zeta}+b_0e^{iq_2\zeta}+g_0(\zeta), \tag{9}$$

$$a_0=\frac{(q_1+i\eta)\tau_1(q_1)}{D'(q_1)},\quad b_0=\frac{(q_2+i\eta)\tau_1(q_2)}{D'(q_2)},\quad D'(q)=\frac{dD(q)}{dq}, \tag{10}$$

$$g_0(\zeta)=\frac{1}{2\pi i}\int_{C_1}\frac{(q+i\eta)\tau_1(q)}{D(q)}e^{iq\zeta}dq. \tag{11}$$

The integration contour $C_1$ (as well as $C_2$) is shown in Fig. 1.

Formulas (9)–(11) determine the spatial distribution of the field. The expression for the surface impedance of a semi-infinite metal is[9]

$$Z_\infty=\frac{4\pi i q_0}{c e_0'(0)}=\frac{4\pi q_0}{c}\left[\frac{i}{2\pi}\int_{-\infty}^{\infty}dq\ln\frac{D(q)}{q^2}\right]^{-1}, \tag{12}$$

where $q_0=\omega u/2\pi c$.

In what follows, we shall need an asymptotic expression for $e_0$ at $\zeta\gg1$. In this case $g_0(\zeta)$ is of the form

$$g_0(\zeta)=c_0g_{sp}(\zeta),\quad c_0={}^1/_2(-1+i\gamma+i\eta)\tau_1(-1+i\gamma),$$
$$g_{sp}(\zeta)=\frac{1}{\pi i}\int_{C_1}\frac{dq}{D(q)}e^{iq\zeta}, \tag{13}$$

where $g_{sp}(\zeta)$ is the Gantmakher-Kaner component (GKC) for specular reflection of the electrons.

3. We now transform the expression $(q+i\eta)\tau_1(q)$ that determines all three coefficients $a_0$, $b_0$, and $c_0$. We integrate in (7) by parts and deform the integration contour to overlap the cut in the lower half-plane. This produces also contributions from the poles at the points $z=-q_1$, $z=-q_2$, and $z=-i\eta$. As a result we get

$$(q+i\eta)\tau_1(q)=-(q+q_1)(q+q_2)e^{I(q)}, \tag{14}$$

$$I(q)=-\frac{1}{2\pi i}\int_{C_1}\frac{D'(z)}{D(z)}\ln(z-q)dz. \tag{15}$$

The calculation can be continued only if the actual form of the nonlocal conductivity $K(z)$ is known.

If $|D(0)|\ll|(dD/dq^2)_{q=0}|$, the small root $q_1$ is easily obtained from the dispersion equation

$$q_1^2=\frac{1}{\alpha}D(0),\quad \alpha=\left.\frac{dD}{dq^2}\right|_{q=0}.$$

This condition is realized in strong magnetic fields ($\xi\ll1$), and also for a compensated metal in the entire range of fields where $\gamma\ll1$. In this case it is convenient to represent the function under the logarithm sign in expression (7) for $\tau_1(q)$ in the form

$$\frac{D(z)}{z^2+\eta^2}=\frac{D(z)}{D_1(z)}\frac{z^2}{z^2+\eta^2}\frac{D_1(z)}{z^2}, \tag{16}$$

$$D_1(z)=D(z)-D(0). \tag{17}$$

As a result of the inequality $|D(0)|\ll|\alpha|$, the logarithm of the first factor in (16) differs noticeably from zero only at small $z$, and can therefore be replaced under the integral sign by $\ln[(z^2-q_1^2)/z^2]$. This makes it possible to calculate the integral of the first two factors in (16):

$$(q+i\eta)\tau_1(q)=(q+q_1)\exp\left\{\frac{1}{2\pi i}\int_{-\infty-i\varepsilon}^{\infty-i\varepsilon}\frac{dz}{z-q}\ln\frac{D_1(z)}{z^2}\right\}. \tag{18}$$

Since the characteristic region of variation of the function $D_1(z)/z^2$ is of the order of unity, it follows that at small $q$ the integral in (18) is approximately equal to its value at $q=0$, and the latter is given by half the residue at the point $z=0$. Taking this residue and recognizing that $D'(q_1)\approx 2q_1\alpha$, we obtain finally for $a_0$

$$a_0=1/\alpha^{1/2}.$$

To calculate the two other coefficients $b_0$ and $c_0$, we integrate in (18) by parts, as before, and deform the contour:

$$(q+i\eta)\tau_1(q)=-(q+q_1)(q+q_2)e^{I_1(q)}, \tag{19}$$

$$I_1(q)=-\frac{1}{2\pi i}\int\limits_{C_1} dz\ln(z-q)\frac{d}{dz}\left[\ln\frac{D_1(z)}{z^2}\right]. \tag{20}$$

An even greater simplification is possible in the case of strong fields. At small $\xi$ the value of $\alpha$ is close to unity, and consequently $a_0\approx 1$. In addition, the derivative of the logarithm in (20) differs noticeably from zero only in a narrow vicinity of the point $z=1-i\gamma$. We can therefore put $z=1-i\gamma$ in the logarithm $\ln(z-q)$ and take the logarithm outside the integral sign. The remaining integrand is an odd function. Next, since the ratio $D_1(z)/z^2$ decreases more rapidly than $1/z$ at large $z$, the integral along the edges of the cut is equal to half the sum of the residues at the poles of the integrand on the first sheet, multiplied by $2\pi i$. Thus, the expression for $I(q)$ takes the form

$$I_1(q)\approx -\ln(1-i\gamma-q). \tag{21}$$

The existence of a second solution of the dispersion equation (6) is due to the abrupt increase of the nonlocal conductivity $s(q)$ in the vicinity of the DSCR, therefore at small $\xi$ the second (doppleron) root $q^2$ is close to $-1$. It follows therefore that we must put $q=-1$ in (21) and (19) when calculating $b_0$ and $c_0$ determined by expressions (10) and (13). As a result we get

$$b_0=-1/D'(q_2),\quad c_0=-{}^1/_2. \tag{22}$$

Since $q_2$ is close to $-1$, and the function $D(q)$ changes abruptly in this region, $b_0$ is much less than unity.

## §2. FIELD IN PLATE AND IMPEDANCE

1. There have been many theoretical studies of the distribution of an electromagnetic field in a plate under conditions of strong spatial dispersion and of diffuse scattering of the electrons. Noteworthy among them are a cycle of original papers by Baraff[10-12] and an interesting paper by Zherebchevskiĭ, Kaner, and Naberezhnykh.[13] Various general methods of solving the problem were developed in these papers. However, just as most general methods, they have a number of shortcomings. All the cited studies end up with iteration procedures, and it is difficult to identify the physical parameter with respect to which the iteration is carried out (an exception is Ref. 10). In addition, it is extremely difficult to obtain concrete results by the methods developed in Refs. 10–13.

In our preceding papers[1-5] we purposefully confined the analysis to limiting cases, with the natural physical parameters of the problem as the guidelines. The first to be considered was the case of a strong magnetic field, for the following reasons. First, the properties of the penetrating components in strong fields are determined by a small group of resonant electrons and depend on the character of the singularities of the nonlocal conductivity, but not on the details of the Fermi surface. A quantitative comparison of the results of the theory and experiment is therefore possible. Second, in strong field both the physical picture of doppleron excitation in the GKC and the final expressions for the field and impedance distributions turn out to be simple and illustrative. Finally the difference between the radiowave penetration in the case of diffuse and specular electron reflection from the surface manifests itself particularly in strong fields.

Another approximation used in our analysis[2-4] concerned the plate thickness $d$. The value of $d$ was assumed large compared with the attenuation lengths of all the field components, so that multiple reflections of the various components from the plate surface could be neglected.

We obtain below a solution free of the foregoing restrictions on the magnetic field and on the plate thickness. Nonetheless, just as in the preceding studies, we use in the solution of the problem a large physical parameter, $L=2\pi d/u$, the ratio of the plate thickness to the characteristic electron displacement during one cyclotron period. Under these conditions the field in the plate is a superposition of different field components in the semi-infinite metal; this is natural, since the components are formed at distances on the order of $u$ from the surface.

The field distribution in the plate of a metal in which the electron trajectories have no points where the longitudinal electron velocity is reversed, other than the points of reflection from the plate surface, is determined by the following integro-differential equation 11:

$$\frac{d^2\mathscr{E}}{d\zeta^2}+\int\limits_0 K(\zeta-\zeta')\mathscr{E}(\zeta')d\zeta'=0. \tag{23}$$

To simplify the solution of this equation we must use the parameter $L$. It is known that it is much more convenient to use iteration procedures in integral equations than in differential equations. However, since a great variety of integral equations can be based on (23), our task is to choose the equation most suitable for our purpose.

2. Our procedure is the following. Obviously, the general solution of (23) can be represented as a linear superposition of a solution symmetrical about the point $\zeta=L/2$ and an antisymmetrical solution. Since the integral equations for both cases are obtained in similar fashion, we construct here the equation satisfied by the antisymmetrical solution.

We consider the antisymmetrical function

$$e_a(\zeta)=e(\zeta)-e(L-\zeta), \tag{24}$$

where $e(\zeta)$ satisfies an equation defined on the straight line $0\leqslant\zeta<\infty$:

$$\frac{d^2e}{d\zeta^2}+\int\limits_{-\infty}^{\infty} K(\zeta-\zeta')e(\zeta')d\zeta'=0. \tag{25}$$

We emphasize that by definition this equation yields the values of the function $e(\zeta)$ at $\zeta \geq 0$ if the value of $e(\zeta)$ at $\zeta < 0$ is specified by an external condition. Since $K(\zeta)$ is an even function, the function $e_a(\zeta)$ on the segment $0 \leq \zeta \leq L$ satisfies a condition that coincides with (25). We choose the values of the function $e(\zeta)$ at $\zeta < 0$ such that the equation satisfied by $e_a(\zeta)$ goes over into Eq. (23) on the segment $0 \leq \zeta \leq L$. It is necessary for this purpose that the function $e_a(\zeta)$ vanish in the regions $\zeta < 0$ and $\zeta > L$. It is obvious from (24) that if we impose on the function $e(\zeta)$ the condition $e(\zeta < 0) = e(L-\zeta)$, our requirement is satisfied. As a result, Eq. (25) takes the form

$$\frac{d^2e}{d\zeta^2}+\int_0^\infty K(\zeta-\zeta')e(\zeta')d\zeta'=-\int_0^\infty K(\zeta+\zeta')e(L+\zeta')d\zeta'; \qquad (26)$$

in the integral with respect to $\zeta'$ from $-\infty$ to 0 we have made the change of variable $\zeta \to -\zeta'$. After changing the integration variable, Eq. (26) now contains only the values of the function at points $\zeta \geq 0$. Therefore this function can be redefined at $\zeta < 0$. As is customary in the Wiener-Hopf method, we specify it in the form

$$e(\zeta<0)=0. \qquad (27)$$

If we now use a Green's function that satisfies the equation

$$\frac{d^2G(\zeta,\zeta_0)}{d\zeta^2}+\int_0^\infty K(\zeta-\zeta')G(\zeta',\zeta_0)d\zeta'=\delta(\zeta-\zeta_0), \qquad (28)$$

and the boundary conditions

$$G(0,\zeta_0)=0,\quad G(\infty,\zeta_0)=0, \qquad (28a)$$

then Eq. (26) can be rewritten in the form

$$e(\zeta)=e_0(\zeta)-\int_0^\infty d\zeta' G(\zeta,\zeta')\int_0^\infty d\zeta'' K(\zeta'+\zeta'')e(L+\zeta''). \qquad (29)$$

Since the function $e_a(\zeta)$ defined by (24) and (29) is constructed to satisfy Eq. (23) on the segment $0 \leq \zeta \leq L$, it constitutes the antisymmetrical solution of this equation.

We note that the symmetrical solution of (23) satisfies the relations

$$e_s(\zeta)=f(\zeta)+f(L-\zeta),$$
$$f(\zeta)=e_0(\zeta)+\int_0^\infty d\zeta' G(\zeta,\zeta')\int_0^\infty d\zeta'' K(\zeta'+\zeta'')f(L+\zeta''). \qquad (30)$$

3. We solve now Eq. (29). Baraff[14] obtained for the Green's function $G(\zeta, \zeta')$ the expression

$$G(\zeta,\zeta')=\int_0^\infty ds[e_0(s+\zeta)e_0(s+\zeta')-e_0(s)e_0(s+\zeta-\zeta')]. \qquad (31)$$

We substitute (31) in (29) and express $e_0(\zeta)$ and $K(\zeta)$ in terms of their Fourier transforms. Calculating the integrals with respect to $\zeta'$ and $\zeta''$ with allowance for the formula

$$i/(q-q')=\int_0^\infty d\zeta e^{i(q-q')\zeta},\quad \mathrm{Im}(q-q')>0,$$

we represent Eq. (29) in the form

$$e(\zeta)=\frac{1}{2\pi}\int dq' e_0(-q')e^{-iq'\zeta}\Big[1+\frac{1}{(2\pi)^2}\int dq'' \times\int dq\frac{e_0(q'')}{q''-q'}\Big(\frac{1}{q''-q}-\frac{1}{q'-q}\Big)K(q)\varphi(q)\Big],$$

where

$$\varphi(q)=\int_0^\infty e(L+\zeta)e^{-iq\zeta}d\zeta \qquad (32)$$

is regular in the lower half-plane and decreases like $1/q$ as $q \to \infty$ with $\mathrm{Im}q'' > \mathrm{Im}q' > \mathrm{Im}q$.

We deform the contour of integration with respect to $q''$ in the lower half-plane. Since $e_0(q'')$ is regular, the integral with respect to $q''$ is determined only by the pole $q'' = q$. As a result we have

$$e(\zeta)=\frac{1}{2\pi}\int dq' e_0(-q')e^{-iq'\zeta}\Big[1+\frac{1}{2\pi i}\int\frac{dq}{q-q'}K(q)\varphi(q)e_0(q)\Big],$$
$$\mathrm{Im}\,q'>\mathrm{Im}\,q. \qquad (33)$$

We deform now the contour of integration with respect to $q$ to overlap the cut in the lower half plane. Since $\varphi(q)$ and $e_0(q)$ are regular in this half-plane, we can add to $K(q)$ any regular function. It is convenient to replace $K(q)$ by $-D(q)+(q-q')(q-q_c)$ where $q_c$ is any convenient constant. After this substitution we return the contour of the integration with respect to $q'$ to its initial form. We move it next above the contour of integration with respect to $q'$. The resultant contribution from the pole at the point $q = q'$ yields zero in the subsequent integration with respect to $q'$, by virtue of the regularity of $\varphi(q)$ in the lower half-plane and by virtue of (8). As a result we represent (33) in the form

$$e(\zeta)=\frac{1}{2\pi}\int dq' e_0(-q')e^{-iq'\zeta}\Big\{1+\frac{1}{2\pi i}\int dq\frac{\varphi(q)e_0(q)}{q-q'} \times[-D(q)+(q-q')(q-q_c)]\Big\}, \qquad (34)$$

where $\mathrm{Im}\ q > \mathrm{Im}\ q'$.

Since the expression in the curly brackets in (34) is a regular function of $q'$ in the lower half-plane, and the function $e_0(-q')$ has poles at the points $-q_1$ and $-q_2$ and a cut from the point $1-i\gamma$, we can express $e(\zeta)$ in the form

$$e(\zeta)=ae^{iq_1\zeta}+be^{iq_2\zeta}+g(\zeta), \qquad (35)$$

where

$$a=a_0\Big\{1+\frac{1}{2\pi i}\int dq\frac{\varphi(q)e_0(q)}{q+q_1}[-D(q)+(q+q_1)(q-q_c)]\Big\}, \qquad (36a)$$

$$b=b_0\Big\{1+\frac{1}{2\pi i}\int dq\frac{\varphi(q)e_0(q)}{q+q_2}[-D(q)+(q+q_2)(q-q_c)]\Big\}, \qquad (36b)$$

$$g(\zeta)=\frac{1}{2\pi}\int_{C_1} dq' e_0(-q')e^{-iq'\zeta}\Big\{1+\frac{1}{2\pi i}\int dq\frac{\varphi(q)e_0(q)}{q-q'}[-D(q)+(q-q')(q-q_c)]\Big\}. \qquad (36c)$$

Substituting (35) in (32), we obtain

$$\varphi(q)=-\frac{ia}{q-q_1}e^{iq_1L}-\frac{ib}{q-q_2}e^{iq_2L}+h(q), \qquad (37)$$

$$h(q)=\frac{1}{2\pi i}\int_{C_1} dq'\frac{e_0(-q')e^{-iq'L}}{q'+q}\Big\{1+\frac{1}{2\pi i}\int dq''\frac{\varphi(q'')e_0(q'')}{q''-q'}[-D(q'')+(q''-q')(q''-q_c)]\Big\}. \qquad (38)$$

We note that all the transformations above were exact. We make now the first approximation, using the fact that $L$ is a large parameter. If $L \gg 1$, only a small range of values of $q'$, close to unity, is significant in

(38), and we obtain

$$h(q)=-iCg_0(L)/(q+1-i\gamma), \tag{39}$$

$$C=1+\frac{1}{2\pi i}\int dq\frac{\varphi(q)e_0(q)}{q-1+i\gamma}[-D(q)+(q-1+i\gamma)(q-q_c)]. \tag{40}$$

We substitute now (39) in (37) and (37) in (40), and write the expression for $C$ in the form

$$C=1-\lim_{\zeta\to0}\frac{1}{2\pi}\int dq\frac{e_0(q)e^{iq\zeta}}{q-1+i\gamma}\left\{\frac{ae^{iq_1L}}{q-q_1}[-D(q)+(q-1+i\gamma)(q-q_1)]\right.$$
$$+\frac{be^{iq_2L}}{q-q_2}[-D(q)+(q-1+i\gamma)(q-q_2)]$$
$$\left.+\frac{Cg_0(L)}{q+1-i\gamma}[-D(q)+(q-1+i\gamma)(q+1-i\gamma)]\right\}, \tag{41}$$

where the arbitrary constant $q_c$ for each of the terms of the function was chosen in the most convenient fashion, namely $q_1$, $q_2$, and $-1+i\gamma$. In addition, we added under the integral sign an coordinate-dependent exponential so as to be able to deform the integration contour in the upper half plane independently for each term.

Because of our choice of the constants $q_c$, the second terms in the square brackets yield upon integration $e_0(\zeta)$ with different coefficients. Inasmuch as in accord with (8) the product $D(q)e_0(q)$ is regular in the upper half-plane, the integrals of the first terms are determined by the respective zeros of the denominators $(q-q_1)$, $(q-q_2)$, and $(q+1-i\gamma)$. Next, since by definition

$$a_0=i\lim(q-q_1)e_0(q),\quad q\to q_1,$$

we have the relation

$$\lim D(q)e_0(q)=-ia_0D'(q_1),\quad q\to q_1. \tag{42}$$

Similar relations must be used also to calculate the residues at the poles $q_2$ and $(-1+i\gamma)$. As a result, (41) takes the form

$$C=1-ae^{iq_1L}\left[1+\frac{a_0D'(q_1)}{1-i\gamma-q_1}\right]-be^{iq_2L}\left[1+\frac{b_0D'(q_2)}{1-i\gamma-q_2}\right]-Cg_0(L)\left[1+\frac{c_0}{1-i\gamma}\right]. \tag{43}$$

Calculating in the same manner the integrals in (36a) and (36b), we obtain two more equations of the system that determines the coefficients $a$, $b$, and $C$:

$$a=a_0\left\{1-ae^{iq_1L}\left[1-\frac{a_0D'(q_1)}{2q_1}\right]-be^{iq_2L}\left[1-\frac{b_0D'(q_2)}{q_2+q_1}\right]\right.$$
$$\left.-Cg_0(L)\left[1+\frac{2c_0}{1-i\gamma-q_1}\right]\right\}, \tag{44}$$

$$b=b_0\left\{1-ae^{iq_1L}\left[1-\frac{a_0D'(q_1)}{q_1+q_2}\right]-be^{iq_2L}\left[1-\frac{b_0D'(q_2)}{2q_2}\right]\right.$$
$$\left.-Cg_0(L)\left[1+\frac{2c_0}{1-i\gamma-q_2}\right]\right\}. \tag{45}$$

Substituting $a$, $b$, and $C$ in (37) and (39), we obtain $\varphi(q)$. Next, substituting $\varphi(q)$ in (34) and using transformations similar to those in the derivation of (43), we get

$$e(\zeta)=\frac{1}{2\pi}\int dqe_0(q)e^{iq\zeta}\left\{1-ae^{iq_1L}\left[1-\frac{a_0D'(q_1)}{q+q_1}\right]\right.$$
$$\left.-be^{iq_2L}\left[1-\frac{b_0D'(q_2)}{q+q_2}\right]-Cg_0(L)\left[1-\frac{2c_0}{q-1+i\gamma}\right]\right\}. \tag{46}$$

This expression, together with (24), determines the field distribution in a plate under antisymmetrical excitation.

We have thus proved that at $\zeta\gg1$ the coordinate dependences of the field components are the same as for the field components in a semi-infinite metal. In addition, the coefficients of the various components depend on $L$ in accord with the coordinate dependence of the penetrating components.

4. From (46) we can calculate the plate impedance

$$Z_a=\frac{8\pi iq_0}{c}\frac{e_a(0)}{e_a'(0)}=\frac{8\pi iq_0}{c}\frac{e(0)-e(L)}{e'(0)-e'(L)}. \tag{47}$$

By virtue of (29) and of the boundary conditions (1a) we get $e(0)=1$ in (28a). The value of $e(L)$ is determined from (46) at $\zeta=L$. Deforming the contour in the upper half-plane and using the analytic properties of $e_0(q)$, the condition $L\ll1$, and Eqs. (43)–(45), we obtain

$$e(L)=ae^{iq_1L}+be^{iq_2L}+Cg_0(L). \tag{48}$$

To calculate the denominator in (47) we need the derivative $e'(\zeta)$, which is obtained by differentiating (46) directly:

$$e'(\zeta)=\frac{i}{2\pi}\int dq\,e_0(q)e^{iq\zeta}\left\{q[1-ae^{iq_1L}-be^{iq_2L}-Cg_0(L)]\right.$$
$$\left.+q\left[ae^{iq_1L}\frac{a_0D'(q_1)}{q+q_1}+be^{iq_2L}\frac{b_0D'(q_2)}{q+q_2}+Cg_0(L)\frac{2c_0}{q-1+i\gamma}\right]\right\}. \tag{49}$$

The value of $e'(L)$ is calculated in analogy with (48):

$$e'(L)=i[q_1ae^{iq_1L}+q_2be^{iq_2L}-(1-i\gamma)Cg_0(L)]. \tag{50}$$

Using (49), we write the expression for $e'(0)$ in the form

$$e'(0)=\lim_{\zeta\to0}\frac{i}{2\pi}\int dqe_0(q)e^{iq\zeta}\left\{Aq+B\right.$$
$$\left.-\left[ae^{iq_1L}\frac{q_1a_0D'(q_1)}{q+q_1}+be^{iq_2L}\frac{q_2b_0D'(q_2)}{q+q_2}-Cg_0(L)\frac{(1-i\gamma)2c_0}{q-1+i\gamma}\right]\right\} \tag{51}$$

$$A=1-ae^{iq_1L}-be^{iq_2L}-Cg_0(L), \tag{52}$$

$$B=a_0D'(q_1)ae^{iq_1L}+b_0D'(q_2)be^{iq_2L}+2c_0Cg_0(L). \tag{53}$$

The term proportional to $A$ is $Ae'_0(0)$, and the term proportional to $B$ equals $ie_0(0)B$, i.e., $iB$. Next, since the product of $e_0(q)$ by the expression in the square brackets decreases like $1/q^2$ as $q\to\infty$, we can put $\zeta=0$ directly in the remaining integral, and deform the integration contour into the lower half plane. Since the function $e_0(q)$ is regular there, this integral is determined by the poles at the points $-q_1$, $-q_2$, and $(1-i\gamma)$. Using (8) and (42), we obtain ultimately

$$e'(0)=Ae_0'(0)+iB-i[q_1ae^{iq_1L}+q_2be^{iq_2L}-(1-i\gamma)Cg_0(L)]. \tag{54}$$

Substitution of (48), (50), and (54) in (47), with allowance for (52), leads to the following expression for the plate impedance:

$$Z_a=\frac{8\pi iq_0}{c}\frac{A}{Ae_0'(0)+iB}=\frac{8\pi q_0}{c}\left[-ie_0'(0)+\frac{B}{A}\right]^{-1}. \tag{55}$$

Thus, expression (55), together with relations (53), (53), (43)–(45), (19), (13)–(15), and (12), solves the plate-impedance problem in general form, subject to the only assumption $L\gg1$. An analysis of the obtained solution for different limiting cases will be presented in the next section.

We have calculated above the wave distribution and the impedance for antisymmetrical excitation of the plate. Obviously, we can solve (30) in similar fashion and obtain the field in the case of symmetric excitation.

One-sided excitation is described by a superposition of the functions $e_a(\zeta)$ and $e_s(\zeta)$:

$$e_1(\zeta)=\frac{1}{2}\left[\frac{e_a(\zeta)}{e_a'(0)}+\frac{e_s(\zeta)}{e_s'(0)}\right].$$

## §3. PLATE IMPEDANCE. LIMITING CASES.

1. *Parabolic lens method.* We compare first the results obtained by the method developed above with the known solution[1,2] for a compensated metal whose electron Fermi surface takes the form of a parabolic lens, and is hole Fermi surface is a cylinder parallel to the lens axis. In this case the function $D(q)$ for minus polarization is given by

$$D(q)=q^2-\xi\left[\frac{1-i\gamma}{(1-i\gamma)^2-q^2}-1\right]. \tag{56}$$

This function has no branch points, so that the GKC defined by (11) is equal to zero. Next, since $D(z)$ has a pole at the point $z=1-i\gamma$, the function $I(q)$ given by (15) is equal to $-\ln(1-i\gamma-q)$. Substituting this function and (14) in (19) and calculating $D'(q_1)$ and $D'(q_2)$, we find

$$a_0=\frac{1-i\gamma+q_1}{q_1-q_2},\qquad b_0=\frac{1-i\gamma+q_2}{q_2-q_1}. \tag{57}$$

The solution of the system (44)–(45) is

$$a=\left[1-\left(\frac{1}{1-i\gamma+q_1}-\frac{1}{1-i\gamma-q_1}e^{iq_1L}\right)\left(\frac{1}{1-i\gamma+q_2}-\frac{1}{1-i\gamma-q_2}e^{iq_2L}\right)^{-1}\right]^{-1},$$
$$b=\left[1-\left(\frac{1}{1-i\gamma+q_2}-\frac{1}{1-i\gamma-q_2}e^{iq_2L}\right)\left(\frac{1}{1-i\gamma+q_1}-\frac{1}{1-i\gamma-q_1}e^{iq_1L}\right)^{-1}\right]^{-1}. \tag{58}$$

Substitution of (52), (53), (58), and (57) in (55) leads to the expression

$$Z_a=\frac{8\pi q_0}{c}\left[(1-e^{iq_1L})\left(\frac{1}{1-i\gamma+q_1}-\frac{1}{1-i\gamma-q_1}e^{iq_1L}\right)^{-1}\right.$$
$$\left.-(1-e^{iq_2L})\left(\frac{1}{1-i\gamma+q_2}-\frac{1}{1-i\gamma-q_2}e^{iq_2L}\right)^{-1}\right]$$
$$\times\left[q_1(1+e^{iq_1L})\bigg/\left(\frac{1}{1-i\gamma+q_1}-\frac{1}{1-i\gamma-q_1}e^{iq_1L}\right)^{-1}\right.$$
$$\left.-q_2(1+e^{iq_2L})\left(\frac{1}{1-i\gamma+q_2}-\frac{1}{1-i\gamma-q_2}e^{iq_2L}\right)^{-1}\right]^{-1}. \tag{59}$$

This expression for the plate impedance coincides with formula (29) from our preceding paper[2] if we put in the latter $p=0$ (diffuse reflection).

2. *Strong magnetic fields* ($\xi\ll 1$). It was shown in §1 that $a_0\approx 1$ and $b_0\ll 1$. Recognizing that on a metal surface the sum $a0+b0+g0(L)$ of all the field components is equal to unity, we obtain $g_0(L)\ll 1$ [since $g_0(L)\ll g_0(0)$]. This enables us to solve the system (43)–(45) by successive approximations. As a result we get

$$a=1,\quad b=b_0(1-e^{iq_1L}),\quad C=1-e^{iq_1L}. \tag{60}$$

Substituting (52), (53), and (60) in (55) and expanding (55) in small terms proportional to $b_0$ and $g_0(L)$, we obtain

$$Z_a=Z_1^0\left\{1+\frac{cZ_1^0}{8\pi q_0}[b_0e^{iq_2L}+c_0g_{sp}(L)]\right\}, \tag{61}$$

$$Z_1^0=\frac{8\pi q_0}{c}\left[-ie_0'(0)+\frac{2q_1e^{iq_1L}}{1-e^{iq_1L}}\right]^{-1}, \tag{62}$$

where $b_0$ and $c_0$ are determined by (22).

Similar formulas for the impedance of a compensated-metal plate were obtained earlier[5] from simple physical considerations. We note in addition that expressions (61) and (62) go over in the limit of a thick plate into formula (13) of Ref. 2. This formula was used also in Refs. 3 and 4.

3. *Thick plate.* We consider the impedance of a plate whose thickness is much larger than the attenuation lengths of all the field components. In this case it is seen from (52) and (53) that $B\ll 1$, and $A\approx 1$, while expression (55) for $Z_a$ takes the form

$$Z_a=2Z_\infty\left(1-\frac{c}{4\pi q_0}Z_\infty B\right). \tag{63}$$

From the system (43)–(45) it follows that $a=a_0$, $b=b_0$, and $C=1$, therefore

$$B=a_0^2D'(q_1)e^{iq_1L}+b_0^2D'(q_2)e^{iq_2L}+2c_0^2g_{sp}(L). \tag{64}$$

Expressions (63) and (64) are valid in a wide range of magnetic fields and go over in the strong-field limit into formula (13) of Ref. 2.

4. *Case of small oscillations.* We consider now the situation when the impedance oscillations connected with the doppleron root $q_2$ and $Cg_0(L)$ are small either because of the small amplitude or because of the strong damping of the corresponding field components. In this case, just as in the preceding section, we solve the system (43)–(45) by successive approximations, and expand the expression for $Z_a$ in series in $b$ and $C$. This yields

$$Z_a=Z_1-\frac{c}{8\pi q_0}Z_1^2\left\{b_0^2D'(q_2)e^{iq_2L}\left[1+a_0^2D'(q_1)e^{iq_1L}\left(\frac{1}{q_1+q_2}-\frac{1}{2q_1}\right)\right]^2\right.$$
$$\left.+2c_0^2g_{sp}(L)\left[1-a_0^2D'(q_1)e^{iq_1L}\left(\frac{1}{2q_1}+\frac{1}{1-i\gamma-q_1}\right)\right]^2\right\}$$
$$\times\left[1-\frac{a_0^2D'(q_1)}{2q_1}e^{iq_1L}\right]^{-2}, \tag{65}$$

$$Z_1=\frac{8\pi q_0}{c}\left\{-ie_0'(0)+a_0^2D'(q_1)e^{iq_1L}\left[1-\frac{a_0^2D'(q_1)}{2q_1}e^{iq_1L}\right]^{-1}\right\}^{-1}. \tag{66}$$

Expressions (65) and (66) are quite general. They describe the most frequently encountered experimental situations.

In the case

$$|q_1|\ll|q_2|,\quad |q_1|\ll 1 \tag{67}$$

expressions (65) and (66) are noticeably simplified:

$$Z_a=Z_1-\frac{c}{8\pi q_0}Z_1^2[b_0^2D'(q_2)e^{iq_2L}+2c_0^2g_{sp}(L)],\quad Z_1=Z_1^0. \tag{68}$$

The conditions (67) are satisfied, for example, for a compensated metal in the entire range of magnetic fields, with the exception of a narrow vicinity of the doppleron threshold. If, however, the plate thickness is such that the effect of Kao and Fisher[15] is observed in it is in a field noticeably stronger than the doppleron threshold field, then formula (68) is valid in the entire range of fields, including the vicinity of the threshold. In strong fields ($\xi\ll 1$) the inequalities (67) are satisfied also for uncompensated metals; in this case (68), naturally, goes over into (61).

Unfortunately, in more general cases the solution of the system of algebraic equations (43)–(45) is quite cumbersome we were unable to present the expression for the impedance in compact form.

## §4. DISCUSSION

1. In § 2 we solved with great degree of rigor the problem of electromagnetic wave propagation in a metal plate, in a rather general case subject to a single restriction—the displacement of the electrons within one cyclotron period along the magnetic field must be much smaller than the plate thickness. We note that in contrast to the previously proposed general methods, ours is noticeably simple. If the distribution of the field in the semi-infinite metal is known, all that is necessary is to solve the system of linear algebraic equations (43)–(45) for the coefficients $a$, $b$, and $C$ and to substitute them in the expressions for the field and for the impedance. From the structure of Eqs. (43)–(45) it is seen that the three field components—skin, doppleron, and GKC—interact with one another, and it is precisely this interaction which determines the ratio of the amplitudes of the different components.

It is simplest to interpret this interaction in the strong field region $\xi \ll 1$, where the wave vectors of the doppleron and of the GKC are quite large, and the amplitude of the skin-effect component exceeds all others. Let us demonstrate this. When waves of amplitude $\mathscr{E}_0$ are incident on the two sides of the plate with oppositely directed electric fields (antisymmetric excitation), the field distribution in the metal is given by

$$\mathscr{E}_a(\zeta)=2iq_0\mathscr{E}_0\frac{e(\zeta)-e(L-\zeta)}{e'(0)+e'(L)}. \tag{69}$$

Substituting here the expression for $e(\zeta)$ and recognizing that $\xi \ll 1$, we get

$$\mathscr{E}_a(\zeta)=2q_0\mathscr{E}_0\frac{cZ_a}{8\pi q_0}\left[\frac{e^{iq_1\zeta}-e^{iq_1(L-\zeta)}}{1-e^{iq_1L}}\right.$$
$$\left.+b_0(e^{iq_2\zeta}-e^{iq_2(L-\zeta)})+c_0(g_{sp}(\zeta)-g_{sp}(L-\zeta))\right], \tag{70}$$

where $Z_a$ is given by (61) and (62). This expression is valid everywhere except in narrow regions near the plate boundaries. We can similarly obtain for the field in the case of symmetric excitation

$$\mathscr{E}_s(\zeta)=2q_0\mathscr{E}_0\frac{cZ_s}{8\pi q_0}\left[\frac{e^{iq_1\zeta}+e^{iq_1(L-\zeta)}}{1+e^{iq_1L}}\right.$$
$$\left.+b_0(e^{iq_2\zeta}+e^{iq_2(L-\zeta)})+c_0(g_{sp}(\zeta)+g_{sp}(L-\zeta))\right], \tag{71}$$

$$Z_s=\frac{8\pi q_0}{c}\left[-ie_0'(0)-\frac{2q_1e^{iq_1L}}{1+e^{iq_1L}}\right]^{-1}$$
$$\times\left\{1+\left[-ie_0'(0)-\frac{2q_1e^{iq_1L}}{1+e^{iq_1L}}\right]^{-1}[b_0e^{iq_2L}+c_0g_{sp}(L)]\right\}. \tag{72}$$

The field distribution under one-sided excitation is $\mathscr{E}_1=(\mathscr{E}_a(\zeta)+\mathscr{E}_s(\zeta))/2$. Let us consider this distribution for a sufficiently thick plate ($\mathrm{Im}\ q_1L > \mathrm{Im}\ q_2L > 1$):

$$\mathscr{E}_1(\zeta)=2q_0\mathscr{E}_0\{[e^{iq_1\zeta}+b_0e^{iq_2\zeta}+c_0g_{sp}(\zeta)][-ie_0'(0)]^{-1}-[b_0e^{iq_2L}$$
$$+c_0g_{sp}(L)][e^{iq_1(L-\zeta)}+b_0e^{iq_2(L-\zeta)}+c_0g_{sp}(L-\zeta)][-ie_0'(0)]^{-2}\}. \tag{73}$$

Expression (73) allows us to draw the following conclusion. A doppleron (as well as a GKC) excited on the left side of the plate, when reflected from the right side, generates a skin-effect component that oscillates at the same phase as the doppleron and has an amplitude much larger than the field of the passing doppleron. It is the field of just this skin-effect component which is registered in experiment. The amplitude of the doppleron oscillations is therefore considerably enhanced by diffuse reflection of the carriers, compared with the specular reflection, and the enhancement increases with increasing magnetic field. We emphasize that, as seen from (73) the enhancement due to the skin-effect component takes place to the same degree in the case of GKC reflection. Similar conclusions can be drawn from (70)–(72) for symmetric and antisymmetric excitation. In Refs. 1 and 2 similar conclusions were drawn starting from a simpler analysis based on the fact that the propagating doppleron and GKC interact in strong fields only with electrons moving in the same directions as the field component, and do not interact with oppositely directed electrons. It is precisely this circumstance which leads to a radical difference between the specular and diffuse reflections of these components.

2. The approach proposed in Refs. 1–3 to the solution of the problem of plate excitation was later criticized in Ref. 6, where it was stated that no consistent account of the plate boundaries was taken in Refs. 1–3, and as a result the two new oscillatory effects were not included. The formulation of the problem in the present paper is no less rigorous than the method proposed in Ref. 13. At the same time the rigor and simplicity of our calculations are greatly superior to the calculation method developed in Ref. 6 on the basis of the method of Ref. 13. Nonetheless, our present results agree with those of Refs. 1–3, but not with those of Ref. 6.

Let us compare in greater detail the results of Refs. 3 and 6, disregarding the terminology employed. In Ref. 3 we considered only pure diffuse and pure specular reflection of the carriers, whereas in Refs. 6 and 7 the character of the reflection was arbitrary. We therefore set the specularity coefficient $p$ in the expressions of Ref. 6 equal to zero, and compare the resultant expressions with the corresponding expressions of Ref. 3 for the diffuse case. No consideration was given in Ref. 3 to the range of fields in which $\xi \sim 1$, while in Ref. 6 only estimates are given for this region. The results of Ref. 6 in the field region $L\xi^2 \ll 1$ agree with those of the much earlier study.[3] In the intermediate field region $1/L \ll \xi^2 \ll 1$ the results of Refs. 6 and 3 are strikingly different. The absence of symmetry in expressions (3.6) and (3.7) of Ref. 6 and the absence of a smooth matching of the expressions (3.6), (3.7) and (3.5) in the field region $L\xi^2 \sim 1$ suggest that errors crept in when the rather cumbersome method of Ref. 13 was used in Ref. 6. A careful study of Ref. 6 shows that the roots of the dispersion equation were not correctly located there in the complex plane. This led apparently to incorrect signs in a large number of expresssions, and ultimately to errors in many of the results of Refs. 6 and 7, in particular, to an incorrect magnetic-field dependence of expressions (3.6) and (3.7) of Ref. 6. At the same time, if the method proposed in Refs. 1 and 2 is used then, knowing the field distribution in a semi-infinite metal, the results of Refs. 6 and 7 for pure diffuse and pure specular reflections in the case of antisymmetric, symmetric, and one-sided excitation can be derived on a single page without any difficulty whatever.

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# Precise measurements of NMR shifts

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A new method for precise measurement of NMR shifts is developed. By stabilizing the specimen temperature against an NMR signal having a temperature sensitive shift (this made it possible to keep the specimen temperature constant to within about 0.01 °C), controlling the stability of the NMR line shape, and regulating the frequency of the modulating oscillator using an automatic frequency control circuit with input from the NMR signal, we achieved an accuracy of $8\times10^{-5}$ ppm (0.006 Hz for a spectrometer working frequency of 80 MHz) in measuring shifts for specimens having an external reference standard, and of $2\times10^{-5}$ ppm (0.0015 Hz) for specimens having an internal reference standard. This accuracy exceeds the resolving power of commercial NMR spectrometers by a factor of about 100. Such accuracy in measuring shifts makes it possible to measure magnetic susceptibilities of substances in solution with a sensitivity 100 times that achievable with the known Evans NMR method, to record contact and pseudocontact shifts that are strongly masked by exchange processes, and to investigate weak temperature dependences of chemical shifts and spin-spin interaction constants. This opens up new prospects for investigating the structures of metal-containing macromolecules (enzymes, nucleic acids, etc.) and coordination compounds, and intermolecular and intramolecular interactions. Apparatus for precise measurement of NMR shifts is described.

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In high resolution nuclear magnetic resonance, a considerable part of the information on the structure of the investigated compound is extracted from the shifts of the resonance signals. By the shift we mean the relative position of the resonance line with respect to that of the reference signal on the frequency axis of a plot of the NMR spectrum.

The reason for differences in the values of the NMR shifts for nuclei of the same isotope may be differences in the chemical structure of the investigated substances (the so-called chemical shift), differences in the natures of the van der Waals interactions of the investigated molecules with one another and with solute molecules, and contact and dipole–dipole interactions of the resonating nuclei with paramagnetic centers (contact and pseudocontact shifts). When the reference substance is contained in an isolated microampoule (a so-called "external reference") the geometry of the specimen and the values of the bulk static magnetic susceptibilities of the principal and reference solutions strongly affect the measured shifts.[1,2] The picture is sometimes complicated by chemical exchange of the resonating nuclei between magnetically inequivalent states.[1,3,4] In accordance with what was said above, measurements of NMR signals are used to identify chemical compounds and to establish structural formulas of substances in organic chemistry, to investigate the conformation of complex molecules, to investigate structures of coordination compounds and the dynamics of chemical exchange, and to measure the static magnetic susceptibilities of substances in solution (the Evans method[2]).

The shifts are usually measured in relative units—in parts per million (ppm). The magnitude of the shift in ppm is calculated from the formula

$$\tau=10^6\Delta f/f_0, \tag{1}$$

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