Nonequilibrium current fluctuations in semiconductors in quantizing magnetic fields

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(Submitted 1 November 1979)
Zh. Eksp. Teor. Fiz. **79**, 497–508 (August 1980)

An equation is obtained for the low-frequency fluctuations of the occupation numbers of the quantum states of the electrons of a semiconductor placed in crossed electric and magnetic fields. The correlators of the current-fluctuation sources are calculated. It is shown that magnetic quantization of the electron motion can lead to a change in the sign of the contribution of the carrier-energy fluctuations to the overall noise.

PACS numbers: 71.45.Gm, 72.20.My

Fluctuations in crossed electric and magnetic fields were investigated in Ref. 1, where a theory was developed for the high-frequency case and the low-frequency fluctuations ($\omega \lesssim \nu_{\rm col}$, where $\nu_{\rm col}$ is the characteristic frequency of the electron-phonon collisions) were not considered at all. Yet it is precisely the low-frequency fluctuations that have maximum intensity, so that their characteristics are easier to measure.

Linear equations for the fluctuations of the occupation numbers of the electron quantum states are obtained after averaging the corresponding equations of motion over a set of close levels $\Delta\nu$. The $\Delta\nu$ interval must be large enough to contain a large number of particles, and at the same time small enough to prevent accuracy loss due to the scatter of the quantum numbers within $\Delta\nu$ when quantum-transition processes are described. The idea of averaging of the equations of motion over physically infinitesimal volumes is not new. In particular, in Klimontovich's monograph[2] the Liouville equation for a multiparticle distribution function is averaged over the spatial coordinates.

No final calculation of the fluctuations can be made without a known distribution function of the carriers over the states. We therefore obtain below an explicit expression for the distribution function, in the form of a one-dimensional integral. In the quantum limit ($\hbar\omega_{cy} \gg \bar{p}_z^2/2m \equiv \bar{\varepsilon}_{p_z}$, where ω_{cy} is the cyclotron frequency, m is the electron mass, $\bar{p}_z$ is the thermal momentum in the direction of the magnetic field **H**, and **H**$\|z$), as well as in classically strong fields ($\bar{\varepsilon}_{p_z} \gg \hbar\omega_{cy} \gg \hbar\nu_{\rm col}$) this function coincides with the known expressions in the form of a Maxwellian distribution with an effective electron temperature.

EQUATIONS FOR THE OCCUPATION-NUMBER FLUCTUATIONS

It is convenient to continue the analysis in a representation in which the Hamiltonian of the electrons in crossed fields is diagonal. If we choose the vector potential of the magnetic field in the form $\mathbf{A} = \{-Hy, 0, 0\}$, then the wave function of the electron at **E**$\|y$ is

$$|\nu\rangle = \exp\left\{\frac{i}{\hbar}(p_z z + p_x x)\right\}\Phi_n(y - y_\nu),$$
$$y_\nu = \frac{l^2}{\hbar}\left(p_x - \frac{|e|}{\omega_{cy}}E\right), \quad l^2 = \frac{c\hbar}{|e|H}, \quad \omega_{cy} = \frac{|e|H}{mc}, \tag{1}$$

where Φ_n are the eigenfunctions of the harmonic oscillator, y_ν is the coordinator of the center of the oscillations, and l is the magnetic length.

The electron energy corresponding to the wave function (1) is expressed in terms of n, p_z and p_x as follows:

$$\varepsilon_\nu = \left(n + \frac{1}{2}\right)\hbar\omega_{cy} + \frac{p_z^2}{2m} + \frac{|e|}{\hbar}El^2 p_x. \tag{2}$$

The choice of the Landau representation for the description of the kinetics of electrons in a quantizing magnetic field is the most convenient and natural. The primary reason is that in the course of the collision act (whose duration is $\hbar/\bar{\varepsilon}$) the magnetic alters noticeably the electron momentum (the period of the Larmor rotation is $1/\omega_{cy}$), provided only that $\hbar/\bar{\varepsilon} \gtrsim \omega_{cy}^{-1}$, i.e., $\hbar\omega_{cy} \gtrsim \bar{\varepsilon}$. Therefore inclusion of the magnetic field in the single-particle spectrum greatly simplifies the description of the electron-phonon scattering in the language of quantum transitions.

The Hamiltonian of the system has in the second-quantization representation the form

$$\mathscr{H} = \sum_\nu \varepsilon_\nu a_\nu^+ a_\nu + \sum_{\mathbf{q}} \omega_{\mathbf{q}} b_{\mathbf{q}}^+ b_{\mathbf{q}} - e\sum_{\nu\nu'} \langle\nu|\delta\mathbf{E}\mathbf{r}|\nu'\rangle a_\nu^+ a_{\nu'}$$
$$+ \sum_{\mathbf{q}\nu\nu'} V_{\mathbf{q}} \langle\nu|e^{i\mathbf{q}\mathbf{r}}|\nu'\rangle (b_{\mathbf{q}} + b_{-\mathbf{q}}^+) a_\nu^+ a_{\nu'}, \tag{3}$$

$a_\nu^+, a_\nu, b_q^+, b_q$ are the electron and phonon creation and annihilation operators (electrons in state ν and phonons in state **q**), ω_q is the phonon energy, and $\delta\mathbf{E}$ is the self-consistent field.

The number of electrons in the state ν, which equals $a_\nu^+ a_\nu$ varies with time because of the interaction with the phonons and with the self-consistent field $\delta\mathbf{E}$:

$$\frac{\partial}{\partial t} f_{\nu\nu} = -e\left(\delta E_z \frac{\partial}{\partial p_z} + \delta E_x \frac{\partial}{\partial p_x}\right) f_{\nu\nu} + \frac{el}{2^{1/2}\hbar}[\delta E_+((n+1)^{1/2} f_{\nu,\nu+1} - n^{1/2} f_{\nu-1,\nu})$$
$$+ \delta E_-((n+1)^{1/2} f_{\nu+1,\nu} - n^{1/2} f_{\nu,\nu-1})] + \frac{1}{i\hbar}\sum_{\mathbf{q},\nu'} V_{\mathbf{q}}(b_{\mathbf{q}} + b_{-\mathbf{q}}^+)(f_{\nu\nu'} I_{\nu\nu'}^{\mathbf{q}} - f_{\nu'\nu} I_{\nu'\nu}^{\mathbf{q}}), \tag{4}$$

where $f_{\nu\nu'} = a_\nu^+ a_{\nu'}$, $\delta E_\pm = \delta E_x \pm i\delta E_y$, the index $\nu \pm 1$ denotes the set of quantum numbers $n \pm 1, p, p_z$,

$$I_{\nu\nu'}^{\mathbf{q}} = \langle\nu|e^{i\mathbf{q}\mathbf{r}}|\nu'\rangle \equiv \exp\{iq_y y_{p_x - \hbar q_x/2}\} I_{nn'}^{\mathbf{q}\perp} \delta_{p_x, p_x' + \hbar q_x} \delta_{p_z, p_z' + \hbar q_z},$$

and $\mathbf{q}_\perp$ is a two-dimensional vector in a plane perpendicular to **H**.

The operator products of the type $b_q f_{\nu\nu}$, which enter

 0038-5646/80/080251-06$02.40

in the right-hand side of (4), must be expressed in terms of the phonon and electron occupation numbers. To this end we write down again the equation of motion

$$\left(\frac{\partial}{\partial t}-\frac{i}{\hbar}\Delta^{\mathbf{q}}_{\nu\nu'}\right)b_{\mathbf{q}}f_{\nu\nu'}=\frac{1}{i\hbar}\sum_{\nu_1}\left\{\sum_{\nu_1'}V_{-\mathbf{q}}I^{-\mathbf{q}}_{\nu_1\nu_1'}a_{\nu_1}{}^{+}a_{\nu_1'}a_{\nu}{}^{+}a_{\nu'}\right.$$

$$\left.+\sum_{\mathbf{q}'}V_{\mathbf{q}'}b_{\mathbf{q}}(b_{\mathbf{q}'}+b^{+}_{-\mathbf{q}'})(I^{\mathbf{q}'}_{\nu'\nu_1}f_{\nu\nu_1}-I^{\mathbf{q}'}_{\nu_1\nu}f_{\nu_1\nu'})\right\}+\frac{1}{i\hbar}[b_{\mathbf{q}}f_{\nu\nu'},\mathcal{H}_{\delta\mathbf{E}}], \qquad (5)$$

where $\Delta^{\mathbf{q}}_{\nu\nu'}=\varepsilon_{\nu}-\varepsilon_{\nu'}-\omega_{q}$ and $\mathcal{H}_{\delta\mathbf{E}}$ is the third term in the right-hand side of (3).

The solution of Eq. (5) can be written in the form

$$b_{\mathbf{q}}f_{\nu\nu'}|_{t}=\exp\left\{-\frac{i}{\hbar}\Delta^{\mathbf{q}}_{\nu\nu'}(t-t_0)\right\}b_{\mathbf{q}}f_{\nu\nu'}|_{t_0}+\int\limits_{t_0}^{t}dt'\exp\left\{-\frac{i}{\hbar}\Delta^{\mathbf{q}}_{\nu\nu'}(t-t')\right\}\psi(t'), \qquad (6)$$

$\psi(t)$ is the right-hand side of (5).

In the calculation of the spatially homogeneous fluctuations of the currents it will be necessary to know the occupation-number fluctuations summed over the quantum number p_x (electrons with different Larmor-rotation centers make equal contributions to the total current of the sample). We therefore sum Eqs. (4) and (6) over p_x after first multiplying the latter of $I^{\mathbf{q}}_{\nu\nu'}$.

The second term in the right-hand side of (6) is then greatly simplified if we discard the "strongly nondiagonal" terms in $\psi(t)$ (for details see Ref. 3) and linearize with respect to the fluctuations of the electron and phonon occupation numbers. It should be noted here that these fluctuations are not small, since their mean square is not small compared with the square of the mean values. Indeed, neglecting the weak electron-phonon interaction, we obtain

$$\langle(\delta f_{\nu\nu})^2\rangle=\langle(f_{\nu\nu}-f_{\nu})^2\rangle=f_{\nu}(1-f_{\nu}), \quad f_{\nu}=\langle a_{\nu}{}^{+}a_{\nu}\rangle$$

and in the general case $f_{\nu}(1-f_{\nu})$ is not small compared with f_{ν}^2.

Nonetheless, the linearization can be justified. To this end we average (4) over the set of states p_z within a small interval Δp_z, and average Eq. (6) over p_z and $\mathbf{q}(\Delta p_z \ll \bar{p}_z, \Delta q_{x,y} \ll \bar{q}_{\perp}, \hbar\Delta q_z \sim \Delta p_z, \bar{q}_{\perp}$ is the characteristic wave vector of the phonons that interact effectively with the electrons; $\bar{q}_{\perp}\sim l^{-1}$ in quantizing magnetic fields). If small volumes with dimensions much smaller than the mean free path contain large numbers of particles whose states are within the averaging interval $\Delta\nu$, then the contribution made to $\psi(t)$ by the quadratic terms compared with the contribution of the linear ones. The corresponding estimates can be easily obtained by calculating their mean values and neglecting the electron-phonon interaction.

Linearization is thus possible if the averaging intervals Δp_z and Δq, as well as the mean free paths of the electrons and phonons are large enough (as indicated above).

The rest of the derivation of the equation for δf_{α}

$$\delta f_{\alpha}=\frac{2\pi l^2}{L_xL_y}\sum_{p_x}[(a_{\nu}{}^{+}a_{\nu})_{\Delta p_z}-f_{\nu}],$$

($\alpha=\{n,p_z\}$, L_x, L_y, L_z are the crystal dimensions, the subscrpt Δp_z denotes averaging over the set of the close levels p_z) is standard (see, e.g., Refs. 3 and 4) and, without dwelling on it, we write down the result forthwith:

$$i\omega\delta f_{\alpha}+e\delta E_z\frac{\partial f_{\alpha}}{\partial p_z}=-\hat{\nu}_{\alpha}'\{\delta f_{\alpha},\delta N\}-\delta\mathbf{E}\frac{\partial\hat{\nu}_{\alpha}}{\partial\mathbf{E}}f_{\alpha}+K_{ef}(\omega,\alpha); \qquad (7)$$

$$\hat{\nu}_{\alpha}f_{\alpha}=\sum_{\alpha'}[W_{\alpha\alpha'}f_{\alpha}(1-f_{\alpha'})-W_{\alpha'\alpha}f_{\alpha'}(1-f_{\alpha})], \qquad (8)$$

$$W_{\alpha\alpha'}=\sum_{\mathbf{q}_{\perp}}W^{\mathbf{q}_{\perp}}_{\alpha\alpha'}=\sum_{\mathbf{q}_{\perp}}\frac{2\pi}{\hbar}|V_{\mathbf{q}}I^{\mathbf{q}_{\perp}}_{nn'}|^2[(N_{\mathbf{q}}+1)\delta(\varepsilon_{\alpha}-\varepsilon_{\alpha}'-\omega_{\mathbf{q}}+|e|l^2[\mathbf{q}\times\mathbf{E}]_z)$$

$$+N_{-\mathbf{q}}\delta(\varepsilon_{\alpha}-\varepsilon_{\alpha'}+\omega_{-\mathbf{q}}+|e|l^2[\mathbf{q}\times\mathbf{E}]_z)], \quad \hbar q_z=p_z-p_{z'}, \qquad (9)$$

$\hat{\nu}_{\alpha}'\{\delta f_{\alpha},\delta N\}$ is the collision integral (8) linearized with respect to δf and δN.

Equation (9) is valid for any orientation of $\mathbf{E}$ in the xy plane, whereas (1) and (2) were derived for a special coordinate system in which $\mathbf{E}\| y$. The equation for δf_{α} at $\delta E_{x,y}=0$ and $\mathbf{E}\|\mathbf{H}$ was used by Rozhkov[5] to investigate the current fluctuations along $\mathbf{H}$. As expected, the fluctuating electric field $\delta E_{x,y}$ influences δf_{α} only in the course of the collisions, so that between the collisions the electron drift perpendicular to $\mathbf{H}$ and to the total field $\mathbf{E}+\delta\mathbf{E}_{\perp}$ without change in their state.

In Eq. (7), $K_{ef}(\omega,\alpha)$—the extraneous flux—is the source of the fluctuations. It stems from allowance for the initial conditions when the collisions of the electron with the phonon are described [the first term in the right-hand side of (6)]. The correlation function of the source can be easily obtained in view of the simple time dependence of the quantities contained in K_{ef}. For quasiclassical fluctuations, in the Born approximation in the interaction constant (it is precisely in this case that the collision integral $\hat{\nu}_{\alpha}'$ has the simple form indicated above) the correlator of the extraneous current is

$$\langle K_{ef}(t,\alpha)K_{ef}(t',\alpha')\rangle_{\omega}=\frac{l^2}{L_xL_y}\left\{\delta_{\alpha,\alpha'}\sum_{\alpha''}[W_{\alpha\alpha''}f_{\alpha}(1-f_{\alpha''})\right.$$

$$\left.+W_{\alpha''\alpha}f_{\alpha''}(1-f_{\alpha})]-W_{\alpha\alpha'}f_{\alpha}(1-f_{\alpha'})-W_{\alpha'\alpha}f_{\alpha'}(1-f_{\alpha})\right\}, \qquad (10)$$

where the Fourier transformation is defined as

$$f_{\omega}=\int\limits_{-\infty}^{+\infty}\frac{dt}{2\pi}e^{-i\omega t}f(t).$$

We disregard hereafter the fluctuations of the phonon distribution function, i.e., we assume that $\delta N=0$ in (7). We note that the influence of the phonon fluctuations on the electrons without a magnetic field was considered earlier in Ref. 6. A generalization of the theory presented their to include the case of quantizing magnetic fields entails no particular difficulty.

Equation (7) in conjunction with the correlator (10) will be used later in the calculation of the current fluctuations in the sample.

CURRENT FLUCTUATIONS

The expression for the spatially homogeneous component of the current density in a crystal is of the form

$$\mathbf{j}=\frac{e}{V}\sum_{i}\dot{\mathbf{r}}_i; \quad \dot{\mathbf{r}}_i=-\frac{i\hbar}{m}\frac{\partial}{\partial\mathbf{r}_i}-\frac{e}{mc}\mathbf{A}(\mathbf{r}_i), \qquad (11)$$

where the subscript i numbers the electrons. In the second-quantization representation we get from (11)

$$j_-=(j_+)^+=-\frac{2^{1/2}e\hbar}{Vml}\sum_\nu\left((n+1)^{1/2}f_{\nu,\nu+1}+\frac{eEl}{2^{1/2}\hbar\omega_{cy}}f_{\nu\nu}\right), \qquad (12)$$

$$j_z=\frac{e}{mV}\sum_\nu p_zf_{\nu\nu}, \quad j_\pm=j_x\pm ij_y.$$

It follows from (12) that the current fluctuations ($\delta\mathbf{j}=\mathbf{j}-\langle\mathbf{j}\rangle$) can be determined if the functions δf_α and $\delta f_{\alpha\pm1,\alpha}$ are known:

$$\delta f_{\alpha\pm1,\alpha}=\frac{2\pi l^2}{L_xL_y}\sum_{p_x}[(f_{\nu\pm1,\nu})_{\Delta p_z}-\langle f_{\nu\pm1,\nu}\rangle].$$

The latter, as will be seen below, can be expressed in terms of δf_α and extraneous current analogous to K_{ef}. This is easily done after writing down the equation for $f_{\nu\pm1,\nu}$

$$\left(\frac{\partial}{\partial t}+i\omega_{cy}+e\delta E_z\frac{\partial}{\partial p_z}+e\delta E_x\frac{\partial}{\partial p_x}\right)f_{\nu,\nu+1}$$
$$=\frac{el}{2^{1/2}\hbar}[\delta E_+((n+2)^{1/2}f_{\nu,\nu+2}-n^{1/2}f_{\nu-1,\nu+1})+\delta E_-(n+1)^{1/2}(f_{\nu+1,\nu+1}-f_{\nu\nu})]$$
$$+\frac{1}{i\hbar}\sum_{\mathbf{q}\nu_1}V_\mathbf{q}(b_\mathbf{q}+b_{-\mathbf{q}}^+)(I^\mathbf{q}_{\nu+1,\nu_1}f_{\nu\nu_1}-I^\mathbf{q}_{\nu_1\nu}f_{\nu_1,\nu+1}). \qquad (13)$$

It is convenient to determine not $f_{\nu,\nu+1}$ but directly the sum

$$\sum_\nu(n+1)^{1/2}f_{\nu,\nu+1},$$

which enters in j_-. We multiply for this purpose (13) by $(n+1)^{1/2}$ and sum both sides of the equation over all ν. The result is

$$\left(\frac{\partial}{\partial t}+i\omega_{cy}\right)\sum_\nu(n+1)^{1/2}f_{\nu,\nu+1}=-\frac{el}{2^{1/2}\hbar}\delta E_-\sum_\nu f_{\nu\nu}$$
$$+\frac{il}{2^{1/2}\hbar}\sum_{\mathbf{q}\nu\nu'}V_\mathbf{q}I^\mathbf{q}_{\nu\nu'}q_-(b_\mathbf{q}+b_{-\mathbf{q}}^+)f_{\nu\nu'}, \quad q_\pm=q_x\pm iq_y. \qquad (14)$$

The products of three operators of the type $b_qf_{\nu\nu'}$, contained in (14), were already determined in the derivation of Eq. (7) [see (6)]. It remains only to substitute the equations obtained there in (14), and substitute (14) in (12). As a result we obtain for the low-frequency case ($\omega\ll\omega_{cy}$)

$$\delta j_-=\left(\delta\mathbf{E}\frac{\partial}{\partial\mathbf{E}}+\delta n_{el}\frac{\partial}{\partial n_{el}}\right)(-i)\frac{e^2n_{el}E_-}{m\omega_{cy}}+\sum_\alpha\frac{\delta\langle j_-\rangle}{\delta f_\alpha}\delta f_\alpha+\delta j_-^{ext}, \qquad (15)$$

where n_{el} is the average electron concentration; the average current $\langle j_-\rangle$ is

$$\langle j_-\rangle=-\frac{ie^2n_{el}E_-}{m\omega_{cy}}+i\frac{e}{2\pi L_z}\sum_{\alpha\alpha'\mathbf{q}_\perp}W^{\mathbf{q}_\perp}_{\alpha\alpha'}q_-f_\alpha(1-f_{\alpha'}), \qquad (16)$$

$\delta\langle j_-\rangle/\delta f_\alpha$ is the variational derivative of the second term in the right-hand side of (16) with respect to the small change of the occupation number f_α; j_-^{ext} is the extraneous current, expressed in terms of the electron and phonon operators as

$$\delta j_-^{ext}(t)=-\frac{e}{Vm\omega_{cy}}\sum_{\nu\nu'\mathbf{q}}V_\mathbf{q}I^\mathbf{q}_{\nu\nu'}q_-\left[\exp\left(-\frac{i}{\hbar}\omega_\mathbf{q}(t-t_0)\right)b_\mathbf{q}|_{t_0}\right.$$
$$\left.+\exp\left(\frac{i}{\hbar}\omega_{-\mathbf{q}}(t-t_0)\right)b_{-\mathbf{q}}^+|_{t_0}\right]\exp\left(\frac{i}{\hbar}(\varepsilon_\nu-\varepsilon_{\nu'})(t-t_0)\right)f_{\nu\nu'}|_{t_0}. \qquad (17)$$

Since all the operators are specified at one and the same instant of time (t_0), we can easily calculate the correlator of the extraneous current. Its value in the quasiclassical case is

$$\langle\delta j_-^{ext}(t)\delta j_+^{ext}(t')\rangle_\omega=\frac{1}{L_zV}\left(\frac{el}{2\pi}\right)^2\sum_{\alpha\alpha'\mathbf{q}_\perp}W^{\mathbf{q}_\perp}_{\alpha\alpha'}q_\perp^2f_\alpha(1-f_{\alpha'}) \qquad (18)$$

(in isotropic scattering the correlators $\langle\delta j_-^{ext}\delta j_-^{ext}\rangle$ and $\langle\delta j_+^{ext}\delta j_+^{ext}\rangle$ are equal to zero).

It follows from the foregoing that the current fluctuations can be easily calculated if the fluctuations of the occupation numbers and the self-consistent field $\delta\mathbf{E}$ are known.

The quantities δf_α and $\delta\mathbf{E}$ are determined from Eq. (7) and from the equations for the current flow through the electric circuit in which the investigated semiconductor is connected. In addition, in concrete calculations it is necessary to know the electron distribution function f_α. An analytic expression for this function is known only in the cae of quasi-elastic scattering by acoustic phonons, when the electrons populate only the lowest Landau level. We obtain here an expression for f_α without confining ourselves to the quantum limit.

ELECTRON DISTRIBUTION FUNCTION

In the nondegenerate case the equation for f_α takes the form

$$\hat{\nu}_\alpha f_\alpha=\sum_{\alpha'}(W_{\alpha\alpha'}f_\alpha-W_{\alpha'\alpha}f_{\alpha'})=0. \qquad (19)$$

Expanding (19) in the small parameters (the phonon energy and the electric field), we obtain the following equation for $f_\alpha=f^n_{p_z}$:

$$\hat{\nu}_\alpha f_\alpha=\frac{2\pi}{\hbar}\sum_{\mathbf{q}n'}|V_\mathbf{q}I^{\mathbf{q}_\perp}_{nn'}|^2\left\{(f_{p_z}^{n}-f^{n'}_{p_z-\hbar q_z})(e^{\delta_1}+1)\cdot\right.$$
$$\left.\cdot\left(1+\frac{\delta_2^2}{2}\frac{\partial^2}{\partial p_z^2}\right)-(f_{p_z}^n+f^{n'}_{p_z-\hbar q_z})\delta_1\delta_2\frac{\partial}{\partial p_z}\right\}\delta(\varepsilon^n_{p_z}-\varepsilon^{n'}_{p_z-\hbar q_z})=0; \qquad (20)$$
$$\delta_1=\frac{\omega_\mathbf{q}}{T},\quad \delta_2=\frac{2m}{\hbar q_z}(\omega_\mathbf{q}-|e|l^2[\mathbf{qE}]_z).$$

where T is the lattice temperature.

We note that the expansion (20) is valid only for energy values such that

$$|\varepsilon_\alpha-\hbar\omega_{cy}(n+1/2)|>\omega_\mathbf{q}.$$

We assume next that the number of electrons in the vicinities of the points $\varepsilon_\alpha=\hbar\omega_{cy}(n+\frac{1}{2})$ is small and that they make no substantial contribution to the kinetic phenomena (for more details see Ref. 7). If we neglect in (20) the terms with small parameters δ_1 and δ_2, then the kinetic equation is satisfied by any function of the energy, i.e.,

$$f_{p_z^n}=f_\varepsilon;\quad \varepsilon=p_z^2/2m+\hbar\omega_{cy}(n+1/2).$$

This part of the collision integral does not take into account the change of the electron energy in the collisions with the phonons.

The explicit form of f_ε can be easily determined once the large terms that describe the elastic collisions are eliminated from (20). This is done by averaging ν_α

over all the values of n and p_n at the given energy ε.[1)] To this end we multiply (20) by $mg_\varepsilon^{-1}/|p_n|$($g_\varepsilon$ is the state density, equal to $2m\sum_{n'}|p_{n'}|^{-1}$) and sum over all values of n and directions of p_n. In the equation obtained in this manner all the terms are of second order in E and ω_q. It follows in addition from (20) that the differences $f_{pn}^n - f_{pn'}^{n'}$ are proportional to $\delta_1\delta_2$ or δ_2^2. Therefore, without loss of accuracy, we can put $f_{pn}^n = f_\varepsilon$ in the obtained equation, which then takes the form

$$-\frac{4\pi}{\hbar}g_\varepsilon^{-1}\frac{\partial}{\partial\varepsilon}\sum_{qnn'}|V_q I_{nn'}^{q_\perp}|^2 N_q\frac{m}{|p_n|}\left\{(e^2l^2[\mathbf{q}\times\mathbf{E}]_z^2+\omega_q^2)\frac{\partial f_\varepsilon}{\partial\varepsilon}+\frac{\omega_q^2}{T}f_\varepsilon\right\}\delta(\varepsilon-\varepsilon_{p_n-\hbar q_z}^{n'})=0 \quad (21)$$

or in abbreviated form $\hat{\nu}_\varepsilon f_\varepsilon = 0$.

With the aid of the operator $\hat{\nu}$ we can express also the correlator (10) averaged over all values of n and p_n for given ε and ε'. We note for this purpose that in the nondegenerate case Eq. (10) is expressed in terms of $\hat{\nu}_\alpha$ in the following manner:

$$\langle K(t,\alpha)K(t',\alpha')\rangle_\omega=\frac{l^2}{L_xL_y}(\hat{\nu}_\alpha+\hat{\nu}_{\alpha'})\delta_{\alpha,\alpha'}f_\alpha. \quad (22)$$

Averaging yields

$$g_\varepsilon^{-1}g_{\varepsilon'}^{-1}\sum_{\substack{nn'\\ p_np_{n'}}}\frac{m^2}{|p_np_{n'}|}\langle K(t,\alpha)K(t',\alpha')\rangle_\omega\equiv\langle K(t,\varepsilon)K(t',\varepsilon')\rangle_\omega$$
$$=\frac{2\pi}{V}\hbar l^2(\hat{\nu}_\varepsilon+\hat{\nu}_{\varepsilon'})\frac{f_\varepsilon}{g_\varepsilon}\delta(\varepsilon-\varepsilon'). \quad (23)$$

Equation (21) with the boundary conditions f_ε, $\partial f_\varepsilon/\partial\varepsilon = 0$ at $\varepsilon=\infty$ can be easily solved, since it reduces after a singe integration with respect to ε to a linear differential equation of first order with variable coefficients. The latter depend on the explicit form of V_q. If $|V_q|^2 = V_0 q$, then (24)

$$\hat{\nu}_\varepsilon=-\frac{4Vm^2s}{\pi\hbar^3l^2}V_0Tg_\varepsilon^{-1}\frac{\partial}{\partial\varepsilon}\left\{g_\varepsilon\sum_n\frac{1}{|p_n|}\left[\varepsilon\left(\frac{\partial}{\partial\varepsilon}+\frac{1}{T}\right)+\tilde{E}^2(2n+1)\frac{\partial}{\partial\varepsilon}\right]\right\},$$

where $\tilde{E}^2=(eEl)^2/4ms^2$ and s is the speed of sound.

In this case the distribution function is

$$f_\varepsilon=\text{const}\exp\left\{-\int_{\hbar\omega_{cy}/2}^{\varepsilon}d\varepsilon'\frac{\varepsilon'}{T}\sum_n|p_n'|^{-1}\Big/\sum_{n'}\frac{\varepsilon'+\tilde{E}^2(2n'+1)}{|p_{n'}'|}\right\},$$

the constant preceding the exponential is determined by the total number of electrons in the sample. At the zero Landau level ($\varepsilon<\frac{3}{2}\hbar\omega_{cy}$) f_ε takes the form

$$f_\varepsilon=\text{const}(\varepsilon+\tilde{E}^2)^{\tilde{E}^2/T}e^{-\varepsilon/T}$$

and coincides with the function obtained by Zlobin and Zyryanov.[8] In the quantum limit ($\bar{\varepsilon}-\frac{1}{2}\hbar\omega_{cy}\ll\hbar\omega_{cy}$), as well as in the case of classically strong fields ($\bar{\varepsilon}\gg\hbar\omega_{cy}$), f_ε is Maxwellian with an effective temperature T_e equal to $T(1+2\tilde{E}^2/\hbar\omega_{cy}$ in the first case and $T(1+4\tilde{E}^2/3\hbar\omega_{cy})$ in the second.

EFFECT CARRIER ENERGY DISTRIBUTION FLUCTUATIONS ON THE CURRENT FLUCTUATIONS

The contribution of the fluctuations of the occupation numbers to $\delta f_{x,y}$ is described by the second term in the right-hand side of (15), and a nonzero contribution is made only by that part of the function δf_α which is even in p_n, i.e.,

$$\delta f_\alpha^c={}^1/_2(\delta f_{p_n}^n+\delta f_{-p_n}^n).$$

We obtain an equation for δf^c by adding (7) to the equation for δf_{-pn}^n

$$i\omega\delta f_\alpha^c=-\hat{\nu}_\alpha\left(\delta f_\alpha^c-\frac{\partial f_\alpha}{\partial\mathbf{E}}\delta\mathbf{E}-cf_\alpha\right)+K_{ef}^c(\omega,\alpha). \quad (25)$$

In the derivation of (25) we used the fact that $\hat{\nu}_\alpha f_\alpha=0$, as well as the identity

$$\frac{\partial\hat{\nu}_\alpha}{\partial\mathbf{E}}f_\alpha+\hat{\nu}_\alpha\frac{\partial f_\alpha}{\partial\mathbf{E}}=\frac{\partial}{\partial\mathbf{E}}(\hat{\nu}_\alpha f_\alpha)=0.$$

We define the arbitrary constant c such as to satisfy the sum rule for the functions on which the operator $\hat{\nu}_\alpha$ acts:

$$\sum_\alpha\left(\delta f_\alpha-\frac{\partial f_\alpha}{\partial\mathbf{E}}\delta\mathbf{E}-cf_\alpha\right)=0.$$

It follows from this condition that $c=\delta n_{el}/n_{el}$.

Equation (25) is easily solved in the case of low frequencies by iteration with respect to the small parameter $\omega\nu_\alpha^{-1}$. This method was extensively used in Ref. 9 to solve similar problems. The smallness of $\omega\nu_\alpha^{-1}$ is ensured if $\omega\tau_\varepsilon$ is small (τ_ε is the characteristic energy relaxation time and is determined by the collision integral $\hat{\nu}_\varepsilon$), since τ_ε is much longer than the elastic-relaxation time ν_{col}^{-1}.

In the zeroth approximation we get

$$\delta f_\alpha^c=\frac{\partial f_\alpha}{\partial\mathbf{E}}\delta\mathbf{E}+\frac{\partial f_\alpha}{\partial n_{el}}\delta n_{el}+\hat{\nu}_\alpha^{-1}\{K_{ef}\}, \quad (26)$$

and the uniqueness of the operation $\hat{\nu}_\alpha^{-1}$ is ensured by the sum rule cited above (for details see Ref. 10). We separate in the solution (26) the part that depends only on energy. To this end, as before, we average (25) over the different states at the specified energy ε. This leaves on the obtained equation only the inelastic part of the collision integral, and in this part we can replace δf_α^c by δf_ε. The rapidly relaxing part, equal to $\delta f_\alpha^c-\delta f_\varepsilon$, is small when $\nu_{col}^{-1}\to 0$. In fact, the difference $\delta f_\alpha^c-\delta f_\varepsilon$ is of the order of

$$\nu_{elast}^{-1}K_{ef}^{elast}\sim\nu_{col}^{-1}K_{ef}^{elast}\sim(\nu_{col}^{-1})^{1/2}.$$

Consequently

$$\delta f_\alpha^c\approx\delta f_\varepsilon=\frac{\partial f_\varepsilon}{\partial\mathbf{E}}\delta\mathbf{E}+\frac{\partial f_\varepsilon}{\partial n_{el}}\delta n_{el}+\hat{\nu}_\varepsilon^{-1}K(\varepsilon). \quad (27)$$

The last term of (27) is the solution of the equation $\nu_\varepsilon\psi_\varepsilon=K(\varepsilon)$, and the sought function ψ_ε satisfies the conditions $\psi_\varepsilon=0$ at $\varepsilon=0$ and $\sum_\alpha\psi_\varepsilon=0$.

Substituting (27) in (15) we obtain

$$\delta j_-=\left(\delta\mathbf{E}\frac{\partial}{\partial\mathbf{E}}+\delta n_{el}\frac{\partial}{\partial n_{el}}\right)\langle j_-\rangle+\delta j_-^E+\delta j_-^{ext}; \quad (28)$$

$$\delta j_-^E=\frac{ie}{2\pi L_z}\sum_{\alpha\alpha' q_\perp}W_{\alpha\alpha'}^{q_\perp}q_-\hat{\nu}_\varepsilon^{-1}K(\varepsilon). \quad (29)$$

It is easily seen that δj_-^E differs from zero only when $E\neq 0$. In the isotropic case δj_-^E is proportional to E_- and the vector $\delta\mathbf{j}_-^E$ is directed along the dissipative component of the current. This term is the contribution made to $\delta\mathbf{j}$ by the fluctuations of the energy distribution of the particles, i.e., the convective noise, which was first investigated in the absence of a magnetic field in Ref. 11. We now compare, in order of magnitude, the correlators $\langle\delta j_+^{ext}(t')\delta j_-^{ext}(t)\rangle$ and $\langle\delta j_+^E(t)\delta j_-^E(t')\rangle$. To this end we replace in (18)

$\sum_{\alpha' q_\perp} W_{\alpha\alpha'}^{q_\perp}$ by ν_{col} and $\sum_\alpha f_\alpha = n_{el} L_z 2\pi l^2$, and in (29)

$\sum_{\alpha' q_\perp} W_{\alpha\alpha'}^{q_\perp} q_-$ by

$$\sum_{\alpha' q_\perp} W_{\alpha\alpha'}^{q_\perp} \frac{el^2[\mathbf{qE}]_z q_-}{\varepsilon_{p_z}} \sim \frac{1}{2}\frac{eE\nu_{col}}{\varepsilon_{p_z}}$$

and $\hat{\nu}_\varepsilon^{-1} \to \tau_\varepsilon$. Then

$$\langle\delta j_+^{ext}\delta j_-^{ext}\rangle_\omega \sim \frac{n_3 l^2 e^2}{2\pi V}\nu_{col}, \quad \langle\delta j_+^E \delta j_-^E\rangle_\omega \sim \frac{n_3 l^2 e^2}{2\pi V}\nu_{col}^2\tau_\varepsilon\frac{(eEl)^2}{\varepsilon_{p_z}^2}.$$

We see that the ratio of the convective current to the noise due to $\delta\mathbf{j}^{ext}$ is of the order of

$$(eEl)^2/[(eEl)^2+\omega_q^2],$$

i.e., it is large enough only in the nonequilibrium case, when electron heating takes place.

Assume that $\mathbf{E}\| y$. Then $\delta j_x^E = 0$, and $\delta j_y^E \neq 0$. It is important to note that the mixed correlators $\langle\delta j_y^E\delta j_y^{ext}\rangle$ differ from zero and contribute, alongside with $\langle(\delta j_y^{ext})^2\rangle$ and $\langle(\delta j_y^E)^2\rangle$ substantially to the total correlation function. We present here a general expression for the correlator of the summary source ($\delta\mathbf{j}^t = \delta\mathbf{j}^{ext} + \delta\mathbf{j}^E$) at an arbitrary degree of inelasticity of the electron-phonon collisions

$$\langle(\delta j_y^t)^2\rangle_\omega = \frac{1}{L_z V}\left(\frac{le}{2\pi}\right)^2 \sum_{\alpha\alpha' q_\perp} W_{\alpha\alpha'}^{q_\perp}\Big[q_x^2 f_\alpha + 2q_x\hat{\nu}_\alpha^{-1}\sum_{q_\perp'\alpha_1} q_x' f_{\alpha_1} \times \Big(W_{\alpha_1\alpha}^{q_\perp'} - f_\alpha(n_{el}\cdot 2\pi l^2 L_z)^{-1}\sum_{\alpha_1'} W_{\alpha_1\alpha_1'}^{q_\perp'}\Big)\Big], \quad (30)$$

$$\langle(\delta j_x^t)^2\rangle_\omega = \frac{1}{L_z V}\left(\frac{le}{2\pi}\right)^2 \sum_{\alpha\alpha' q_\perp} W_{\alpha\alpha'}^{q_\perp} q_y^2 f_\alpha. \quad (31)$$

The second term in the square brackets of (30) is the contribution of the fluctuations of the symmetrical part of the distribution function. We obtain its relative value in quasielastic scattering of the type described above:

$$\frac{\langle\delta j_y^E(\delta j_y^E + 2\delta j_y^{ext})\rangle_\omega}{\langle(\delta j_y^{ext})^2\rangle_\omega} = \frac{T_e - T}{T_e}\begin{cases}\left(1-\frac{8}{3\pi}\right)z + \frac{4}{3} - 4\ln 2 + \frac{2-\pi^2/6}{z}, & \hbar\omega_{cy} \gg T_e,\\ 1{,}16, & \hbar\omega_{cy} \ll T_e\end{cases} \quad (32)$$

where $z = \ln(4\sqrt{2}T_e/|e|El) - 1 - \frac{1}{2}C, C$, and C is the Euler constant.

When integrating with respect to p_z in (30) we have neglected the phonon energy, but in the probabilities $W_{\alpha\alpha}^{q_\perp'}$ we took exact account of the change of the electron energy on account of its displacement along the electric field in the collision act, since δj_y^E differs from zero only at $E_y \neq 0$.

It follows from (32) that in the classical case the fluctuations of the electron energy make a positive contribution to the total noise, whereas in the quantum region this contribution is determined by the value of the parameter $T_e/|e|El$. The convective term is significant only at $(T_e - T)/T_e \sim 1$ (when $|e|El \gtrsim \bar{\omega}_q$) and it can therefore be assumed in the estimate that the parameter $T_e/|e|El$ does not exceed

$$\frac{T_e}{\bar{\omega}_q} \sim \left(\frac{T_e}{\hbar\omega_t}\right)^{1/2}\frac{\bar{v}_z}{s}$$

and that in real situation z is equal to several times one. The allowance for the energy fluctuations then decreases the total noise.

Thus, the quantization of the electron motion in the magnetic field changes the relative positive of the low-frequency ($\omega < \tau_\varepsilon^{-1}$) and high-frequency ($\nu_{col} > \omega > \tau_\varepsilon^{-1}$) plateaus. The physical cause of the reversal of the sign of the convective increment is obvious: in the classical region the total current is proportional to $T_e^{1/2}$, as against $-T_e^{-3/2}$ in the quantum region. Therefore the convective increment to the noise current, which is of the order of $(\partial j_y/\partial T_e)\delta T_e$($\delta T_e$ is the fluctuation of the electron temperature) has different signs in these two cases.

Experimental observation of convective noise in monopolar semiconductors is made difficult by the appearance of Hall fields and currents. To simplify the interpretation of the experiments one can use long samples in the direction of the current-carrying contacts (in the x direction). Then in regions far enough from the contacts the electric field E_y, which is equal to

$$-\frac{\sigma_{yx}}{\sigma_{yy}}E_x \sim \frac{\omega_{cy}}{\nu_{col}}E_x,$$

is much stronger than the applied field E_x. Assume that the total number of the carriers does not fluctuate ($\delta n_{el} = 0$), and the impedance of the external circuit to the alternating fluctuation current is infinitely large, i.e., $\delta j_x = \delta j_y = 0$. We then obtain that the fluctuations $\delta E_\perp$ are equal to

$$\delta E_x = -\frac{1}{\sigma_{yx}}(\delta j_y^{ext} + \delta j_y^E), \qquad \delta E_y = \frac{\delta j_x^{ext}}{\sigma_{yx}}, \quad (33)$$

i.e., under the same experimental setup the convective noise contributes to the fluctuations of the field along the flowing direct current. On the other hand if $\delta j_y = 0$, $\delta E_x = 0$ (the sample is short circuited with respect to the ac component in the x direction), then

$$\delta E_y = -(\delta j_y^{ext} + \delta j_y^E)\left[\frac{\partial}{\partial E_y}\sigma_{yy}E_y\right]^{-1}, \quad \delta j_x = \sigma_{xy}\delta E_y. \quad (34)$$

In this case the convective noise is present in the field fluctuations δE_y and in the current fluctuations δj_x. Comparing the expressions for δE_y in (33) and in (34), we see that their correlators in the second case are $(\omega_{cy}/\nu_{col})^2$ times larger than in the first. The reason is that the electron flux that produces the current δj_x is acted upon by a Lorentz force in the y direction. But since the current δj_y does not flow, a fluctuating field δE_y is produced in the sample and balances the Lorentz force.

If the sample is in a waveguide having a wave resistance equal to the weak-signal resistance of the sample, then the total current will equal half the short-circuit current, i.e., δj_x will equal hafl the value given by Eq. (34) (see Ref. 12).

We note in conclusion that the expressions obtained above for the correlators of the extraneous currents can be used to determine the Hermitian components of the diffusion tensor with the aid of the relation

$$\langle \delta j_\alpha{}^t \delta j_\alpha{}^t \rangle_\omega = \frac{n_{el} e^2}{\pi V} \Delta_{\alpha\alpha}.$$

[1)]For the sake of convenience we shall write p_n in place of p_z in the transformations that follow.

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Translated by J. G. Adashko

Domain structure of uniaxial ferrimagnets with a compensation point in strong magnetic fields

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(Submitted 6 November 1979)
Zh. Eksp. Teor. Fiz. **79**, 509–527 (August 1980)

Results are presented of theoretical and experimental investigations of the domain structure of uniaxial ferrimagnets of finite dimensions during second-order phase transitions in the vicinity of the magnetic compensation point. The theoretical analysis is carried out for the magnetic-field range from zero to the "flip" field of the magnetization vectors of the sublattices. Domains in fields above 100 kOe were observed on the "Solenoid" installation at the P.N. Lebedev Physical Institute of the Academy of Sciences, USSR.

PACS numbers: 75.50.Gg, 75.60.Ch

INTRODUCTION

Investigation of orientational second-order phase transitions induced in uniaxial magnetic materials by a magnetic field H perpendicular to the axis of easy magnetization (AEM) is of great interest both for physics and for applications. In theoretical papers[1,2] devoted to the analysis of the nucleation of a domain structure in the vicinity of such transitions in plates of uniaxial ferromagnets (with **n**‖AEM, where **n** is the normal to the plate surface), it was shown that because the static susceptibility of the crystal increases on approach to the phase-transition point (line), the effect of the demagnetizing field on the distribution of magnetization within the plate is considerably enhanced. Nevertheless, in the vicinity of lines of second-order phase transition the presence of a small parameter (the amplitude of the magnetization within a domain) makes it possible to find all the parameters of the domain structure directly from the equations of state and the equations of magnetostatics, with allowance for the boundary conditions on the magnetization and on the magnetic field, without resorting to any model-type assumptions. It should be noted that allowance for the nonuniformity both along the length and along the thickness of the plate is in principle important both for determination of the type of phase transition and for calculation of the temperature and field dependences of the parameters that characterize the nonuniform state. In other theoretical papers in this direction, the authors have determined the limits of stability of the uniform magnetic state[3-6] and have also, on the basis of model representations of the nature of the nucleating domain structure, similar to those of Ref. 7, calculated certain parameters of the structure. Obviously, according to the considerations indicated above, the last-mentioned results are correct only at a sufficient distance from the transition point.

In the present paper, we carry out a theoretical and experimental investigation of the domain structure of uniaxial ferrimagnets with a compensation point. In such magnets, as was first shown in Ref. 8 (see also Ref. 9), for an infinite medium a noncollinear state originates at arbitrarily small fields; this shows up especially clearly in the vicinity of the magnetic com-

 0038-5646/80/080256-05$02.40