and γ_2.

The principal spectral characteristic in each magnetic band with number j is the density of the number of states per unit area in the interval dE, which we designate by $\mu_j(E)dE$, where

$$\int\left(\sum_j \mu_j\right)dE=\frac{e\Phi}{2\pi|K|}=\frac{1}{4\pi^2}|K'|M.$$

Here M is the multiplicity of the degeneracy at fixed p_1 and p_2. Irrational numbers $(2\pi)^{-1}e\Phi$ can be approximated by rational ones $N_iM_i^{-1}$, where the numerator N_i and the denominator M_i increase if $N_iM_i^{-1}\to(2\pi)^{-1}e\Phi$, $i\to\infty$. It is easily seen that with increasing N_i and M_i the dispersion law breaks up into more and more magnetic bands, so that all the characteristics, with the exception of the total density of the number of states per unit area

$$\sum_j \mu_j\,dE,$$

become meaningless.

At large N it is possible to pose the natural problem of calculating the statistical weights of various numbers of magnetic bands produced in the decay, bearing in mind the percentage of the configurations γ_1 and γ_2 with the number of bands in the given interval $k\pm\Delta k$ (i.e., the common cycles of a pair of commuting permutations) relative to the total number of configurations, i.e., classes of conjugacy of the commuting permutations. This is a purely combinatorial problem.

[1)]The reader must be warned that the paper by Aharonov and Casher[3] contains wrong references to the known work of Atiyah and Singer. Fortunately, these references have no bearing on the matter.

[2)]The quadrature non-integrability of one of the eigenfunctions in Ref. 3 points to the absence of a gap in this case.

[1]J. Zak, Phys. Rev. **136A**, 776 (1964).
[2]M. F. Atiyah and I. M. Singer, Bull. Am. Math. Soc. **69**, 422 (1963).
[3]Y. Aharonov and A. Casher, Phys. Rev. A **19**, 2461 (1979).
[4]J. Zak, in: Solid State Physics, H. Eherenreich, ed. Academic, 1972, Vol. 17.
[5]J. Zak, Phys. Rev. A **16**, 4154 (1977).
[6]A. Erdelyi, ed. Higher Transcendental Functions, Vol. 3, Chap. 13, McGraw, 1967.
[7]L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Chap. 15, Nauka, 1974 (Pergamon).
[8]B. A. Durbovin, S. P. Novikov, and A. T. Fomenko, Sovremennaya geometriya (Modern Geometry); part 2, Chap. 5, Nauka, 1979.

Translated by J. G. Adashko

Nonlinearity of current-voltage characteristic and magnetoresistance of the interface between the superconducting and normal phases

B. I. Ivlev, N. B. Konin, and C. J. Pethick[1)]

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

(Submitted 5 March 1980)
Zh. Eksp. Teor. Fiz. **79**, 1017–1024 (September 1980)

We consider the dc resistance of the interface between the normal and superconducting phases at current densities of the order of the Ginzburg-Landau critical value. Account is taken of the effect of an external constant magnetic field on the interface resistance.

PACS numbers: 74.30.Ci

1. Measurements of the resistance of superconductors in the intermediate state have shown that it exceeds the resistance of the purely normal phase.[1,2] It was subsequently established that the additional resistance is due to the finite depth of penetration of the electric field, l_E, into the superconductor. This depth greatly exceeds the coherence length $\xi(T)$ and the depth $\lambda(T)$ of penetration of the electric field in the super conductor, and is due to processes that are peculiar to superconductors and are connected with the relaxation of the unbalance of the populations of electron-like and hole-like branches of the excitation energy spectrum.[3] The population difference between the electron and hole branches of the spectrum leads to a difference between the chemical potential of the pairs, $\mu_p=(1/2)\partial\chi/\partial t$ (per particle) from the chemical potential $\mu_e=-e\varphi$ of the quasiparticles (χ is the phase shift of the order parameter and φ is the scalar potential of the electric field. As a result, the gauge-invariant scalar potential

$$\Phi=\varphi+\frac{1}{2e}\frac{\partial\chi}{\partial t}$$

differs from zero. The characteristic length over which the difference between the populations of the spectrum branches in the interior of the superconductor relaxes is in fact the depth of penetration of the electric field. Near the critical temperature, the depth

[1)]NORDITA, Denmark.

0038-5646/80/090516-05$02.40

of penetration of the electric field exceeds the diffusion length $l_\varepsilon=(D/\gamma)^{1/2}$ of the quasiparticles (D is the diffusion coefficient and $\gamma\sim T^3/\Theta_D^2$ is the reciprocal time of the electron-phonon collisions). As a result, at lengths on the order of l_ε, there is established an energy dependence of the distribution function, characterized by a chemical potential $-e\Phi$ that relaxes slowly in turn over lengths on the order of $l_E \gg l_\varepsilon$.

In the case of weak currents and magnetic fields, the depth of penetration at $T_c-T\ll T_c$ is $l_E^{(0)}=l\,(4T/\pi\Delta)^{1/2}$ (Refs. 4 and 5) and is determined by the electron-phonon interaction. The electron-phonon interaction, however, is not the only mechanism of relaxation of the electron-hole unbalance. The relaxation can be accelerated by any factor that leads to pair breaking and facilitates particle exchange between the excitations and the condensate. The role of the paramagnetic impurities in this case is well known[4,6] (see, e.g., the equation for Φ in Ref. 6). An equation for that part of the distribution function which is even in energy and is connected with the unbalance of the electron and hole branches was obtained earlier.[7,8] This equation offers evidence that an important role can be played here by the vector potential of the electromagnetic field. An equation that describes the behavior of the scalar electric potential with allowance for this circumstance was obtained near the critical temperature in Ref. 9. In a certain temperature range that will be defined below the vector potential can substantially influence the electric-field penetration depth, which can therefore depend strongly on the superconducting current as well as on the external magnetic field. It is important to emphasize that this influence begins to manifest itself at magnetic field and current values substantially lower than the corresponding critical values. The additional resistance introduced by the superconducting phase is proportional to the resistivity of the normal metal multiplied by the depth of penetration of the electric field. The indicated effects should lead thus to nonlinearities of the current-voltage characteristic and to a strong dependence of the resistance of the interface between the normal and superconducting phases (S–N interface) on the applied magnetic field. A definite idea of the influence of the magnetic field and of the current on the rate of relaxation of the electron-hole unbalance and on the depth of penetration of the electric field can be obtained by using the results of Refs. 4 and 5 for alloys with paramagnetic impurities, where the role of the pair-breaking factor is assumed by the reciprocal time τ_s^{-1} of the electron mean free path with respect to spin flip. By replacing, in order of magnitude, τ_s^{-1} by $c^{-2}e^2DQ^2$, as was done in Refs. 4 and 10, it is possible to understand qualitatively the main features of the behavior of the depth of penetration. In this paper these effects are considered consistently on the basis of the exact equations of Ref. 9.

In addition to the finite depth of penetration, the S–N interface is characterized also by a jump (i.e., by a rapid change in scales of the order of ξ and l_ε) of the electric field. This question has been studied earlier.[4,11] In the temperature range considered below, however, the jump of the field is small and will not be taken into account.

We assume for simplicity that the sample is a cylinder of radius $r_0\ll\xi,\lambda$ and is made of dirty superconductor with $l_{\rm imp}\ll\xi(T)$. It is assumed that the sample is divided into a superconducting and a normal region by an interface with a thickness of the order of $\xi(T)\ll l_E$ which can therefore be regarded as infinitely thin. Such a sample can be placed in a longitudinal magnetic field **H**.

2. In the stationary state, the superconductor is described by the following equations, which are valid at temperatures close to critical:

$$\frac{T_c-T}{T_c}\Delta-\frac{7\zeta(3)}{8\pi^2T^2}\Delta^3-\frac{\pi D}{2T}\left(\frac{e}{c}\mathbf{Q}\right)^2\Delta+\frac{\pi D}{8T}\nabla^2\Delta=0, \tag{1}$$

$$\mathbf{j}=-\sigma\nabla\Phi-\frac{\pi\sigma\Delta^2}{2cT}\mathbf{Q}, \tag{2}$$

where Δ is the modulus of the order parameter, while $\mathbf{Q}=\mathbf{A}-(c/2e)\nabla\chi$ and Φ are respectively the gauge invariant vector and scalar potentials. The vector potential Q satisfies Maxwell's equation

$$\operatorname{rot}\operatorname{rot}\mathbf{Q}=4\pi\mathbf{j}/c,$$

and the distribution of the scalar potential Φ should be found with the aid of the kinetic equation. The electroneutrality condition connects the scalar potential Φ with that part of the distribution function which is even in energy, $\alpha=n_\varepsilon+n_{-\varepsilon}-1$, which describes the unbalance of the electron and hole branches:

$$2e\Phi=\int_{-\infty}^{\infty}\alpha_\varepsilon\,d\varepsilon.$$

In turn, α_ε satisfies the kinetic equation[4,7-9]

$$D\nabla^2\alpha_\varepsilon-D\left[\left(\frac{\nabla\Delta}{\varepsilon}\right)^2u_\varepsilon^4+\left(\frac{2e}{c}\mathbf{Q}\right)^2\frac{\Delta^2}{\varepsilon^2}u_\varepsilon^2\right]\alpha_\varepsilon=J_{ph}, \tag{3}$$

where $u_\varepsilon=\varepsilon/(\varepsilon^2-\Delta^2)^{1/2}$ and $J_{\rm ph}$ is the integral of the collision with the phonons, whose explicit form will not be written out here (see, e.g., Ref. 9). The term with $(\nabla\Delta)^2$ in the left-hand side of (3) is significant only at distances on the order of ξ from the S–N interface and leads to the appearance of a jump in the electric field.[4] From the results of Artemenko, Volkov, and Zaĭtsev[1] it is seen that in the temperature region $(T_c-T)/T\ll(\gamma/T)^{1/2}$ the jump of the electric field is small. We shall use below for Φ an equation that is valid precisely in this temperature range, so that the term with $(\nabla\Delta)^2$ in Eq. (3) can be neglected.

We align the z axis with the cylinder axis. If $\mathbf{H}\parallel z$, then the components of the vector potential $\mathbf{Q}=\{Q_\rho,Q_\varphi,Q_z\}$ take the form $Q_\varphi=\frac{1}{2}H\rho$, $Q_\rho=0$. Since the cylinder radius $r_0\ll\xi,\delta$, it follows that Q_z and Δ are practically independent of ρ. To obtain the equations which these quantities satisfy, we must average Eqs. (1)–(3) over the thickness of the sample. The quantity Q^2 in Eqs. (1) and (3) is then replaced by

$$\overline{Q^2}=\frac{1}{\pi r_0^2}\int_0^{r_0}Q_\varphi^2\,dS+Q_z^2=\frac{H^2r_0^2}{8}+Q_z^2.$$

Equation (3) takes the form

$$D\frac{\partial^2\alpha_\varepsilon}{\partial z^2}-D\frac{4e^2}{c^2}\overline{Q^2}\frac{\Delta^2}{\varepsilon^2}u_\varepsilon^2\alpha_\varepsilon=J_{ph}.$$

As shown in Ref. 9, the function α can now be sought in the form

$$\alpha_\varepsilon=\frac{e\Phi}{2T}\left[u_\varepsilon\,\mathrm{ch}^2\left(\frac{\varepsilon}{2T}\right)\left(1+\frac{4e^2}{c^2}\frac{D\overline{Q^2}}{\gamma}\frac{\Delta^2}{\varepsilon^2}u_\varepsilon\right)\right]^{-1},$$

and for Φ we obtain the equation

$$D\frac{\partial^2\Phi}{\partial z^2}-\frac{\pi\Delta}{4T}\gamma(1+I)\Phi=0, \tag{4}$$

where

$$I(p)=\frac{2p}{\pi}\int_1^\infty\frac{(x^2-1)^{1/2}\,dx}{x[x(x^2-1)^{1/2}+p]}=\begin{cases}2p/\pi, & p\ll 1,\\ p^{1/2}, & 1\ll p\ll T^2/\Delta^2,\end{cases} \tag{5}$$

and $p=4e^2D\overline{Q}^2/\gamma c^2$. At small $\overline{Q}^2$, when $p\ll 1$, we arrive at the result of Atremenko *et al.*[4] Equation (4) is valid in the temperature range

$$(\gamma/T)^2\ll(T_c-T)/T_c\ll(\gamma/T)^{1/2}.$$

The depth of penetration of the electric field into the superconductor, as follows from (4), is proportional to $l_\varepsilon[4T/\pi\Delta(1+I)]^{1/2}$ and depends strongly on the magnetic field and on the current.

We change over to the following dimensionless variables:

$$z=\xi z',\quad \Delta=\bar\Delta\Delta',\quad Q=\frac{c}{2e\xi}Q',\quad H=2^{1/2}\varkappa H_cH',$$

$$\Phi=\frac{\pi\bar\Delta^2}{4eT}\Phi',\quad j=\frac{\pi\sigma\bar\Delta^2}{4eT\xi}j',$$

where

$$\xi=\frac{\pi D}{8(T_c-T)},\quad \bar\Delta^2=\frac{8\pi^2T^2}{7\zeta(3)}\frac{T_c-T}{T_c},$$

and $\varkappa$ is the Ginzburg-Landau parameter. Equations (1), (2), and (4) then take the form

$$\Delta^2+\overline{Q^2}=1, \tag{6}$$

$$j=-\partial\Phi/\partial z-\Delta^2Q_z,\quad j=\mathrm{const}, \tag{7}$$

$$\partial^2\Phi/\partial z^2=l_b^{-2}\Phi. \tag{8}$$

We have left out here the primes of all the new variables, and in (6) we have also left out the term with the second derivative of Δ, since the characteristic scale of variation of all the quantities is $l_b\gg 1$, where

$$l_b^2=a/(1+I)\Delta,\quad a=(l_E^{(0)}/\xi)^2=32T(T_c-T)/\pi\bar\Delta\gamma. \tag{9}$$

The quantity $I(p)$ is determined by Eq. (5), in which now

$$p=\beta\overline{Q}^2=\beta(1-\Delta^2),\quad \beta=8(T_c-T)/\pi\gamma.$$

We put $j_s=-\Delta^2Q_z$, and then we have from (6)–(8)

$$\partial j_s/\partial z=l_b^{-2}\Phi,\quad j=j_s-\partial\Phi/\partial z,\quad \Delta^2+j_s^2/\Delta^4=\eta^2, \tag{10}$$

where $\eta^2=1-H^2/H_0^2$, and $H_0=8^{1/2}/r_0$ is the critical field for a cylinder with a small radius $r_0\ll\lambda$ (in ordinary units we have $H_0=4\lambda H_c/r_0$).

In the interior of the superconducting region (see Fig. 1) the normal current at $z\gg l_b$ is $j_n=-\partial\Phi/\partial z=0$, therefore

$$j_s=j\le j_c(H),\quad \Delta=\Delta_0,\quad \Phi=0 \tag{11}$$

[the critical current is $j_c(H)=2\eta^3/3\cdot 3^{1/2}$]. The quantities j and Δ_0 are connected by the third equation of (10). As the S–N interface, where $z=0$, is approach-

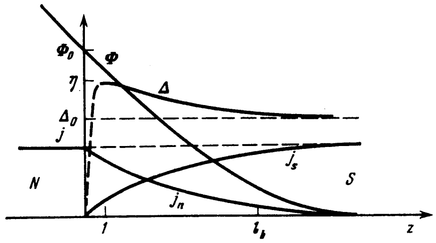

FIG. 1.

ed, the normal current increases, the superconducting current decreases, and the order parameter increases. Over distances $1\ll z\ll l_b$ we have, by virtue of the continuity of E (the jump of E is small), $j_n=j$ and

$$j_s=0,\quad \Delta=\eta,\quad \Phi=\Phi_0. \tag{12}$$

At still smaller distances from the interface ($z\sim 1$), Δ begins to decrease. When the problem is considered at scales on the order of l_b, we can use (11) and (12) as the boundary conditions at $z=\infty$ and $z=0$, respectively.

Equations (10) with boundary conditions (11) and (12) can be easily integrated:

$$\Phi_0^2-\Phi^2=2\int_0^{j_s}l_b^2(j-j_s)\,dj_s, \tag{13}$$

$$z=\int_0^{j_s}l_b^2\frac{dj_s}{\Phi}. \tag{14}$$

The total potential drop Φ_0 across the S–N interface is obtained from (13), in which we must put $\Phi=0$ and $j_s=j$. Changing over to integration with respect to $d\Delta$, we get

$$\Phi_0^2=2a\int_{\Delta_0}^{\eta}\frac{[j-\Delta^2(\eta^2-\Delta^2)^{1/2}](3\Delta^2-2\eta^2)}{[1+I(p)](\eta^2-\Delta^2)^{1/2}}\,d\Delta. \tag{15}$$

The quantity Δ_0 is connected with the current j by the relation $j=\Delta_0^2(\eta^2-\Delta_0^2)^{1/2}$ and lies in the range $(\frac{2}{3})^{1/2}\eta\le\Delta_0\le\eta$.

3. Expression (15) describes completely the characteristics of the S–N interface at the employed values of the magnetic fields and currents, but the explicit form is quite complicated, so that it is useful to consider several limiting cases.

A. Assume first no magnetic field, $H=0$ and $\eta=1$.

a) We consider the temperature region $T_c-T\ll\gamma$, where $\beta\ll 1$. Here $I\ll 1$ and the integral in (15) can be easily calculated:

$$\Phi_0^2=aj^2f(\Delta_0),$$

where

$$f(x)=\frac{2}{x^4(1-x^2)}\left\{\frac{3}{2}x^3(1-x^2)-\frac{1}{2}x^2(1-x^2)^{1/2}\arcsin[(1-x^2)^{1/2}]+\frac{2}{3}(1-x^3)-\frac{3}{5}(1-x^5)\right\}. \tag{16}$$

At low currents, Δ_0 is close to 1 and from (16) we have $f(\Delta_0)\approx 1$,

$$\Phi_0=a^{1/2}j \tag{17}$$

meaning in ordinary units a voltage $V=jR_0$ across the S–N interface, whose resistance is then $R_0=l_E^{(0)}/\sigma$, where $l_E^{(0)}=l\,(4T/\pi\Delta)^{1/2}$. This agrees with the result of Ref. 5.

The function (16) for large currents was tabulated numerically. It turned out that it is approximated with good accuracy by unity, so that the function (17) retains the same form all the way to the critical current. This circumstance is quite unexpected, inasmuch as when the current is increased to the critical value $j_c=2/3\sqrt{3}$ the order parameter Δ_0 in the interior of the superconducting region is suppressed to a value $\Delta_0=\sqrt{2/3}$, and this should lead to an increase of the depth of penetration of the electric field and to an increase of the resistance. For purely numerical reasons this effect, however, is small, and l_E is practically constant. The linear dependence of the voltage V on the current was confirmed in experiment[12] up to critical values of the latter.[12]

b) We turn now to the temperature range

$$\gamma/T\ll(T_c-T)/T_c\ll(\gamma/T)^{1/2},$$

where $\beta\gg1$. We consider currents that are not very weak, so that $p=\beta(1-\Delta^2)\gg1$, and $I=p^{1/2}$. From (15) we obtain with logarithmic accuracy

$$\Phi_0=j^{1/2}\left\{\frac{a}{\beta^{1/2}}\ln(j^2\beta)\right\}^{1/2}. \tag{18}$$

In ordinary units we have for the voltage on the $S-N$ interface

$$V=\frac{\Delta_0^2}{2eT}\left(\frac{j}{j_c}\right)^{1/2}\left(\frac{28\zeta(3)}{27\pi}\right)^{1/4}\left(\frac{T}{\gamma}\right)^{1/4}\ln^{1/2}\left[\frac{j^2}{j_c^2}\frac{T_c-T}{\gamma}\right].$$

We see that in this temperature region the current voltage characteristic (CVC) of the $S-N$ interface is essentially nonlinear. This singularity of the CVC can be easily understood qualitatively if it is noted that when the current is increased then, in accordance with the statements made in Sec. 1, the relaxation of the electron-hole unbalance is accelerated. The depth of penetration of the electric field (9) then decreases in proportion to $Q^{-1/2}\sim j^{-1/2}$, and the resistance $R\sim l_E/\sigma\propto j^{-1/2}$ decreases with it. For weak currents $j/j_c\sim[\gamma/(T_c-T)]^{1/2}\ll1$, the CVC assumes the linear form (17). Figure 2 shows schematically the CVC in this temperature range.

B. Let now the sample be placed in a parallel magnetic field H.

a) We consider first the region of small currents $j/j_c(H)\ll1$. In this case $p=\beta(1-\eta^2)=\beta H^2/H_0^2$ and

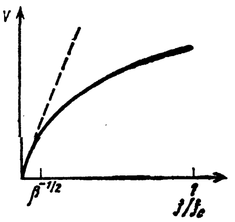

FIG. 2. Schematic current-voltage characteristic of $S-N$ interface in the temperature range $\gamma/T\ll(T_c-T)/T\ll(\gamma/T)^{1/2}$. The dashed line shows the linear dependence characterized by the resistance $R_0=l_E^{(0)}/\sigma$.

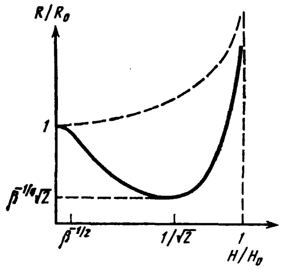

FIG. 3. Schematic dependence of the resistance of $S-N$ interface on the magnetic field in the temperature range $\gamma/T\ll(T_c-T)/T\ll(\gamma/T)^{1/2}$.

$$\Phi_0=j\left\{\frac{a}{\eta[1+I(p)]}\right\}^{1/2}.$$

This linear dependence of the voltage on the current is characterized by a resistance (in ordinary units)

$$R=R_0\left(1-\frac{H^2}{H_0^2}\right)^{-1/4}\left[1+I\left(\beta\frac{H^2}{H_0^2}\right)\right]^{-1/2}. \tag{19}$$

In the temperature region $T_c-T\ll\gamma$, where $\beta\ll1$, we have

$$R=R_0\left(1-\frac{H^2}{H_0^2}\right)^{-1/4}\left(1-\frac{\beta}{\pi}\frac{H^2}{H_0^2}\right). \tag{20}$$

The factor $(1-H^2/H_0^2)^{1/2}$ describes here the suppression of the superconductivity by the magnetic field.

We consider the temperature range $\gamma/T\ll(T_c-T)/T_c\ll(\gamma/T)^{1/2}$. Here

$$\beta=8(T_c-T)/\pi\gamma\gg1.$$

In very weak magnetic fields, where $H^2/H_0^2\ll\beta^{-1}\ll1$, the resistance is

$$R=R_0\left(1-\frac{\beta}{\pi}\frac{H^2}{H_0^2}\right). \tag{21}$$

When the field is increased to values $H/H_0\sim\beta^{-1/2}\ll1$ the resistance decreases sharply and at $H/H_0\gg\beta^{-1/2}$ we have

$$R=R_0\beta^{-1/4}\left(1-\frac{H^2}{H_0^2}\right)^{-1/4}\left(\frac{H}{H_0}\right)^{-1/2}. \tag{22}$$

The decrease of the resistance continues to the value $H/H_0=1/\sqrt{2}$, where $R/R_0=\beta^{-1/4}\sqrt{2}$, after which it increases. The dependence of the resistance on the magnetic field is shown schematically in Fig. 3. Kadin, Skocpol, and Tinkham[10] investigated the influence of the magnetic field on the effective depth of penetration $l_E=R\sigma$ of the electric field. The $l_E(H)$ dependence they obtained agrees qualitatively with the dependence described by formulas (19), (21), and (22) and shown in Fig. 3.

b) We consider now large values of the magnetic field, $\eta\gg1$. Here $\Delta\ll1$ and $p=\beta$. From (15) we obtain

$$\Phi_0=j\left\{\frac{a}{\eta[1+I(\beta)]}f\left(\frac{\Delta_0}{\eta}\right)\right\}^{1/2},$$

where $f(x)$ is defined in (16). As indicated above, $f(x)\approx1$, so that in this case the resistance of the $S-N$ interface is approximately

$$R\approx R_0\left(1-\frac{H^2}{H_0^2}\right)^{-1/4}[1+I(\beta)]^{-1/2}$$

in the entire range of currents $0\leqslant j\leqslant j_c(H)$.

The authors are grateful to A. F. Volkov and Yu. N. Ovchinnikov for helpful discussions.

[1] I. L. Landau, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 437 (1970) [JETP Lett. **11**, 295 (1970)].
[2] A. B. Pippard, J. G. Shepherd, and D. A. Tindall, Proc. Roy. Soc. Ser. A **324**, 17 (1971).
[3] M. Tinkham, Phys. Rev. B **6**, 1747 (1972).
[4] S. N. Artemenko, A. F. Volkov, and A. V. Zaĭtsev, Low Temp. Phys. **30**, 478 (1978).
[5] A. Schmid and G. Schön, J. Low Temp. Phys. **20**, 207 (1975).
[6] L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
[7] L. P. Gor'kov and N. B. Kopnin, *ibid.* **64**, 356 (1974) [**37**, 183 (1973)].
[8] V. P. Galaĭko, *ibid.* **68**, 223 (1975) [**41**, 108 (1975)].
[9] I. E. Bulyzhenkov and B. I. Ivlev, Fiz. Tverd. Tela (Leningrad) **21**, 2325 (1979) [Sov. Phys. Solid State **21**, 1339 (1979)].
[10] A. M. Kadin, W. J. Skocpol, and M. Tinkham, J. Low Temp. Phys. **33**, 481 (1978).
[11] Yu. N. Ovchinnikov, *ibid.* **31**, 785 (1978).
[12] M. L. Yu and J. E. Mercereau, Phys. Rev. Lett. **28**, 1117 (1972).

Translated by J. G. Adashko

Temperature dependence of the conductivity of point junctions between a superconductor and a normal metal

S. I. Dorozhkin

Institute of Solid State Physics, USSR Academy of Sciences

(Submitted 6 March 1980)
Zh. Eksp. Teor. Fiz. **79**, 1025–1035 (September 1980)

The temperature dependences of the current-voltage characteristics of point junctions between a superconductor (aluminum) and a normal metal (silver) were investigated experimentally and estimates were made of the parameters of the junctions and of the mean free path l of the electrons in the region of the junction. The measured temperature dependences are close to those predicted by Zaĭtsev [Sov. Phys. JETP **51**, 111 (1978)] for junctions with diameter $d \ll (l,\xi_0)$, although the assumptions of the theory were not realized in the experiments. A simple model is proposed for the conduction of point junctions with $\xi(T) \ll d \ll l_\varepsilon$, where l_ε is the energy relaxation length of the electrons in the normal state of the superconductor. It is shown on the basis of this model that Zaĭtsev's results are valid also for such junctions.

PACS numbers: 74.70.Gj, 74.70.Lp, 73.40.Jn

The current-voltage characteristics of point junctions between normal metals are linear if the junction voltage U is such that $eU \ll k\Theta_D$ (Θ_D is the Debye temperature),[2] and do not depend on the temperature at sufficiently low temperature. The transition of one of the electrodes of the junction into the superconducting state leads to the appearance of noticeable nonlinearities (see Fig. 1). Below the superconducting-transition temperature T_c, a change takes place in the differential resistance $r = dU/dI$ at $U=0$, and the CVC takes in an appreciable current interval the form

$$I = U/R + I_0 \operatorname{sign} U,$$

where R is the resistance of the junction when both electrodes are in the normal state, I_0 does not depend on U and is called the excess current. These features of the CVC were first noted by Pancove.[3]

The values of r and I_0 depend on temperature below T_c. Chien and Farrell[4] observed a differential-resistance temperature dependence of the form $r \approx R - R_1\Delta(T)/\Delta(0)$, where R_1 is a constant and $\Delta(T)$ and $\Delta(0)$ are the energy gaps in the superconductor at temperatures T and $0K$, respectively. Gubankov and Margolin[3] obtained an experimental dependence that agreed with the corresponding dependence of the energy gap in the superconductor, while the differential resistance varied nonmonotonically with temperature.

Recent theoretical papers[1,6] dealing with the conductivity of S-c-N (superconductor–constriction–normal metal) junctions, which include also the point junctions, have shown that the results depend on the relations between the electron mean free path l, the super-conductor coherence length ξ_0, and the geometric dimensions d and l of the constriction. (The constriction is regarded as a cylinder of diameter d and length L, which joins massive materials.) Both references are devoted to the study of junctions with $(d, L) \ll \xi(T)(1-T/T_c)^{1/4}$ [$\xi(T)$ is the coherence length in the superconductor at the temperature T], but the restrictions on the electron mean

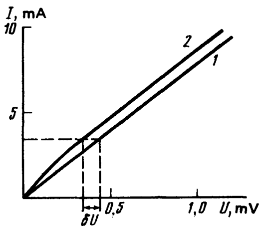

FIG. 1. Current-voltage characteristics of a point junction between aluminum and silver. $R = 0.128\Omega$, 1) $T = 1.4$ K, both electrodes in the normal state; 2) $T = 0.65$ K, the aluminum is superconducting. δU is the change of the junction voltage when the aluminum becomes superconducting (the current through the junction is fixed.

