$$W(E)=\frac{\pi^2 g(\gamma)A_0}{g(\gamma_0)z^2(1+3^{1/2}\omega_0{}^3n^5/z^2c^3)}. \quad (13)$$

The spectral intensity of the resonance bremsstrahlung of slow electrons turns out to be practically independent of the frequency:

$$dW/d\omega=\omega d\sigma/d\omega=W/E.$$

It is convenient to put Eq. (13) in the form

$$W(E)=\frac{g(\gamma)W_\kappa}{g(\gamma_0)(1+3^{1/2}\omega_0{}^3n^5/z^2c^3)},$$

$$W_\kappa=\frac{8\pi z^2}{3^{3/2}c^3},$$

where W_K is the total bremsstrahlung intensity, in the Kramers approximation, of an electron in the Coulomb field of a center with charge z. The term $\omega_0^3n^5/z^2c^3$ is not small in comparison with unity only when $E\to\omega_0$, i.e., $n\to\infty$. Therefore the intensity of resonance bremsstrahlung, in the case when it is possible, is of the order of or larger than that of the bremsstrahlung from potential scattering. Figure 1 shows the total intensity W_T of resonance and potential bremsstrahlung for the collision of an electron with the lithiumlike oxygen ion $O^{5+}(1s^22s)$, with ω_0 equal to the energy of the transition $2s\to2p$ (the contribution of potential bremsstrahlung is shown by a dashed line).

In conclusion we point out that the total radiation from the whole system also includes a contribution from recombination radiation, and for $E\geqslant\omega_0$ (the inequality is to the accuracy of the level width) also a contribution from excitation of the ion. Therefore the total intensity of the radiation from the whole system does not have a slight jump downward at $E\geqslant\omega_0$, but a sharp jump upward, because the excitation channel opens up. A separate treatment of the bremsstrahlung is nevertheless not without meaning, since it has its own peculiar spectral characteristics.

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Effect of mass renormalization in the stochastic acceleration of particles

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It is shown that an effect of mass renormalization leads to an additional energy loss in stochastic acceleration of particles. A kinetic description is given, and the collision integral associated with this effect is found.

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1. INTRODUCTION. STATEMENT OF THE PROBLEM

1. Stochastic acceleration of particles is possible both as the Fermi type of process[1] and also as a result of resonance interaction with waves and oscillations.[2] The two mechanisms are similar in their physical meaning (they correspond to a heating-up of particles either in collisions with heavy clouds or else with oscillations of large effective temperature[3]), although there is also a definite difference between them.

To illustrate the relation between these mechanisms we can use the following model. Let a fast particle move in a random manner in regions (clouds) with a field E which is constant in magnitude but not in direction. The size of a region occupied by the field is l, and the average distance between regions is $l_0\gg l$. We note that resonance particles are acted on by a constant field and such a model correctly reflects the main features of the interaction of fast particles with a gas of solitons; it is most similar to the Fermi model of collisions with magnetic clouds (in the Fermi model, a particle is also acted on during a collision by a constant electric field $-\mathbf{u}\times\mathbf{H}_0$, where $\mathbf{u}$ is the velocity of the cloud).

Let us consider the simplest one-dimensional case. In each collision with a region occupied by an electric field a fast particle acquires or loses (depending on the sign of the field E_1) an energy[1)] $\Delta\varepsilon=eE_1l$. But a particle that encounters a favoring field (pointing along its velocity) and acquires an energy $\Delta\varepsilon$ has a larger ve-

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locity $v_0+\Delta v, \Delta v=eE_1 l/mv_0$, after the collision, and consequently takes less time, $l_0/(v_0+\Delta v)$, with $l_0 \gg l$, in reaching the next collision. The average rate of gain of energy by the particle is

$$\frac{d\varepsilon}{dt}=\frac{eE_1l}{l_0}(v_0+\Delta v)-\frac{eE_1l}{l_0}(v_0-\Delta v)=\frac{2eE_1l\Delta v}{l_0}=\frac{2e^2E_1^2l^2}{mv_0l_0}=\frac{2e^2l\overline{E_1^2}}{mv_0}, \quad (1)$$

where $\overline{E_1^2}=E_1^2 l/l_0$ is the average energy of the field (times a factor 8π).

It is interesting that apart from an unimportant numerical factor of the order of unity, Eq. (1) agrees with the basic result of the theory of stochastic acceleration,[2] if instead of l we insert in Eq. (1) the mean correlation length

$$\bar{l}=\int |E_{k_1}|^2\frac{dk_1}{k_1}\Big/\int |E_{k_1}|^2 dk_1, \quad \overline{E_1^2}=\int |E_{k_1}|^2 dk_1.$$

This model (and also, incidentally, a more rigorous analysis) shows that the mechanism of stochastic acceleration is not sensitive to whether the random field corresponds to strong turbulence (a gas of solitons) or to weak turbulence (a gas of waves, in which case $l_0\approx l$), and depends only on the turbulence spectrum of the mean square field. Fermi acceleration corresponds to reflection from the cloud, while in resonance acceleration the particle passes through the field region, being only weakly perturbed.

We note that the tails of accelerated particles observed in many plasma experiments are, as is well known, due precisely to stochastic acceleration effected by various plasma oscillations (of the Langmuir, ionic-acoustic, lower-hybrid, and other types). This mechanism is so widely encountered, and also more effective than the Fermi mechanism, that it seems quite natural to regard it as the specific source of cosmic rays.[5] The main problem is that of explaining the energy distribution of cosmic rays, since the effectiveness of the acceleration is quite sufficient to account for their average energy.

The problem is in fact to find an effective braking force which could secure a balance between acceleration and deceleration and give a stationary spectrum. The present paper is devoted to the search for such a force. If we consider the resonance acceleration of fast but nonrelativistic particles, this problem has a bearing on the heating of a plasma to obtain controlled thermonuclear fusion. Most methods used for the heating of the plasma by means of waves of various frequency ranges reduce in the final analysis to a quasilinear increase of the energy of the resonance particles, provided, of course, that effects of binary collisions are negligible. The resonance may be of Cerenkov or of cyclotron type, and may be either between an injected wave and the plasma particles or between the particles and secondary waves that arise owing to linear or nonlinear conversion. The physical essence of the effect, which is the object of our present study, is that the stochastic acceleration (or stochastic heating) of the plasma must naturally be accompanied by an increase of the electromagnetic mass of a particle. This effect of "renormalization" of the electromagnetic mass is observable and is proportional to the rate of acceleration ($\sim\overline{E_1^2}$). Because energy is expended in increasing the electromagnetic mass there is a further deceleration of the particles. The main result of the present paper is that, in the framework of a classical (nonquantum) treatment the magnitude of the braking force owing to the effect in question is infinite. This is due to the large contribution of large wave vectors (momenta) of the proper field of the particle (ultraviolet divergence). A definite analogy can be traced between this divergence and the divergence of the Rayleigh-Jeans distribution for thermal radiation.

The divergence of the classical expression for the deceleration clearly indicates that the effect is important and rather large, which is also indicated by estimates obtained with a reasonable quantum cut-off of the divergence. When the effect of mass renormalization is taken into account one gets power-law energy spectra for the particles. Usually resonance stochastic acceleration can be described with a quasilinear collision integral. In the present paper we find an additional collision integral owing to the mass renormalization effect. In the framework of the classical description the coefficients of this integral diverge. With a reasonable cut-off they give additional terms of very significant size, which in many practically interesting cases are larger than those that come from binary collisions of the particles. This means that the use of the quasilinear integral together with the integral for binary collisions (a widely used scheme in many theoretical investigations) without the collision integral due to the effect of mass renormalization of the particles is not legitimate. The present paper contains the result of a classical, nonquantum, treatment of the problem for the general case of particles with any velocities (relativistic and nonrelativistic).

To simplify the exposition we shall suppose that a resonance quasilinear field E_{k1} is given and is longitudinal, so that

$$\langle E_{k_1,i}E_{k_1',j}\rangle=|E_{k_1}|^2\frac{k_{1i}k_{1j}}{k_1^2}\delta(\mathbf{k}_1+\mathbf{k}_1')\delta(\omega_1+\omega_1'), \quad (2)$$

and shall also assume that this field is isotropic (which suffices for the analysis of the problem of acceleration and heating, since an anisotropy would lead only to an additional angular scatter of the particles). Corresponding to our assumption that the field E_1 is longitudinal, we assume that its resonance with the particles is of the Cerenkov type, i.e., the resonance condition is of the form

$$\Omega_1\equiv\omega_1-\mathbf{k}_1\mathbf{v}=0.$$

Cyclotron resonance can be examined in a similar way.

The quasilinear collision integral takes the form[6]

$$\frac{d\Phi_\mathbf{p}}{dt}=I_{\text{qu.l.}}=\frac{\partial}{\partial p_i}D_{ij}^{\text{qu.l.}}\frac{\partial\Phi_\mathbf{p}}{\partial p_j},$$
$$D_{ij}^{\text{qu.l.}}=q^2\pi\int\frac{|E_{k_1}|^2k_{1i}k_{1j}}{k_1^2}\delta(\Omega_1)\,d\mathbf{k}_1\,d\omega_1, \quad (3)$$

where q is the charge of the resonating particles, and

Φ_p is the regular part of their distribution function ($k_1 = \{\mathbf{k}_1, \omega_1\}$, $dk_1 = d\mathbf{k}_1 d\omega_1$).

2. EFFECT OF RENORMALIZATION OF THE ELECTROMAGNETIC MASS ON THE CHANGE OF ENERGY OF A RESONANCE TEST PARTICLE

Each resonance particle in the given random field (2) acquires an energy which changes not only the stochastic component, but also the regular component of its velocity. Therefore, strictly speaking, we cannot prescribe its velocity at $-\omega$ (adiabatic turning on), since in this case it would formally acquire an infinite energy up to the time t considered. Therefore we pose the problem with the initial conditions that we assign to the particle a velocity, say, $\mathbf{v}$ at time $t=0$ (and for simplicity's sake also set $\mathbf{r}=0$ at $t=0$). Naturally this will not mean that the charge q of the particle is created at $t=0$; the velocity of the particle is simply fixed (measured) at $t=0$, and its field exists for $t<0$ as well as for $t>0$. The change of energy of the particle will be given by

$$\begin{aligned}\frac{d\varepsilon}{dt} &= q\langle \mathbf{v}(t)\,(\mathbf{E}_1(t)+\mathbf{E}^q(t))\rangle \\ &= q\int \langle \mathbf{v}(t)\mathbf{E}_{k_1}\rangle \exp(-i\Omega_1 t + i\mathbf{k}_1\mathbf{r}(t))\,d\mathbf{k}_1\,d\omega_1 \\ &\quad + q\int \langle \mathbf{v}(t)\mathbf{E}_k^q(t)\rangle \exp(i\mathbf{k}\mathbf{v}t + i\mathbf{k}\mathbf{r}(t))\,d\mathbf{k},\end{aligned} \tag{4}$$

where $\mathbf{E}_1(t)$ is the resonance field (1) at the position of the particle and $\mathbf{E}^q(t)$ is the proper field which the particle produces at its position.

Then in the general case the motion of a relativistic particle is described by the equation

$$\begin{aligned}\frac{d\mathbf{v}(t)}{dt} &= \frac{q}{\varepsilon}\int (\mathbf{E}_{k_1} - \mathbf{v}(t)\,(\mathbf{v}(t)\mathbf{E}_{k_1}))\exp(-i\Omega_1 t + i\mathbf{k}_1\mathbf{r}(t))\,d\mathbf{k}_1\,d\omega_1 \\ &+ \frac{q}{\varepsilon}\int (\mathbf{E}_{\mathbf{k}}^q(t) - \mathbf{v}(t)\,(\mathbf{v}(t)\mathbf{E}_{\mathbf{k}}^q(t)) + [\mathbf{v}(t)\mathbf{H}_{\mathbf{k}}^q(t)])\exp(i\mathbf{k}\mathbf{v}t + i\mathbf{k}\mathbf{r}(t))\,d\mathbf{k},\end{aligned} \tag{5}$$

where $\varepsilon = m/(1-v^2)^{1/2}$, $c=1$, $\mathbf{E}^q = \int E_{\mathbf{k}}^q \exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{k}$, and the coordinate of the particle is $\mathbf{v}t + \mathbf{r}(t)$, so that here and from now on $\mathbf{r}(t)$ describes the deviation from the rectilinear motion with the velocity $\mathbf{v}$. Finally,

$$i\mathbf{H}_{\mathbf{k}}^q(t) = \int_{-\infty}^{t} [\mathbf{k}\mathbf{E}_{\mathbf{k}}^q(t')]\,dt'.$$

We shall hereafter concern ourselves only with resonance interactions $\sim\delta(\Omega_1)$ and go to the limit $t\to\infty$ in Eq. (4), using the fact that

$$\frac{\sin\Omega_1 t}{\Omega_1} \to \pi\delta(\Omega_1), \quad t\to\infty. \tag{6}$$

We obtain the expression for the quasilinear increase of the energy of the particle by neglecting the proper field of the particle in Eqs. (4) and (5), and also neglecting the deviation of the trajectory from the rectilinear one. Then

$$\begin{aligned}\mathbf{v}(t) &\approx \mathbf{v}^{(1)}(t) = \frac{q}{\varepsilon}\int (\mathbf{E}_{k_1} - \mathbf{v}(\mathbf{v}\mathbf{E}_{k_1}))\frac{\exp(-i\Omega_1 t) - 1}{-i\Omega_1}\,d\mathbf{k}_1\,d\omega_1, \\ \mathbf{r}(t) &\approx \mathbf{r}^{(1)}(t) = \frac{q}{\varepsilon}\int (\mathbf{E}_{k_1} - \mathbf{v}(\mathbf{v}\mathbf{E}_{k_1}))\frac{\exp(-i\Omega_1 t) - 1 + i\Omega_1 t}{-\Omega_1^2}\,d\mathbf{k}_1\,d\omega_1.\end{aligned} \tag{7}$$

From this we get

$$\begin{aligned}\left(\frac{d\varepsilon}{dt}\right)_{\text{qu.l.}} &\approx q\int \mathbf{E}_{k_1}(\mathbf{v}^{(1)}(t) + i\mathbf{v}(\mathbf{k}_1\mathbf{r}^{(1)}(t)))\exp(-i\Omega_1 t)\,d\mathbf{k}_1\,d\omega_1 \\ &= \operatorname*{Re}_{t\to\infty}\frac{q^2}{\varepsilon}\int \frac{|E_{k_1}|^2}{k_1^2}\Big\{(k_1^2 - (\mathbf{k}_1\mathbf{v})^2)\frac{1-\exp(i\Omega_1 t)}{-i\Omega_1} \\ &\quad + (\mathbf{k}_1\mathbf{v})\,(k_1^2 - (\mathbf{k}_1\mathbf{v})^2)\frac{1-(1-i\Omega_1 t)\exp(i\Omega_1 t)}{-i\Omega_1^2}\Big\}d\mathbf{k}_1\,d\omega_1 \\ &= \frac{q^2}{\varepsilon}\pi\int \frac{|E_{k_1}|^2}{k_1^2}(k_1^2 - (\mathbf{k}_1\mathbf{v})^2)\,(\delta(\Omega_1) - (\mathbf{k}_1\mathbf{v})\,\delta'(\Omega_1))\,d\mathbf{k}_1\,d\omega_1.\end{aligned} \tag{8}$$

This result agrees exactly with that found from the expression (3) for the mean particle energy

$$\varepsilon = \int \varepsilon\frac{d\Phi_p}{dt}\frac{d\mathbf{p}}{(2\pi)^3},$$

namely

$$\frac{d\bar{\varepsilon}}{dt} = \int \left(\frac{d\varepsilon}{dt}\right)_{\text{qu.l.}}\Phi_p\frac{d\mathbf{p}}{(2\pi)^3}, \tag{9}$$

where $(d\varepsilon/dt)_{\text{qu.l.}}$ is given by the right-hand side of Eq. (8).

The mass renormalization effect is given by the particle's proper fields $\mathbf{E}^q$ and $\mathbf{H}^q$. Keeping in mind the divergence which we have discussed, with the main contribution coming from the largest admissible wave number of the proper fields, we must include the proper fields $\mathbf{E}^q$ and $\mathbf{H}^q$ in the limit of small wavelengths, where the dielectric constant is practically equal to unity ($|\mathbf{k}| \gg \omega_1, \omega_{pe}$; $c=1$). Then the field of the particle can be broken up into longitudinal and transverse components, $E_{\mathbf{k}}^{ql}(t)$ and $E_{\mathbf{k}}^{qt}(t)$. In this approximation the longitudinal field

$$\mathbf{E}_{\mathbf{k}}^{ql}(t) = \frac{4\pi\mathbf{k}q}{ik^2}\exp(-i\mathbf{k}\mathbf{r}(t) - i\mathbf{k}\mathbf{v}t) \tag{10}$$

is local in time, and, as can easily be seen, makes no contribution to the change of the particle's energy. Therefore we can regard the field $\mathbf{E}^q$ in Eq. (4) as transverse:

$$\mathbf{E}_{\mathbf{k}}^q(t) = \mathbf{E}_{\mathbf{k}}^{qt}(t) = -\frac{q}{2\pi^2}\int_{-\infty}^{t} \mathbf{v}^t(t')\exp(-i\mathbf{k}\mathbf{v}t' - i\mathbf{k}\mathbf{r}(t'))\cos|\mathbf{k}|\,(t-t')\,dt', \tag{11}$$

where $\mathbf{v}^t = \mathbf{v} - \mathbf{k}(\mathbf{k}\cdot\mathbf{v})/k^2$.

In first approximation in the field $\mathbf{E}^q$ and the intensity $|E_{k1}|^2$ of the resonant field, the rate of change of the particle's energy can be written in the form of two terms:

$$\left(\frac{d\varepsilon}{dt}\right)_q = \left(\frac{d\varepsilon}{dt}\right)_q^{\,t} + \left(\frac{d\varepsilon}{dt}\right)_q^{\,t}, \tag{12}$$

where the first term is due to the work done by the proper field, and the second, to the field $\mathbf{E}_1$ (more exactly, to the change of the work of the field $\mathbf{E}_1$ which is linear in the field $\mathbf{E}^q$).[2)] Namely,

$$\begin{aligned}\left(\frac{d\varepsilon}{dt}\right)_q^{\,t} &= q\int \langle \mathbf{v}(t)\mathbf{E}_{\mathbf{k}}^q(t)\rangle \exp(i\mathbf{k}\mathbf{v}t + i\mathbf{k}\mathbf{r}(t))\,d\mathbf{k} \\ &\approx -\operatorname{Re}\frac{q^2}{2\pi^2}\int d\mathbf{k}\int_{-\infty}^{t} dt'\,\langle \mathbf{v}^t(t)\mathbf{v}^t(t')\,(1 + i\mathbf{k}(\mathbf{r}(t) - \mathbf{r}(t'))) \\ &\quad - \frac{1}{2}(\mathbf{k}(\mathbf{r}(t) - \mathbf{r}(t')))^2)\exp(-i\Omega(t-t'))\rangle.\end{aligned} \tag{13}$$

Here $\Omega = |\mathbf{k}| - \mathbf{k}\cdot\mathbf{v}$.

Equation (13) includes only terms quadratic in $\mathbf{r}(t)$, which describes the deviation from rectilinear motion. We must insert into Eq. (13) for $\mathbf{r}(t)$, $\mathbf{v}(t)$ and $\mathbf{r}(t')$, $\mathbf{v}(t')$ their expansions to and including terms quadratic in the field $\mathbf{E}_1$:

$$\mathbf{r}(t)=\mathbf{r}^{(1)}(t)+\mathbf{r}^{(2)}(t), \qquad \mathbf{v}(t)=\mathbf{v}+\mathbf{v}^{(1)}(t)+\mathbf{v}^{(2)}(t), \tag{14}$$

where $\mathbf{r}^{(1)}(t)$ and $\mathbf{v}^{(1)}(t)$ are given by Eqs. (7) and (8), and $\mathbf{r}^{(2)}(t)$ and $\mathbf{v}^{(2)}(t)$ are found from the next approximation in terms of the field $\mathbf{E}_1$:

$$\begin{aligned}
\mathbf{v}^{(2)}(t)=&\frac{q^2}{\varepsilon^2}\int\frac{d\omega_1|E_{k_1}|^2\,d\mathbf{k}_1}{k_1^2}(2(\mathbf{k}_1-\mathbf{v}(\mathbf{k}_1\mathbf{v}))(\mathbf{k}_1\mathbf{v})\\
&+\mathbf{v}(k_1^2-(\mathbf{k}_1\mathbf{v})^2)\frac{(\exp(-i\Omega_1 t)-1)}{\Omega_1^2}-\frac{q^2}{\varepsilon^2}\int\frac{|E_{k_1}|^2}{k_1^2\Omega_1^3}(\mathbf{k}_1-\mathbf{v}(\mathbf{k}_1\mathbf{v}))\\
&\times(k_1^2-(\mathbf{k}_1\mathbf{v})^2)((\exp(-i\Omega_1 t))(2+i\Omega_1 t)-2)\,d\mathbf{k}_1\,d\omega_1,\\
\mathbf{r}^{(2)}(t)=&\frac{q^2}{\varepsilon^2}\int\frac{|E_{k_1}|^2\,d\mathbf{k}_1\,d\omega_1}{k_1^2(-i\Omega_1^3)}(2(\mathbf{k}_1-\mathbf{v}(\mathbf{k}_1\mathbf{v}))(\mathbf{k}_1\mathbf{v})\\
&+\mathbf{v}(k_1^2-(\mathbf{k}_1\mathbf{v})^2)(i\Omega_1 t-1+\exp(-i\Omega_1 t))+\frac{q^2}{\varepsilon^2}\int\frac{|E_{k_1}|^2}{k_1^2 i\Omega_1^4}(\mathbf{k}_1-\mathbf{v}(\mathbf{k}_1\mathbf{v}))\cdot\\
&\times(k_1^2-(\mathbf{k}_1\mathbf{v})^2)(-3-2i\Omega_1 t+(3+i\Omega_1 t)\exp(-i\Omega_1 t))\,d\mathbf{k}_1\,d\omega_1.
\end{aligned}\tag{15}$$

Using these expressions and substituting the expansions (14) in Eq. (13), keeping only terms of order E_1^2, and going to the limit $t\to\infty$, using Eq. (6), after rather cumbersome calculations we get the following result:

$$\begin{aligned}
\left(\frac{d\varepsilon}{dt}\right)_q^t=&-\frac{q^4}{2\pi\varepsilon^2}\int\frac{|E_{k_1}|^2\delta(\Omega_1)}{k_1^2(|\mathbf{k}|-\mathbf{kv})^4}\,d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}\Big\{(|\mathbf{k}|-\mathbf{kv})^2[(\mathbf{k}_1^t-\mathbf{v}^t(\mathbf{k}_1\mathbf{v}))^2\\
&-2(\mathbf{k}_1\mathbf{v})((\mathbf{k}_1^t\mathbf{v}^t)-(\mathbf{v}^t)^2(\mathbf{k}_1\mathbf{v}))-(\mathbf{v}^t)^2(k_1^2-(\mathbf{k}_1\mathbf{v})^2)]\\
&+(|\mathbf{k}|-\mathbf{kv})[4((\mathbf{k}_1^t\mathbf{v}^t)-(\mathbf{v}^t)^2(\mathbf{k}_1\mathbf{v}))((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))\\
&-2(\mathbf{k}_1\mathbf{v})((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))-(\mathbf{kv})(k_1^2-(\mathbf{k}_1\mathbf{v})^2)]+3((\mathbf{kk}_1)\\
&-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))^2\Big\}+\frac{q^4}{2\pi\varepsilon^2}\int\frac{|E_{k_1}|^2\delta'(\Omega_1)\,d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}}{k_1^2(|\mathbf{k}|-\mathbf{kv})^3}\\
&\times\{(|\mathbf{k}|-\mathbf{kv})(k_1^2-(\mathbf{k}_1\mathbf{v})^2)((\mathbf{k}_1^t\mathbf{v}^t)-(\mathbf{v}^t)^2(\mathbf{k}_1\mathbf{v}))+[\mathbf{k}\times\mathbf{v}]^2(k_1^2-(\mathbf{k}_1\mathbf{v})^2)\\
&\cdot((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))/k^2\};\qquad \mathbf{k}_1^t=\mathbf{k}_1-\mathbf{k}(\mathbf{kk}_1)/k^2.
\end{aligned}\tag{15a}$$

This expression, like the formula (8) for the resonance acceleration, contains a term in $\delta'(\Omega_1)$ in addition to the one in $\delta(\Omega_1)$.

For an isotropic field E_{k1} the result becomes much simpler:

$$\begin{aligned}
\left(\frac{d\varepsilon}{dt}\right)_q^t=&\frac{q^4}{2\pi\varepsilon^2}\int\frac{|E_{k_1}|^2}{k_1^2k^2}\,d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}\,\delta(\Omega_1)\frac{(\mathbf{k}_1\mathbf{v})^2}{v^4}\\
&\times\left\{3-4v^2-(1-v^2)(3-2v^2)\frac{1}{2v}\ln\frac{1+v}{1-v}\right\}\\
&-\frac{q^4}{2\pi\varepsilon^2}\int|E_{k_1}|^2\Big|_{\omega_1=k_1v}\,d\mathbf{k}_1\frac{d\mathbf{k}}{k^2}\frac{(1-v^2)}{v^2}\left\{1-\frac{(1-v^2)}{2v}\ln\frac{1+v}{1-v}\right\}.
\end{aligned}\tag{16}$$

The second term in Eq. (16) comes formally from $\delta'(\Omega_1)$ when we integrate by parts, and is analogous to the well known corresponding term in quasilinear acceleration (heating up), as described by Eq. (9):

$$\left(\frac{d\varepsilon}{dt}\right)_{\text{qu.l.}}=\frac{2q^2}{\varepsilon}\pi\int\frac{|E_{k_1}|^2}{k_1^2}(\mathbf{k}_1\mathbf{v})^2\delta(\Omega_1)\,d\mathbf{k}_1\,d\omega_1+\frac{q^2}{\varepsilon}\pi(1-v^2)\int|E_{k_1}|^2\Big|_{\omega_1=k_1v}\,d\mathbf{k}_1.$$

In both cases this term is small in the ultrarelativistic limit $v\to 1$, and is large, on the average, for nonrelativistic velocities. It is easily verified that the sign in Eq. (16) is opposite to that in Eq. (9); that is, the expression (16) describes a loss of energy by the particle.

The second part of the work done by the forces is

$$\left(\frac{d\varepsilon}{dt}\right)_q^l=q\int\langle\mathbf{v}(t)\mathbf{E}_{k_1}\rangle\,d\mathbf{k}_1\,d\omega_1\exp(-i\Omega_1 t+i\mathbf{k}_1\mathbf{r}(t)). \tag{17}$$

On substituting the values of $\mathbf{v}(t)$ and $\mathbf{r}(t)$ in this equation, we again include only the contribution linear in $\mathbf{E}^q$. We also keep only the contribution linear in E_1 [since the expression (17) already contains one factor E_1]. The result of a rather cumbersome calculation can be written in the form

$$\left(\frac{d\varepsilon}{dt}\right)_q^l=\left(\frac{d\varepsilon}{dt}\right)_q^t-\frac{q^4}{\pi\varepsilon^2}\int\frac{|E_{k_1}|^2(\mathbf{k}_1\mathbf{v})^2}{k_1^2v^2k^2}\delta(\Omega_1)\,d\mathbf{k}\,d\mathbf{k}_1\,d\omega_1\left(\frac{1}{2v}\ln\frac{1+v}{1-v}-1\right), \tag{18}$$

where $(d\varepsilon/dt)_q^t$ is given by Eq. (15).

The result [Eq. (18)] holds both for anisotropic and isotropic fields $\mathbf{E}_1$. If $\mathbf{E}_1$ is isotropic, we can use Eq. (16) for the first term. We note that the total loss (12) contains twice the term $(d\varepsilon/dt)_q^t$ plus the second term of Eq. (16). In the limiting cases, nonrelativistic ($v\ll 1$) and ultrarelativistic ($v\to 1$) we get the respective results

$$\left(\frac{d\varepsilon}{dt}\right)_q\approx-\frac{19q^4}{15\pi\varepsilon^2}\int\frac{|E_{k_1}|^2(\mathbf{k}_1\mathbf{v})^2}{k_1^2k^2}\delta(\Omega_1)\,d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}-\frac{q^4}{3\pi\varepsilon^2}\int|E_{k_1}|^2\Big|_{\omega_1=k_1v}\frac{d\mathbf{k}_1\,d\mathbf{k}}{k^2}, \tag{19}$$

$$\left(\frac{d\varepsilon}{dt}\right)_q\approx-\frac{q^4}{\pi\varepsilon^2}\int\frac{|E_{k_1}|^2}{k_1^2}(\mathbf{k}_1\mathbf{v})^2\left(\ln\frac{2\varepsilon}{m}-\frac{1}{2}\right)\delta(\Omega_1)\frac{d\mathbf{k}\,d\mathbf{k}_1}{k^2}. \tag{20}$$

Like the general expressions (16) and (18), the limiting formulas (19) and (20) diverge for large k, since $d\mathbf{k}/k^2=4\pi d|\mathbf{k}|$. We shall discuss this result later.

3. THE CHANGE OF THE ENERGY OF THE PROPER FIELD OF THE PARTICLE

We shall give a proof that this effect of deceleration of the particle is due to the change of its proper field. For this purpose it is convenient to consider the change with time of the high-frequency field (see Ref. 7):

$$\frac{\partial W^t}{\partial t}=\frac{\partial}{\partial t}\int\frac{(\mathbf{E}^t)^2+\mathbf{H}^2}{8\pi}d\mathbf{r}=\frac{\partial}{\partial t}\pi^2\int(\mathbf{E}_{\mathbf{k}}^t(t)\mathbf{E}_{-\mathbf{k}}^t(t)+\mathbf{H}_{\mathbf{k}}(t)\mathbf{H}_{-\mathbf{k}}(t))\,d\mathbf{k}, \tag{21}$$

$$\frac{\partial W^l}{\partial t}=\frac{\partial}{\partial t}\int\frac{(\mathbf{E}^l)^2}{8\pi}d\mathbf{r}=\frac{\partial}{\partial t}\pi^2\int\mathbf{E}_{\mathbf{k}}^l\mathbf{E}_{-\mathbf{k}}^l\,d\mathbf{k}. \tag{22}$$

In calculating the expressions (21) and (22) we must keep terms to and including those quadratic in E_1, i.e., those of types $E^{(0)}E^{(2)}$ and $E^{(1)}E^{(1)}$, and similarly for the magnetic field (the upper index indicates the degree in E_1). The required fields are calculated from Eqs. (10) and (11). It is easy to show that if in Eq. (22) both longitudinal fields are given by Eq. (10) the corresponding change of the energy is zero. If in Eq. (22) one of the longitudinal fields is $\mathbf{E}_1$ and the other is $\mathbf{E}^{ql}$, then the result of the calculation of Eq. (22) is equal, except for sign, to $(d\varepsilon/dt)_q^l$:

$$dW^l/\partial t=-(d\varepsilon/dt)_q^l.$$

Accordingly, $(d\varepsilon/dt)_q^l$ comes from the change of the energy of the longitudinal field owing to the action of the transverse high-frequency field of the particle. In precisely the same way one can derive that

$$\partial W^t/\partial t = -(d\varepsilon/dt)_q{}^t.$$

Thus the complete energy balance is established: The (negative) work done by the field on the particle goes into the increase of the energy of the field.

We now need only to see what kind of field we are concerned with, the radiation field or the proper field of the particle. Let us write out, for example, $E_k^{qt(1)}$ as found from Eq. (11):

$$\begin{aligned}E_k^{tq(1)}(t) = &\frac{q^2}{4\pi^2\varepsilon}\int\frac{(\mathbf{E}_{k_1}{}^t - \mathbf{v}^t(\mathbf{E}_{k_1}\mathbf{v}))}{\Omega_1}e^{-i\mathbf{k}\mathbf{v}t}d\mathbf{k}_1\,d\omega_1\\ &\times\left\{e^{-i\Omega_1 t}\left(\frac{1}{\mathbf{kv}+|\mathbf{k}|+\Omega_1}+\frac{1}{\Omega_1-|\mathbf{k}|+\mathbf{kv}}\right)\right.\\ &\left.-\left(\frac{1}{\mathbf{kv}+|\mathbf{k}|}+\frac{1}{\mathbf{kv}-|\mathbf{k}|}\right)\right\}\\ &+\frac{q^2}{4\pi^2\varepsilon}\int\frac{\mathbf{v}^t((\mathbf{kE}_{k_1})-(\mathbf{kv})(\mathbf{E}_{k_1}\mathbf{v}))}{\Omega_1{}^2}e^{-i\mathbf{k}\mathbf{v}t}\\ &\times\left\{e^{-i\Omega_1 t}\left(\frac{1}{\Omega_1+(\mathbf{kv})+|\mathbf{k}|}+\frac{1}{\Omega_1+(\mathbf{kv})-|\mathbf{k}|}\right)\right.\\ &-(1-i\Omega_1 t)\left(\frac{1}{(\mathbf{kv})+|\mathbf{k}|}+\frac{1}{(\mathbf{kv})-|\mathbf{k}|}\right)\\ &\left.-\Omega_1\left(\frac{1}{((\mathbf{kv})+|\mathbf{k}|)^2}+\frac{1}{((\mathbf{kv})-|\mathbf{k}|)^2}\right)\right\}d\mathbf{k}_1\,d\omega_1.\end{aligned}\tag{23}$$

Since the effect in question is proportional to $\delta(\Omega_1)$, so that $\Omega_1=0$, the field $\mathbf{E}_k^{qt(1)}$ is propagated with the frequency $\mathbf{k}\cdot\mathbf{v}$, i.e., is displaced at the particle velocity $\mathbf{v}$, and not at the speed of light, at which an electromagnetic wave is propagated. Accordingly, what we have is a change of the proper speed of the particle, and not of the radiation field. Accordingly, the physics of the effect is that the quasilinear acceleration (heating up) leads to an increase of the energy of the particle. It must also be accompanied by an increase of the proper field of the particle (a mass renormalization). The expenditure of energy to increase the proper field leads to an added energy loss. The mass renormalization, like transition radiation, has a spectral density that does not depend on frequency.[7] We can verify that there is no radiation at high frequencies $\omega\approx|\mathbf{k}|\gg\omega_1$ by calculating the power radiated by the particle as the integral of the Poynting vector over a cylindrical surface with its axis along the velocity v. The random field $\mathbf{E}_1$ is a set of wave trains of random lengths, on which there can be transition radiation. It is exponentially small, however, in the limit with which we are concerned here, that of wavelengths $2\pi/|\mathbf{k}|$ much smaller than the size of the inhomogeneities.

4. THE KINETIC EQUATION WITH MASS RENORMALIZATION INCLUDED

To describe the kinetics of a system of accelerated (heated) particles, we shall, as usual, introduce the random component $f_p(\mathbf{r},t)$ and the regular component $\Phi_p(\mathbf{r},t)$ of the distribution of the particles. We shall use the spatial Fourier component $f_{p,k}(t)$ of the random distribution function,

$$f_p(\mathbf{r},t)=\int f_{p,k}(t)e^{i\mathbf{kr}}d\mathbf{k}.$$

For it we have the equation

$$\begin{aligned}\frac{\partial f_{p,k}(t)}{\partial t}+i(\mathbf{kv})f_{p,k}(t) = &-\left(\mathbf{F}_k(t)\frac{\partial}{\partial\mathbf{p}}\right)\Phi_p(t)\\ &-\int d\mathbf{k}_1\left\{\left(\mathbf{F}_{k_1}\frac{\partial}{\partial\mathbf{p}}\right)f_{p,k-k_1}(t)-\left\langle\left(\mathbf{F}_{k_1}\frac{\partial}{\partial\mathbf{p}}\right)f_{p,k-k_1}(t)\right\rangle\right\}.\end{aligned}\tag{24}$$

Here

$$\mathbf{F}_k(t)=q\{\mathbf{E}_k(t)+[\mathbf{v}(t)\mathbf{H}_k(t)]\}$$

and for simplicity we regard the regular component as uniform in space,

$$\frac{d\Phi_p(t)}{dt}=\frac{\partial}{\partial\mathbf{p}}\mathbf{J}_p,\qquad \mathbf{J}_p=-q\int\langle\mathbf{F}_k(t)f_{p,k'}(t)\rangle\exp\{i(\mathbf{k}+\mathbf{k}')\mathbf{r}\}d\mathbf{k}\,d\mathbf{k}',\tag{25}$$

where $\mathbf{J}_p$ is the flux of particles in momentum space.

We again break up the field $\mathbf{E}_k$ into two components

$$\mathbf{E}=\mathbf{E}_1+\mathbf{E}^q.$$

The field $\mathbf{E}_k^{qt}$ of the particles is found from the expression

$$\mathbf{E}_k{}^{qt}(t)=-4\pi\int\frac{\mathbf{v}^t\,d\mathbf{p}}{(2\pi)^3}\int_{-\infty}^{t}dt' f_{p,k}(t')\cos\{|\mathbf{k}|(t-t')\},\tag{26}$$

and the longitudinal field produced by the particles is found in a similar way. The formal solution of Eq. (24) can be written in the form

$$\begin{aligned}f_{p,k}(t)=f_{p,k}(0)&-\exp\{-i(\mathbf{kv})t\}\int_0^t dt'\exp\{i(\mathbf{kv})t'\}\left(\mathbf{F}_k(t')\frac{\partial}{\partial\mathbf{p}}\right)\Phi_p(t')\\ &-\exp\{-i(\mathbf{kv})t\}\int d\mathbf{k}_1\int_0^t dt'\exp\{i(\mathbf{kv})t'\}\left\{\left(\mathbf{F}_{k_1}(t')\frac{\partial}{\partial\mathbf{p}}\right)f_{p,k-k_1}(t')\right.\\ &\left.-\left\langle\left(\mathbf{F}_{k_1}(t')\frac{\partial}{\partial\mathbf{p}}\right)f_{p,k-k_1}(t')\right\rangle\right\}.\end{aligned}\tag{27}$$

We can separate the integral (25) into three terms:

$$\begin{aligned}&\mathbf{J}_p=\mathbf{J}_p^{qt}+\mathbf{J}_p^{ql}+\mathbf{J}_p^{\text{qu.l.}},\\ &\mathbf{J}_p{}^{qt}=-q\int\langle\mathbf{F}_k{}^q(t)f_{p,k'}(t)\rangle\exp\{i(\mathbf{k}+\mathbf{k}')\mathbf{r}\}d\mathbf{k}\,d\mathbf{k}';\\ &\mathbf{J}_p^{\text{qu.l.}}+\mathbf{J}_p{}^{ql}=-q\int\langle\mathbf{E}_{k_1,\omega_1}f_{p,k_1'}(t)\rangle\exp\{-i\omega_1 t+i(\mathbf{k}_1+\mathbf{k}_1')\mathbf{r}\}d\mathbf{k}_1\,d\mathbf{k}_1'\,d\omega_1,\end{aligned}\tag{28}$$

which are analogous to $(d\varepsilon/dt)_q^t$ and $(d\varepsilon/dt)_q^l$ in the previous calculations. We conduct our further calculations in such a way that only terms linear in $\mathbf{F}^q$ and quadratic in $\mathbf{E}_1$ are taken into account. For this purpose we iterate Eq. (27) the necessary number of times and drop terms of higher orders than those indicated. Finally, being interested only in terms linear in Φ_p, we keep only terms quadratic in $f_{p,k}(0)$. To the needed accuracy (since the expansion with respect to the necessary parameters has been carried out), we can calculate the quadratic combinations of the $f_{p,k}(0)$ by regarding the particles as uncorrelated[8]:

$$\langle f_{p,k}(0)f_{p',k'}(0)\rangle=\Phi_p\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{k}+\mathbf{k}').$$

After rather cumbersome calculations we get the result

$$\frac{d\Phi_p}{dt}=\frac{\partial}{\partial p_i}(D_{ij}^{\text{qu.l.}}+D_{ij}{}^q)\frac{\partial\Phi_p}{\partial p_j}+\frac{\partial}{\partial p_i}F_i{}^q\Phi_p.\tag{29}$$

Unlike the quasilinear integral, the mass renormalization effects (index q) lead not only to diffusion (coefficient D_{ij}^q), but also to a friction force (the term in F_i^q):

$$D_{ij}^{q}=D_{ij}^{q,E}+D_{ij}^{q,H},$$

$$D_{ij}^{q,E}=-\frac{q^4}{2\pi}\int\frac{|E_{k_1}|^2\,d\mathbf{k}\,d\mathbf{k}_1\,d\omega_1\delta(\Omega_1)}{k_1^2(|\mathbf{k}|-(\mathbf{kv}))}\times\left\{k_{1i}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}}\right)v_j^t+k_{1j}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}}\right)v_i^t\right\}\frac{1}{(|\mathbf{k}|-(\mathbf{kv}))}\quad(30)$$

or in a different form

$$D_{ij}^{q,E}=-\frac{q^4}{2\pi\varepsilon}\int\frac{|E_{k_1}|^2}{k_1^2}\frac{d\mathbf{k}\,d\mathbf{k}_1\,d\omega_1\delta(\Omega_1)}{(|\mathbf{k}|-(\mathbf{kv}))^3}\times\{(|\mathbf{k}|-(\mathbf{kv}))[k_{1i}(k_{1j}^t-v_j^t(\mathbf{k}_1\mathbf{v}))+k_{1j}(k_{1i}^t-v_i^t(\mathbf{k}_1\mathbf{v}))]+(v_j^tk_{1i}+v_i^tk_{1j})((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))\}.\quad(30a)$$

Furthermore,

$$D_{ij}^{q,H}=-\frac{q^4}{2\pi\varepsilon}\int\frac{|\mathbf{E}_{k_1}|^2\,d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}\delta(\Omega_1)}{k_1^2|\mathbf{k}|(|\mathbf{k}|-(\mathbf{kv}))^3}\{(|\mathbf{k}|-(\mathbf{kv}))\times(k_{1j}[\mathbf{v}\times[\mathbf{k}\times\mathbf{k}_1]]_i+k_{1j}[\mathbf{v}\times[\mathbf{k}\times\mathbf{k}_1]]_j)+((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))\times(k_{1j}[\mathbf{v}\times[\mathbf{k}\times\mathbf{v}]]_i+k_{1i}[\mathbf{v}[\mathbf{k}\times\mathbf{v}]]_j)\}.\quad(31)$$

In the compact form (30) of the expression, the operators $\partial/\partial_{\mathbf{p}}$ act only on the functions depending on p in the diffusion coefficient [and not on the distribution function $\Phi_{\mathbf{p}}$ in Eq. (29)]. This is also reflected in the other way of writing the diffusion coefficient $D_{ij}^{q;E}$ [Eq. (30a)], in which the differentiation has been done explicitly. In just the same way the expression (31) can be written more compactly. The diffusion coefficient $D_{ij}^{q;H}$ is due to the magnetic force; the terms containing k_{ij} in Eq. (31) arise from the Lorentz force in $\mathbf{J}_{\mathbf{p}}^{qt}$, while the terms that contain k_{ii} arise from the inclusion of the Lorentz force in $f_{\mathbf{p},\mathbf{k}}$, which appears in $\mathbf{J}_{\mathbf{p}}^{ql}$ [see Eq. (28)]. The diffusion coefficient (31) does not lead to any change of the mean energy of the particles.

The term in the coefficient D_{ij}^{qE} proportional to k_{ii} arises from the electric force in $\mathbf{J}_{\mathbf{p}}^{qt}$, and that proportional to k_{ij} comes from $\mathbf{J}_{\mathbf{p}}^{ql}$. These two terms make equal contributions to the change (9) of the energy of a particle, each being equal on the average to $(d\varepsilon/dt)_q^t$ [the total $(d\varepsilon/dt)_q$ contains $2(d\varepsilon/dt)_q^t$, just as it is contained in Eqs. (12) and (18)]. Finally, the calculations give for the force of friction:

$$\mathbf{F}^q=-\frac{q^4}{2\pi}\int\frac{|E_{k_1}|^2}{k_1^2}\mathbf{k}_1\delta(\Omega_1)\frac{\partial}{\partial p_i}\frac{1}{(|\mathbf{k}|-(\mathbf{kv}))^3}\times\left\{(|\mathbf{k}|-(\mathbf{kv}))\left(k_{1i}^t-v_i^t(\mathbf{k}_1\mathbf{v})+\frac{1}{|\mathbf{k}|}([\mathbf{v}\times[\mathbf{k}\times\mathbf{k}_1]]_i-(\mathbf{k}_1\mathbf{v})[\mathbf{v}\times[\mathbf{k}\times\mathbf{v}]]_i)\right)+\left(v_i^t+\frac{1}{|\mathbf{k}|}[\mathbf{v}\times[\mathbf{k}\times\mathbf{v}]]_i\right)((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))\right\}\frac{1}{\varepsilon}d\mathbf{k}\,d\mathbf{k}_1\,d\omega_1$$
$$=\frac{q^4}{2\pi\varepsilon^2}\int\frac{|E_{k_1}|^2\,d\mathbf{k}\,d\mathbf{k}_1\,d\omega_1}{k_1^2k^2v^2}\delta(\Omega_1)\left(\frac{1}{2v}\ln\frac{1+v}{1-v}-1\right).\quad(32)$$

In the expression (9) this force leads to the change of energy described by the second term of Eq. (18). Accordingly, by still another method we have found exactly the same expressions for the energy lost by particles owing to the effect of mass renormalization.

We note that all of these coefficients D_{ij}^q and F_j^q found in this way diverge at large k (in proportion to $d\mathbf{k}/k^2=4\pi dk$). This sort of divergence is of much greater significance than that which has been discussed in connection with the so-called turbulent broadening of resonances.[6] The resonance denominators have divergences in higher orders near resonances, while the terms found here diverge independently of the resonance conditions, and the broadening of the resonance $\Omega_1\approx 0$ does not remove this divergence. In the framework of the present approach we can also calculate the change of the field amplitude of the transverse electromagnetic waves, and verify that there is no emission of radiation in the high-frequency limit. In other words, not only do the individual particles not radiate, but also the system of particles does not occasion emission from the plasma, although very high frequencies are present in the proper fields of the particles.

5. DISCUSSION OF THE RESULTS

We must discuss the limits of applicability of our results. In the estimates we must keep in mind the pleasant condition $|\mathbf{k}|\gg\omega_1$. It can be seen from Eq. (13) that times $\tau=t-t'\leqslant 1/\Omega\approx 1/|\mathbf{k}|$ contribute to this expression (the fractional contribution of angles with $\cos\theta=\mathbf{k}\cdot\mathbf{v}/kv\to 1$ turns out to be small). Consequently, for $|\mathbf{k}|\gg\omega_1$ we have $\mathbf{r}(t')\approx\mathbf{r}(t)-\tau\mathbf{v}(t)$, i.e.,

$$\mathbf{kr}(t)-\mathbf{kr}(t-\tau)\approx q\int\frac{E_{k_1}}{\varepsilon\Omega_1}d\mathbf{k}_1\,d\omega_1.$$

Accordingly the use of the expression (13) puts a restriction on the field amplitude E, but not on the quantity $|\mathbf{k}|$. The restriction on E is not a severe one (the velocity of the oscillations of the particle in the field of the wave must be smaller than the speed of the particle or of light). It is important that the classical nonquantum restrictions on the quantity $|\mathbf{k}|$ do not arise here. It can be seen from the preceding section that the divergence arises from the fluctuations brought in by $f_{\mathbf{p},\mathbf{k}}(0)$. The other well known divergences, due, for example, to spontaneous emission, also[8] come from $f_{\mathbf{p},\mathbf{k}}(0)$. Therefore at any rate this divergence is not the only one of its kind.

As for the quantum cut-off of the divergence, in principle, since we are concerned with the cutting off of a virtual field, we could use $k_{\max}\approx\varkappa\varepsilon/\hbar$, where $\varkappa$ is some numerical constant. In the case of isotropic distribution of particles and of turbulence, Eq. (29) with such a cut-off is of the form

$$\frac{d\Phi_{\mathbf{p}}}{dt}=\frac{1}{p^2}\frac{\partial}{\partial p}p^2D^{\text{qu.l.}}\left\{\left[1-\frac{4\alpha\varkappa}{\pi v^2}\left(1-\frac{(1-v^2)}{2v}\ln\frac{1+v}{1-v}\right)\right]\frac{\partial\Phi_{\mathbf{p}}}{\partial p}+\frac{4\varkappa\alpha}{\pi\varepsilon v}\left(\frac{1}{2v}\ln\frac{1+v}{1-v}-1\right)\Phi_{\mathbf{p}}\right\},\qquad\alpha=\frac{q^2}{\hbar c},\quad(33)$$
$$D^{\text{qu.l.}}=\pi q^2\int|E_{k_1}|^2d\mathbf{k}_1d\omega_1\delta(\Omega_1)\,\omega_1^2/k_1^2v^2.$$

It can be seen from this formula that the effect of the mass renormalization is relatively small, $\sim\alpha=q^2/\hbar c$; i.e., it is of the nature of the radiative corrections to the quasilinear collision integral. This means also that the mass renormalization effect can greatly exceed the effect of binary collisions, which has as its small factor $4\pi nmv^2/N_d|E_1|^2$, considering $N_d>4\pi nmv^2/\alpha|E_1|^2$. The widespread use of the quasilinear integral along with the collision integral with the mass renormalization effect neglected is illegitimate over a broad range of practically interesting values of N_d, and especially for the relaxation of relativistic beams (N_d is the number of particles in the Debye sphere).

The new effect of the appearance of a friction force is qualitatively important (in comparison with quasilinearity). In the nonrelativistic limit its relative importance among renormalization effects is of the order $v^2 \ll 1$, but it is larger than the friction forces owing to binary collisions for $|E_1|^2/4\pi nT > c^4\hbar\omega_1/v_T^4 T$. In the ultrarelativistic limit its contribution to the renormalization effects is sizable or even predominant [larger by a factor $\ln(\varepsilon/m)$ than the terms that describe the dispersion in energy). The presence of the friction force makes possible the existence of equilibrium distributions of the particles. In the region of nonrelativistic velocities such distributions are Maxwellian with the relativistic temperature $T_{eff} \approx 3mc^2/4\alpha\varkappa$; i.e., they do not completely forbid escape (for example for ionic-acoustic oscillations[9]) at nonrelativistic energies. For $v \to 1$, i.e., in the ultrarelativistic limit, the equilibrium distribution will be a power law, $1/\varepsilon^\gamma$, both for ions and for electrons.

To answer the question as to whether one can explain in this way the observed power-law spectrum of cosmic rays, it is necessary to obtain a quantum-theoretical expression for $\varkappa = \varkappa(\gamma)$. It is important that this is almost the only mechanism which, when one uses resonance acceleration,[2] can give a power-law spectrum for ions (for electrons there also exists the model of the so-called turbulent plasma reactor[10,11]). The mass renormalization effect can also affect the propagation of transverse electromagnetic waves of high frequency, since an electromagnetic wave can vibrate particles at a frequency corresponding to those that contribute to the mass renormalization. For isotropic particles there is an additional anomalous absorption, which was calculated earlier in Ref. 12.

[1] Here the resonance field is given a subscript 1 throughout.

[2] The physical meaning of these parts of the work done by the forces, which parts are due respectively to the transverse (superscript t) and longitudinal (superscript l) mass renormalizations, will be explained in Sec. 3.

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Vibrational bistability in an optically excited nonequilibrium molecular gas

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It is shown that a molecular gas subjected to sufficiently strong optical excitation of the electron transition can exist in two stationary states with different degree of excitation of the vibrational degrees of freedom. The transition from one state to the other is discontinuous when the parameters of the external action reach their critical values, and is an analog of first-order phase transitions. The case of a collision gas of molecules with few atoms and of a rarefied gas of complex polyatomic molecules are considered separately. The vibrational bistability of a molecular gas manifests itself, in particular, in ambiguity and jumplike changes of the optical characteristics of the gas. The effect is subject to hysteresis.

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§1. INTRODUCTION

The optics of electron-vibrational molecular systems in weak electromagnetic fields has been well investigated both experimentally and theoretically. If the probability of optical excitation is much smaller than the reciprocal times of the different relaxation processes, then the absorbing electron-vibrational system is in a state of thermodynamic equilibrium, as is usually assumed in the study of the optics of these objects. In modern spectroscopy, however, a situation arises wherein, owing to the large power of the absorbed radiation, the molecular gas is in an essentially nonequilibrium state. The disequilibrium of the absorbing system should in turn exert an influence on the absorption process itself. This change in the properties of the medium under the influence of the radiation can manifest itself in a large number of specific nonlinear effects, to which the present paper is devoted.

Consider a gas of molecules having two electronic states, 1 and 2, and N vibrational degrees of freedom