Temperature and frequency dependence of the electron conductivity in a two-band model with impurities

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We propose a method for evaluating the electron conductivity $\sigma(\omega_0,T)$ as function of the frequency ω_0, the temperature T, and the Fermi energy E_F for the case when the electron motion is described by a one-dimensional nonrelativistic Dirac equation with a Gaussian random δ-correlated potential. We show that at $T=0$ the conductivity is given by the Kubo-Greenwood formula $\sigma\propto\omega_0^2\rho^2(E_F)$, if $\omega_0\ll E_F\ll\Delta_0$ and $\sigma(\omega_0)\propto\omega_0^2\ln^2\omega_0$ when $E_F\gg\Delta_0$, where $2\Delta_0$ is the width of the impurity band. The last expression for $\sigma(\omega_0)$ is the same as the one which was obtained earlier [N. F. Mott and E. A. Davis, Electronic processes in noncrystalline materials, Clarendon Press, Oxford, 1971; Yu. A. Bychkov, Sov. Phys. JETP **38**, 209 (1974); V. L. Berezinskiĭ, Sov. Phys. JETP **38**, 620 (1974); A. A. Gogolin, V. I. Mel'nikov, and É. I. Rashba, Sov. Phys. JETP **42**, 168 (1975); A. A. Abrikosov and I. A. Ryzhkin, Sov. Phys. JETP **44**, 630 (1976)]. We find an analytical expression for the conductivity $\sigma(T)$ in the low- and high-temperature limits: $\sigma(T)\propto T^2\rho(T)$ as $T\to0$ and $E_F=0$ and $\sigma(T)=\sigma(0)+bT^2$, if $\omega_0\ll E_F$ where $b>0$, if $E_F\ll\Delta_0$ and $b<0$, if $E_F\gg\Delta_0$; as $T\to\infty$, $\sigma(T)\propto T^{-3}\ln^2T$. Taking it into account that the level density $\rho(E)\propto E^{\nu-1}$ as $E\to0$ [A. A. Ovchinnikov and N. S. Érikhman, Sov. Phys. JETP **46**, 340 (1977)] we obtain a power-law dependence for the conductivity $\sigma(T)\propto T^{\nu+2}$ as $T\to0$, and not an exponential one as for a pure semiconductor ($E_F=0$). The results indicate a sharp decrease of the conductivity at electron energies $E_F\sim\Delta_0$ (a jump in the conductivity).

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INTRODUCTION

The problem of describing the kinetic properties of one-dimensional (quasi-one-dimensional) systems with impurities is a timely one. Most of all because the destruction, even though small, of strict periodicity leads in the one-dimensional case to a sudden change in the electronic properties of the system. It was shown already in the first papers on this problem that the presence of impurities leads to the localization of eigenstates,[1] and to vanishing of the static conductivity.[2,3] In Mott and Davis' monograph[4] a wide range of theoretical and experimental studies of the electronic properties of non-crystalline substances is discussed. However, in view of the complexity of obtaining quantitative results for disordered systems, many conclusions reached in Ref. 4 have a semi-empirical character, and this refers in particular to the formulae for the temperature and frequency dependence of the conductivity $\sigma(\omega_0,T)$ given in this monograph. It is therefore of interest to consider models which enable us to obtain quantitative results.

Many papers[5-8] have recently been devoted to this problem. In Refs. 6–8 of the presence of impurities is simulated by a random "white noise" type potential, and in such a model asymptotically exact expressions were found for the frequency dependence of the conductivity $\sigma(\omega_0)$ in the limit of low frequencies ω_0 and for large Fermi energies E_F, thus confirming Mott's ideas. In those papers systems were considered which gave a model of the situation in a metal [Schrödinger equation,[6] Dirac equation with zero gap,[8] single-band approximation with an arbitrary energy spectrum[7] $\varepsilon(k)$]. In this connection an analysis of the situation in semiconductors when there is a gap in the energy spectrum is of interest. The present paper is devoted to that problem.

We choose as our mathematical model in the present paper a non-realativistic Dirac equation with a δ-correlated random Gaussian potential. Such a model enables us to describe a one-dimensional (quasi-one-dimensional) semiconductor or metal with a narrow forbidden band when the mean distance between the impurity centers is much larger than the Bohr radius of the eigenfunctions of an electron.[4]

It is well known that for the calculation of the conductivity $\sigma(\omega_0,T)$ one needs to average a product of Green functions over the realizations of the random potential. This problem turns out to be very complicated as in the given case perturbation theory is unsuitable. In the evaluation of averages it was therefore necessary in Refs. 6–8 to sum infinite classes of divergent diagrams in order to obtain asymptotically exactly ($\omega_0\to0$) soluble recurrence relations even for finite quantities.

We propose in the present paper a new method for evaluating averages of Green functions without using a diagram technique. The representations obtained for the averaged Green function enable us to obtain asymptotically exact expressions for the conductivity $\sigma(\omega_0)$ as $\omega_0\to0$ in the case when the Fermi energy E_F lies sufficiently far from the edge of the impurity band [see (26)].

§1. STATEMENT OF THE PROBLEM AND DERIVATION OF THE BASIC EQUATIONS FOR AVERAGED QUANTITIES

We consider a system which is described by a Dirac-type equation in the segment $(-L,L)$ with a random potential $\xi(x)$:

$$-i\frac{\partial\psi_1}{\partial x}+[\Delta_0+\xi(x)]\psi_2=E\psi_1,\qquad\langle\xi(x)\rangle=0;$$

$$i\frac{\partial\psi_2}{\partial x}+[\Delta_0+\xi(x)]\psi_1=E\psi_2,\tag{1}$$

where

$$\langle \xi(x), \xi(y)\rangle = 2\mu^2\delta(x-y).$$

Here $\langle\ldots\rangle$ indicates statistical averages. We choose the boundary conditions in the form

$$\psi_1(-L)=\psi_2(-L)=1, \quad \psi_1(L)/\psi_2(L)=1. \tag{2}$$

One verifies easily that conditions (2) guarantee the Hermiticity of the Hamiltonian (1). From (1) and (2) follows the symmetry condition

$$\psi_1(x)=\psi_2^*(x). \tag{3}$$

Moreover as $L\to\infty$ we have the symmetry

$$\hat H\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=E\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad \hat H\begin{pmatrix}i\psi_2\\ -i\psi_1\end{pmatrix}=-E\begin{pmatrix}i\psi_2\\ -i\psi_1\end{pmatrix}. \tag{4}$$

Equations (3) and (4) turn out to be useful to establish the symmetry properties of the Green functions.

Equation (1) has the integral

$$\psi_{E1}\tilde\psi_{E2}-\psi_{E2}\tilde\psi_{E1}=\text{const}, \tag{5}$$

where ψ_E and $\tilde\psi_E$ are any two solutions of (1).

Using (3) and (4) we write down the symmetry conditions for the Green functions $G^\pm$ as $L\to\infty$:

$$\begin{gathered}[G_{11}^\pm(x,y|E)]^*=G_{11}^\mp(y,x|E)=G_{22}^\mp(x,y|E),\\ [G_{12}^\pm(x,y|E)]^*=G_{21}^\mp(y,x|E)=G_{21}^\mp(x,y|E)\end{gathered} \tag{6}$$

and

$$\begin{gathered}G_{11}^\pm(x,y|-E)=-G_{22}^\mp(x,y|E),\\ G_{12}^\pm(x,y|-E)=G_{21}^\mp(x,y|E).\end{gathered} \tag{7}$$

We note that the symmetry (6) holds also for finite L, as it is a consequence of (3).

We write down the expression for the conductivity when the frequency of the alternating current is ω_0. To do that we use the standard method (see, e.g., Ref. 9) and as a result we get

$$\sigma(\omega_0,T)=\frac{1}{\pi\omega_0}\int_0^\infty dE\{f(E,T)+f(-E,T)\}\int_y^\infty dx\, j(E+\omega_0,E|x-y), \tag{8}$$

where $f(E,T)$ is the average number of electrons or holes with energies E at a temperature T, while the quantity $j(E+\omega_0,E|x-y)$ is given by the following expression:

$$\begin{aligned}j(E+\omega_0,E|x-y)=&[j^{+-}(E+\omega_0,E|x-y)-j^{++}(E+\omega_0,E|x-y)]\\ &-[j^{+-}(E,E-\omega_0|x-y)-j^{++}(E,E-\omega_0|x-y)]\\ &+[j^{--}(E,E-\omega_0|x-y)-j^{++}(E,E-\omega_0|x-y)].\end{aligned} \tag{9}$$

The quantities $j^{\pm\pm}$ are given by the relation

$$\begin{aligned}j^{\pm\pm}(E',E|x-y)=&\langle\{G_{11}^\pm(y,x|E')G_{11}^\pm(x,y|E)\\ &+G_{22}^\pm(y,x|E')G_{22}^\pm(x,y|E)-G_{12}^\pm(y,x|E')G_{21}^\pm(x,y|)E)\\ &-G_{21}^\pm(y,x|E')G_{12}^\pm(x,y|E)\}\rangle, \quad E'>E,\ x>y.\end{aligned} \tag{10}$$

In obtaining Eq.(9) we used the symmetry properties (6) and (7) and also the fact that the probability flux and the electrical current for holes ($E<0$) have the opposite direction. We note that the last term in (9) is purely imaginary. This follows easily from (10), (6), and (7). For the determination of the real part of the conductivity—and that is the only one we shall be interested in—it is therefore sufficient to be able to evaluate the quantity $j^{+-}-j^{++}$. The averaging methods differ somewhat depending on whether the Fermi level E_F is situated in the impurity band or in the conduction band. We consider first the case $E_F>\Delta_0$ (a metal).

§2. EVALUATION OF THE CONDUCTIVITY AT $T=0$ FOR THE CASE WHEN $E_F>\Delta_0$

We write the Green function in the following form:

$$G(x,y|E)=\frac{i}{\psi_{E1}\tilde\psi_{E2}-\psi_{E2}\tilde\psi_{E1}}\begin{pmatrix}\psi_{E1}(x)\tilde\psi_{E2}(y) & \psi_{E1}(x)\tilde\psi_{E1}(y)\\ \psi_{E2}(x)\tilde\psi_{E2}(y) & \psi_{E2}(x)\tilde\psi_{E1}(y)\end{pmatrix} \tag{11}$$

when $x<y$,

where the functions $\psi_E(x)$ and $\tilde\psi_E(x)$ satisfy Eq. (1) and the boundary conditions for $x=-L$ and $x=L$, respectively. Using (5) we verify easily that (11) is a Green function. To obtain the retarded and advanced Green functions $G^\pm(x,y|E)$ it is sufficient to add in the representation (11) an infinitesimally small imaginary part to the energy $E\pm i_\gamma$, and after that go to the limit $L\to\infty$.

To elucidate the idea of the averaging method proposed in what follows, it is convenient to represent the procedure for getting the Green functions G^+ and G^- from Eq. (11) in a special way. To do that we introduce new variables. We write the solutions of Eq. (1) with the boundary condition for $x=-L$ and $x=L$ in the form

$$\begin{gathered}\psi_{E1,2}(x)=\exp\{u_x\}\left\{\cos[\omega(x+L)+\alpha_x]\pm\frac{i\varepsilon}{\omega}\sin[\omega(x+L)+\alpha_x]\right\},\\ \tilde\psi_{E1,2}(x)=\exp\{\tilde u_x\}\left\{\cos[\omega(x-L)+\tilde\alpha_x]\pm\frac{i\varepsilon}{\omega}\sin[\omega(x-L)+\tilde\alpha_x]\right\},\\ \alpha(-L)=u(-L)=\tilde\alpha(L)=\tilde u(L)=0,\end{gathered} \tag{12}$$

where

$$\omega=(E^2-\Delta_0^2)^{1/2}, \quad \varepsilon=E-\Delta_0. \tag{13}$$

Substituting (12) into (1) we get an equation for α_x and U_x:

$$\begin{gathered}\frac{d\alpha_x}{dx}=-\xi(x)\left\{\frac{\omega}{\varepsilon}\cos^2[\omega(x+L)+\alpha_x]-\frac{\varepsilon}{\omega}\sin^2[\omega(x+L)+\alpha_x]\right\},\\ \frac{du_x}{dx}=-\frac{1}{2}\xi(x)\left(\frac{\omega}{\varepsilon}+\frac{\varepsilon}{\omega}\right)\sin[2\omega(x+L)+2\alpha_x],\\ \alpha(-L)=u(-L)=0\end{gathered} \tag{14}$$

and an analogous equation for $\tilde\alpha_x$ and $\tilde U_x$ with the boundary condition at $x=L$. Substituting (12) into the denominator of Eq. (11) we get

$$\psi_{E1}\tilde\psi_{E2}-\psi_{E2}\tilde\psi_{E1}=\frac{2i\varepsilon}{\omega}\exp\{u_x+\tilde u_x\}\sin[2\omega L+\alpha_x-\tilde\alpha_x]. \tag{15}$$

As the Hamiltonian is Hermitian, Eq. (15) is non-vanishing if the energy E has an imaginary correction. Moreover, the quantity $2\omega L+\alpha_x-\tilde\alpha_x$ is, generally speaking, complex when $\operatorname{Im}E\neq0$. Therefore, when $\operatorname{Im}E>0$

$$|\exp[-2i\omega L-i\alpha_x+i\tilde\alpha_x]|>|\exp[2i\omega L+i\alpha_x-i\tilde\alpha_x]|,$$

and when $\operatorname{Im}E<0$ the opposite inequality holds [this relation between the moduli of exponentials is dictated by the correspondence with the case when $\xi(x)=0$]. Taking into account what has been said we can write (15) in the form of a series:

$$\begin{aligned}i[\psi_{E1}(y)\tilde\psi_{E2}(y)-\psi_{E2}(y)\tilde\psi_{E1}(y)]^{-1}&=\frac{\omega}{2\varepsilon}\exp\{-u_y-\tilde u_y\}[\sin(2\omega L+\alpha_y-\tilde\alpha_y)]^{-1}\\ &=\mp i\frac{\omega}{\varepsilon}\exp\{-u_y-\tilde u_y\pm2i\omega L\pm i(\alpha_y-\tilde\alpha_y)\}\sum_{n=0}^\infty\exp\{\pm i4\omega Ln\pm2i(\alpha_y-\tilde\alpha_y)n\}.\end{aligned} \tag{16}$$

Substituting (12) and (16) into (11) we obtain the following representations for the matrix elements of the Green functions:

$$
\begin{aligned}
G_{11}^{\pm}(x,y|E) = &\mp\frac{i\omega}{4\varepsilon}\left(1\mp\frac{\varepsilon}{\omega}\right)^2\exp\{u_x-u_y\mp i\omega(x-y)\\
&\mp i(\alpha_x-\alpha_y)\}\left[1+\frac{\pm\omega+\varepsilon}{\pm\omega-\varepsilon}\exp\{\pm 2i\omega(x+L)\pm 2i\alpha_x\}\right]\\
&\times\left[1+\frac{\pm\omega+\varepsilon}{\pm\omega-\varepsilon}\exp\{\mp 2i\omega(y-L)\mp 2i\tilde\alpha_y\}\right]\\
&\times\sum_{n=0}^{\infty}\exp\{\pm 2i[\omega(x+L)+\alpha_x]n\}\exp\{\mp 2i[\omega(y-L)+\tilde\alpha_y]n\}\\
&\times\exp\{\mp 2i\omega(x-y)n\mp 2i(\alpha_x-\alpha_y)n\},\qquad x<y,\\
G_{12}^{\pm}(x,y|E) = &\mp\frac{i\omega}{4\varepsilon}\left(1-\frac{\varepsilon^2}{\omega^2}\right)\exp\{u_x-u_y\mp i\omega(x-y)\mp i(\alpha_x-\alpha_y)\}\\
&\times\left[1+\frac{\pm\omega+\varepsilon}{\pm\omega-\varepsilon}\exp\{\pm 2i\omega(x+L)\pm 2i\alpha_x\}\right]\\
&\times\left[1+\frac{\pm\omega-\varepsilon}{\pm\omega+\varepsilon}\exp\{\mp 2i\omega(y-L)\mp 2i\tilde\alpha_y\}\right]\\
&\times\sum_{n=0}^{\infty}\exp\{\pm 2i\omega(x+L)n\pm 2i\alpha_x n\}\exp\{\mp 2i\omega(y-L)n\mp 2i\tilde\alpha_y n\}\\
&\times\exp\{\mp 2i\omega(x-y)n\mp 2i(\alpha_x-\alpha_y)n\},\qquad x<y.
\end{aligned}
\tag{17}
$$

The matrix elements $G_{22}^{\pm}$ and $G_{21}^{\pm}$, and also the values of the Green functions for $x>y$ are obtained from (17) using the symmetry (6). Before taking the limit $L\to\infty$ we obtain from (17) a series expansion of the Green function (11) in the energy region $\operatorname{Im}E>0$ (+ sign) and in the energy region $\operatorname{Im}E<0$ (− sign). After taking the limit $L\to\infty$ and afterwards $\operatorname{Im}E\to 0$ these expansions go, respectively, over into G^+ and G^-.

When $\xi(x)=0$ this is immediately clear. Indeed, in that case $\alpha_x=U_x=\tilde\alpha_x=\tilde U_x=0$ and as $L\to\infty$ we get the following expression for $G_0^{\pm}$:

$$
G_0^{\pm}(x,y|E)=\frac{\mp i\exp\{\pm i(E^2-\Delta_0^2)^{1/2}|x-y|\}}{2(E^2-\Delta_0^2)^{1/2}}
\times\begin{pmatrix} E\pm\varepsilon(x-y)(E^2-\Delta_0^2)^{1/2} & \Delta_0 \\ \Delta_0 & E\mp\varepsilon(x-y)(E^2-\Delta_0^2)^{1/2}\end{pmatrix},
\tag{18}
$$

where $\varepsilon(x)=1$ when $x>0$ and $\varepsilon(x)=-1$ when $x<0$. In obtaining (18) we used (13).

We note that in obtaining the unperturbed Green functions it is important to observe the order of taking first the limit $L\to\infty$ and afterwards $\operatorname{Im}E\to 0$, for in the opposite case $\operatorname{Im}E\to 0$ and $L\to\infty$ the terms in the representation (17) oscillate and have no limit. It turns out that when there is a random potential present the calculation of the averaged Green functions can be performed by taking the energy to be real right from the beginning, $\operatorname{Im}E=0$. This is connected with the fact that, firstly, the averaged terms of the series (17) have a limit as $L\to\infty$ even if the energy is real right from the beginning (vide infra) and, secondly, the series obtained of the averaged quantities is convergent. Taking what has been said into account we shall in what follows use for the evaluation of the averaged Green functions and their products the representation (17) and assume the energy to be real. The representation (17) is convenient because one can easily obtain for the averaged terms of the series (17) and their products recurrence relations which are asymptotically exactly soluble in the limit

$$\mu^2(E^2-\Delta_0^2)^{-1/2}\ll 1.$$

For the calculation of the real part of the conductivity at $T=0$ we must find the averaged correlator of the current $j^{+-}-j^{++}$ [see Eqs. (8) to (10)]. Substituting (17) into (10) we get for that quantity the following representation:

$$
\begin{aligned}
&j^{+-}(E+\omega_0,E|x-y)-j^{++}(E+\omega_0,E|x-y)\\
&=\frac{\omega'\omega}{4\varepsilon'\varepsilon}\sum_{m,n=0}^{\infty}\Big\langle\Big\{\Big(\frac{\varepsilon}{\omega}+\frac{\varepsilon'}{\omega'}\Big)[1-F_{22}^{00}(y)]\\
&+\Big(\frac{\varepsilon}{\omega}-\frac{\varepsilon'}{\omega'}\Big)[F_{20}^{00}(y)-F_{02}^{00}(y)]\Big\}F_{2m,2n}^{00}(y)F_{-(2m+1),-(2n+1)}^{-1,-1}(y)F_{2m+1,2n+1}^{11}(x)\Big\rangle\\
&\times\Big\langle\tilde F_{2m,2n}^{00}(x)\Big\{\Big(\frac{\varepsilon}{\omega}+\frac{\varepsilon'}{\omega'}\Big)[1-\tilde F_{22}^{00}(x)]+\Big(\frac{\varepsilon}{\omega}-\frac{\varepsilon'}{\omega'}\Big)[\tilde F_{20}^{00}(x)\\
&-\tilde F_{02}^{00}(x)]\Big\}\Big\rangle+\frac{\omega'\omega}{4\varepsilon'\varepsilon}\sum_{m,n=0}^{\infty}\Big\langle\Big\{\Big(\frac{\varepsilon}{\omega}-\frac{\varepsilon'}{\omega'}\Big)[1-F_{2,-2}^{00}(y)]\\
&+\Big(\frac{\varepsilon}{\omega}+\frac{\varepsilon'}{\omega'}\Big)[F_{20}^{00}(y)-F_{0,-2}^{00}(y)]\Big\}F_{2m,-2n}^{00}(y)F_{-(2m+1),(2n+1)}^{-1,-1}(y)\\
&\times F_{2m+1,-(2n+1)}^{1,1}(x)\Big\rangle\Big\langle\tilde F_{2m,-2n}^{00}(x)\Big\{\Big(\frac{\varepsilon}{\omega}-\frac{\varepsilon'}{\omega'}\Big)[1-\tilde F_{2,-2}^{00}(x)]\\
&+\Big(\frac{\varepsilon}{\omega}+\frac{\varepsilon'}{\omega'}\Big)[\tilde F_{2,0}^{00}(x)-\tilde F_{0,-2}^{00}(x)]\Big\}\Big\rangle,\qquad x>y,
\end{aligned}
\tag{19}
$$

where

$$
\begin{aligned}
F_{mn}^{pq}(x)&=\exp\{-pu_x'-qu_x+i(m\omega'-n\omega)(x+L)+i(m\alpha_x'-n\alpha_x)\},\\
\tilde F_{mn}^{pq}(x)&=\exp\{-p\tilde u_x'-q\tilde u_x-i(m\omega'-n\omega)(x-L)-i(m\tilde\alpha_x'-n\tilde\alpha_x)\},
\end{aligned}
\tag{20}
$$

while $E'=E+\omega_0$. When obtaining (19) we took it into account that the quantities α_x, α_y, U_x, and U_y, on the one hand, and $\tilde\alpha_x$ and $\tilde U_x$, on the other hand, are statistically independent, as follows from (1) and (14).

We note that as $\omega_0\to 0$ the terms of the series in (19) tend to zero as ω_0^2. This does, however, not enable us to reach any conclusions about the law $\sigma(\omega_0)\propto\omega_0^2$, as the double series in (19) diverges for $\omega_0=0$. An estimate of expression (19) requires therefore more accurate calculations.

We introduce the equations for the averages $\langle F_{mn}^{pq}(x)\rangle$. Using (14) we get

$$
\begin{aligned}
\frac{dF_{mn}^{pq}(x)}{dx}=&\,i\Big\{(m\omega'-n\omega)+\frac{\xi(x)}{2}\Big[m\Big(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\Big)\\
&-n\Big(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\Big)\Big]\Big\}F_{mn}^{pq}(x)-\frac{i}{4}\xi(x)\Big(\frac{\omega'}{\varepsilon'}+\frac{\varepsilon'}{\omega'}\Big)(m+p)F_{m+2,n}^{pq}\\
&-\frac{i}{4}\xi(x)\Big(\frac{\omega'}{\varepsilon'}+\frac{\varepsilon'}{\omega'}\Big)(m-p)F_{m-2,n}^{pq}+\frac{i}{4}\xi(x)\Big(\frac{\omega}{\varepsilon}+\frac{\varepsilon}{\omega}\Big)(n+q)F_{m,n+2}^{pq}\\
&+\frac{i}{4}\xi(x)\Big(\frac{\omega}{\varepsilon}+\frac{\varepsilon}{\omega}\Big)(n-q)F_{m,n-2}^{pq}.
\end{aligned}
\tag{21}
$$

To obtain closed equations for the average quantities $\langle F_{mn}^{pq}\rangle$ we use a technique discussed in a paper by Klyatskin and Tatarskiĭ.[10] Taking the functional derivative of both sides of Eq. (21) and integrating the resulting equation we find

$$
\begin{aligned}
\frac{\delta F_{mn}^{pq}(x)}{\delta\xi(x-0)}=&\,\frac{i}{2}\Big[m\Big(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\Big)-n\Big(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\Big)\Big]F_{mn}^{pq}(x)\\
&-\frac{i}{4}\Big(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\Big)(m+p)F_{m+2,n}^{pq}-\frac{i}{4}\Big(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\Big)(m-p)F_{m-2,n}^{pq}\\
&+\frac{i}{4}\Big(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\Big)(n+q)F_{m,n+2}^{pq}+\frac{i}{4}\Big(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\Big)(n-q)F_{m,n-2}^{pq}.
\end{aligned}
\tag{22}
$$

Averaging both sides of Eq. (21) and using the fact that $\xi(x)$ is a Gaussian random function we decouple the resulting averages of the form $\langle \xi F_{mn}^{pq}\rangle$ using Novikov's formula[10]:

$$\langle \xi(x) F_{mn}{}^{pq}(x)\rangle = \int_{-\infty}^{+\infty} \langle \xi(x)\xi(y)\rangle \left\langle \frac{\delta F_{mn}{}^{pq}(x)}{\delta \xi(y)} \right\rangle dy. \tag{23}$$

Moreover, using (1) we transform (23) to the form

$$\langle \xi(x) F_{mn}{}^{pq}(x)\rangle = \mu^2 \left\langle \frac{\delta F_{mn}{}^{pq}(x)}{\delta \xi(x-0)} \right\rangle. \tag{24}$$

After this the averaged Eq. (21) takes the following form, if we use (22) and (24),[2)]

$$\frac{d\langle F_{mn}{}^{pq}\rangle}{dx} = \left\{ i(m\omega'-n\omega) - \frac{\mu^2}{4}\left[m\left(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\right) - n\left(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\right)\right]^2 \right.$$
$$\left. -\frac{\mu^2}{8}\left(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\right)^2 (m^2-p^2+2p) - \frac{\mu^2}{8}\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)^2 (n^2-q^2+2q) \right\} \langle F_{mn}{}^{pq}\rangle$$
$$+\frac{\mu^2(m+p)}{4}\left(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\right)\left[(m+1)\left(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\right) - n\left(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\right)\right] \langle F^{pq}_{m+2,n}\rangle$$
$$+\frac{\mu^2(m-p)}{4}\left(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\right)\left[(m-1)\left(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\right) - n\left(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\right)\right] \langle F^{pq}_{m-2,n}\rangle$$
$$+\frac{\mu^2(n+q)}{4}\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)\left[(n+1)\left(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\right) - m\left(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\right)\right] \langle F^{pq}_{m,n+2}\rangle$$
$$+\frac{\mu^2(n-q)}{4}\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)\left[(n-1)\left(\frac{\varepsilon}{\omega}-\frac{\omega}{\varepsilon}\right) - m\left(\frac{\varepsilon'}{\omega'}-\frac{\omega'}{\varepsilon'}\right)\right] \langle F^{pq}_{m,n-2}\rangle$$
$$+\frac{\mu^2}{8}\left(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\right)\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)\{(m+p)(n+q)\langle F^{pq}_{m+2,n+2}\rangle$$
$$+(m-p)(n-q)\langle F^{pq}_{m-2,n-2}\rangle + (m+p)(n-q)\langle F^{pq}_{m+2,n-2}\rangle$$
$$+(m-p)(n+q)\langle F^{pq}_{m-2,n+2}\rangle\} - \frac{\mu^2}{16}\left(\frac{\varepsilon'}{\omega'}+\frac{\omega'}{\varepsilon'}\right)^2\{(m+p)(m+p+2)\langle F^{pq}_{m+4,n}\rangle$$
$$+(m-p)(m-p-2)\langle F^{pq}_{m-4,n}\rangle\} - \frac{\mu^2}{16}\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)^2\{(n+q)(n+q+2)\langle F^{pq}_{m,n+4}\rangle$$
$$+(n-q)(n-q-2)\langle F^{pq}_{m,n-4}\rangle\}. \tag{25}$$

We note that Eq. (25) is satisfied also by any function of the form $\langle F_{mn}^{pq}(x) F_{m'n'}^{p'q'}(y)\rangle$, if $x>y$. We shall use this fact in what follows.

From (25) it is clear that if the condition

$$\lambda = (\mu^2/\omega)(\omega/\varepsilon+\varepsilon/\omega)^2 \ll 1 \tag{26}$$

holds, the quantities $\langle F_{mn}^{pq}(x)\rangle$ for $m\neq n$ contain fast oscillating factors $\exp\{i(m-n)\omega x\}$ and as $x\to\infty$ have the magnitude $\sim\lambda\langle F_{mm}^{pq}(x)\rangle$. This fact enables us to neglect in Eq. (25) and in expression (19) all terms with $m\neq n$. In particular, this means that in that approximation we can neglect the correlator of the currents j^{++}, i.e., omit the second sum in (19). This is just the kind of approximation used by Berezinskiĭ[6] when calculating the current correlator in the Schrödinger equation case.[3)] Taking into account what has been said we can write Eq. (19) in the form (see footnote 2)

$$j^{+-}(E+\omega_0, E|x-y) = \frac{\omega'\omega}{4\varepsilon\varepsilon'}\left(\frac{\varepsilon'}{\omega'}+\frac{\varepsilon}{\omega}\right)^2 \sum_{n=0}^{\infty} (B_n-B_{n+1})C_n(x-y), \tag{27}$$

where

$$B_n = \lim\langle F^{00}_{2n,2n}(y)\rangle = \lim\langle F^{00}_{2n,2n}(y)\rangle, \tag{28}$$

$$C_n(x-y) = \lim_{L\to\infty}\langle [F^{00}_{2n,2n}(y) - F^{00}_{2n+2,2n+2}(y)]$$
$$\times F^{-1,-1}_{-(2n+1),-(2n+1)}(y) F^{1,1}_{2n+1,2n+1}(x)\rangle, \qquad x>y; \tag{29a}$$

$$C_n(0) = \lim_{L\to\infty}\langle [F^{00}_{2n,2n}(y) - F^{00}_{2n+2,2n+2}(y)]\rangle = B_n - B_{n+1}. \tag{29b}$$

Using (20) and (25) we get for the quantities B_n and C_n the equations

$$i\beta n B_n + n^2(B_{n+1}+B_{n-1}-2B_n) = 0, \qquad B_0 = 1, \quad \lim_{n\to\infty} B_n = 0; \tag{30}$$

$$dC_n/dt = i\beta(n+1/2)C_n + \{C_{n+1}(n+1)^2 + C_{n-1}n^2 - C_n(2n^2+2n+1)\}, \tag{31a}$$
$$C_n(0) = B_n - B_{n+1}, \tag{31b}$$

where

$$t = \frac{\mu^2}{2}\left(\frac{\varepsilon}{\omega}+\frac{\omega}{\varepsilon}\right)^2 (x-y), \qquad \beta = \frac{4(\omega'-\omega)}{\mu^2(\varepsilon/\omega+\omega/\varepsilon)^2}. \tag{32}$$

When obtaining (30) and (31) from (25) we neglected, in accordance of what was said above, all quantities $\langle F_{mn}^{pq}\rangle$ with $m\neq n$. Moreover, as $B_n = \lim B_n(x+L), L\to\infty$, the derivative dB_n/dt is put equal to zero in Eq. (30). And finally, in all coefficients in Eqs. (30) and (31) we retained only the zeroth and first order contributions in $\omega'-\omega\sim\omega_0$, since $\omega_0/\mu^2\ll 1$.

The quantities B_n are easily found, if we introduce the generating function

$$B(z) = \sum_{n=1}^{\infty} B_n z^{n-1}, \qquad |z|<1. \tag{33}$$

Substituting (33) into (30) and using the condition $B_n\to 0$ as $n\to\infty$, which follows from the definition (28), we find for $B(z)$ the following expression:

$$B(z) = \frac{1}{(1-z)^2}\exp\left\{\frac{-i\beta}{1-z}\right\}\int_z^1 \exp\left\{\frac{i\beta}{1-z'}\right\}dz'. \tag{34}$$

Knowing $B(z)$, and hence also the B_n, we can, in principle, find the coefficients $C_n(t)$ from Eq. (31). The conductivity $\sigma(\omega_0)$ is determined by the integral over x of the correlator of the currents j^{+-} [see (27)]. To find this integral we note that the generating function (34) is exactly the same as the function $r(u)$ [see Eq. (2.73) of Ref. 11], if we reduce them to the same variables. Hence, the coefficients B_n found in the present paper and in the paper by Abrikosov and Ryzhkin[11] will also be the same. Moreover, the equation for the $C_n(t)$, (31), is also exactly the same as the equations for the $C_n(t)$ found in Ref. 11 [see Eq. (2.76)]. We can thus obtain the integral of the correlator of the currents (27) by using the results of Ref. 11, where it was found [see (2.81), (2.83), (2.84), (2.96), and (2.95)] that

$$\int_0^{\infty} dt \sum_{n=0}^{\infty} (B_n - B_{n+1})C_n(t) = \{-2i\zeta(3)\beta + \beta^2\ln^2\beta\}. \tag{35}$$

Using (8), (9), (27), (32), and (13), and the fact that at $T=0$ the Fermi function is a step function we get from (35) for the real part of the conductivity the following expression:

$$\sigma(\omega_0, E_F, T=0) = \frac{\omega_0^2}{2\pi\mu^6}\left(1-\frac{\Delta_0^2}{E_F^2}\right)^2 \ln^2\left\{\frac{\mu^2}{\omega_0}\left(1-\frac{\Delta_0^2}{E_F^2}\right)^{-1/2}\right\}. \tag{36}$$

It is clear from (36) that as $E_F\to\infty$ the conductivity, increasing monotonically, tends to a finite limit

$$\sigma_\infty(\omega_0) = \frac{\omega_0^2}{2\pi\mu^6}\ln^2\left(\frac{\mu^2}{\omega_0}\right). \tag{37}$$

We recall that Eq. (35) is valid under the condition (26).

§3. EVALUATION OF THE CONDUCTIVITY WHEN $E_F < \Delta_0$ IN THE $T=0$ CASE

The situation in the $E_F < \Delta_0$ case is in an essential way different from the $E_F > \Delta_0$ case considered above. The reason for this is that for energies inside the impurity band we have no zeroth approximation for the retarded and advanced Green functions, since they coincide in that case. This leads to the fact that the representation (17) for G^+ and G^- turns out to be inconvenient, since the functions α_x and U_x introduced above [see (12)] become complex for $E_F < \Delta_0$. We therefore proceed differently. We write the solution of Eq. (1) in the form

$$\psi_1=\exp[u_x+i\alpha_x],\quad \psi_2=\exp[u_x-i\alpha_x],\quad \alpha(-L)=u(-L)=0;$$
$$\tilde\psi_1=\exp[\tilde u_x+i\tilde\alpha_x],\quad \tilde\psi_2=\exp[\tilde u_x-i\tilde\alpha_x],\quad \tilde\alpha(L)=\tilde u(L)=0. \tag{38}$$

The condition that $\psi=\text{const.}\,\tilde\psi$ are eigenfunctions has the form

$$\alpha(L)=\pi n,\quad \tilde\alpha(-L)=\pi m. \tag{39}$$

Substituting (38) into (1) we get the following equation for α_x and U_x:

$$d\alpha/dx=E-[\Delta_0+\xi(x)]\cos 2\alpha,$$
$$du/dx=-\sin(2\alpha)[\Delta_0+\xi(x)],\quad \alpha(-L)=u(-L)=0. \tag{40}$$

The equations for $\tilde\alpha_x$ and $\tilde U_x$ are the same, but the boundary conditions are at $x=L$.

We write down the solutions of Eq. (40) for $\xi(x)=0$

$$\alpha_x=-\operatorname{arctg}\left\{\frac{\Delta_0-E}{(\Delta_0^2-E^2)^{1/2}}\operatorname{th}[(\Delta_0^2-E^2)^{1/2}(x+L)]\right\},$$
$$u_x=\frac{1}{2}\ln\left\{1+\frac{2\Delta_0}{\Delta_0+E}\operatorname{sh}^2[(\Delta_0^2-E^2)^{1/2}(x+L)]\right\}. \tag{41}$$

Similar formulae are valid for $\tilde\alpha_x$ and $\tilde U_x$ with the one difference, that $L\to -L$.

It is clear from (41) that $E<\Delta_0$ cannot be an eigenvalue, since

$$\alpha_\infty=-\operatorname{arctg}\left(\frac{\Delta_0-E}{\Delta_0+E}\right)^{1/2},\quad u_{x_{L\to\infty}}=(\Delta_0^2-E^2)^{1/2}(x+L), \tag{42}$$

and hence condition (39) is not satisfied for such E.

Substituting (38) into (11) we get for the Green function the expression

$$G(x,y|E)=\frac{i\exp\{u_y-u_x\}}{\exp\{i(\alpha_x-\tilde\alpha_x)\}-\exp\{-i(\alpha_x-\tilde\alpha_x)\}}\times\begin{pmatrix}\exp\{i(\tilde\alpha_x-\alpha_y)\} & \exp\{i(\tilde\alpha_x+\alpha_y)\}\\ \exp\{-i(\tilde\alpha_x+\alpha_y)\} & \exp\{-i(\tilde\alpha_x-\alpha_y)\}\end{pmatrix},\quad x>y. \tag{43}$$

To obtain G^+ and G^- we use the method described above. We assume in the first case that $\operatorname{Im}E>0$, and in the second that $\operatorname{Im}E<0$ and we have then in the denominator of (43)

$$|\exp\{-i(\alpha_x-\tilde\alpha_x)\}|>|\exp\{i(\alpha_x-\tilde\alpha_x)\}|\ \text{when}\ \operatorname{Im}E>0,$$

$$|\exp\{i(\alpha_x-\tilde\alpha_x)\}|>|\exp\{-i(\alpha_x-\tilde\alpha_x)\}|\ \text{when}\ \operatorname{Im}E<0$$

respectively, for G^+ and G^-. After that we expand the denominator in a series and we substitute the resultant expression into the expression (10) of the correlator of the currents $j^{+-}-j^{++}$.[4)]

If we follow this procedure we get $(x>y)$

$$2\operatorname{Re}[j^{+-}(E',E|x-y)-j^{++}(E',E|x-y)]=\exp[u_y'-u_x'+u_y-u_x]$$
$$\times\exp\{i(\alpha_x'-\tilde\alpha_x')-i(\alpha_x-\tilde\alpha_x)\}[\exp\{i(\alpha_y'-\alpha_y)\}-\exp\{-i(\alpha_y'-\alpha_y)\}]$$
$$\times[\exp\{-i(\tilde\alpha_x'-\tilde\alpha_x)\}-\exp\{i(\tilde\alpha_x'-\tilde\alpha_x)\}]$$
$$\times\sum_{m=-\infty}^{\infty}\exp[2im(\alpha_x'-\tilde\alpha_x')]\sum_{m=-\infty}^{\pm\infty}\exp[-2im(\alpha_x-\tilde\alpha_x)]. \tag{44}$$

One verifies easily that the sums in (44) give after averaging the level density $\pi\rho(E)$.[5)] They are non-vanishing (for finite L) only when E and E' are eigenenergies. The quantization condition

$$2(\alpha_x-\tilde\alpha_x)|_{E=E_i}=2\pi n,\quad 2(\alpha_x'-\tilde\alpha_x')|_{E=E_j}=2\pi m, \tag{45}$$

which follows from Eqs. (38) and (39) is then satisfied. We can therefore assume before the averaging that the phases in (43) satisfy condition (45). This enables us to regroup the factors in front of the sums in (44) in such a way that we get for $j^{+-}-j^{++}$ the expression

$$2\operatorname{Re}\{j^{+-}(E',E|x-y)-j^{++}(E',E|x-y)\}=\exp\{u_y'-u_x'+u_y-u_x\}$$
$$\times[\exp\{i(\alpha_y'-\alpha_x')-i(\alpha_y-\alpha_x)\}[1-\exp\{2i(\tilde\alpha_x'-\tilde\alpha_x)\}]+\exp\{-i(\alpha_y'-\alpha_x')$$
$$+i(\alpha_y-\alpha_x)\}[1-\exp\{-2i(\tilde\alpha_x'-\tilde\alpha_x)\}]]$$
$$\times\sum_{m=-\infty}^{+\infty}\exp\{2im(\alpha_x'-\tilde\alpha_x')\}\sum_{m=-\infty}^{+\infty}\exp\{-2im(\alpha_x-\tilde\alpha_x)\},\quad x>y. \tag{46}$$

Expression (46) must be averaged.

One can prove (see the Appendix) that the series in (46) are self-averaged quantities as $L\to\infty$ and $\mu^2 \ll(\Delta_0^2-E^2)^{1/2}$ and, hence when averaging they can be taken out of the average sign and replaced by the quantity $\pi\rho(E)$.[12] It is thus sufficient to average in (46) the factor in front of the sums. However, if we take it into account that the parameter μ^2 is small we can replace them by their unperturbed values (41) when $L\to\infty$. As a result we get the following expression for the current correlators when $E'-E=\omega_0\ll\mu^2$:[6)]

$$\operatorname{Re}\{j^{+-}(E+\omega_0,E|x-y)-j^{++}(E+\omega_0,E|x-y)\}$$
$$=\frac{\pi^2\omega_0^2}{2(\Delta_0^2-E^2)}\rho(E+\omega_0)\rho(E)\exp\{-2(\Delta_0^2-E^2)^{1/2}(x-y)\},\quad x>y. \tag{47}$$

Substituting (47) into (49) and then into (8) and using the fact that the temperature $T=0$ we find for the conductivity when $E_F\gg\Delta_0$ the following expression:

$$\sigma(\omega_0,E_F,T=0)=\frac{\pi\omega_0^2\rho(E_F)\rho(E_F+\omega_0)}{4(\Delta_0^2-E_F^2)^{1/2}}\sim\frac{\pi\omega_0^2}{4\Delta_0^3}\rho^2(E_F),\quad E_F\ll\Delta_0. \tag{48}$$

The level density $\rho(E)$ has a power-law behavior $E^{\nu-1}$ with $\nu=\Delta_0/\mu^2$ when $E\ll\Delta_0$.[13] Hence it is clear that the conductivity is anomalously small in the impurity band as compared with the conduction band.

Generally speaking, we must add to Eq. (48) the result of integrating over the energy to the lower limit [see (8) and (9)]:

$$\delta\sigma=-\frac{1}{\pi\omega_0}\int_0^{\omega_0}dE\int_y^{\infty}\{j^{+-}(E,E-\omega_0|x-y)-j^{++}(E,E-\omega_0|x-y)\}dx.$$

This integral gives, when we take into account (47) and the symmetry $\rho(E)=\rho(-E)$, the contribution

$$|\delta\sigma|\sim\frac{\pi\omega_0^2}{4\Delta_0^3}\rho^2\left(\frac{\omega_0}{2}\right)\ll\frac{\pi\omega_0^2}{4\Delta_0^3}\rho^2(E_F)\ \text{when}\ \omega_0\ll E_F, \tag{49}$$

i.e., we can neglect it. An exception is the case $E_F \ll\Delta_0$ when both contributions are comparable. In that case, substituting (47) into (8) we get

$$\sigma(\omega_0, E_F, T=0) = \frac{\pi\omega_0}{4\Delta_0^3}\int_0^{E_F} dE\{\rho(\omega_0+E)-\rho(\omega_0-E)\}\rho(E)$$

$$= \frac{\pi E_F^2}{2\Delta_0^3}\rho(E_F)\rho(\omega_0), \qquad \omega_0 \gg E_F. \tag{50}$$

In deriving (50) we used the fact that $\rho(E) \propto E^{\nu-1}$.

§4. TEMPERATURE DEPENDENCE OF $\sigma(T)$

The distribution function of non-interacting electrons and holes has the Fermi form and the chemical potential χ is determined from the equation

$$\int_0^\infty \frac{\rho(E)\,dE}{\exp\{(E-\chi)/T\}+1} - \int_{-\infty}^0 \frac{\rho(E)\,dE}{\exp\{(-E+\chi)/T\}+1} = N(E_F), \tag{51}$$

where $N(E_F)$ is the number of electrons at $T=0 (E_F>0)$. From (51) we obtain easily the relations needed in what follows:

$$\chi(T) = E_F + 2\int_0^\infty \frac{z\,dz}{e^z+1}\frac{-\rho'(E_F)}{\rho(E_F)}T^2 \quad \text{as} \quad T\to 0, \tag{52}$$

$$\chi(T) \to \pi N(E_F) \approx E_F \ (E_F \gg \Delta_0) \quad \text{as} \quad T\to\infty. \tag{53}$$

In deriving (53) we used the fact that $\rho(E) = \pi^{-1}$ as $T\to\infty$.

As $T\to 0$ it follows from (8) and (9) that

$$\sigma(\omega_0, T) = \sigma(\omega_0, \chi(T)) + 2\int_0^\infty \frac{z\,dz}{e^z+1}\frac{\partial^2\sigma(\omega_0, E_F)}{\partial E_F^2}T^2, \tag{54}$$

where $\sigma\ \omega_0, \chi(T)]$ is the conductivity at $T=0$, if we substitute for the energy E_F the chemical potential $\chi(T)$.

We consider three cases: $E_F \gg \Delta_0$, $\omega_0 \ll E_F \ll \Delta_0$, $E_F = 0$. Substituting (36) and (52) into (54) and using the fact that $E_F \gg \Delta_0$ we get

$$\sigma(\omega_0, T) = \sigma(\omega_0, E_F, T=0) - 24\int_0^\infty \frac{z\,dz}{e^z+1}\sigma_\infty(\omega_0)\frac{\Delta_0^2 T^2}{E_F^4}, \tag{55}$$

where $\sigma_\infty(\omega_0)$ is given by Eq. (37). In the case $\omega_0 \ll E_F \ll \Delta_0$ we find by substituting (48) and (52) into (54)

$$\sigma(T) = \sigma(\omega_0, E_F, T=0)\left\{1 + 4\int_0^\infty \frac{z\,dz}{e^z+1}\frac{\rho''(E_F)}{\rho(E_F)}T^2\right\}. \tag{56}$$

Using the fact that $\rho''(E) > 0$ when $E \ll \Delta_0$ and $\mu^2 \ll \Delta_0$,[13] we are led to the conclusion that the functions $\sigma(T)$ increase at small T.

Finally, we consider the case $E_F = 0$. In that case $\sigma(T)$ is given by the integral (50) in which the upper limit is chosen to be infinite and the integrand is multiplied by a factor $[\exp(E_F/T)+1]^{-1}$ as for $E_F = 0$ the chemical potential $\chi(T) \equiv 0$. This leads to the following expression for

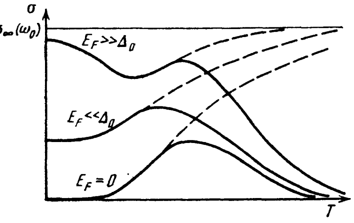

FIG. 1. Typical profiles of the function $\sigma(T)$ for different values of the Fermi energy E_F. The dashed lines show the behavior of $\sigma(T)$ in the case of purely static impurities ($\lim \sigma(T) = \sigma_\infty(\omega_0)$, $T\to\infty$).

the conductivity:

$$\sigma(T) = \pi\nu\int_0^\infty \frac{x^{\nu-1}}{e^x+1}dx$$

$$\times \rho(\omega_0)\rho(T)\frac{T^2}{\Delta_0^3}. \tag{57}$$

In obtaining (57) we used the fact that as $E\to 0$, $\rho(E) \propto E^{\nu-1}$.[13] It is clear from (57) that the presence of impurities leads to a power law behavior $\sigma(T) \propto T^{\nu+1}$ as $T\to 0$ and at $E_F = 0$.

We now consider the case $T\to\infty$. Using (8), (9), and (53) we get

$$\sigma(\omega_0, T) = \frac{1}{\pi}\int_0^\infty dx\int_0^\infty dE\left\{\frac{\partial}{\partial E}[j^{+-}(E, E-\omega_0|x-y)\right.$$

$$\left. - j^{++}(E, E-\omega_0|x-y)]\right\}\left(1 - \frac{E}{2T}\right) = \sigma_\infty(\omega_0)\left[1 + O\left(\frac{1}{T}\right)\right], \tag{58}$$

where $\sigma_\infty(\omega_0)$ is given by Eq. (37). It follows from (58) that the conductivity $\sigma(T)$ tends to a finite limit as $T\to\infty$. We must note here that at high temperatures static phonons play the role of impurities and the strength of the random potential will be $\mu^2 \sim gT$, where g is the dimensionless electron-phonon interaction constant. Using (37) this leads to the law $\sigma(T) \propto T^{-3}\ln^2 T$. Such a law of a decrease indicates that as $T\to\infty$ the main contribution to the conductivity comes from the interaction with the dynamic phonons which, as is well known, leads to the law $\sigma(T) \propto T^{-2}$.[14,15] The formulae given here enable us to obtain the qualitative form of the function $\sigma(T)$, and we show this in the figure.

In conclusion we note that from the symmetry properties (7) and Eqs. (8) and (51) if follows that

$$\sigma(E_F) = \sigma(-E_F), \tag{59}$$

as should be the case, if we pay attention to the symmetry between holes and electrons.

APPENDIX

We shall prove that the quantity

$$R(E, x) = \sum_{m=-\infty}^{+\infty} \exp[2im(\alpha_x - \tilde{\alpha}_x)] \tag{I}$$

after averaging with arbitrary functionals $F\{\xi(x)\}$ can be taken out of the averaging sign, provided the strength μ^2 of the correlator is sufficiently small. To do this it is sufficient to show the validity of the following relation[7]:

$$R(E, x) = \langle R(E, x)\rangle \quad \text{as} \quad L\to\infty. \tag{II}$$

We introduce the quantities $f_{mn}(x)$ through the equation

$$f_{mn}(x) = \exp[2i(m\alpha_x - n\tilde{\alpha}_x)]. \tag{III}$$

It follows from (40) that the quantities $f_{mn}(x)$ satisfy the following chain of equations

$$\frac{df_{mn}}{dx} = 2iE(m-n)f_{mn} - i(\Delta_0+\xi)[m(f_{m+1,n}+f_{m-1,n}) - n(f_{m,n+1}+f_{m,n-1})]. \tag{IV}$$

Putting $m=n$ in (IV) we get

$$\frac{df_{nn}}{dx} = -i(\Delta_0+\xi)\{[nf_{n+1,n} - (n-1)f_{n,n-1}] + [nf_{n-1,n} - (n+1)f_{n,n+1}]\}$$

$$+ i(\Delta_0+\xi)[f_{n,n-1} - f_{n,n+1}]. \tag{V}$$

One must take all expressions which follow below in the sense that they must be integrated over a small energy interval $\delta E(L\delta E \gg 1)$ which tends to zero as the length of the system $L\to\infty$. Summing both sides of Eq. (V) over n from $-\infty$ to $+\infty$ we get an equation for $R(E,x)$:

$$\frac{dR(E,x)}{dx}=i[\Delta_0+\xi(x)][\exp(2i\tilde{\alpha}_x)-\exp(-2i\alpha_x)]R(E,x). \qquad \text{(VI)}$$

Using the fact that the function $R(E,x)$ is real [see (1)] we write (VI) in the form

$$\frac{dR(E,x)}{dx}=-[\Delta_0+\xi(x)][\sin(2\alpha_x)+\sin(2\tilde{\alpha}_x)]R(E,x), \qquad \text{(VII)}$$

whence

$$R(E,x)=R(E,L)\exp\left\{-\int_L^x[\Delta_0+\xi(y)][\sin(2\alpha_y)+\sin(2\tilde{\alpha}_y)]dy\right\}. \qquad \text{(VIII)}$$

Taking the second Eq. (40) into consideration we rewrite Eq. (VIII) as follows:

$$R(E,x)=R(E,L)\exp[-u_L-\tilde{u}_L+u_x+\tilde{u}_x]. \qquad \text{(IX)}$$

Using the fact that the perturbing potential $\xi(x)$ is small we can replace the functions u_x and $\tilde{u}_x$ by their unperturbed values (41). As a result of this we get

$$R(E,x)=\text{const} \quad \text{as} \quad L\to\infty. \qquad \text{(X)}$$

We thus see that the function $R(E,x)$ is independent of x as $L\to\infty$ and for sufficiently small μ^2. On the other hand, the integral of the quantity $R(E,x)$ gives the total number of levels of the system per unit energy inteval (see footnote 5) i.e.,

$$(\delta E)\int_{-L}^{L}R(E,x)dx=2L\cdot\text{const}\cdot\delta E=2L\pi\rho(E)\delta E_0. \qquad \text{(XI)}$$

The number of levels $\pi L\rho(E)\delta E$ is, apart from a term $\sim\sqrt{L}$ independent of the actual realization of the process $\xi(x)$[12,13], i.e., it is a self-averaged quantity. Hence it follows that also the quantity $R(E,x)$ which equals

$$R(E,x)=2\pi L\rho(E)/2L=\pi\rho(E), \qquad \text{(XII)}$$

will be a self-averaged quantity. This means that $R(E,x)$ satisfies Eq. (II) and therefore when averaging the expression (46) we can take the product $R(E,x)R(E',x)$ out from under the average sign and replace it by the product $\pi^2\rho(E)\rho(E')$.

1)When $x>y$ the function $G(x,y)$ is found by using (6).

2)The equation for $\langle\tilde{F}_{mn}^{pq}\rangle$ is obtained from (25) by changing the sign of the right-hand side.

3)Abrikosov and Ryzhkin used in Ref. 8 a model which is similar to ours but with $\Delta_0=0$ and with a random potential of the form $\xi(x)+i\eta(x)$, where $\xi(x)$ and $\eta(x)$ have the following correlators:

$$\langle\xi(x)\eta(y)\rangle=0, \qquad \langle\xi(x)\xi(y)\rangle=\langle\eta(x)\eta(y)\rangle=\mu^2\delta(x-y).$$

One can easily show that in this case, using the technique given above, Eq. (25) breaks up into a class of equations, each of which contains only quantities $\langle F_{mn}^{pq}(x)\rangle$ with a constant difference $m-n=\text{const}$, which at $m=n$ are exactly the same as the equations obtained in Ref. 8.

4)It is important to note that when $E<\Delta_0$ both correlators give important contributions and neither of them can be neglected.

5)For this it is sufficient to use the formula $\rho(E)=-\pi^{-1}\text{Im}(\text{Tr}\, G^+)$ and to take for G^+ the corresponding series obtained from (43) at $\text{Im}\, E>0$.

6)Using the relations obtained in that case, which are analogous to Eqs. (25), one can show that this approximation leads to a relative error $\sim\mu^2(\Delta_0^2-E^2)^{-1/2}\ll 1$.

7)Equation (II) means that $R(E,x)$ is independent of the actual realization of the process $\xi(x)$, if the length of the system $L\to\infty$.

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