Decay of a weakly bound level in a monochromatic field

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(Submitted 29 September 1979)
Zh. Eksp. Teor. Fiz. 79, 751–762 (September 1980)

The problem of the decay of a weakly bound level (in the zero-range-potential model) in the presence of an elliptically polarized monochromatic field with an arbitrary degree of polarization ellipticity is solved using the formalism of quasistationary quasienergy states. Numerical results are presented for the strong-field case, and various limiting cases are investigated analytically and the results compared with those of earlier quasiclassical calculations. Simple expressions are obtained for the probability for N-photon ionization and for the corrections to it that determine the conditions for the applicability of the first nonvanishing order of perturbation theory. The effect of a static electric field on the decay of the system in the field of a nonresonant light wave is also examined, and it is shown that considerable enhancement of the decay is possible in the case of few-photon processes as a result of "tunneling from a quasienergy-state harmonic."

PACS numbers: 41.10. – j, 32.80.Kf

1. INTRODUCTION

The decay of a system under the action of an intense light field plays an important part in many processes involving the interaction of laser light with matter. A general treatment of the ionization problem with arbitrary relations between the properties of the wave and the characteristic parameters of the system turns out to be extremely complicated, even for the simplest system capable of decay—a short-range potential well with a single bound level (a negative-ion model). Only the case of a weak low-frequency field ($\hbar\omega \ll E_0$, where E_0 is the energy of the bound state),[1-4] together with the special case of a circularly polarized wave,[5,6] has been studied adequately. The methods developed in Refs. 1–4 cannot be used to study decays in strong fields having "optical" frequencies $\hbar\omega \lesssim E_0$, even if only because they do not take account of the change in the position of the bound level due to the action of the field, which is not small in strong fields and substantially affects the level width (decay probability).[5]

The study of quasistationary (decaying) states that arise from discrete levels on exposing the system to a monochromatic field can be substantially simplified by using the quasistationary-quasienergy-state (QQES) formalism,[7,8] which is a generalization of the usual quasienergy-state (QES) formalism[9] to a group of problems in which field broadening of the levels is significant. In this approach the position and width of the level are determined in a unified manner as the real and imaginary parts of the complex quasienergy $\varepsilon = \mathrm{Re}\,\varepsilon - i\Gamma/2$, $\Gamma/\hbar$ being the ionization probability per unit time in the exponential decay region.[8] In this paper we use the QQES formalism to solve the problem of calculating the complex quasienergy of a particle with a low binding energy (in the zero-range potential approximation) in the field of a plane wave of arbitrary polarization.[1)] Unlike the case of circular polarization, in which the problem reduces to a stationary problem in a rotating coordinate system,[5] the general case does not rule out a periodic time dependence of the Hamiltonian, and the problem is essentially of quasienergy type from the very beginning. The solution obtained generalizes the results of Refs. 1–4 to the case of arbitrary frequencies and strong fields, and thereby completely solves the problem of the action of an intense monochromatic field on a weakly bound system. The presence of an exact solution also made it possible to clarify the accuracy of the approximations used in Refs. 1–4 and to investigate the problems, which are important for practical applications, of the limits of applicability of the first nonvanishing order of perturbation theory for the width of the level and of the dependence of the decay rate on the polarization of the wave.

2. EQUATION FOR THE COMPLEX QUASIENERGY AND PERTURBATION THEORY

The Schrödinger equation for the QQES wave function

$$\Psi_\varepsilon(\mathbf{r},t)=e^{-i\varepsilon t/\hbar}\Phi_\varepsilon(r,t),\ \Phi(t+2\pi/\omega)=\Phi(t)$$

for a particle in the potential $U(r)$ in the presence of the monochromatic field

$$\mathbf{F}(t)=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}=F(1+\xi^2)^{-1/2}\{\cos\omega t,\ \xi\sin\omega t,\ 0\}$$

($|\xi|\leq 1$ is the ellipticity of the polarization) can be written in integral form as[2)]

$$\Phi_\varepsilon(r,t)=\int\limits_{-\infty}^{t}dt'e^{-i\varepsilon(t-t')/\hbar}\int d\mathbf{r}'G(\mathbf{r},t;\mathbf{r}',t')U(\mathbf{r}')\Phi_\varepsilon(\mathbf{r}',t'). \qquad (1)$$

Here G is the retarded Green's function for a free particle in the field $\mathbf{F}(t)$; it has the well-known form[11]

$$i\hbar G(\mathbf{r},t;\mathbf{r}',t')=\theta(t-t')\left[\frac{m}{2\pi i\hbar(t-t')}\right]^{3/2}\exp\left\{\frac{i}{\hbar}S_{\rm cl}(\mathbf{r},t;\mathbf{r}',t')\right\}, \qquad (2)$$

$$S_{\rm cl}(\mathbf{r},t;\mathbf{r}',t')=\frac{m[\mathbf{r}-\mathbf{r}'-\boldsymbol{\alpha}(t)+\boldsymbol{\alpha}(t')]^2}{2(t-t')}-\int\limits_{t'}^{t}\frac{e^2\mathbf{A}^2(\tau)}{2mc^2}d\tau,\quad \boldsymbol{\alpha}(t)=-\frac{e\mathbf{F}(t)}{m\omega^2}.$$

As in problems on decay in a static field, an important circumstance that simplifies the problem for the case of the zero-range potential

$$U(\mathbf{r})=\frac{1}{r^2}\delta(r)\left(1+r\frac{\partial}{\partial r}\right),$$

is the well-known behavior[12] of the function $\Phi\varepsilon(r,t)$ as $r\to 0$:

$$\Phi_\varepsilon(\mathbf{r},t)\approx\left(\frac{1}{r}-1+2i\frac{(\mathbf{A}\mathbf{r})}{r}\right)f_\varepsilon(t)+O(r),\quad f\left(t+\frac{2\pi}{\omega}\right)=f(t). \qquad (3)$$

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Here and below we use "Rydberg" units ($e=m=\hbar=1$). Energies are measured in units of the binding energy $|E_0|$ of the unperturbed level in the potential well, and the field amplitude F is measured in units of $F_0 = \sqrt{8m|E_0|^3}/e\hbar$. The relation

$$U(\mathbf{r})\Phi(\mathbf{r},t) = -4\pi\delta(\mathbf{r})f(t)$$

makes it possible to perform the integration over r' in Eq. (1) at once. Taking $r \to 0$ in that equation and selecting the term proportional to r^{-1} in the integration over t' (in which it is convenient first to transform from t' to the new variable $\tau = t - t'$) we obtain, with the aid of (3), the equation for ε and the periodic function $f\varepsilon(t)$:

$$(\sqrt{E}-1)f(t) = (4\pi i)^{-1}\int_0^\infty \frac{dt'}{t'^{3/2}}\exp(-iEt')\{f(t-t')\exp iS(t,t') - f(t)\}, \qquad (4)$$

where

$$S(t,t') \equiv S_{cl}(0,t;0,t-t')$$
$$= \frac{4\gamma^2}{\omega^2 t'}\sin^2\frac{\omega t'}{2}[1 - l\cos\omega(2t-t')] + \frac{l\gamma^2}{\omega}\sin\omega t'\cos\omega(2t-t'), \qquad (5)$$

$E = \gamma^2 - \varepsilon$, $l = (1-\xi^2)/(1+\xi^2)$ is the degree of linear polarization of the field,[13] and $\gamma^2 = 2F^2/\omega^2$ or, in absolute units, $\gamma^2 = E_F/|E_0|$, where $E_F = e^2F^2/4m\omega^2$ is the average kinetic energy of the oscillations of the electron in the field.

The substitution

$$f(t) = \varphi(t)\exp\left\{i\frac{l\gamma^2}{2\omega}\sin 2\omega t\right\} \qquad (6)$$

(which corresponds to a unitary transformation that eliminates the term proportional to $\mathbf{A}^2$ in the interaction $V(\mathbf{r},t)$ eliminates the last term in (5) and somewhat simplifies the equation:

$$(\sqrt{E}-1)\varphi(t) = (4\pi i)^{-1}\int_0^\infty \frac{d\tau}{\tau^{3/2}}e^{-iE\tau}\Big[\varphi(t-\tau)\exp\Big\{i\frac{4\gamma^2}{\omega^2\tau}$$
$$\times\sin^2\frac{\omega\tau}{2}[1 - l\cos\omega(2t-\tau)]\Big\} - \varphi(t)\Big]. \qquad (7)$$

The kernel of the integral equation (7) is periodic in τ with the period $T_1 = T/2 = \pi/\omega$; from this it follows that $\varphi(t)$ contains Fourier components of only one parity. This is due to the fact that $\varphi(t)$ is determined by the asymptotic behavior as $r \to 0$ of the s-wave part of $\Phi(\mathbf{r},t)$ alone and, according to the dipole selection rules, the kth and $(k+1)$-th harmonics of Φ have opposite parities and so only one of them can contribute to $\varphi(t)$. In a circularly polarized field ($l=0$) the space part of the kth Fourier component of $\Phi\varepsilon(\mathbf{r},t)$ corresponds to the orbital angular momentum projection $M=k$, so only the Fourier component with $k=0$ contributes to $\varphi(t)$; this means that Eq. (7) has the trivial solution $\varphi(t) = \text{const}$, which leads to a transcendental equation for E. It is easy to verify that, in accordance with the ambiguity in the definition of the quasienergy, Eq. (7) has not only the solution $\{E, \varphi(t)\}$, but also the solutions

$$\{E+k\omega,\ \varphi(t)\exp(ik\omega t)\},\ k=\pm1, \pm2, \ldots.$$

We eliminate this ambiguity in the usual manner by selecting the solution that reduces to the unperturbed solution $\{E=1, \varphi(t)=1\}$ for $F=0$.

Expressing φ in the form

$$\varphi(t) = \sum_{k=-\infty}^{\infty}\varphi_k e^{2ik\omega t},$$

we obtain the following homogeneous equations for the φ_k:

$$[(E+2k\omega)^{1/2} - 1]\varphi_k = \sum_{n=-\infty}^{\infty} M_{kn}(E)\varphi_n, \qquad (8)$$

where

$$M_{kn}(E) = (4\pi i)^{-1/2}\int_0^\infty \frac{d\tau}{\tau^{3/2}}\exp[-i(E+(n+k)\omega)\tau]$$
$$\times\{(-i)^{k-n}e^{ix}J_{k-n}(lx) - \delta_{k,n}\}, \quad x = \frac{4\gamma^2}{\omega^2\tau}\sin^2\frac{\omega\tau}{2}, \qquad (9)$$

and, as can be easily verified, the M_{kn} satisfy the relations

$$M_{kn}(E) = M_{nk}(E) = M_{0,\,n-k}(E+2k\omega) = M_{0,\,k-n}(E+2n\omega). \qquad (10)$$

Using Poisson's integral representation of the Bessel function and calculating (9) by expanding the integrand in powers of γ, we obtain for the M_{0k} with $k \geq 0$ the following power series in γ, which is absolutely convergent for all $\omega \neq 0$:

$$M_{0,k}(E) = -\left(-\frac{2l\gamma}{\omega^2}\right)^k\sum_{n=\delta_{k,0}}^{\infty}\left(\frac{4\gamma^2}{\omega^2}\right)^n\frac{(n+k)!}{n!k!(2n+2k+1)}$$
$$\times {}_2F_1\left(-\frac{n}{2};\frac{1-n}{2};k+1;l^2\right)\sum_{m=-(n+k)}^{n+k}(-1)^m\frac{[E+(m+k)\omega]^{n+k+1/2}}{(n+k+m)!(n+k-m)!}. \qquad (11)$$

Here ${}_2F_1$ is a hypergeometric polynomial. The values of the roots $p_k \equiv \sqrt{E-k\omega}$ are so chosen that the QQES function $\Phi_\varepsilon(\mathbf{r},t)$ in (1) will behave asymptotically as outgoing waves in "open" channels ($\text{Re}E > k\omega$) and will damp out in closed channels ($\text{Re}E < k\omega$):

$\text{Re}\,p_k > 0$ when $k > \text{Re}\,E/\omega$,
$\text{Im}\,p_k < 0$ when $k > \text{Re}\,E/\omega$.

The equation for E,

$$\det\|(\sqrt{E+2k\omega}-1)\delta_{k,n} - M_{kn}(E)\| = 0,$$

which follows from (8) is comparatively easy to solve numerically on a computer. Some results of such numerical calculations are presented in Fig. 1. While the level shift $\text{Re}\,\varepsilon+1$ remains quadratic in F up to $F\sim0.1$, the width begins to deviate from the power-law dependence $F^{2(1/\omega+1)}$ in considerably weaker fields $F\sim F_{cr}$; moreover, F_{cr} decreases with decreasing ω. The ionization probability rises substantially as l is increased from 0 to 1 with fixed values of F and ω. We note, further, that when $\gamma \leq 1$ the functional dependences $\Gamma(F)$ of Γ on F for $l=0$ and $l=1$ differ considerably from one another (see Fig. 1) and that the deviations of $\Gamma(F)$ from the power-law dependence set in at considerably lower values of F in a circularly polarized field than in a linearly polarized one (also see Sec. 4).

The matrix elements M_{kn} are small when γ is small ($M_{kn}\sim\gamma^{2(|n-k|+\delta_{n,k})}$) and to investigate the dependence of ε on the wave parameters one can use the following expansion [the "Brillouin-Wigner series" for Eqs. (8)]:

$$\sqrt{E} = 1 + M_{00}(E) + \sum_{k\neq0}\frac{M_{0k}(E)M_{k0}(E)}{(E+2k\omega)^{1/2}-1}$$
$$+ \sum_{k,r\neq0}\frac{M_{0k}(E)M_{kr}(E)M_{r0}(E)}{[(E+2k\omega)^{1/2}-1][(E+2r\omega)^{1/2}-1]} + \ldots. \qquad (12)$$

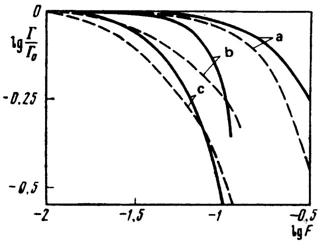

FIG. 1. The level width Γ as a function of the field amplitude F. The full (dashed) curves are for linear (circular) polarization of the field. Curves a, b, and c are for field frequencies ω of 0.8, 0.4, and 0.3, respectively, and correspond to two-, three-, and four-photon ionization, respectively. Γ_0 is the width in the first nonvanishing order of perturbation theory.

When $l=0$ we have $M_{kn}\sim\delta_{kn}$ and Eq. (12) reduces to the following equation for ε, which was obtained earlier in Refs. 5 and 6:

$$\sqrt{E}=1+M_{00}(E;\ l=0). \tag{12a}$$

These equations are suitable for calculating the terms of the perturbation series

$$\varepsilon=-1+\varepsilon_2+\varepsilon_4+\ldots,\quad \varepsilon_{2n}\sim F^{2n}. \tag{13}$$

In particular, the expressions for ε_2 and ε_4 obtained from (12) are the same as those obtained by direct calculations using QQES perturbation theory[8] (the amount of calculations required, however, is much less than when using the formulas of Ref. 8). When $\omega\leqslant 1/N$ the terms in the perturbation series (13) with $n\leqslant N$ are real. When $1/N<\omega\leqslant 1/(N-1)$ the quantity $\Gamma_N=-2\,\mathrm{Im}\,\varepsilon_{2N}$ is the probability for N-photon ionization in the first nonvanishing order of F, and when $\omega>1/(N-1)$, Γ_N determines the correction to the width calculated in lower orders in the field. Formulas for Γ_1 and Γ_2 will be found in Ref. 8, and an expression for Γ_3 is given below [Eq. (20)]. The expressions for Γ_N become extremely cumbersome for large N, but in that case one can devise an approximation that takes account of the presence of a small parameter: $\omega\ll 1$. We shall examine such an approximation below.

3. THE LOW-FREQUENCY CASE; COMPARISON WITH QUASICLASSICAL CALCULATIONS

When F is small, the function $f(t)$ in Eq. (4) is only weakly dependent on ω, whereas the t dependence of φ may be strong when $\omega\ll 1$ [see Eq. (6)]. In place of Eqs. (8), therefore, it is convenient to use the corresponding equations for the Fourier components of $f(t)$:

$$[(E+2k\omega)^{1/2}-1]f_k=\sum_{n=-\infty}^{\infty}\tilde{M}_{kn}(E)f_n. \tag{14}$$

The $\tilde{M}_{kn}$ also satisfy relations (10) and, using Eq. (4), we can obtain an expansion of $\tilde{M}_{0k}$ ($k\geqslant 0$) in a convergent series, analogous to expansion (11):

$$\tilde{M}_{0k}(E)=-\left(\frac{-2l\gamma^2}{\omega^2}\right)^k\sum_{n=\delta_{k,0}}^{\infty}\left(\frac{4\gamma^2}{\omega^2}\right)^n\sum_{s=0}^{[n/2]}\left(\frac{l}{2}\right)^{2s}\frac{(k!)^2(k+2s)!}{(n-2s)!s!(s+k)!}$$
$$\times\sum_{p=0}^{2s+k}\left(\frac{\omega}{8}\right)^p\frac{(n+k-p)!(2n+2k-p)!}{(2s+k-p)!(2n+2k-2p+1)!}\sum_{r_1=0}^{p}\sum_{r_2=0}^{2n+2k-p}$$
$$\times\frac{(-1)^{r_2}[E+(r_1+r_2-n)\omega]^{n+k-p+1/2}}{(r_1)!(r_2)!(2n+2k-p-r_2)!(p-r_1)!}. \tag{15}$$

Although the expression for $\tilde{M}_{0k}$ is more cumbersome than that for M_{0k}, when ω is small we can limit ourselves in the expansion analogous to (12) to the approximation

$$\sqrt{E}=1+\tilde{M}_{00}(E). \tag{16a}$$

An estimate of the first englected term shows that this approximation allows us to determine the N-photon width Γ_N with a relative accuracy of $\sim\omega/16$ when ω is small (see (19) below). In Eq. (4), approximation (16a) corresponds to setting $f(t)\approx$ const and averaging the right-hand side with respect to t over the period $T=2\pi/\omega$:

$$\sqrt{E}=1+\frac{\omega}{2\pi}(4\pi i)^{-1/2}\int_0^T dt\int_0^\infty\frac{d\tau}{\tau^{3/2}}e^{-iE\tau}\{e^{iS(t,\tau)}-1\}. \tag{16b}$$

The accuracy of this approximation can be estimated by averaging, after first taking account of the Stark effect on the adiabatic in $f(t)$, i.e., writing

$$f(t)=\exp\left\{i\int^t[\varepsilon_2(F(t'))-\bar{\varepsilon}_2]dt'\right\},$$
$$\varepsilon_2(F)=-\frac{1}{4}F^2,\qquad \bar{\varepsilon}_2=\frac{1}{T}\int_0^T\varepsilon_2(F(t))dt.$$

This estimate again gives the relative accuracy of approximation (16) as $\sim\omega/16$.

Now let us examine the relation between the equations obtained above and the quasiclassical approximation.[1-4] When $F\ll 1$, we can put $\varepsilon=-i$, i.e., $E=1+\gamma^2$, on the right in (16) and take

$$\Gamma=2\,\mathrm{Im}\,E=2\,\mathrm{Im}(2\tilde{M}_{00}+\tilde{M}_{00}^2)\approx 4\,\mathrm{Im}\,\tilde{M}_{00}\ (E=1+\gamma^2).$$

To extract the imaginary part of $\tilde{M}_{00}$ we do not use expression (2) for the Green's function $G(0,t;0,t-t')$ in (16b), but its expansion

$$iG(\mathbf{r},t;\mathbf{r}',t-t')=\theta(t')\int d\mathbf{p}\varphi_{\mathbf{p}}(\mathbf{r},t)$$
$$\times\varphi_{\mathbf{p}}^*(\mathbf{r}',t-t')\exp\left\{-i\left(\frac{\mathbf{p}^2}{2m}+E_F\right)t'\right\}$$

in the wave functions $\varphi_{\mathbf{p}}(\mathbf{r},t)\exp\{-i(\mathbf{p}^2/2m+E_F)t\}$ for a free particle in the field $\mathbf{F}(t)$. Then we expand $\varphi_{\mathbf{p}}(\mathbf{r}=0,t)$ in a Fourier series in t:

$$\varphi_{\mathbf{p}}(\mathbf{r}=0,t)=(4\pi)^{-1}\sum_{n=-\infty}^{\infty}F_n(\mathbf{p})e^{-in\omega t},$$
$$F_n(\mathbf{p})=\frac{1}{T}\int_0^T dt\exp\left\{-i\left[(\mathbf{p}\alpha(t))+\int^t(\beta^2(t')-\gamma^2)dt'-n\omega t\right]\right\},$$
$$\alpha(t)=-4F(t)/\omega^2,\quad \beta={}^1\!/_2\,d\alpha/dt.$$

Now, using the relation

$$\int_0^\infty e^{ix\tau}d\tau=i\frac{P}{x}+\pi\delta(x)$$

we can easily perform the τ integration in (16b) and extract the imaginary part. As a result, we obtain

$$\mathrm{Im}\,\tilde{M}_{00}(1+\gamma^2)=2\pi\sum_n\int d\mathbf{p}|F_n(\mathbf{p})|^2\delta(\mathbf{p}^2+1+\gamma^2-n\omega). \tag{17}$$

These expressions for Γ correspond to the initial formulas (10)–(15) of Ref. 4, where they were investigated in detail in the low-frequency ($\omega\ll 1$) approximation where the integration in $F_n(\mathbf{p})$ can be performed by the saddle point method.

In the "antiadiabatic" limit $\gamma \ll 1$, the perturbation-theory expression for the ionization probability for the case $N \gg 1$ can be easily obtained from relations (15) and (16a) without introducing any additional error due to the approximations involved in calculating $F_n(\mathbf{p})$. Thus, to the first nonvanishing order in F we obtain

$$\Gamma_N = 4\sqrt{\omega}\left(\frac{4\gamma^2}{\omega}\right)^N \sum_{s=0}^{2[N/2]} a_s^{(0)}(N-\nu)^{N-s+1/2}, \quad \nu = \frac{1}{\omega}, \quad N > \nu > N-1, \tag{18}$$

where

$$a_s^{(0)} = \left(-\frac{1}{8}\right)^s \frac{(N-s)!}{s!(2N-2s+1)!} \sum_{k=[(s+1)/2]}^{[N/2]} \left(\frac{l}{2}\right)^{2k} \frac{(2k)!}{(N-2k)!(2k-s)!(k!)^2}$$

As follows from an analysis of the terms rejected in deriving (16a), $a_s^{(0)}$ is the first term in an expansion of the coefficients a_s in powers of ω; specifically,

$$a_s(\omega) = a_s^{(0)} + \frac{\omega^2}{16} s a_s^{(0)} + \dots . \tag{19}$$

We note that even for $N=3$, the difference between (18) and the accurate expression derived from (12),

$$\Gamma_3 = \frac{1}{6}\left(\frac{8F^2}{\omega^4}\sqrt{3\omega-1}\right)^3 \left\{ \Phi^2 l^2 + \frac{1}{5}(3\omega-1)\Phi l^2 + \frac{1}{210}\left(1+\frac{3}{2}l^2\right)(3\omega-1)^2 \right\}, \tag{20}$$

$$\Phi = \frac{1}{12\omega}[(1-2\omega)^{1/2}+1] \times [2(1-\omega)^{1/2}-1-(1-2\omega)^{1/2}],$$

amounts to only ~5%.

For $l=0$ (circular polarization) formula (18) becomes exact,

$$\Gamma_N = \frac{4}{(2N+1)!}\left(\frac{8F^2}{\omega^4}\right)^N (N\omega-1)^{N+1/2} \tag{21}$$

and agrees with the expression obtained in Refs. 5 and 6. As numerical calculations show, in a linearly polarized field it is convenient to express Γ_N in the form

$$\Gamma_N(l=1) = C_N(F^2/23\omega^3)^N N^6 f_N(N-\nu). \tag{22}$$

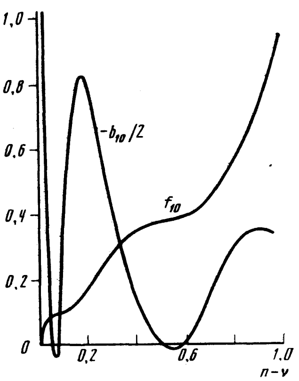

FIG. 2. Frequency dependence of the parameters $F_N(N-\nu)$ and $b_N(N-\nu)$ that determine the probability for N-photon ionization in a linearly polarized field for the case of 10-photon ionization ($N=10$); f_N is the result obtained in the first nonvanishing order of perturbation theory, and b_N is the correction to it.

Here $f_N(N-\nu)$ determines the dependence of Γ_N on the deviation of the frequency from the N-photon threshold $[f(1)=1]$ and C_N is a numerical factor that remains virtually constant ($C_N \approx 2-3$) for the values $N \le 20$ that we are considering. The behavior of f_N is qualitatively the same for all N. The graph of $f_{N=10}$ is shown in Fig. 2 as an example.

As is evident from Eqs. (21) and (22), for large N the ionization probability Γ_N is considerably higher in a linearly polarized field than in a circularly polarized one, even in the most favorable case $N-\nu=1$. We also note that the N dependence of Γ_N for $l=1$ differs significantly from the results given by the quasiclassical approximation[1-4] in the antiadiabatic limit $\gamma \ll 1$. In other words, in the limit in which the level decays via the multiphoton-ionization mechanism, the quasiclassical approach correctly reproduces only the functional dependence of Γ on F: $\Gamma \sim F^{2N}$.

4. LIMITS OF APPLICABILITY OF THE FIRST NONVANISHING ORDER OF PERTURBATION THEORY FOR THE LEVEL WIDTH

The question of the limits of applicability of the first nonvanishing order of perturbation theory for Γ is very important for the analysis of experiments on multiphoton ionization. As was shown in Ref. 10, the perturbation series in an alternating field is convergent, the radius of convergence F_c in the low-frequency case being

$$F_c \lessdot (\omega/2)^{3/2}. \tag{23}$$

However, the critical field F_{cr}, which determines the condition for the applicability of the first term in the expansion of Γ_N with respect to F, depends on the rate of convergence of the series and, generally speaking, is not the same as F_c. The results of the preceding section permit us to investigate this problem using the δ-function well as an example.

The expression for the first correction to Γ_N follows from (15) with $k=0$:

$$\Gamma_N^{(1)} = \left[4\,\mathrm{Im}\,R_{N+1}(E) + 2R_1(E)\frac{\partial\Gamma_N}{\partial E} + \Gamma_N\frac{\partial}{\partial E}(E^2R_1(E))\right]_{E=1}, \tag{24}$$

where $R_n \sim \gamma^{2n}$ is the nth term of expansion (15) with $k=0$. $\Gamma_N^{(1)}$ has a simple form only in the case of a circularly polarized field. Writing

$$\Gamma_N^{(1)} = \frac{8F^2}{\omega^3}\Gamma_N b_N \tag{25}$$

and neglecting the last term in (24), which is small as compared with the preceding terms for all $N>1$, we obtain

$$b_N(l=0) = \frac{[1+(N-\nu)^{-1}]^{N+1/2}(N+1-\nu)}{(2N+2)(2N+3)} - \frac{2N+1}{8(N-\nu)} . \tag{25a}$$

The b_N for $l=1$ were calculated numerically [Fig. 2 shows an example of the dependence of b_N on $(N-\nu)$] and is was established that the N dependence of b_N is weak when $N \le 20$ and that we may assume that

$$|b_N(l=1)| \approx 1. \tag{25b}$$

for all ν except for a narrow near-threshold region where $N-\nu$ is close to zero. The conditions for the applicability of the first nonvanishing order of perturbation theory for the imaginary part of the quasienergy in the case of ionization from a δ-function well follow from Eqs. (25). In a linearly polarized field we have

$$F \ll F_{cr}{}^{l} = (\omega/2)^{3/2} \approx (2N)^{-3/2}, \tag{26a}$$

i.e., here we have exactly the same parameter as determines the radius of convergence (23) of the perturbation series. For a circularly polarized field it follows from (25a) that when $(N-\nu)=1$ and N is large we have

$$F \ll F_{cr}{}^{A} \approx {}^{1}/_{2}(2^{N}N)^{-1/2}, \tag{26b}$$

i.e., the critical field for large N decreases much faster for $l=0$ than for $l=1$. The difference becomes even more substantial when $N-\nu$ is decreased. These results are confirmed by accurate calculations (see Fig. 1).

Thus, the rate of convergence of the perturbation series depends not only on the nature of the specific system, but also on the wave parameters l and ω. Thus, for a circularly polarized field when $N \gtrsim 8$ we have the inequality $F_{cr}^{A} < F_c$, which follows from (23) and (26b). In fields such that $F_{cr}^{A} < F < F_c$, therefore, the correction $\Gamma_N^{(1)}$ exceeds the value of Γ_N given by the first nonvanishing order of perturbation theory, although the entire perturbation series still converges. At the same time, in a linearly polarized field the violation of the conditions for the applicability of $\Gamma_N^{(1)}$ means that perturbation theory cannot be used to calculate the level width when $F \gtrsim F_{cr}^{l}$.

Let us make another remark concerning the applicability of perturbation series to the description of specific experiments. Since the value of Γ_N as measured experimentally cannot be smaller than a certain value Γ_{min} because of instrumental factors, background effects, etc., there is a lower bound to the field strength F:

$$\Gamma_N(F) \geqslant \Gamma_{min}.$$

On the other hand, Γ_N falls off rapidly with increasing N [see Eqs. (21) and (22)] and the greatest permissible field strength (for a given frequency ω) is $F_{max}^{(N)} \approx F_c \sim (2N)^{-3/2}$. Hence perturbation theory for Γ_N cannot be used for all values of N, but only for values smaller than a certain value N_{max} such that

$$\Gamma_N(F_{max}^{(N)}) \geqslant \Gamma_{min}. \tag{27}$$

When $N > N_{max}$ condition (27) is satisfied only for $F > F_c$, so perturbation theory is certainly not applicable to the description of actual experiments involving a high photon multiplicity N. Taking $\Gamma_{min} \sim 10^{-15}|E_0|$ as an estimate and using formulas (21) and (22) for a δ-function well we obtain $N_{max} \approx 10$ for a linearly polarized field and $N_{max} \approx 8$ for a circularly polarized one. The maximum value of N for which inequality (27) still holds is, generally speaking, larger for atoms than for a δ-function well, since the value of Γ_N for atoms substantially exceeds the value of Γ_N for a short-range potential (for the same value of F/F_0). This is also confirmed by the fact that ionization of atoms in fields of strength $F \lesssim F_c$ has been observed experimentally for values of N up to ~20. Thus, we would expect deviations from the power-law dependence of Γ_N on F to be observed at lower photon multiplicities N in experiments on the breakup of negative ions than in experiments on atoms.

5. THE EFFECT OF AN ELECTRIC FIELD ON DECAY STIMULATED BY A NONRESONANT LIGHT WAVE

The problem of decay in the presence of an additional static electromagnetic field can be treated in much the same way as the decay problem in the absence of a static field was treated in Sec. 2. The only complication is a somewhat more cumbersome expression for the Green's function G in Eq. (1). Thus, if the direction of the field vector $\mathcal{F}$ of the static electric field is determined by the angles θ and φ in a coordinate system in which the z and x axes are parallel to the wave propagation direction n and to the major axis of the polarization ellipse of the wave, respectively, the equation for E will again have the form of Eq. (4), but with the following more complicated expression for $S(t,t')$:

$$S_{\mathcal{F}}(t,t') = S_{\mathcal{F}=0}(t,t') - \frac{1}{3}\mathcal{F}^2 t'^3 + \frac{4\gamma}{\omega}\mathcal{F}\Lambda\cos\left(\omega t + \varphi_1 - \frac{\omega t'}{2}\right)\left(t'\cos\frac{\omega t'}{2} - \frac{2}{\omega}\sin\frac{\omega t'}{2}\right), \tag{28}$$

$$\Lambda = \sin\theta(1 + l\cos 2\varphi)^{1/2}, \quad \mathrm{tg}\,\varphi_1 = \xi\,\mathrm{tg}\,\varphi.$$

Expression (28) will be considerably simpler if the two fields are orthogonal ($\Lambda=0$), and if, further, the field $F(t)$ is circularly polarized, the equation for E reduces from an integral equation to a transcendental equation.

The effect of a static field on multiphoton ionization was first discussed qualitatively in Ref. 14. A quantitative treatment for the case of a short-range potential was given in Ref. 15 in the weak-field low-frequency ($F \ll 1$, $\omega \ll 1$) approximation, which, however, cannot be applied to few-photon processes ($\omega \lesssim 1$), for which the effects of a static field turn out to be most important. The use of an additional static field in experiments on the laser spectroscopy of atoms leads to a substantial change in the characteristics of the resonant processes (e.g., in the ion yield in the case of ionization), especially when highly excited states, which are easily disturbed or destroyed even by weak static fields, are involved in the process. Such a scheme was used in experiments[16] on selective ionization, in which high-lying levels are populated by a sequence of resonant cascade transitions from the ground state.

Analysis of the exact equations (4) and (28) shows that the application of a weak field $\mathcal{F}$ may considerably alter the ion yield even in the case of nonresonant ionization, provided the frequency of one or more of the laser photons is close enough to the edge of the continuous spectrum. Then decay can take place via the mechanism of "tunneling from a virtual state": an electron absorbs several (k) photons and tunnels in the field $\mathcal{F}$ with the energy $(E_0 + k\omega) < 0$ close to the top of the barrier. In QES language this mechanism implies tunneling from a QES harmonic. We emphasize that this mechanism can operate only in the presence of two fields—a high-frequency field and a static field—having different amplitudes, and cannot operate when $\mathscr{F}=0$, when tunneling and multiphoton ionization take place only in the mutually exclusive limiting cases $\gamma \gg 1$ and $\gamma \ll 1$, and cannot interfere at all.

The presence of terms corresponding to tunneling from QES harmonics in the expression for the total width $\Gamma = 2\,\mathrm{Im}\,E$ determined by Eqs. (4) and (28) can be easily seen in the case $F \ll 1$, in which the field F can be taken into account by perturbation theory up to the order $N-1$ (by expanding the exponential in (4) in powers of γ) and the remaining integral over t' can be calculated by the saddle point method (provided $\mathscr{F} \ll |(N-1)\omega - 1|^{3/2}$). If, in the equations written below, we retain only terms of the first nonvanishing order in F, we can express the total decay probability Γ in the form

$$\Gamma = \Gamma_N + \sum_{k=1}^{N-1} \Gamma_{k,\mathscr{F}} + \Gamma_{\mathscr{F}}. \tag{29}$$

Here Γ_N is the multiphoton-ionization probability in the field $F(t)$ (see Sec. 3), $\Gamma_{\mathscr{F}} = \mathscr{F}\exp\{-\frac{2}{3}\mathscr{F}\}$ is the tunneling probability in the static field $\mathscr{F}$ (for $F=0$), and the $\Gamma_{k,\mathscr{F}} \sim F^{2k}$ are "interference" terms corresponding to tunneling with the absorption of just $k<N$ photons. The expression for $\Gamma_{k,\mathscr{F}}$ takes the simplest and clearest form in the case of circular polarization of the field $F(t)$ and a static field $\mathscr{F}$ orthogonal to it:

$$\Gamma_{k,\mathscr{F}} = |C^{(k)}|^2 \Gamma_{\mathscr{F}}(E_0 + k\omega),$$
$$|C^{(k)}|^2 = \frac{4}{(2k+1)!}\left(\frac{8F^2}{\omega^4}\right)^k (1-k\omega)^{k+1/2}.$$
$$\Gamma_{\mathscr{F}}(E_0+k\omega) = 4\frac{(2k+1)!}{k!}\left(\frac{\nu_k^3\mathscr{F}}{4}\right)^{1+k} \exp\left(-\frac{2}{3\nu_k^3\mathscr{F}}\right), \quad \nu_k = (1-k\omega)^{-1/2}. \tag{30}$$

Here $|C^{(k)}|^2$ represents the probability for populating the "virtual" state by the action of the wave field, and $\Gamma_{\mathscr{F}}(E_0 + k\omega)$ represents the probability for its decay in the static field $\mathscr{F}$. Expressions analogous to (29) and (30) can also be obtained for a general potential well $U(r)$ by taking the light field into account by perturbation theory out to the $(n-1)$-th order in the QES wave function

$$\Phi(\mathbf{r}, t) = \Psi_{E_0}(\mathbf{r}) + \sum_{k=1}^{N-1} \Phi^{(k)}(\mathbf{r}) e^{-ik\omega t},$$

and then using the quasiclassical approximation to calculate the tunneling probability in the field $\mathscr{F}$.

In the case of a nondegenerate initial (unperturbed) state Ψ_{E_0}, the angular dependence of $\Phi^{(k)}(\mathbf{r})$ can be expressed as a superposition of spherical functions $Y_{LM}(\mathbf{r})$ with $0 \leq L \leq k$. The coefficient $C_{LM}^{(k)} \sim F^k$ in (30) is determined for given L and M by the asymptotic behavior as $r \to \infty$ of the kth harmonic $\Phi_{LM}^{(k)}$ of the QES function (the projection of $\Phi^{(k)}(r)$ onto Y_{LM}):

$$\Phi_{LM}^{(k)}(\mathbf{r}) \sim C_{LM}^{(k)} \Psi_{\varepsilon_k, L}(\mathbf{r}) Y_{LM}(\mathbf{r}),$$

where $\Psi_{\varepsilon_K L} Y_{LM}$ is the solution for the potential $U(r)$ for the energy $\varepsilon_k = E_0 + k\omega$ that is regular at infinity and reduces to the normalized wave function for one of the real excited states when ε_k is equal to the energy of that state. $\Gamma_{\mathscr{F}}(E_0 + k\omega)$ is the probability for tunneling in the field $\mathscr{F}$ from the "virtual" state $\Psi_{k,L}(r) Y_{LM}(\mathbf{r})$, whose dependence on ε_k, L, and M in the field $\mathscr{F}$ is determined by well-known formulas, which are given for the Coulomb and short-range potentials, for example, in Ref. 3. We note that in the general case the probability amplitudes $C_{LM}^{(k)}$ for ionization via states with different L and M values may interfere. The specific values of the angular momentum L and its component M in the direction of the static field that occur in the amplitude $C_{LM}^{(k)}$ are determined by selection rules that depend on the type of polarization of the field $F(t)$ and on the mutual orientation of the fields [in particular, $L=M=k$ in Eq. (30)]. When ε_k is equal to the energy E_i of a real state, $C^{(k)}$ has a pole and $\Gamma_{k,\mathscr{F}}$ reduces to the product of the probability for resonant population of the real level Ψ_{E_1} and the probability for the subsequent decay of that level in the field $\mathscr{F}$ in accordance with the cascade mechanism investigated in Ref. 16.

As an example, let us examine the breakup of the ground state of hydrogen in a linearly polarized field $F(t)$ whose direction makes the angle θ with that of the static field $\mathscr{F}$. For simplicity we shall limit ourselves to the term $\Gamma_{K\mathscr{F}}$ with $k=1$; this will allow us to investigate the most interesting case, in which the direct two-photon ionization channel Γ_2 is open:

$$\Gamma_{1\mathscr{F}}(E_0+\omega) = F^2 \frac{64\nu^3}{(1+\nu)^{12}}\left(\frac{\nu-1}{\nu+1}\right)^{2\nu-6} \Gamma^2(2-\nu) \tag{31}$$
$$\times\{\cos^2\theta + {}^1/_2\mathscr{F}\nu^3\sin^2\theta\}\left(\frac{\nu^3\mathscr{F}}{4}\right)^{1-2\nu} \exp\left\{-\frac{2}{3\nu^3\mathscr{F}}\right\}, \quad \nu = (1-\omega)^{-1/2}.$$

Here the presence of the term proportional to $\mathscr{F}\sin^2\theta$ is associated with the fact that when $\theta = \pi/2$ the angular momentum of the electron after the photon is absorbed has the projection $M = \pm 1$ in the direction of the static field $\mathscr{F}$ while the tunneling probability is proportional to $\mathscr{F}^{|M|}$. The presence of the factor $\mathscr{F}^k$ in Eq. (30) is associated with the same circumstance. The ionization is therefore most efficient when $\theta = 0$.

Let us estimate the enhancement factor for hydrogen. Typical values of Γ_2 for frequencies $1-\omega \approx 0.012$ (≈ 1300 cm^{-1}) in the absence of resonances are $\sim 10^2 F^4$.[17] At these frequencies, as follows from (31), $\nu = 9.1$ and $\Gamma_{1,\mathscr{F}}$ amounts to $\sim 1.0F^2$ in a static field $\mathscr{F} = 45$ kV/cm. In fields $F \sim 10^{-3}-10^{-4}$, which are typical for nonresonant ionization experiments, the enhancement therefore amounts to 4–6 orders of magnitude, i.e., to obtain a given number of ions by applying an additional static field $\mathscr{F}$ it is sufficient to use a field F that is 2–3 orders of magnitude weaker than would be required in the absence of a static field. The difference between $(F)_{\mathscr{F}}$ and $(F)_{\mathscr{F}=0}$ obviously becomes less important in the many-photon case $N \gg 1$ in which $\Gamma_{N-1,\mathscr{F}} \sim F^{2N-2}$ (since a considerable increase in Γ_N can be achieved by a small change in F); hence the mechanism under discussion is most effective for few-photon processes.

[1)]Some of the results of this paper are included in an earlier note.[10]

[2)]The interaction with the field is taken into account in the dipole approximation.

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Translated by E. Brunner

Interference of synchrotron radiation

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(Submitted 21 November 1979)
Zh. Eksp. Teor. Fiz 79, 763–774 (September 1980)

Interference of synchrotron radiation of relativistic electrons, which is synchronized consecutively by the particle beam itself at two points separated by a long straight gap is investigated. The spectral and polarization-angular characteristics of the radiation are studied. Satisfactory agreement between theory and experiment is obtained. It is shown that the interference of synchrotron radiation in installations in which the magnetic field drops off sharply in the straight gap, and in which the electron beam has a small variance of the angular spread of the particles, can be of independent significance, on top of synchrotron and wiggler radiation, for the solution of many scientific and applied problems.

PACS numbers: 41.70. + t

INTRODUCTION

In experiments[1,2] on the properties of wiggler radiation (WR) of relativistic electrons in a magnetic wiggler mounted in one of the straight sections of a synchrotron, it was noted that the synchrotron radiation (R) of the particles in the stray magnetic field at the entrance and exit of the straight gap of the accelerator constitute an undersirable "background" in the observation of the polarization-angular characteristics of the WR (especially at high electron energies). A more detailed investigation of this background has revealed the effect of interference of synchrotron radiation[3] (ISR) in the region where the radiation of the electrons from the far and near ends of the straight gap of the synchrotron overlap. The radiation from these ends of the gap can be represented as the radiation of two quasi-pointlike SR sources the distance between which is L (comparable with the length of the gap), synchronized successively by the electron itself, which moves with velocity $v=\beta c$ (c is the speed of light).

The properties of SR are particles moving along a closed circle were investigated in sufficient detail both theoretically[4,5] and experimentally.[6,7] It turned out, however, that the radiation of the electrons in the direction of the straight gap has certain singularities connected with the interference of the radiation. No attention was paid to this circumstance before, and this question remained uninvestigated. Interest in ISR is raised also by the fact that at the present time more and more attention is being paid to the use of straight gaps of electron synchrotrons and storage rings in order to place in them special magnetic systems (e.g., wigglers), and to generate in these systems, by a relativistic electron beam, radiation that can be used together with SR to solve a large group of scientific and applied problems. It is clear that in such investigations the ISR must also be taken into account. This article presents the results of an investigation of the ISR phenomenon.

1. EXPERIMENTAL PROCEDURE AND SETUP

The ISR was experimentally investigated with the 1.5-GeV "Sirius" electron synchrotron,[8] whose straight

 0038-5646/80/090388-07$02.40