Resonance bremsstrahlung of an electron in collision with an ion

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Resonance bremsstrahlung of an electron in collision with an ion is due to emission by the electron during its lifetime in an autoionization state. Formulas are derived which describe this phenomenon. It is shown that the resonance bremsstrahlung can exceed the bremsstrahlung occasioned by potential scattering.

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1. Recently considerable attention has been given[1–5] to the effects on bremsstrahlung of the perturbation of the bound electrons of an atom (neutral or partially ionized) by an incident nonrelativistic electron. It follows from Refs. 1–5 that when we take into account the perturbation of the bound electrons we obtain a description of the radiation of the whole system, many-electron atom + incident electron, in the framework of a single formalism. In this way the separation of the total radiation into radiation owing to excitation of bound electrons (when this is energetically possible) by the incident electron, and bremsstrahlung proper, becomes to some extent a matter of convention. At course, at frequencies close to the excitation energies of atomic levels the main contribution to the emission from the entire system is that of emission owing to excitation of the atom. It was pointed out in Ref. 4 that in this region, according to the general formalism, the spectral intensity $dW/d\omega$ of the radiation is described, as usual, by the product of an excitation cross section $\sigma(E)$ and a Lorentz line shape:

$$\frac{dW}{d\omega}=\frac{\sigma(E)\omega_0\Gamma}{(\omega-\omega_0)^2+\Gamma^2/4},$$

where ω_0 is the excitation energy, Γ is the width of the level, E is the energy of the incident electron, and ω is the frequency emitted. In Refs. 1 and 2 the radiation in this region is called "resonance bremsstrahlung." Obviously this name for the radiation in this region must be regarded as erroneous, since physically and by generally accepted usage this radiation is to be ascribed to emission from the excited atom.

From Refs. 1–5 it can be concluded that for a measure of the suitability of the static approximation for describing the radiation from a given group of bound electrons one should consider the ratio of the frequency of the radiation to the frequency of the motion of the bound electrons in the atom. The static approximation can be used to describe strongly bound electrons (with binding energies larger than the energy of the incident electron), which make the main contribution to the screening of the nuclear charge. If, on the other hand, the frequency of the radiation is of the order of the energy of the incident electron and is larger than the binding energy of a weakly bound electron, the static approximation does not hold for the description of such electrons' contribution to the total radiation from the system. Only rough estimates[4] can now be made as to the conditions for applicability of the static approximation to describe the radiation of the whole system far from frequencies corresponding to excitation energies of the atom.

In the present paper we discuss yet another question about the influence of the perturbation of the bound electrons on the radiation from the incident electron, namely the effect of resonances in the scattering of a slow electron colliding with a partially ionized ion. By a slow electron we mean one whose energy is smaller than the energy for excitation of the first level. Thus the channel corresponding to excitation of the ion is closed. To within the reservations mentioned in the preceding paragraphs we can call this sort of radiation resonance bremsstrahlung. One feature of ions is that there are large numbers (Rydberg series) of autoionization states. Up to now it has been well known that the presence of these autoionization states leads under certain conditions to a marked increase of the cross sections for the excitation of multiply charged ions by electron collisions (resonance excitation[6]) and for photorecombination (dielectric recombination[7,8]). It will be shown here that the existence of the autoionization states can also leads to a considerable increase of the bremsstrahlung intensity. Ordinarily the bremsstrahlung of a slow electron colliding with an ion is small in comparison with the recombination radiation, and therefore the effect in question is interesting mainly from the point of view of examining the mechanisms of resonance phenomena and the limits of the applicability of the static approximation to the description of bremsstrahlung.

We note that the resonance bremsstrahlung of a slow electron in collision with a neutral atom has been studied earlier by D'yachkov, Kobzev, and Norman.[9] The feature that the phenomena these authors considered and those treated here have in common is their resonance character. The difference is in the nature of the resonances. In the present paper many two-electron resonances are considered, whereas in Ref. 9 only one one-electron resonance was dealt with.

2. The mechanism of the resonance phenomena is as follows:

$$A^z(\gamma_0)+e(E)\to A^{z-1}(\gamma_2,nl), \quad (1)$$

$$A^{z-1}(\gamma_2,nl)\to\begin{cases} A^z(\gamma_0)+e(E), & (2)\\ A^z(\gamma_1)+e(E-\hbar\omega_{\gamma_1\gamma_2}), & (3)\\ A^{z-1}(\gamma_0,nl)+\hbar\omega_{\gamma_0\gamma_2}, & (4)\\ A^{z-1}(\gamma_1,nl)+\hbar\omega_{\gamma_1\gamma_2}, & (5)\\ A^{z-1}(\gamma_2,n'l')+\hbar\omega_{nn'}\to A^z(\gamma_0)+e(E-\hbar\omega_{nn'})+\hbar\omega_{nn'}. & (6)\end{cases}$$

Here z is the charge of the ion $A^z(\gamma_0)$; $\gamma_{0,1,2}$ are the quantum numbers of the ground state and two excited states of the ion A^z; $e(E)$ is an electron with the energy E; $\hbar\omega$ is the energy of the transitions. An autoionization state $A^{z-1}(\gamma_2,nl)$ is formed in the collision of the electron with the ion. The possible further processes are the inverse decay (2), resonance excitation (3) of the transition $\gamma_0\to\gamma_1$, dielectric recombination (4) and (5), and resonance bremsstrahlung (6). Accordingly, resonance bremsstrahlung is emission from the electron during its lifetime in the autoionization state.

If in channel (5) the state $A^z(\gamma_1,nl)$ is itself an autoionization state, then its decay also leads to resonance bremsstralung:

$$A^{z-1}(\gamma_1,nl)+\hbar\omega_{\gamma_1\gamma_2}\to A^z(\gamma_0)+e(E-\hbar\omega_{\gamma_1\gamma_2})+\hbar\omega_{\gamma_1\gamma_2}. \quad (7)$$

Apart from the notations, the schemes (6) and (7) describe exactly the same mechanism. In fact, resonance bremsstrahlung is possible with either of the two excited electrons of the autoionization state $A^{z-1}(\gamma_2,nl)$ doing the emitting of the radiation. The less strongly excited electron which is in the state γ_2 radiates more strongly than the more strongly excited electron which is in the state nl. Therefore the channel (7), when it is open, makes a larger contribution to the bremstrahlung than does channel (6). Channel (7) is open if the electron's energy is larger than $h\omega_{\gamma_1\gamma_2}$.

As has already been stated, we are confining ourselves to consideration of collisions of a slow electron with an ion in cases when channels associated with ionization, and also channel (7), are closed. In this case the states that are most important are the autoionization states $A^{z-1}(\gamma_1,nl)$ where γ_1 is the first excited state of the ion A^z, denoted hereafter as γ. We shall also suppose that ω_0, the energy of the transition $\gamma_0\to\gamma$ is much smaller than the ionization potential of the ion $A^z(\gamma_0)$. Furthermore the autoionization states $A^{z-1}(\gamma,nl)$ are Rydberg states.

We point out that in Eqs. (3)–(6) the energy of the photon is fixed, of course to the accuracy of the width of the autoionization states. We shall not be interested further in the structure of individual resonances. We consider only the cross section averaged over the resonances.

3. Using the fact that the density of the autoionization states is $\rho(E)=n^3/z^2$, in first-order perturbation theory we get the following expression for the cross section for formation of an autoionization state, averaged over the resonances:

$$\bar\sigma_{nl}(E)=(2l+1)\frac{\pi^2 g(\gamma)n^3}{Eg(\gamma_0)z^2}\Gamma_{nl}, \quad (8)$$

$$\Gamma_{nl}=\frac{2\pi}{(2l+1)g(\gamma)}\sum_{\gamma\gamma_0 ml'm'}\left|\left\langle\psi_\gamma(r_1)\varphi_{nlm}(r_2)\left|\frac{1}{r_{12}}\right|\psi_{\gamma_0}(r_1)\varphi_{El'm'}(r_2)\right\rangle\right|^2, \quad (9)$$

$$E=\omega_0-z^2/2n^2. \quad (10)$$

Here $g(\gamma)$ is the statistical weight of the state γ, Γ_{nl} is the autoionization width, and Σ_γ means summation over the quantum numbers of the state γ. The relation (10) determines the connection of the principal quantum number of the autoionization state with the energy of the incident electron, as fixed by the law of energy conservation.

The total intensity of the resonance bremsstrahlung is the product of the cross section (8), the intensity of the emitted radiation I_{nl}, and the lifetime $1/(\Gamma_{nl}+\Gamma_r)$ of the autoionization state.

$$W(E)=\sum_l(2l+1)\frac{\pi^2 g(\gamma)n^3}{Eg(\gamma_0)z^2}\frac{\Gamma_{nl}I_{nl}}{\Gamma_{nl}+\Gamma_r}. \quad (11)$$

Here $\Gamma_r=2\omega_0^2 f_{\gamma\gamma 0}/c^3$ is the radiation width of the transition $\gamma\to\gamma_0$, and I_{nl} is the intensity of emission from the level nl to all levels with principal quantum number larger than n_E. n_E is determined from the equation $\frac{1}{2}z^2(1/n_E^2-1/n^2)=E$.

The spectral density $dW/d\omega$ and the cross section $d\sigma/d\omega$ of resonance brèmsstrahlung are obviously given by the relation

$$\frac{dW}{d\omega}=\omega\frac{d\sigma}{d\omega}=\sum_l(2l+1)\frac{\pi^2 g(\gamma)n^3 n_\omega^3}{Eg(\gamma_0)z^4}\frac{\Gamma_{nl}I_{nl,n_\omega}}{\Gamma_{nl}+\Gamma_r}, \quad (12)$$

where $I_{nl,n\omega}$ is the intensity of emission from the level nl to the level whose principal quantum number n_ω satisfies the equation $\frac{1}{2}z^2(1/n_\omega^2-1/n^2)=\omega$.

In the case of slow electrons we can use averaged values of Γ_{nl} and I_{nl} without loss of accuracy. In the Kramers approximation we have

$$\bar I_{nl,n_1}=\frac{1}{n^2}\sum_l(2l+1)I_{nl,n_1}=\frac{z^2A_0}{n^5n_1^3},$$

$$\bar I_{nl}=\frac{1}{n^2}\sum_l(2l+1)I_{nl}=\frac{EA_0}{n^5}, \qquad A_0=\frac{8z^4}{\pi 3^{3/2}c^3}.$$

The average width of the autoionization decay is given by[8]

$$\bar\Gamma_{nl}=\frac{1}{n^2}\sum_l(2l+1)\Gamma_{nl}=\frac{2f_{\gamma\gamma_0}z^2}{3^{1/2}\omega_0 n^5};$$

from this we get

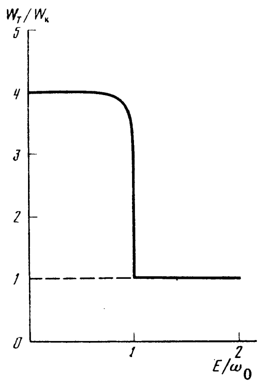

FIG. 1.

$$W(E)=\frac{\pi^2 g(\gamma)A_0}{g(\gamma_0)z^2(1+3^{1/2}\omega_0{}^3n^5/z^2c^3)}. \qquad (13)$$

The spectral intensity of the resonance bremsstrahlung of slow electrons turns out to be practically independent of the frequency:

$$dW/d\omega=\omega d\sigma/d\omega=W/E.$$

It is convenient to put Eq. (13) in the form

$$W(E)=\frac{g(\gamma)W_\text{к}}{g(\gamma_0)(1+3^{1/2}\omega_0{}^3n^5/z^2c^3)},$$

$$W_\text{к}=\frac{8\pi z^2}{3^{3/2}c^3},$$

where W_K is the total bremsstrahlung intensity, in the Kramers approximation, of an electron in the Coulomb field of a center with charge z. The term $\omega_0^3 n^5/z^2c^3$ is not small in comparison with unity only when $E\to\omega_0$, i.e., $n\to\infty$. Therefore the intensity of resonance bremsstrahlung, in the case when it is possible, is of the order of or larger than that of the bremsstrahlung from potential scattering. Figure 1 shows the total intensity W_T of resonance and potential bremsstrahlung for the collision of an electron with the lithiumlike oxygen ion $O^{5+}(1s^22s)$, with ω_0 equal to the energy of the transition $2s\to 2p$ (the contribution of potential bremsstrahlung is shown by a dashed line).

In conclusion we point out that the total radiation from the whole system also includes a contribution from recombination radiation, and for $E\geqslant\omega_0$ (the inequality is to the accuracy of the level width) also a contribution from excitation of the ion. Therefore the total intensity of the radiation from the whole system does not have a slight jump downward at $E\geqslant\omega_0$, but a sharp jump upward, because the excitation channel opens up. A separate treatment of the bremsstrahlung is nevertheless not without meaning, since it has its own peculiar spectral characteristics.

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Effect of mass renormalization in the stochastic acceleration of particles

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It is shown that an effect of mass renormalization leads to an additional energy loss in stochastic acceleration of particles. A kinetic description is given, and the collision integral associated with this effect is found.

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1. INTRODUCTION. STATEMENT OF THE PROBLEM

1. Stochastic acceleration of particles is possible both as the Fermi type of process[1] and also as a result of resonance interaction with waves and oscillations.[2] The two mechanisms are similar in their physical meaning (they correspond to a heating-up of particles either in collisions with heavy clouds or else with oscillations of large effective temperature[3]), although there is also a definite difference between them.

To illustrate the relation between these mechanisms we can use the following model. Let a fast particle move in a random manner in regions (clouds) with a field E which is constant in magnitude but not in direction. The size of a region occupied by the field is l, and the average distance between regions is $l_0\gg l$. We note that resonance particles are acted on by a constant field and such a model correctly reflects the main features of the interaction of fast particles with a gas of solitons; it is most similar to the Fermi model of collisions with magnetic clouds (in the Fermi model, a particle is also acted on during a collision by a constant electric field $-\mathbf{u}\times\mathbf{H}_0$, where $\mathbf{u}$ is the velocity of the cloud).

Let us consider the simplest one-dimensional case. In each collision with a region occupied by an electric field a fast particle acquires or loses (depending on the sign of the field E_1) an energy[1)] $\Delta\varepsilon=eE_1l$. But a particle that encounters a favoring field (pointing along its velocity) and acquires an energy $\Delta\varepsilon$ has a larger ve-

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