# Beats due to superposition of "white" electromagnetic noise on spontaneous radiation from a resonant medium

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The passage of a "white" electromagnetic signal through a resonant medium imposes a characteristic structure on the noise spectrum of this signal and this is due to the nonlinear properties of the medium. The structure can be used to determine the atomic constants and it is formed as a result of two effect: beats due to superposition of the initial on the induced radiation and beats due to superposition of the initial on the spontaneous radiation. The former effect has been investigated experimentally and theoretically [E.V. Aleksandrov, O.V. Konstantinov, V.N. Kulyasov, A.B. Mamyrin, and V. I. Perel', Sov. Phys. JETP **34**, 1210 (1972)]. Here, we shall discuss the latter effect. We shall show that in the optical range this effect can have a decisive influence on the structure of the noise spectrum.

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We shall assume that a spectrally wide electromagnetic signal crossed an optically thin gas layer subjected to a static longitudinal magnetic field. The transmitted radiation is applied, without any preliminary monochromatization, to a photodetector and the photocurrent produced $i(t)$ is subjected to a spectral analysis. The output characteristic of such an experiment is

$$i^2(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\langle i(t)i(t+\tau)\rangle e^{-i\omega\tau}d\tau. \qquad (1)$$

Aleksandrov *et al.*[1] showed by calculation that in this case the nonlinear properties of the medium convert the initially "white" noise into a structure characteristic of the medium and this structure can be used, in principle, to determine the constants of both levels of an atom interacting with the external field. The experiments of Aleksandrov *et al.*[1] revealed a structure which was attributed to the upper atomic level. However, the structure of the lower level could not be observed. Estimates indicated that this could be explained by the proposed theory because the predicted structure of the lower level should be much weaker than that of the upper level. Nevertheless, this conclusion could be questioned in view of the following circumstance. The calculations carried out in Ref. 1 were based on classical representations of the field and the spontaneous emission was not allowed for. The spectrum therefore represented of beats resulting from superposition of the original noise on the induced radiation ("induced" effect).

We shall show that the spectrum of beats resulting from superposition of the original noise on the spontaneous radiation ("induced" effect) contains the structure of the upper level and, moreover, the effect is not less but greater than the induced effect. However, one should point out immediately that this was not true under the experimental conditions of Ref. 1, i.e., the induced effect was considerably greater than the spontaneous one and, therefore, the latter was ignored. However, the spontaneous effect is of interest for its own sake. In practice, it is easily separated from the induced effect because it is observed at all directions in the case of suitable mixing of the original noise in the photodetector, whereas, the induced effect can only be observed along the direction of propagation of the original signal.

The spectrum of the photocurrent (1) can be expressed in terms of the fourth correlation function of the electromagnetic field on the photodetector surface[2,3]:

$$i^2(\omega)=\int_{-\infty}^{\infty}d\tau\iint_{(S)}d^2r_1\,d^2r_2\,\langle E^+(r_1,t)E^+(r_2,t+\tau)E(r_2,t+\tau)E(r_1,t)\rangle e^{-i\omega\tau}. \qquad (2)$$

Here, $E$ and $E^+$ are the positive- and negative-frequency parts of the electromagnetic field operators in the Heisenberg representation; $S$ is the photocathode area. The quantity (2) is called the excess noise since in practice it is observed against the background of the Poisson shot noise which carries no information on the field and reflects only the discrete nature of the absorption of the field in the photocathode. We shall ignore this shot noise because under the experimental conditions of Ref. 1 this noise was negligible compared with the excess component.

We shall represent the operator $E(r,t)$ as a sum of the original field operator $E_0(r,t)$ and a correction $E'(r,t)$ associated with the presence of the medium. We shall assume that the change in the field due to the thin gas layer in question is small. Therefore, the correlation function $\langle E^+(1)E^+(2)E(2)E(1)\rangle$, which should be calculated in accordance with Eq. (2), can be expressed conveniently as a series in powers of $E'$. The zeroth term, in relation to $E'$, corresponds to the noise spectrum of the original radiation and terms of the first order are beats resulting from superposition of the original and induced radiation. These beats were calculated by Aleksandrov *et al.*[1] but without a constant factor which is important in the determination of the relationship between the stimulated and spontaneous effects. The terms of the second order in $E'$ can be of the $\langle E_0^+E_0^+E'E'\rangle$ or $\langle E_0^+E'^+E'E_0\rangle$ type. The former correspond to beats due to superposition of the stimulated radiation on itself and can be ignored because of their smallness, whereas the latter are beats due to superposition of the spontaneous and the original radiation,

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i.e., they represent the effect of interest to us. We shall carry out our calculations in a resonance approximation. It follows from quantum electrodynamics that the operator $E_0$ can be regarded as a $c$-number random function and the operator $E'$ as the quantized field operator in the interaction representation. This allows us to obtain the Perel'–Knostantinov graphs[4] for the required correlation functions exactly as was done in Ref. 5, when the original signal was taken in the form of a monochromatic wave. However, the final results should be averaged over the random original field.

The correlation function $\langle E_0^*(1)E'^*(2)E'(2)E_0(1)\rangle$ is a sum of the graphs shown in Fig. 1. The graphs differ from one another by the location on the atomic horizontal lines of the points of interaction of the original field with an atom in the resonant medium. The straight vertical arrows entering and leaving these points (and also the photocathode points identified by circles) represent the factors $E_0$ and $E_0^*$, respectively. The wavy lines drawn from an atomic line to the photocathode or from the photocathode to an atomic line represent the photon propagators.

The expressions given below apply to the case when the width of the original spectrum is considerably greater than the Doppler width of the resonant medium. In practice this is not normally true and the widths are of the same order but this does not affect significantly the results of calculations. It is also assumed that the Doppler broadening of the spectral line is considerably greater than the natural width.

The graphs in Fig. 1 describe the spontaneous effect. The induced effect is described by graphs with a single wavy line. Typically, this gives the results shown in Fig. 2.

Omitting standard calculations, we obtain the final result in the form

$$i^2(\omega)=i_0^2(1+Q_{\text{ind}}(\omega)+Q_{\text{sp}}(\omega)).$$

Here, $i_0^2$ is the spectrum of the photocurrent (2) in the absence of the resonant medium. It is assumed in this calculation that $i_0^2$ is independent of the frequency. The quantity $i_0^2Q_{\text{ind}}$ determines the nature of beats due to the interaction between the original and induced fields. Then, $Q_{\text{ind}}$ is described by the following explicit expression:

$$Q_{\text{ind}}(\omega)=\frac{\pi}{2}Ld^4k^2P\left(\frac{n_b}{2j_b+1}-\frac{n_a}{2j_a+1}\right)\left[K_{a0}\frac{\gamma_a}{\gamma_a^2+\omega^2}\right.$$
$$\left.+K_{a1}\frac{\gamma_a}{\gamma_a^2+(2|\Omega_a|-\omega)^2}+K_{b0}\frac{\gamma_b}{\gamma_b^2+\omega^2}+K_{b1}\frac{\gamma_b}{\gamma_b^2+(2|\Omega_b|-\omega)^2}\right]+(\omega\to-\omega). \quad (3)$$

Here, $L$ is the thickness of the resonant medium; $n_a$ and $n_b$ are the steady-state populations of the upper ($a$) and lower ($b$) levels in the absence of the original field

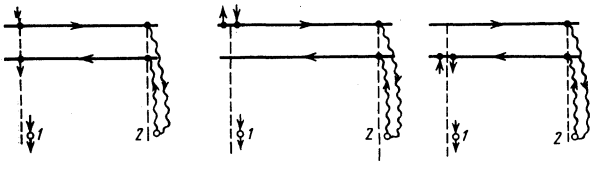

FIG. 1.

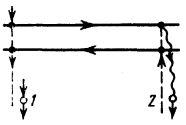

FIG. 2.

$E_0$; $j_a$ and $j_b$ are the momenta of the levels; $d$ is the reduced matrix element of the dipole moment of the transition; $\gamma_a$ and $\gamma_b$ are the widths of the levels assumed in these calculations to be due to the spontaneous decay to other levels; $\Omega_a$ and $\Omega_b$ are the values of the Zeeman splitting of the levels; $P$ is the power of the original radiation per unit spectral interval and unit surface area of a photocathode. The constants $K$ are expressed in terms of the $6j$ symbols[6]:

$$K_{a0}=\frac{1}{3}\begin{Bmatrix}1&1&0\\ j_a&j_a&j_b\end{Bmatrix}^2+\frac{1}{6}\begin{Bmatrix}1&1&2\\ j_a&j_a&j_b\end{Bmatrix}^2,$$
$$K_{a1}=\begin{Bmatrix}1&1&2\\ j_a&j_a&j_b\end{Bmatrix}^2;$$

$K_{b0}$ and $K_{a1}$, are obtained from $K_{a0}$ and $K_{b1}$, respectively, by the substitution $a\rightleftarrows b$.

Equation (3) holds for the case when the original radiation is linearly polarized and observations are carried out in the same polarization. It corresponds fully to the situation investigated in Ref. 1.

The quantity $Q_{\text{sp}}(\omega)$ is governed by the beats occurring on superposition of the spontaneous and the original radiations and it differs basically from $Q_{\text{ind}}$ by the absence of the profiles associated with the lower level $b$. The constant in front of the brackets, which is a quantitative characteristic, is also different. The ratio of the spontaneous and induced constants is $\hbar ck^3\Omega/\pi P$ ($\Omega$ is the solid angle for the collection of the radiation).

Under the experimental conditions of Ref. 1 the source of radiation is a gaseous xenon discharge due to the 5.57 $\mu$ transition. The spectral width of this radiation is of the order of 60 MHz and the total power is $10^{-4}$–$10^{-5}$ W. The maximum value of the ratio of the spontaneous and induced effects is then of the order of the solid angle $\Omega$. Since under the experimental conditions the solid angle did not exceed $10^{-2}$, the conclusions reached in Ref. 1 should be regarded as valid. Aleksandrov *et al.* did indeed observe the stimulated effect and, therefore, the absence of the structure of the lower atomic level could be explained only by its weakness. However, these estimates indicate that similar experiments on other gases may give quite different results. For example, if instead of the radiation of wavelength 5.57 $\mu$ we use radiation of wavelength an order of magnitude less, i.e., if we go over from the infrared to the optical part of the spectrum, we find that the ratio of the spontaneous and induced effects increases by three orders of magnitude and becomes $10^3\Omega$ with all the consequences that follow from this.

As pointed out earlier, the formulas given above apply to the case when the original radiation is linearly polarized and the observations are carried out in this polarization. However, if a filter is placed behind a cell with the investigated gas and this filter transmits

circularly but not linearly polarized light, the contributions to the noise spectrum due to the spontaneous and induced effects are now different: the contribution of the induced effect remains unchanged (apart from a constant factor), whereas the Zeeman line disappears from the spontaneous radiation. This can be used to distinguish the effects in doubtful cases.

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[1]E. V. Aleksandrov, O. V. Konstantinov, V. N. Kulyasov, A. B. Mamyrin, and V. I. Perel', Zh. Eksp. Teor. Fiz. 61, 2259 (1971) [Sov. Phys. JETP 34, 1210 (1972)].
[2]R. Glauber, in: Kvantovaya optika i kvantovaya radiofizika, (Quantum Optics and Quantum Radiophysics), Russ. Transl., ed. by O. V. Bogdankevich and O. N. Krokhin, Mir, M., 1966.
[3]D. F. Smirnov, I. V. Sokolov, and A. S. Troshin, Vestn. Leningr. Univ. Fiz. Khim, No. 2, 36 (1977).
[4]O. V. Konstantinov and V. I. Perel', Zh. Eksp. Teor. Fiz. 39, 197 (1960) [Sov. Phys. JETP 12, 142 (1961)].
[5]Yu. M. Golubev and L. I. Plimak, Opt. Spektrosk. 48, 222 (1980) [Opt. Spectrosc. (USSR) 48, 126, (1980)].
[6]D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonskiĭ, Kvantovaya teoriya uglovogo momenta (Quantum Theory of Angular Momentum), Nauka, L., 1975.

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# Effect of pressure on the parameters of the energy spectrum of graphite

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The dependence of the extremal sections and of the effective masses of the carriers of pyrolytic graphite were investigated at pressures up to 17 kbar and temperatures 2–4 K. A nonlinear variation of the extremal sections $S$ of the Fermi surface was observed at high pressures, and it is shown that this variation becomes linear when plotted in the coordinates $S$ and $V$, where $S$ is the specimen volume. The logarithmic derivatives of the extremal sections of the Fermi surface and of the effective masses of the electrons and holes with respect to pressure are determined. The logarithmic derivatives with respect to pressure are calculated for the parameters of the electron energy spectrum of graphite, and a comparison is made with the results of other experimental and theoretical studies.

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## INTRODUCTION

Graphite and synthetic carbon materials on its base have a number of unique properties and have recently found extensive practical use. The distinct layered crystal structure of graphite causes all its properties to be highly anisotropic, since the interaction of the carbon atoms within each layer exceeds by one order of magnitude and interaction between atoms in neighboring layers.

A universally accepted model of the energy spectrum of graphite is at present the Slonczewski–Weiss model,[1] in which the interaction between layers is taken into account by perturbation theory. The semimetallic properties are due to the presence of a weak interaction between atoms of different layers, an interaction described by the parameters $\gamma_i$ $(i=1-6)$,[1] which are small compared with the parameter $\gamma_0$ that corresponds to the interaction of the atoms in the layer. The Fermi surface of graphite consists of electron and hole parts that are strongly elongated along the $H-K-H$ edge of the Brillouin zone.

In the Slonczewski–Weiss model, the effective masses $m^*$ and the extremal sections of the Fermi surface in the basal plane perpendicular to the hexagonal $c$ axis of the crystal lattice are respectively described by the following formulas:

$$m_e^*(\Psi)=\frac{4}{3}\left(\frac{\hbar}{a_0}\right)^2\frac{\gamma_1}{\gamma_0^2}\cos\Psi\Big/\left(1+\frac{4\gamma_4}{\gamma_0}\cos\Psi\right), \tag{1}$$

$$m_h^*(\Psi)=\frac{4}{3}\left(\frac{\hbar}{a_0}\right)^2\frac{\gamma_1}{\gamma_0^2}\cos\Psi\Big/\left(1-\frac{4\gamma_4}{\gamma_0}\cos\Psi\right), \tag{2}$$

$$S_e=\frac{3\pi\hbar^2}{4\gamma_0^2a_0^2}\frac{2\gamma_2-\varepsilon_F}{(1+2\gamma_4/\gamma_0)^2}(\Delta-2\gamma_1+2\gamma_5+\varepsilon_F), \tag{3}$$

$$S_h=\frac{3\pi\hbar^2}{4\gamma_0^2a_0^2}\frac{2\gamma_2\cos^2\Psi_0-\varepsilon_F}{(1-2\gamma_4\cos\Psi_0/\gamma_0)^2}(\Delta+2\gamma_1\cos\Psi_0+2\gamma_5\cos^2\Psi_0-\varepsilon_F), \tag{4}$$

$$S_m=\frac{3\pi\hbar^2}{4\gamma_0^2a_0^2}\varepsilon_F(\varepsilon_F-\Delta), \tag{5}$$

where $\Psi=k_zc_0/2$, $k_z$ is the wave number reckoned from the point $K$ of the edge of the Brillouin zone, $c_0$ and $a_0$ are the parameters of the graphite crystal lattice ($c_0=6.708$ Å, $a_0=2.462$ Å for single-crystal graphite), $S_e$ is the maximum section of the electron part of the Fermi surface, $S_h$ is the maximum section of the hole part, $S_m$ is the minimum section of the hole part of the Fermi surface of graphite near the point $H$, $\varepsilon_F$ is the Fermi energy, $\Delta\equiv\gamma_6$, $\cos\Psi_0\approx|\varepsilon_F/6\gamma_2|^{1/2}$.

The accuracy of the cited expressions is ~10%. The

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