provided the trapping interval Δk_x covers the whole of the energy-bearing part of the spectrum. In the case of small soliton amplitudes the trapping interval decreases rapidly and the attentuation of a soliton is described by the relationship found in Ref. 2.

We have considered only the attenuation of an ion-acoustic soliton under the action of plasma waves. Clearly, this action may also amplify a soliton if the plasma contains high-intensity short Langmuir waves with $k_x > k_*$.

We shall conclude by noting that we can consider similarly the problems of transformation of Langmuir waves under the action of collisionless shock waves, and also the interaction of ion-acoustic and electromagnetic waves.

The authors are grateful to V. I. Karpman and to G. M. Fraĭman for discussing the results.

[1)]We are considering here the initial-value problem. However, similar results can also be obtained in the boundary-value problem.

[2)]Similar problems have been considered also for other types of waves, in particular, for electromagnetic waves in a plasma.[4]

[3)]In the case of a single reflection of plasma waves from an ion-acoustic soliton there is a considerable increase in k only for waves with a small value of $|k_x|$. However, if the reflection process is of multiple nature (as is the case for two ion-acoustic solitons moving toward one another), monotonic pumping of energy of Langmuir plasmons up the spectrum is possible.

[4)]The procedure of going over from Eq. (15) to the explicit form of the law of conservation of the momentum (17) was found in cooperation with V. I. Karpman.

[1]A. A. Galeev, R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 25 (1976) [JETP Lett. **24**, 21 (1976)].

[2]A. Ya. Basovich and E. M. Gromov, Fiz. Plasmy **5**, 833 (1979) [Sov. J. Plasma Phys. **5**, 466 (1979)].

[3]A. Ya. Basovich and E. M. Gromov, Tezisy dokladov V Vsesoyuznoĭ konferentsii po fizike nizkotemperaturnoĭ plazmy (Abstracts of Papers Presented at Fifth All-Union Conf. on Physics of Low-Temperature Plasma), Vol. 1, Kiev, 1979, p. 72.

[4]Yu. A. Kravtsov, L. A. Ostrovskiĭ (Ostrovsky), and N. S. Stepanov, Proc. IEEE **62**, 1492 (1974).

[5]P. S. Epstein, Proc. Nat. Acad. Sci. U.S.A. **16**, 658 (1930).

[6]V. I. Karpman and E. M. Maslov, Zh. Eksp. Teor. Fiz. 73, 537 (1977) [Sov. Phys. JETP **46**, 281 (1977)].

[7]L. I. Rudakov and V. N. Tsytovich, Phys. Rep. C **40**, 1 (1978).

[8]V. N. Tsytovich, Nelineĭnye effekty v plazme, Nauka, M., 1964 (Nonlinear Effects in Plasma, Plenum Press, New York, 1970), Chap. VIII.

[9]V. I. Karpman, Zh. Eksp. Teor. Fiz. **77**, 1382 (1979) [Sov. Phys. JETP **50**, 695 (1979)].

Translated by A. Tybulewicz

Nonlinear theory of penetration of *p*-polarized electromagnetic waves in a plasma

A. G. Boev

Scientific-Research Institute for Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR, Kharkov

(Submitted 8 January 1980)
Zh. Eksp. Teor. Fiz. **79**, 134–142 (July 1980)

The approximation of the normal skin effect is used to consider the problem of penetration of a high-frequency p-polarized nonlinear ionizing electromagnetic wave into a plasma under conditions when the plasma permittivity changes its sign in the field of the wave. The change in the permittivity from positive to negative values is abrupt. The limits of this abrupt change are found and their behavior is studied as a function of the angle of incidence and frequency of the wave. The abrupt change in the permittivity of the plasma at its boundary results in a jump in the amplitude dependence of the reflection coefficient of the wave.

PACS numbers: 52.25.Mq, 52.40.Db

The problem of investigating the properties of a p-polarized electromagnetic wave propagating in a nonlinear medium is of considerable interest in theory and in practical applications. The problem is considerably more difficult to tackle than the corresponding problem of the behavior of an s-polarized wave because of an increase in the number of components of the electric field vector and because of the possibility of a discontinuity (jump) in the permittivity of the medium when the sign of this permittivity changes in the field of the wave.[1] Therefore, the solution of the problem should allow for the jump in the field and permittivity; the boundaries of this jump are not known in advance and should be found in the process of solution.

The structure of continuous spatial distributions of the field of a high-frequency p-polarized electromagnetic wave in a locally nonlinear medium was studied by Eleonskiĭ and Silin.[2,3] Our task will be to determine the structure of discontinuous distributions of the field of such a wave which appear when the permittivity of the medium changes its sign in the field of the wave. We shall concentrate mainly on a nonlinear wave which ionizes a plasma with a permittivity $\varepsilon_0 > 0$ in the absence of the field because this problem is of interest in practical tasks of generating a plasma in a microwave discharge. We shall find the boundaries of a permittivity jump, and study their behavior as a function of the angle of incidence and frequency of the wave. We shall show that under certain conditions such a wave can only create a plasma with a negative permittivity. Under the same conditions there may be a finite value of the electric field component parallel to the electron density gradient at a plasma resonance point and a discontinuity of the permittivity for a normally incident wave. The abrupt change in the permittivity of a plasma at its boundary results in a jump in the amplitude dependence of the reflection coefficient of the wave.

1. We shall assume that a half-space $z \geqslant 0$ is filled with a plasma whose permittivity in the absence of the field is ε_0 and which adjoins a dielectric (permittivity ε_2). A plane p-polarized wave is incident from this dielectric at an angle θ to the normal and the components of the electric and magnetic fields are $\mathbf{E}_0\{0, E_y, E_z\}$ and $\mathbf{H}_0\{H_0, 0, 0\}$; it is assumed that the wave is capable of ionizing a gas. The frequency of this wave is $\omega \gg \nu$ (ν is the effective collision frequency). We shall ignore the dissipation of the wave energy associated with collisions of electrons on other particles and with the transformation into plasma waves.

We shall assume that the depth of penetration of the field L_E is considerably greater than the characteristic lengths of diffusion, heat conduction, and redistribution of the density of electrons because of their heating by the inhomogeneous field. Then, the dependence of the permittivity on the field amplitude is local. We shall select this dependence in the form

$$\varepsilon=\varepsilon_0-\varkappa(|\mathbf{E}|^2/E_q^2-1), \quad |\mathbf{E}|>E_q, \qquad \varepsilon=\varepsilon_0, \quad |\mathbf{E}|\leqslant E_q. \tag{1.1}$$

The dependence (1.1) allows for the possibility of existence of a threshold field E_q due to allowance, in the equation describing the plasma particle balance, for the trapping of electrons by molecules.[4] When the electric field is less than E_q, there are no perturbations in the plasma and the behavior of the field represents a linear problem.

For simplicity, we shall assume that the field dependence of the permittivity is quadratic: the qualitative behavior of the solutions is not affected by a more complex dependence. The practical range of the validity of this dependence will be discussed at the end of the paper.

We shall use the following equation for the mangetic field of the wave[5]:

$$\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}-\frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial y}\frac{\partial H}{\partial y}-\frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial z}\frac{\partial H}{\partial z}+\frac{\omega^2}{c^2}\varepsilon H=0. \tag{1.2}$$

If we seek the field in the dielectric in the form of a sum of the incident and reflected fields, omitting the factor $\exp(i\omega t)$,

$$H_2=H_0\{\exp(ik_3z)+R\exp[i(\psi-k_3z)]\}\exp(ik_2y),$$

$$k_2=\frac{\omega}{c}\varepsilon_2^{1/2}\sin\theta, \quad k_3=\frac{\omega}{c}\varepsilon_2^{1/2}\cos\theta,$$

and the field in the plasma in the form

$$H=E_qb(z)\exp i[\varphi(z)+k_2y],$$

we find that the conditions of continuity of the tangential components of the fields at the $z = 0$ boundary yield the following equations for the determination of the modulus of the reflection coefficient R and its phase ψ, as well as of the modulus b_1 and the phase φ_1 of the transmitted wave at the boundary:

$$\begin{gathered} R^2=1+4\frac{(\varepsilon_2-\eta)^{1/2}}{\varepsilon_2}\frac{\varphi_1'}{\varepsilon_1}\left\{\left[\frac{(\varepsilon_2-\eta)^{1/2}}{\varepsilon_2}-\frac{\varphi_1'}{\varepsilon_1}\right]^2+u_1^2\right\}^{-1}, \\ \operatorname{tg}\psi=2\frac{(\varepsilon_2-\eta)^{1/2}}{\varepsilon_2}u_1\left\{\left[\frac{(\varepsilon_2-\eta)^{1/2}}{\varepsilon_2}\right]^2-\left(\frac{\varphi_1}{\varepsilon_1}\right)^2+u_1^2\right\}^{-1}, \\ \operatorname{tg}\varphi_1=\frac{u_1}{(\varepsilon_2-\eta)^{1/2}/\varepsilon_2-\varphi_1'/\varepsilon_1}, \quad \frac{\varphi_1'}{\varepsilon_1}=\frac{-(\varepsilon_2-\eta)^{1/2}}{b_1^2}, \\ b_1^2=H_0^2(1+R^2+2R\cos\psi), \quad u_1=b_1'/\varepsilon_1b_1, \end{gathered} \tag{1.3}$$

where

$$\eta=\varepsilon_2\sin^2\theta, \quad \varepsilon_1=\varepsilon(+0).$$

The amplitude of the magnetic field in the plasma b and its phase φ are given by

$$\varepsilon\frac{d}{d\zeta}\left[\frac{1}{\varepsilon}\frac{db}{d\zeta}\right]=(\eta+\varphi'^2-\varepsilon)b, \quad \varphi'=\frac{M\varepsilon}{b^2} \tag{1.4}$$

($\zeta = \omega z/c$), where the integration constant M represents the energy flux of the wave into the plasma; $M = -4\pi S/c$. If far from the boundary the plasma is opaque ($\varepsilon_0 - \eta \leqslant 0$), then $M = 0$.

The moduli of the components of the electric field ($E_\perp = E_z/E_q$, $E_\parallel = E_y/E_q$) normalized to E_q are calculated from

$$E_\perp=\left|\frac{\eta^{1/2}}{\varepsilon}be^{i\varphi}\right|, \quad E_\parallel=\left|\frac{i}{\varepsilon}\frac{d}{d\zeta}(be^{i\varphi})\right|. \tag{1.5}$$

We then have

$$|E|^2=\frac{1}{\varepsilon^2}\left\{(\eta+\varphi'^2)b^2+\left(\frac{db}{d\zeta}\right)^2\right\}. \tag{1.6}$$

The dependence (1.1) introduces also the boundary condition in the interior of the plasma: perturbations should disappear on reduction of the electric field of the wave to E_q. Therefore, where

$$|E|=E_q, \quad \varepsilon=\varepsilon_0, \tag{1.7}$$

the solution of the nonlinear problem should join the solution of the linear problem.

Under transparency ($\varepsilon_0 > \eta$) conditions, the solution of the linear problem is a plane wave with the electric field amplitude E_q traveling into the interior and, therefore,

$$b^2=\varepsilon_0, \quad \varphi'=-(\varepsilon_0-\eta)^{1/2}, \quad \varepsilon=\varepsilon_0. \tag{1.8}$$

If $\varepsilon_0 \leqslant \eta$, the solution of the linear problem decays with depth and, therefore,

$$b^2=\varepsilon_0^2/(2\eta-\varepsilon_0), \quad \varphi'=0, \quad \varepsilon=\varepsilon_0. \tag{1.9}$$

2. The solution of the first equation in Eq. (4) is

$$b^2=\frac{\varepsilon}{2\eta-\varepsilon}\left[\frac{\varepsilon_0^2-\varepsilon^2+C_1}{2\varkappa}\right], \tag{2.1}$$

where C_1 is the integration constant.

The dependence of the permittivity ε on the coordinate ζ is given by

$$\zeta=\int\frac{db}{d\varepsilon}\left\{\left[\frac{\varepsilon_0-\varepsilon}{\varkappa}+1\right]\varepsilon^2-\left(\eta+\frac{M^2\varepsilon^2}{b^4}\right)b^2\right\}^{-1/2}d\varepsilon. \tag{2.2}$$

The radicand in the integral should be nonnegative, i.e.,

$$F(\varepsilon)\equiv\left[\frac{\varepsilon_0-\varepsilon}{\varkappa}+1\right]\varepsilon^2-\left(\eta+\frac{M^2\varepsilon^2}{b^4}\right)b^2\geqslant 0. \tag{2.3}$$

The formula (2.2) is structurally similar to an expression which gives the time in the problem of motion of a point in the field of a central force.[6] However, in contrast to the problem in the mechanics when the boundaries of the region of motion away from the force center are given by the roots of an equation of the (2.3) type, the presence of the factor $db/d\varepsilon$ capable of vanishing in the integral in Eq. (2.2) makes the problem of determination of the range of permissible values somewhat more complex.

We can see from Eq. (2.1) that generally three values of the plasma permittivity correspond to the same magnetic field b. The equation describing these values differs from the corresponding equation in the nonlinear theory of a longitudinal field near a plasma resonance,[1] but the qualitative result is similar. However, it is important to note that the parameter in this equation is not the amplitude of the incident field but some value of the field in the plasma allowed by the inequality (2.3).

Let us now assume that the wave amplitude is such that the plasma permittivity ε_1 at the boundary is negative. Then, the value of ε in the bulk of the plasma changes its sign going abruptly through the value $\varepsilon = 0$. Consequently, we shall solve the problem separately for the positive and negative values of ε and then match the solutions at the boundaries of the jump using the conditions of continuity of the tangential component of the fields. Each of these solutions should satisfy the inequality (2.3); the function F in the range of positive values of ε will be denoted by F_+ and that in the range of negative values by F_-.

We shall first consider the case when a wave ionizes a transparent plasma ($\varepsilon_0 - \eta > 0$). Selecting C_1 in Eq. (2.1), we find that the boundary-value condition (1.8) yields the following expression for the range of positive values of the permittivity

$$b^2 = \frac{\varepsilon}{2\eta - \varepsilon} \frac{\varepsilon_0^2[1 + 2(\varkappa/\varepsilon_0)(2\eta/\varepsilon_0 - 1)] - \varepsilon^2}{2\varkappa}. \tag{2.4}$$

The inequality (2.3) [where $b^2(\varepsilon)$ is given by Eq. (2.4)], which defines the range of physically permissible positive values of ε, can be transformed to

$$F(x) = -x^2(1-x)^2\left\{8\frac{\varkappa}{\varepsilon_0}y^2 + y\left[4\frac{\varkappa}{\varepsilon_0} + (1+x)\left(1 - 3x + 4\frac{\varkappa}{\varepsilon_0}\right)\right] + 2x^2\left(1 + x - \frac{\varkappa}{\varepsilon_0}\right)\right\} \geqslant 0, \tag{2.5}$$

where $x = \varepsilon/\varepsilon_0$, $y = \eta/\varepsilon_0$. The allowed region of variation x must include the point $x = 1$.

It is found that the structure of the solution of the inequality (2.5) depends strongly on the value of parameter $\varkappa/\varepsilon_0$. If $\varkappa/\varepsilon_0 < 2$, the allowed range of variation of ε consists only of the point $\varepsilon = \varepsilon_0$ irrespective of the angle of incidence from the transparent region ($0 \leqslant \eta \leqslant \varepsilon_0$). If $\varkappa/\varepsilon_0 \geqslant 2$, the range of allowed values of ε depends strongly on η. When η is defined by $0 < \eta \leqslant \eta_1(\varkappa)$, where

$$\frac{\eta_1}{\varepsilon_0} = \left(\left\{\left[\left(\frac{\varkappa}{\varepsilon_0} + 1\right)^2 - 2\right]^2 + 4\frac{\varkappa}{\varepsilon_0}\left(\frac{\varkappa}{\varepsilon_0} + 2\right)\right\}^{1/2} - \left[\left(\frac{\varkappa}{\varepsilon_0} + 1\right)^2 - 2\right]\right)\frac{\varepsilon_0}{4\varkappa}, \tag{2.6}$$

the range of allowed positive values of ε represent the interval $[\varepsilon_m, \varepsilon_0]$ whose left-hand boundary ε_m varies on increase in η from zero to ε_0.

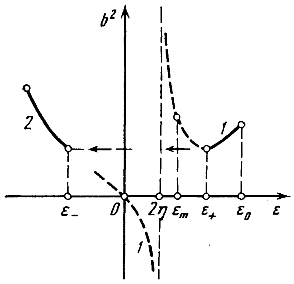

FIG. 1. Dependence of the magnetic field on the plasma permittivity $[2\eta/\varepsilon_0 < (2\varkappa - \varepsilon_0)/2\varkappa, \varkappa \geqslant 2\varepsilon_0]$. Curve 1 represents the dependence $b^2(\varepsilon)$ corresponding to Eq. (2.1); curve 2 represents the dependence $b^2(\varepsilon)$ corresponding to Eq. (3.1).

The value of η_1 considered as a function of the parameter $\varkappa/\varepsilon_0$ has a maximum at $\varkappa = 4.5\varepsilon_0$ amounting to $0.044\varepsilon_0$, so that for $\varepsilon_0 = 1$ the value $\varepsilon_2 = 4$ corresponds to the maximum angle of incidence $\theta \approx 6°$. If $\eta \geqslant \eta_1(\varkappa)$, the allowed range of variation consists only of the point $\varepsilon = \varepsilon_0$. Thus, the positive boundary of the jump in the permittivity for $\varkappa < 2\varepsilon_0$ and for all values of η in the transparency range when $\varkappa \geqslant 2\varepsilon_0$ and $\eta \geqslant \eta_1(\varkappa)$ is $\varepsilon_+ = \varepsilon_0$. Under these conditions a plasma with a positive permittivity can exist only in the unperturbed state; an ionizing electromagnetic field transforms abruptly this plasma to a state with a negative permittivity.

A more detailed discussion is needed to determine the positive boundary of the jump in the case when $\varkappa \geqslant 2\varepsilon_0$ and $0 \leqslant \eta \leqslant \eta_1$.

Curve 1 in Fig. 1 shows the dependence $b^2(\varepsilon)$ [Eq. (2.1)] in the case when $2\eta/\varepsilon_0 < (2\varkappa - \varepsilon_0)/2\varkappa$. At the value $\varepsilon = \varepsilon_+$ satisfying the equation

$$\varepsilon_+^3 - 3\eta\varepsilon_+^2 + \eta\varepsilon_0^2\left[2\frac{\varkappa}{\varepsilon_0}\left(\frac{2\eta}{\varepsilon_0} - 1\right) + 1\right] = 0, \tag{2.7}$$

the derivative $db/d\varepsilon$ changes its sign. We can show that in the range of variation of η of interest to us the value of ε_+ lies between ε_m and ε_0. At the point ε_+ the derivative $d\varepsilon/d\zeta$ vanishes and the range of permissible positive values of ε should be limited on the left by the point ε_+. The dependence $b^2(\varepsilon)$ observed in the interval $[\varepsilon_+, \varepsilon_0]$ is represented by the continuous curve 1 in Fig. 1. Thus, in this case the value of ε_+ is the positive boundary of the jump in the permittivity. The direction of the jump in the range of negative values of ε is indicated by arrows in Fig. 1; curve 2 is the dependence $b^2(\varepsilon)$ in the range of negative values of ε. It will be discussed below.

In the case under consideration the magnetic field in the range of positive values of ε increases with depth, whereas the modulus of the electric field decreases tending to E_q. It follows from Eq. (2.7) that in the case of small angles of incidence, we have

$$\varepsilon_+ \approx \eta^{1/3}[\varepsilon_0(2\varkappa - \varepsilon_0)]^{1/3}. \tag{2.8}$$

3. The solution in the range of negative values of ε will be sought in the form

$$b^2 = \frac{\varepsilon}{2\varkappa(2\eta - \varepsilon)}\left\{\varepsilon_0^2\left[1 + 2\frac{\varkappa}{\varepsilon_0}\left(\frac{2\eta}{\varepsilon_0} - 1\right)\right] - \varepsilon^2 + C\right\}, \tag{3.1}$$

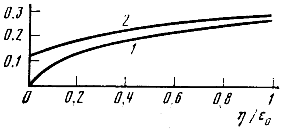

FIG. 2. Dependence of $|\varepsilon_-|$ (curve 1) and of $0.1|C|$ (curve 2) on the parameter $\eta/\varepsilon_0(\varkappa=0.1\varepsilon_0)$.

where C is (at this stage) an arbitrary constant. Its value and the highest negative permittivity ε_-, representing the negative boundary of the jump, can be found from the conditions

$$b^2(\varepsilon_-)=b^2(\varepsilon_+),\quad E_\parallel^2(\varepsilon_-)=E_\parallel^2(\varepsilon_+).$$

In the case when $\varkappa<2\varepsilon_0$ and η has any value in the transparency range or when $\varkappa\geq 2\varepsilon_0$ and $\eta\geq\eta_1$, we obtain

$$\varepsilon_-=\frac{\varkappa\eta}{2\varepsilon_0}-\left[\left(\frac{\varkappa\eta}{2\varepsilon_0}\right)^2+\varkappa\eta\right]^{1/2},\quad C=2\varkappa(2\eta-\varepsilon_-)/\varepsilon_- -(\varepsilon_0^2-\varepsilon_-^2).\qquad(3.2)$$

We can see that the value of C is negative like ε_-.

At low angles of incidence ($\eta\ll\varkappa/4$), we have

$$|\varepsilon_-|\approx(\varkappa\eta)^{1/2},\quad |C|\approx1+2\varkappa/\varepsilon_0.\qquad(3.3)$$

The range of validity of the first of the above formulas is given by the inequality $\eta\ll4\varepsilon_0^2/\varkappa$ so that at low values of $\varkappa$ this formula can be used in a wide range of angles of incidence. The values of $|\varepsilon_-|$ (curve 1) and $|C|$ (curve 2) are plotted in Fig. 2 as a function of η/ε_0 for $\varkappa/\varepsilon_0 = 0.1$.

A study of the dependence $b^2(\varepsilon)$ [Eq. (3.1)] in the range of negative values of ε shows that the magnetic field is a monotonically rising function of $|\varepsilon|$. The fields $E_\perp$ and $E_\parallel$ are characterized by a considerable variation in the vicinity of ε_-. Then, $E_\perp$ decreases in the region $\tilde\varepsilon\leq\varepsilon\leq\varepsilon_-$, where

$$|\tilde\varepsilon|=\frac{(C-1)\varepsilon_0^2}{2\eta}+\left[\frac{(C-1)^2\varepsilon_0^4}{4\eta^2}+(C-1)\varepsilon_0\right]^{1/2}.$$

At the point $\tilde\varepsilon$ the field $E_\perp$ has a flat minimum and then it rises slowly approaching asymptotically the value $(\eta/2\varkappa)^{1/2}$ in the limit $|\varepsilon|\to\infty$. The electric field component $E_\parallel$ increases on increase in $|\varepsilon|$.

If $\varkappa<2\varepsilon_0$, then near the boundary of the jump ($\varepsilon\sim\varepsilon_-$) the field $E_\perp$ predominates over the field $E_\parallel$ irrespective of the angle of incidence from the transparent region. The ratio of these two components is maximal for $\varepsilon=\varepsilon_-$ and is given by

$$E_\perp^2/E_\parallel^2=\eta\varepsilon_0^2/\varepsilon_-^2(\varepsilon_0-\eta).$$

Away from the jump the ratio of the field components is reversed and $E_\parallel$ begins to predominate so that in the bulk of the plasma the electric field vector rotates.

The most interesting field pattern is observed in the case of normal incidence of the wave ($\eta=0$). According to Eq. (3.3) the negative boundary of the jump ε_- tends to zero together with η as $\eta^{1/2}$ and, therefore, the field component $E_\perp$ parallel to the electron density gradient remains finite at the boundary of the jump in the limit $\eta\to0$ ($b_-^2=\varepsilon_0$):

$$E_\perp^2|_{\varepsilon_-}=\eta b_-^2/\varepsilon_-^2=\varepsilon_0/\varkappa.\qquad(3.4)$$

For nonzero values of ε the component $E_\perp$ vanishes for $\eta=0$ and the wave is transverse.

We thus find that for $\varkappa<2\varepsilon_0$ and normal wave incidence a transverse electromagnetic wave is excited by the longitudinal component of the field whose amplitude is $E_q(\varepsilon_0/\varkappa)^{1/2}$ at the plasma resonance point. This effect is nonlinear: it is associated with the specific nature of the dependence of the plasma permittivity created by the wave field on the angle of incidence of the wave.

The inequality (2.3), where $b^2(\varepsilon)$ is given by Eq. (3.1), determines the range of permissible negative values of the permittivity and it is obeyed by all negative values of ε beginning from ε_-. Since the range of permissible positive values of ε consists of the point $\varepsilon=\varepsilon_0$, the reverse jump from the negative ε to the positive values on reduction in the amplitude of the incident wave is possible only at the point $\varepsilon=\varepsilon_-$ and, consequently, there is no hysteresis in respect of the incident wave amplitude.

For $\varkappa>2\varepsilon_0$ and small angles of incidence the positive boundary of the jump is the value ε_+ given by Eq. (2.8). In the limit $\eta\to0$ we have $\varepsilon_+\sim\eta^{1/3}\to0$ and the field component $E_\perp$ disappears at the point $\varepsilon=0$.

$$E_\perp^2|_{\varepsilon_+}=\eta b_+^2/\varepsilon_+^2\sim\eta^{1/3}\to0.$$

The negative boundary of the jump and the constant C can be found from

$$\frac{\varepsilon_-}{\varepsilon_+}=\frac{\varkappa\eta b_+^2}{2\varepsilon_+^3}-\left\{\left[\frac{\varkappa\eta b_+^2}{2\varepsilon_+^3}\right]^2+\frac{\varkappa\eta b_+^2}{\varepsilon_+^3}\right\}^{1/2},$$
$$C=\frac{(2\eta-\varepsilon_-)\varepsilon_+}{(2\eta-\varepsilon_+)\varepsilon_-}\left[\varepsilon_0^2\left(1+\frac{2\varkappa}{\varepsilon_0}\right)-\varepsilon_+^2\right]-\left[\varepsilon_0^2\left(1+\frac{2\varkappa}{\varepsilon_0}\right)-\varepsilon_-^2\right],\qquad(3.5)$$

where b_+ is the modulus of the magnetic field at $\varepsilon=\varepsilon_+$.

In the limit $\eta\to0$, we have $\varepsilon_-\to0$ and $C\to0$ so that the boundaries of the jump merge at the point $\varepsilon=0$. The field and permittivity vary continuously across the plasma resonance point $\varepsilon=0$ and everywhere in the plasma the following formula applies:

$$b^2=\frac{\varepsilon_0^2(2\varkappa/\varepsilon_0-1)-\varepsilon^2}{2\varkappa}.$$

It should be noted that in this case the field component $E_\perp$ vanishes everywhere including the point $\varepsilon=0$.

The transition from the negative values of ε to the positive ones occurs in the reverse order when the amplitude of the incident wave is reduced: the transition is from the point ε_- to the point ε_+. This can be seen in Fig. 1 since ε_- is the smallest negative value from which we can go to curve 1 retaining the tangential components of the field.

The above solution allows us to calculate, with the aid of the formulas in Eq. (1.3), the reflection coefficient of a wave incident on a plasma. It is interesting to consider the behavior of this coefficient in the case when there is a jump of the field and permittivity. Since the plasma permittivity at the boundary changes abruptly in a certain range of amplitudes, there is a corresponding jump in the dependence of the reflection coefficient of the wave on its amplitude.

For simplicity, we shall consider the case $\varkappa(\omega)<2\varepsilon_0$.

The formulas in Eq. (1.3) are expressed in terms of quantities which are conserved after transition across the jump so that the modulus and the phase of the reflection coefficient are equal to the values R_0 and ψ_0 found from the linear theory for variation of the field amplitude at the boundary from E_q right up to E_-, where the amplitude at the boundary corresponding to the permittivity ε_- at the same boundary is

$$\frac{E_-^2}{E_q^2}=1+\frac{\varepsilon_0}{\varkappa}\left[1+\frac{|\varepsilon_-|}{\varepsilon_0}\right]. \tag{3.6}$$

If $E \geqslant E_-$, the permittivity at the boundary ε_1 is negative and the reflected field is strongly enhanced. Thus, for $E = E_-$ the modulus and phase of the reflection coefficient have a discontinuity. The magnitudes of the discontinuities are

$$\langle R\rangle=R(E_-+0)-R_0,\quad \langle\psi\rangle=\psi(E_-+0)-\psi_0,$$

(the symbol +0 in the arguments means that the amplitude approaches the value E_- from above).

According to Eq. (1.3), $\tan\psi$ is proportional to $db/d\zeta \equiv F_-(\varepsilon)$ from Eq. (2.3). We can show that in this case the value of ε_- is given by the root of the equation $F_-(\varepsilon) = 0$ and, consequently, we have $\psi(\varepsilon_-) = 0$.

Figure 3 shows the dependences of the modulus R (curve 1) and of the phase ψ (curve 2) of the reflection coefficient on the amplitude of the incident wave in the case when $\varepsilon_0 = 1$ and the initial state of the plasma is a neutral gas with $R_0 = \psi_0 = 0$ ($\varkappa = 0.07$, $\varepsilon_2 \approx 1$, $\theta = 10°$). If allowance is made for the diffusion of electrons and spatial dispersion, it is found that the reflection coefficient varies continuously though rapidly in the region of the jump (dashed part of curve 1). Calculations show that the dependences of R and ψ on the angle of incidence in the range of fields $E \geqslant E_-$ are relatively weak, as expected for high values of the electron density in the plasma.

The solution of the problem in the $\varepsilon_0 = 0$ presents no difficulties and we shall not consider it in detail. We shall simply note that the plasma exists in the range where $\varepsilon < 0$ and the fields vary continuously in this region.

If a wave is incident on a plasma boundary at an angle $\theta \geqslant \theta_c$, where θ_c is the angle of total internal reflection from the unperturbed plasma ($\eta \geqslant \varepsilon_0$), it follows from Eqs. (2.1) and (1.9) that

$$b^2=\frac{\varepsilon[\varepsilon_0^2(1+2\varkappa/\varepsilon_0)-\varepsilon^2]}{2\varkappa(2\eta-\varepsilon)}. \tag{3.7}$$

With exception of the range of variation of η, the solution (3.7) is similar to the solution for a surface electromagnetic wave[7] and, therefore, we shall not discuss it in detail. The range of allowed positive values of ε corresponding to all $\varkappa$ represents the interval $[\varepsilon_+, \varepsilon_0]$, where ε_+ is the root of the equation $db/d\varepsilon = 0$.

We shall now consider the problem of the practical range of validity of the ionization law (1.1). We begin by noting that a dependence of this type may be valid also in strong fields. If in the electron energy balance the main losses are inelastic and the ionization process is multistage, the ionization frequency depends quadratically on the field.

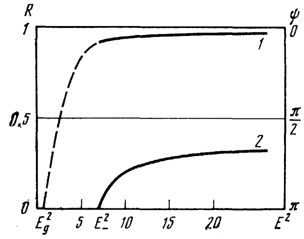

FIG. 3. Dependence of the reflection coefficient of the wave on its amplitude: 1) modulus R; 2) phase ψ.

Under conditions typical of microwave discharges in inert gases ($\omega \sim 10^{10} - 10^{11}\ \mathrm{sec}^{-1}$, $n \sim 10^{12} - 10^{14}\ \mathrm{cm}^{-3}$, $T_e \sim 1$-3 eV, $E \sim 10^2$ V/cm) a dependence of the (1.1) type is valid in the range of pressures $p \sim 1$-30 Torr. We then obtain

$$\varkappa=\alpha/\rho n_c,\quad E_q=E_p[U\alpha/kT\delta\nu]^{1/2},$$

where α and ρ are the coefficients representing the trapping of electrons by molecules and electron recombination, respectively; U is the excitation potential; T is the temperature of the gas; $\delta\nu$ is the energy frequency of collisions of electrons with atoms; E_p is the plasma field.

In the case of argon of special purity (containing $\sim 10^{-3}$% of electronegative impurities), we find that estimates give

$$\varkappa\sim 10^{-2},\quad E_q\sim 10^{-1}\ \mathrm{V/cm}.$$

If the energy balance of electrons is dominated by losses due to elastic processes, the ionization frequency depends on the field exponentially and then Eq. (1.1) is valid only in weak fields defined by $E \ll (kT/U)^{1/2}E_p$.

The author is grateful to F.G. Bass for his interest.

[1]A. V. Gurevich and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. 45, 1243 (1963) [Sov. Phys. JETP 18, 855 (1964)].

[2]V. P. Silin, Zh. Eksp. Teor. Fiz. 53, 1662 (1967) [Sov. Phys. JETP 26, 955 (1968)].

[3]V. M. Eleonskiĭ, and V. P. Silin, Zh. Eksp. Teor. Fiz. 60, 1927 (1971) [Sov. Phys. JETP 33, 1039 (1971)]; Pis'ma Zh. Eksp. Teor. Fiz. 13, 167 (1971) [Sov. Tech. Phys. Lett. 13, 117 (1971)].

[4]V. E. Golant, Usp. Fiz. Nauk 65, 39 (1958).

[5]V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme, Nauka, M., 1967 (The Propagation of Electromagnetic Waves in Plasmas, 2nd ed., Pergamon Press, Oxford, 1970).

[6]L. D. Landau and E. M. Lifshitz, Mekhanika, Fizmatgiz, M., 1958 (Mechanics, Pergamon Press, Oxford, 1960).

[7]A. G. Boev, Zh. Eksp. Teor. Fiz. 77, 92 (1979) [Sov. Phys. JETP 50, 47 (1979)].

Translated by A. Tybulewicz