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# Interaction of ion-acoustic solitons with Langmuir waves

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An analysis is made of the interaction of strong ion-acoustic solitons with Langmuir waves considered in the approximation of slow changes in the soliton parameters compared with the rate of evolution of the wave spectrum. The scattering matrix is found for the interaction of a Langmuir wave with an ion-acoustic soliton of given amplitude and a determination is made of the change in the frequency and wave number due to the Doppler effect. The change in the soliton parameters under the action of the waves is found. Even when the Langmuir noise level is low, the characteristic attenuation time of a soliton interacting with waves may be less than the attenuation time for the interaction with resonant particles.

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## 1. INTRODUCTION. FORMULATION OF THE PROBLEM

The interaction between Langmuir waves and intense ion-acoustic waves transforms the energy of a Langmuir turbulence shifting it to shorter wavelengths where collisionless dissipation is important. The first analysis of the transformation (conversion) of Langmuir waves in a random field of ion-acoustic waves is given in Ref. 1. The interaction of ion-acoustic waves (periodic and solitary) with Langmuir waves is considered in Ref. 2 allowing for the reflection of the latter from a density perturbation created by an ion-acoustic wave. The use of the geometric-optics approximation in Ref. 2 makes it possible to consider only relatively weak ion-acoustic waves which are of considerable intrinsic width. In this approximation the transformation interval of Langmuir waves is narrow.

In contrast to Ref. 2, we shall consider the interaction of Langmuir waves with a strong ion-acoustic soliton without the above restriction in the case when the width of an ion-acoustic soliton and the Langmuir wavelength are both arbitrary.[3] We shall initially find rigorously the scattering matrix for a Langmuir wave interacting with a given soliton (as represented by the reflection $R$ and transmission $T$ coefficients), and we shall then determine slow changes in the parameters of the soliton under the action of Langmuir waves.

We shall assume that an ion-acoustic soliton appears in a plasma with a weak Langmuir turbulence[1)] at a moment $t=0$. In the presence of a moving density perturbation the field of plasma waves is described by

$$i\frac{\partial \mathbf{E}}{\partial t}+\omega_p\left(1+\frac{1}{2}\beta(x,t)\right)\mathbf{E}-\frac{3}{2}\frac{V_T^2}{\omega_p}\Delta\mathbf{E}=0, \quad (1)$$

where $\beta=\tilde{n}(x,t)/n_0=v(x,t)/C\ll 1$; $n_0$ and $\tilde{n}$ are the particle density and its perturbation; $v$ is the velocity of particles in an ion-acoustic wave; $C$ is the velocity of a perturbation; $\omega_p$ is the plasma frequency of electrons; $V_T=(\varkappa T_e/m_e)^{1/2}$ is the thermal velocity of electrons.

The equation for nonlinear ion-acoustic waves traveling in the direction of $x$ has the following form when the striction force is taken into account:

$$\frac{\partial\beta}{\partial t}+c_s(1+\beta)\frac{\partial\beta}{\partial x}+\frac{1}{2}c_sD^2\frac{\partial^3\beta}{\partial x^3}=-\frac{1}{32\pi\rho_0c_s}\frac{\partial}{\partial x}\langle|\mathbf{E}(\mathbf{r},t)|^2\rangle, \quad (2)$$

where $D=V_T/\omega_p$ is the Debye radius; $\rho_0=m_in_0$ is the plasma density; $c_s$ is the velocity of ion sound; the angular brackets on the right-hand side of Eq. (2) denote averaging over the ensemble. Since the field of plasma waves is homogeneous at right-angles to the $x$ axis and the spectral components of the field are regarded as δ-correlated, only the derivative of the high-frequency potential with respect to $x$ differs from zero in Eq. (2). In considering the interaction between plasma waves and an ion-acoustic soliton, we shall ignore other mechanisms of the nonlinearity of plasma waves and particularly their interaction with particles and decay processes. We shall determine the validity conditions of this approximation later.

## 2. TRANSFORMATION OF LANGMUIR WAVES IN THE FIELD OF AN ION-ACOUSTIC SOLITON

We shall first consider the reflection of a harmonic (in respect of time) plasma wave by a traveling density profile created by an ion-acoustic soliton; $\beta=\beta_0\cosh^{-2}(\xi/\Delta)$, where $\xi=x-Ct$; $C=c_s(1+\frac{1}{3}\beta_0)$ is the soliton velocity; $\Delta=6^{1/2}D\beta_0^{-1/2}$ is the soliton width.[2)]

An incident Langmuir wave of frequency $\omega$ may, in a laboratory coordinate system, either catch up with the soliton or run away from it, depending on whether the projection of its group velocity $V_{gx}$ on the $x$ axis is greater or smaller than $C$. Since the parameters of the medium in a reference system linked to the soliton (we shall call it the system $C$) are constant in the approximation of a given soliton, the frequency of an incident wave $\Omega$ is equal to the frequency of the transmitted and reflected waves. The dispersion relationship for Langmuir waves in the system $C$ is

$$\Omega=\omega_p+\mathbf{k}^2/2m^*-k_xC\equiv\Omega^*+\tilde{k}_x^2/2m^*+\mathbf{k}_\perp^2/2m^*, \quad (3)$$

where $m^*=\omega_p/3V_T^2$ is the plasmon "mass"; $\Omega^*=\omega_p-m^*C^2/2$; $\mathbf{k}_\perp$ is the component of the wave vector perpendicular to the $x$ axis; $\tilde{k}_x=k_x-k_*$, $k_x$ is the projection of the wave vector of the $x$ axis; $k_*=m^*C$ is the projection of the $x$ axis of that wave vector for which $V_{gx}=C$.

It follows from the one-dimensional nature of the investigated ion-acoustic solitons that $\mathbf{k}_\perp=\text{const}$ and that $|\tilde{k}_x|$ is also conserved on reflection. Therefore, if a soliton catches up with a Langmuir wave ($k_x<k_*$), transformation accompanied by an increase in $k_x$ and $\omega$ occurs in the front edge of the soliton. If a Langmuir wave catches up with a soliton ($k_x>k_*$), this wave transforms at its rear edge into a Langmuir wave with reduced $k_x$ and $\omega$. The equation for the field in the coordinate system $C$ is obtained by the substitutions $t'=t$, $\xi=x-Ct$ and $\mathbf{E}=\tilde{\mathbf{E}}\exp\{-im^*C\xi\}$. Transforming Eq. (1), we have

$$i\frac{\partial\tilde{\mathbf{E}}}{\partial t}+[\Omega^*+s\beta(\xi)]\tilde{\mathbf{E}}-\frac{1}{2m^*}\Delta\tilde{\mathbf{E}}=0, \quad (4)$$

where $s=\omega_p/2$. Since the parameters of the medium in the system $C$ depend only on $\xi$, we shall seek the solution of Eq. (4) in the form

$$\tilde{\mathbf{E}}=\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)\exp\{i(\Omega t-\mathbf{k}_\perp\mathbf{r}_\perp)\}, \quad (5)$$

where $\Omega$ and $\mathbf{k}_\perp$ are related by Eq. (3), and $\mathbf{r}_\perp$ is the coordinate transverse to the $x$ axis. Substituting Eq. (5) in Eq. (4) and using Eq. (3), we obtain

$$\frac{d^2\mathbf{E}_0}{d\xi^2}+\tilde{k}_x^2\left(1-\frac{\beta(\xi)}{3D^2\tilde{k}_x^2}\right)\mathbf{E}_0=0. \quad (6)$$

Equation (6) with $\beta=\beta_0\cosh^{-2}(\xi/\Delta)$ is investigated in Ref. 5. Using the results of Ref. 5, we can find the solutions of Eq. (6) stationary in the system $C$ and corresponding, far from a soliton, to incident, reflected, and transmitted Langmuir waves.

According to Ref. 5, the field $\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)$ is a linear combination of two solutions of Eq. (6):

$$\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)=\mathbf{A}_1Q_1(\xi,\Omega,\mathbf{k}_\perp)+\mathbf{A}_2Q_2(\xi,\Omega,\mathbf{k}_\perp)\ \text{for}\ \xi>0, \quad (7a)$$

$$\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)=\mathbf{B}_1P_1(\xi,\Omega,\mathbf{k}_\perp)+\mathbf{B}_2P_2(\xi,\Omega,\mathbf{k}_\perp)\ \text{for}\ \xi<0, \quad (7b)$$

where

$$Q_{1,2}(\xi,\Omega,\mathbf{k}_\perp)=P_{2,1}(-\xi,\Omega,\mathbf{k}_\perp)=\exp(\mp i|\tilde{k}_x|\xi)\times(1+\exp(-2\xi/\Delta))F(d,d\pm 2a,1\pm 2a,-\exp(-2\xi/\Delta)); \quad (8)$$

$F$ is the hypergeometric function; $a=i\Delta|\tilde{k}_x|$, $d=\frac{1}{2}(1-i\sqrt{7})$. The functions $Q_2$ and $P_2$ in the system $C$ correspond to waves incident on the leading and rear edges of a soliton, respectively, whereas $Q_1$ and $P_1$ are waves reflected from a soliton or transmitted by it. The coefficient for a reflection of waves from a soliton is

$$R=\frac{\Gamma(2a)\Gamma(d-2a)\Gamma(1-d-2a)}{\Gamma(-2a)\Gamma(d)\Gamma(1-d)}, \quad |R|^2=\frac{\text{ch}^2(\pi\,\text{Im}\,d)}{\text{ch}^2(\pi\,\text{Im}\,d)+\text{sh}^2(2\pi|a|)}. \quad (9)$$

When an ion-acoustic soliton appears at some moment in a plasma with a random field of Langmuir waves, these waves begin to be reflected by the front and rear edges of the soliton. For each spectral component $\Omega$ the dimensions of the region with reflected waves increase with time in accordance with the law $l\propto|\partial\Omega/\partial\tilde{k}_x|t$, where $\partial\Omega/\partial\tilde{k}_x$ is the group velocity of waves in the system $C$. Within the limits of this region in the case of sufficiently long times $t$ so that $l\gg\tilde{k}_x^{-1}$, we find that for all the spectral components a solution close to stationary is established:

$$\mathbf{E}(\mathbf{r},t)=\int_0^{+\infty}d\Omega\int_{-\infty}^{+\infty}d\mathbf{k}_\perp\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)\exp\{i(\Omega t-k_*\xi-\mathbf{k}_\perp\mathbf{r}_\perp)\}, \quad (10)$$

where $d\mathbf{k}_\perp=dk_ydk_z$ and the field $\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)$ is of the form

$$\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)=\mathbf{E}_+(\Omega,\mathbf{k}_\perp)[Q_2(\xi,\Omega,\mathbf{k}_\perp)+R(\Omega,\mathbf{k}_\perp)Q_1(\xi,\Omega,\mathbf{k}_\perp)]+\mathbf{E}_-(\Omega,\mathbf{k}_\perp)T(\Omega,\mathbf{k}_\perp)Q_1(\xi,\Omega,\mathbf{k}_\perp)\ \text{for}\ \xi>0, \quad (11a)$$

$$\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)=\mathbf{E}_-(\Omega,\mathbf{k}_\perp)[P_2(\xi,\Omega,\mathbf{k}_\perp)+R(\Omega,\mathbf{k}_\perp)P_1(\xi,\Omega,\mathbf{k}_\perp)]+\mathbf{E}_+(\Omega,\mathbf{k}_\perp)T(\Omega,\mathbf{k}_\perp)P_1(\xi,\Omega,\mathbf{k}_\perp)\ \text{for}\ \xi<0. \quad (11b)$$

Here, $T(\Omega,\mathbf{k}_\perp)$ is the coefficient describing the coefficient of plasma waves by a soliton. The functions $Q_{1,2}$ and $P_{1,2}$ corresponding to the limit $|\xi|\to\infty$ transform asymptotically into plane waves $\exp\{\pm i|\tilde{k}_x|\xi\}$. The quantities $\mathbf{E}_+(\Omega,\mathbf{k}_\perp)$ and $\mathbf{E}_-(\Omega,\mathbf{k}_\perp)$ are the spectral amplitudes of the field corresponding to, respectively, $\tilde{k}_x<0$ in the limit $\xi\to\infty$ and to $\tilde{k}_x>0$ in the limit $\xi\to-\infty$.

In the subsequent determination of the law governing changes in the soliton amplitude we shall require only the asymptotic form of the Langmuir wave field in the limit $|\xi|\to\infty$. The complete expressions (10) and (11) may be useful in a more detailed analysis of the soliton structure, in particular, in a study of the behavior of its tails.

The dependence of $|R|$ on $k_x$ is shown in Fig. 1. For $k_x=k_*$ the modulus of the reflection coefficient is unity and it falls on both sides of this value. We shall define the boundary of the characteristic interval of wave numbers where significant reflection of Langmuir waves takes place by selecting the value of $\Delta\tilde{k}_x$ corresponding to $|R|^2=\frac{1}{2}$. Then, using the relationship between the

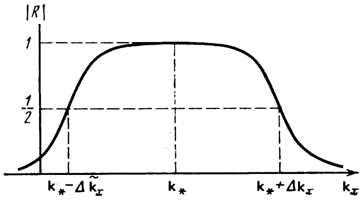


FIG. 1. Dependence of the Langmuir-wave reflection coefficient on $k_x$.

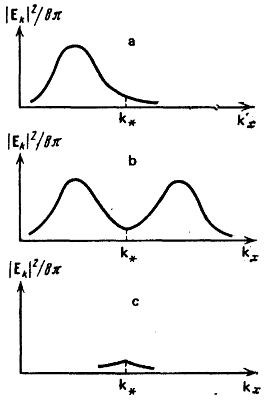


FIG. 2. a) Unperturbed Langmuir-wave spectrum, b) Langmuir-wave spectrum ahead of the soliton, c) Langmuir-wave spectrum behind the soliton.

soliton amplitude and its width, we find from Eq. (9) that

$$\Delta\tilde{k}_x \approx 0.54 D^{-1}\beta_0^{1/2}. \tag{12}$$

The quantity $\Delta\tilde{k}_x$ is close to the interval of trapping of Langmuir plasmons found in Ref. 2 in the geometric-optics approximation. It follows from Eq. (12) and from Ref. 2 that the width of the trapping interval rises rapidly on increase in $\beta_0$ and even for $\beta_0 \sim 2\times 10^{-4}$ we have $\Delta\tilde{k}_x \approx k_*$. However, for $\beta_0 \sim 0.25$, we have $\Delta\tilde{k}_x \sim 0.27 D^{-1}$ and the boundary of the transformation interval reaches the region where the Landau damping is important.[3] Thus, even in the case of a fairly low amplitude of an ion-acoustic perturbation the reflection coefficient of Langmuir waves is close to unity in a wide range of wave numbers of Langmuir waves. In particular, the interaction with a soliton can result in transformation of other than just plasma waves, for example, spatially homogeneous oscillations with $|\mathbf{k}| = 0$ are transformed by a traveling soliton into a wave whose wave number is $k_x = 2k_*$.

The changes in the Langmuir wave spectrum created by an intense ion-acoustic soliton (with a wide trapping interval) are illustrated in Fig. 2. Figure 2a shows an unperturbed spectrum of a Langmuir turbulence. Since the trapping interval covers the energy-carrying part of the spectrum, all the plasma waves are reflected from a soliton with $|R|$ close to unity and the energy density of two plasma waves in front of the soliton (Fig. 2b) is much higher than behind it (Fig. 2c).

## 3. CHANGES IN SOLITON PARAMETERS

We shall determine the changes in the parameters of an ion-acoustic soliton under the action of Langmuir waves on the basis of Eq. (2). Transformations similar to those in Ref. 2 give

$$\frac{d}{dt}\int_{-\infty}^{+\infty}\frac{\beta^2}{2}d\xi = -\frac{1}{32\pi\rho_0 c_s}\int_{-\infty}^{\infty}\beta\frac{\partial}{\partial\xi}\langle|\mathbf{E}(\mathbf{r},t)|^2\rangle d\xi. \tag{13}$$

The following conditions for conservation of the soliton shape are assumed to be satisfied during the interaction process:[6]

$$\beta = \beta_0(t)\,\mathrm{ch}^{-2}(\xi/\Delta(t)), \quad \beta_0(t)\Delta^2(t) = 6V_T^2/\omega_p^2. \tag{14}$$

We shall seek the solution of Eq. (13) in a quasistationary approximation when the distribution of the Langmuir wave field at a soliton is governed at each moment $t$ by the soliton amplitude at the same moment. We shall determine the relationship governing changes in the soliton amplitude by substituting Eqs. (10) and (11) in Eq. (13); then, assuming that the spectral components of the field are $\delta$-correlated, we obtain the following equation

$$\frac{d}{dt}\int_{-\infty}^{+\infty}\frac{\beta^2}{2}d\xi = -\frac{1}{32\pi\rho_0 c_s}\int_0^{\infty}d\Omega\int_{-\infty}^{\infty}d\mathbf{k}_\perp\int_{-\infty}^{\infty}d\xi\,\beta\frac{d|\mathbf{E}_0(\xi,\Omega,\mathbf{k}_\perp)|^2}{d\xi}. \tag{15}$$

We shall transform the right-hand side of Eq. (15) using Eq. (6). We shall multiply Eq. (6) by $d\mathbf{E}_0^*/d\xi$, where $\mathbf{E}_0^*$ is the complex conjugate of $\mathbf{E}_0$ and we shall add to the resultant expression its complex conjugate. This gives the following relationship for the integrand in Eq. (15):

$$\beta(\xi)\frac{d|\mathbf{E}_0|^2}{d\xi} = 3D^2\frac{d}{d\xi}\left(\tilde{k}_x^2|\mathbf{E}_0|^2 + \left|\frac{d\mathbf{E}_0}{d\xi}\right|^2\right). \tag{16}$$

Substituting Eq. (16) in Eq. (15), integrating with respect to $\psi$, and applying (12), we obtain

$$\frac{d}{dt}\left\{\rho_0 c_s\int_{-\infty}^{\infty}\beta^2 d\xi\right\} = -\int_0^{\infty}d\Omega\int_{-\infty}^{\infty}d\mathbf{k}_\perp\cdot 2|R|^2\frac{3D^2\tilde{k}_x^2}{8\pi}(|\mathbf{E}_+|^2 - |\mathbf{E}_-|^2). \tag{17}$$

This relationship represents the law of conservation of the momentum of an ion-acoustic soliton and of Langmuir waves in the coordinate system $C$.[4] In the absence of a perturbation, the velocity of plasmon motion in the system $C$ is $-C$, whereas in the presence of a soliton the velocity is $V_C = -C + v$. The soliton momentum is the difference between the momenta of the perturbed and unperturbed media:

$$P = \int_{-\infty}^{\infty}\rho V_C\,d\xi - \int_{-\infty}^{\infty}\rho_0 C\,d\xi = \int_{-\infty}^{\infty}\tilde{\rho} v\,d\xi \approx \rho_0 c_s\int_{-\infty}^{\infty}\beta^2 d\xi,$$

where $\tilde{\rho} = \tilde{n} m_i$, $\rho = \rho_0 + \tilde{\rho}$. The last equality is derived on the assumption that the difference between $C$ and $c_s$ is small.

We can thus see that on the left-hand side of Eq. (17) we have the rate of change of the soliton momentum, whereas the right-hand side gives the change in the momentum flux of Langmuir waves reflected from a soliton:

$$\Pi_\pm = \frac{k_x|\mathbf{E}_\pm|^2}{8\pi\omega_p}V_{g_x}^{(C)} = \frac{k_x|\mathbf{E}_\pm|^2}{8\pi\omega_p}\frac{3V_T^2}{\omega_p}\tilde{k}_x^2 = \frac{3}{8\pi}D^2 k_x\tilde{k}_x|\mathbf{E}_\pm|^2.$$

Substituting Eq. (14) in Eq. (17), assuming the unperturbed Langmuir wave field to be statistically and spatially homogeneous with a spectrum $\mathbf{E}_\mathbf{k}(\mathbf{k})$, and going over from integration with respect to the variables $\Omega$ and $\mathbf{k}$, to integration with respect to $k_x$ and $\mathbf{k}_\perp$, we obtain the following equation for the soliton amplitude:

$$\frac{d\beta_0}{dt} = -\frac{6^{1/2}}{4\pi}\frac{V_T}{\rho_0 c_s\omega_p\sqrt{\beta_0}}\left[\int_{k_x<k_*}d\mathbf{k}_\perp|\mathbf{E}_k(\mathbf{k})|^2|R(\mathbf{k})|^2\tilde{k}_x^2 - \int_{k_x>k_*}d\mathbf{k}|\tilde{\mathbf{E}}_k(\mathbf{k})|^2|R(\mathbf{k})|^2\tilde{k}_x^2\right]. \tag{18}$$

The relationship (18) describes attenuation or amplification of an ion-acoustic soliton under the action of Langmuir waves (the first and second terms in the brackets). It is valid both at low soliton amplitudes when the trapping interval of Langmuir waves $\Delta\bar{k}_x$ is small, as well as at high amplitudes when all the Langmuir waves are reflected. If $\beta_0$ is small, we can go over from Eq. (18) to the limiting case investigated in Ref. 2 in the geometric-optics approximation. We shall analyze the other limiting case of considerable values of $\beta_0$ such that $\Delta\bar{k}_x \gg |\bar{k}_x|$, and the reflection coefficient of all the Langmuir wave is close to unity. Without specifying the form of the Langmuir wave spectrum, we shall simply assume that the energy of these waves is concentrated in the long-wavelength region $|k_x| \ll k_*$. Allowing for this factor, we find that Eq. (18) yields a simple equation for the amplitude of an ion-acoustic soliton:

$$\frac{d\beta_0}{dt} = -\frac{1}{3\cdot 6^{1/2}}\left(\frac{m_e}{m_i}\right)^{3/2}\omega_p\beta_0^{-1/2}\frac{\mathscr{E}}{n\varkappa T_e}, \quad \mathscr{E} = \frac{1}{8\pi}\int |\mathbf{E}_k(\mathbf{k})|^2\, d\mathbf{k}, \quad (19)$$

where $\mathscr{E}$ is the unperturbed energy density of plasma waves. The solution of Eq. (19) is

$$\beta_0(t) = \beta_0(0)\,(1 - t/\tau_1)^{2/3}, \quad (20)$$

where

$$\tau_1 = \omega_p^{-1}\, 2\cdot 6^{1/2}\frac{n\varkappa T_e}{\mathscr{E}}\left(\frac{m_i}{m_e}\right)^{3/2}\beta_0^{3/2}(0)$$

is the characteristic time of the change in the parameters of an ion-acoustic soliton and $\beta_0(0)$ is the value of $\beta_0$ at $t = 0$. The relationship (20) holds, in the case of sufficiently large $\beta_0$.

We shall now determine the limits of validity of the results obtained. The above discussion is valid if the characteristic time of the change in the Langmuir wave field under the action of an ion-acoustic perturbation $\tau_{N_1}$ is much shorter than the time $\tau_1$ of changes in the soliton parameters. We shall take $\tau_{N_1}$ to be the time during which the plasma wave with the $a$ wave vector $|\bar{k}_x| \approx k_*$, travels a distance $L = 10 \times 2\pi/|\bar{k}_x|$:

$$\tau_{N_1} = \frac{L}{\partial\Omega/\partial\bar{k}_x} = 60\pi\frac{m_i}{m_e}\omega_p^{-1}.$$

The inequality $\tau_1 \gg \tau_{N_1}$ imposes restrictions on the initial amplitude of a soliton:

$$\beta_0(0) \gg (m_e/m_i)^{1/3}(30\pi/6^{1/2})^{2/3}(\mathscr{E}/n\varkappa T_e)^{2/3}; \quad (21)$$

in the case of a hydrogen plasma, it follows from Eq. (21) that $\beta_0(0) \gg 0.93(\mathscr{E}/n\varkappa T_e)^{2/3}$.

The maximum energy density of Langmuir turbulence at which our analysis still holds is found from the condition of absence of a modulation instability:[7]

$$\frac{\mathscr{E}}{n\varkappa T_e} < (k_* D)^2 \approx 5\cdot 10^{-5}. \quad (22)$$

The condition for conservation of the shape of a soliton, which is that the change in $\beta_0$ should be small in the time $\tau_s = \Delta/\beta_0 c_s$, i.e., $\tau_1 \gg \tau_s$, corresponds to the inequality

$$\beta_0(0) \gg \left(\frac{m_e}{2m_i}\frac{\mathscr{E}}{n\varkappa T_e}\right)^{1/3} \approx 6.5\cdot 10^{-2}\left(\frac{\mathscr{E}}{n\varkappa T_e}\right)^{1/3}, \quad (23)$$

which is stronger than the inequality (21) if $\mathscr{E}/n\varkappa T_e < 10^{-4}$. Equation (19) is derived on the assumption that $|\bar{k}_x| \ll \Delta\bar{k}_x$. Assuming that $|\bar{k}_x| \approx k_*$, we find that

$$\beta_0(0) \gg 2\cdot 10^{-4}. \quad (24)$$

We can neglect other nonlinear mechanisms resulting in the diffusion of plasmons in the wave-vector space if $\tau_{N_1} \ll \tau_k$, where $\tau_k$ is the characteristic time of the most important of the above processes. In the range $|\mathbf{k}| \lesssim 2k_*$ this process is the induced scattering of Langmuir waves on ions[8] with a time $\tau_k \sim n\varkappa T_e m_i/\mathscr{E} m_e \omega_p$. The corresponding inequality is $\mathscr{E}/n\varkappa T_e \ll 10^{-4}$.

One should also find in which case the attenuation of an ion-acoustic soliton by Langmuir waves is considerably greater than its attenuation due to the interaction with particles. Electrons are trapped by the soliton field and mixing of their initial phases takes place during the motion along trajectories, and attenuation ceases rapidly. Ions are reflected in the field of a soliton, acquiring continuously energy from the soliton. The soliton is attenuated in accordance with the law[9]

$$\beta_0(t) = \beta_0(0)\,(1 + t/\tau_i)^{-2}, \quad (25)$$

where

$$\tau_i = (12\pi)^{1/2}(m_i/m_e)^{1/2}(T_i/T_e)^{3/2}\exp\left(\tfrac{1}{2}T_e/T_i\right)(\beta_0(0))^{-1/2}\omega_p^{-1}.$$

Equation (25) is derived assuming a Maxwellian distribution function of the particle velocities.

We can ignore the attenuation by ions if $\tau_1 \ll \tau_i$, which leads to the condition

$$\beta_0(0) \ll \left(\frac{\pi}{2}\right)^{1/2}\left(\frac{m_e}{m_i}\frac{\mathscr{E}}{n\varkappa T_e}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/4}\exp\left(\frac{1}{4}\frac{T_e}{T_i}\right). \quad (26)$$

In a plasma with the electron-to-ion temperature ratio $T_e/T_i \sim 10^2$, the condition (26) becomes

$$\beta_0(0) \ll 6.7\cdot 10^7\left(\frac{\mathscr{E}}{n\varkappa T_e}\right)^{1/2}. \quad (27)$$

If the initial soliton amplitude is $\beta_0 \sim 10^{-2}$, this condition is violated only for $\mathscr{E}/n\varkappa T_e < 10^{-18}$.

Figure 3 shows the limits of the range of validity of this analysis in the plane of the parameters $\beta_0(0)$, $\mathscr{E}/n\varkappa T_e$. The main restrictions are imposed as follows: line 1 corresponds to Eq. (22), line 2 corresponds to Eq. (23), and line 3 corresponds to Eq. (24). By way of example, we shall obtain an estimate for a plasma with the parameters $T_e \sim 10^5\,°$K, $n \sim 10^{12}$ cm$^{-3}$, $T_e \sim 10^2 T_i$. For $\mathscr{E}/n\varkappa T_e \sim 10^{-5}$, an ion-acoustic soliton with an initial amplitude $\beta_0 \sim 10^{-2}$ is attenuated over a distance of the order of a few tens of centimeters in a time $\tau_1 \sim 7.5 \times 10^{-4}$ sec. As the soliton amplitude decreases with time, the relationship (20) describing changes in this amplitude is no longer obeyed, since Eq. (20) is valid

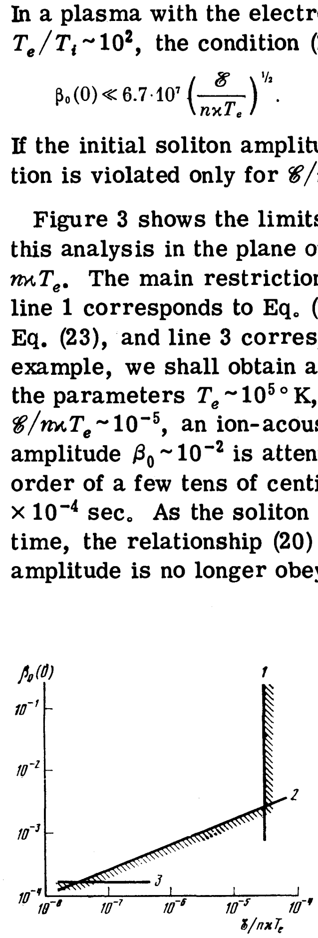


FIG. 3.

provided the trapping interval $\Delta k_x$ covers the whole of the energy-bearing part of the spectrum. In the case of small soliton amplitudes the trapping interval decreases rapidly and the attentuation of a soliton is described by the relationship found in Ref. 2.

We have considered only the attenuation of an ion-acoustic soliton under the action of plasma waves. Clearly, this action may also amplify a soliton if the plasma contains high-intensity short Langmuir waves with $k_x > k_*$.

We shall conclude by noting that we can consider similarly the problems of transformation of Langmuir waves under the action of collisionless shock waves, and also the interaction of ion-acoustic and electromagnetic waves.

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[1)]We are considering here the initial-value problem. However, similar results can also be obtained in the boundary-value problem.

[2)]Similar problems have been considered also for other types of waves, in particular, for electromagnetic waves in a plasma.[4]

[3)]In the case of a single reflection of plasma waves from an ion-acoustic soliton there is a considerable increase in $k$ only for waves with a small value of $|k_x|$. However, if the reflection process is of multiple nature (as is the case for two ion-acoustic solitons moving toward one another), monotonic pumping of energy of Langmuir plasmons up the spectrum is possible.

[4)]The procedure of going over from Eq. (15) to the explicit form of the law of conservation of the momentum (17) was found in cooperation with V. I. Karpman.


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# Nonlinear theory of penetration of *p*-polarized electromagnetic waves in a plasma

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The approximation of the normal skin effect is used to consider the problem of penetration of a high-frequency $p$-polarized nonlinear ionizing electromagnetic wave into a plasma under conditions when the plasma permittivity changes its sign in the field of the wave. The change in the permittivity from positive to negative values is abrupt. The limits of this abrupt change are found and their behavior is studied as a function of the angle of incidence and frequency of the wave. The abrupt change in the permittivity of the plasma at its boundary results in a jump in the amplitude dependence of the reflection coefficient of the wave.

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The problem of investigating the properties of a $p$-polarized electromagnetic wave propagating in a nonlinear medium is of considerable interest in theory and in practical applications. The problem is considerably more difficult to tackle than the corresponding problem of the behavior of an $s$-polarized wave because of an in crease in the number of components of the electric field vector and because of the possibility of a discontinuity (jump) in the permittivity of the medium when the sign of this permittivity changes in the field of the wave.[1] Therefore, the solution of the problem should allow for the jump in the field and permittivity; the boundaries of this jump are not known in advance and should be found in the process of solution.

The structure of continuous spatial distributions of the field of a high-frequency $p$-polarized electromagnetic wave in a locally nonlinear medium was studied by Eleonskiĭ and Silin.[2,3] Our task will be to determine the structure of discontinuous distributions of the field of such a wave which appear when the permittivity of the medium changes its sign in the field of the wave. We shall concentrate mainly on a nonlinear wave which ionizes a plasma with a permittivity $\varepsilon_0 > 0$ in the absence of the field because this problem is of interest in practical tasks of generating a plasma in a microwave discharge. We shall find the boundaries of a permittivity jump, and study their behavior as a function of the angle of incidence and frequency of the wave. We shall show that under certain conditions such a wave can only create a plasma with a negative permittivity. Under the same conditions there may be a finite value of the electric

  