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Translated by J. G. Adashko

Dielectric fluctuations in ultrathin metallic filaments

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(Submitted 20 September 1979)

Zh. Eksp. Teor. Fiz. **78**, 1261-1275 (March 1980)

The Cooper and Peierls instabilities in a metallic filament with a cross section of several atomic units are investigated. It follows in the self-consistent field approximation that although the Peierls transition temperature T_p decreases with the filament diameter exponentially, the transition can be observable because of the large pre-exponential factor. The transition itself is represented as a sequence of transitions of the separate bands. Allowance for the fluctuations in second order of the renormalization-group method, within the framework of the multicomponent Fermi-gas model, shows that below the corresponding T_c there is only short-range order in the system. The superconducting fluctuations suppress the dielectric ones near T_c . It is shown with the aid of the "bosonization" method that this effect is due to the difference between the influence of the long-wave fluctuations of the electron density on these two types of instability. The different susceptibilities and critical exponents are calculated.

PACS numbers: 71.30. + h, 74.40. + k

1. INTRODUCTION

Metallic filaments with diameters of several atomic units occupy, with respect to their physical properties, a position intermediate between one-dimensional and three-dimensional systems. The spacing of the discrete energy levels corresponding to different states of the transverse motion turns out in such filaments to be larger than or equal to some energy scale that is a characteristic of some considered phenomenon. This phenomenon has therefore a different behavior in a filament than in the bulk material, and turns out to be close to what should be observed in the one-dimensional case. At the same time, the large number of different transverse-motion states that participate in the phenomenon make thin filaments different from purely one-dimensional systems.

A similar physical situation is realized also in a quasi-one-dimensional system made up of one-dimensional metallic filaments that are weakly coupled.¹ There are known experimental facts that point to a similarity between the physical properties of the two systems.² The three-dimensionality parameter for a quasi-one-dimensional system is the transverse cou-

pling between the filaments. In a filament, this parameter is its diameter d . When d is large we have the well-investigated object—the bulk metal. By decreasing the filament diameter we can track the variation of certain physical properties and see how new ones appear as the filament becomes one-dimensional.

A specific feature of a one-dimensional system is its inherent Peierls instability.³ It manifests itself in the appearance in the system of a superstructure with a period $\pi/p_0(\hbar=1)$ and is accompanied by a restructuring of the electron spectrum. A gap opens on the Fermi surface and some of the electrons lower the kinetic energy. The corresponding energy gain compensates for the loss of the elastic energy of the lattice and makes such a transition energywise favored.

In the case of a filament, the appearance in the filament of a superstructure with a period corresponding to the Fermi momentum of one of the transverse-motion states produces an energy gain mainly on account of the electrons corresponding to this state of transverse motion, since the Fermi momenta of the different transverse-motion states are not commensurate, whereas the loss in the lattice elastic energy is proportional to the number of atoms in the filament

cross section. Therefore the order parameter corresponding to the Peierls splitting decreases exponentially with the filament diameter. However, in contrast to the superconducting transition, we have here a large pre-exponential factor of the order of the Fermi energy. Therefore the Peierls singularity must still be taken into account for filaments with a diameter of about several atomic units.

The Peierls instability manifests itself differently in a quasi-one-dimensional system.¹ In this case Peierls instability takes place in a large region of transverse coupling. The effect of the fluctuations in these systems is different. In a thin filament the fluctuations upset the long-range order,⁴ whereas in a quasi-one-dimensional system, at nonzero coupling and finite temperature, long range order can be present.¹ The impossibility of a phase transition at finite temperature in a thin filament is physically due to the destructive action of the long-wave collective excitations.⁴ As will be shown below, they affect the Peierls and the Cooper instabilities differently, suppressing the former and enhancing the latter with increasing d . In the limit of large d we obtain a bulky superconducting metal.

2. PEIERLS TRANSITION IN A THIN FILAMENT IN THE MEAN-FIELD APPROXIMATION

We consider a thin metallic filament. The single-electron states in it are determined by the longitudinal quasimomentum p and by the number of the band l , quantities characterizing the different transverse-motion states. Correspondingly, the electron energy $\varepsilon_l(p)$ near the Fermi surface can be written in the form

$$\varepsilon_l(p) = v_l(|p| - p_l), \quad (1)$$

where v_l and p_l are the longitudinal velocity and longitudinal momentum of the electron on the surface in the l -th band.

The position of the energy level η_l of the transverse motion, and hence v_l and p_l , depends strongly on the transverse dimension d of the filament. In the actual analysis we consider a filament with a square cross section, for which we choose

$$\eta_l = \frac{1}{2m} \left(\frac{\pi}{d} \right)^2 l^2, \quad l^2 = l_x^2 + l_y^2, \quad l_{x,y} = 1, 2, 3, \dots, \quad (2)$$

$$p_l = v_l m = \varepsilon_l^{1/2}, \quad \varepsilon_l = \mu - \eta_l. \quad (3)$$

The chemical potential μ is obtained from the equation

$$nd^2 = \frac{2}{\pi} \sum_i p_i, \quad (4)$$

where n is the volume density of the electrons and does not depend on d . The dependence of μ on the filament size, which follows from (2)–(4), is shown in Fig. 1.

The dimensionless parameter that characterizes the thickness of the filament is the quantity N , which is proportional (with a coefficient of the order of unity) to the number of atoms that can be placed in the filament

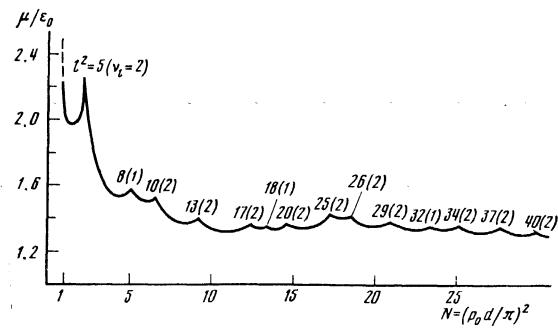


FIG. 1. Dependence of the chemical potential on the transverse dimension of the filament.

cross section, and is equal to

$$N = (p_0 d / \pi)^2, \quad (5)$$

where p_0 is the Fermi momentum of the bulk metal. The peaks on the curve of Fig. 1 correspond to the start of the filling of the next band with increasing d . In Fig. 1, ν_l denotes the degeneracy multiplicity of the corresponding band. One-dimensional filaments correspond to $N \lesssim 3(2/\pi)^{2/3} \approx 2.2$; in this case only the first band with $l^2 = 2$ is filled.

We consider first, in the self-consistent-field approximation, the Peierls transition in a thin filament. It consists in the appearance of a static distortion wave in the lattice. The equation that determines the phonon spectrum in the system is of the form

$$\omega^2 = \Omega_L^2(Q) \left\{ 1 - \frac{2\lambda_L^2(Q)}{\Omega_L(Q)} \Pi_L^0(Q, \omega, T) \right\}, \quad (6)$$

$$\Pi_L^0(Q, \omega, T) = 2 \frac{T}{\pi} \sum_i \frac{1}{d^2} \sum_l \int \frac{dq}{2\pi} G_i(q, \omega) G_{i+L}(q+Q, \omega+\omega). \quad (7)$$

The temperature at which the distortion first appears in the lattice can be obtained from the equation

$$1 = \frac{2\lambda_L^2(Q)}{\Omega_L(Q)} \Pi_L^0(Q, 0, T_p). \quad (8)$$

The polarization operator Π^0 in (7) goes over in the limit as $d \rightarrow \infty$ into the known Lindhard function. At finite d we can represent Π^0 with logarithmic accuracy in the form

$$\Pi_L(Q, 0, T) = \frac{1}{\pi d^2} \sum_i \frac{2}{v_i + v_{i+L}} \ln \frac{\max(T, \Delta E)}{\min(\varepsilon_i, \varepsilon_{i+L})}, \quad (9)$$

$$\Delta E = |\varepsilon_i(Q/2 - p_i) + \varepsilon_{i+L}(Q/2 - p_{i+L})|. \quad (10)$$

At small values of d the distance $\Delta\varepsilon$ between neighboring bands may turn out to be larger¹ than T_p . For the upper bands

$$\Delta\varepsilon \approx v_0/d. \quad (11)$$

Therefore the indicated relation means also that

$$\xi_p \gg d, \quad (12)$$

where $\xi_p^0 = v_0 T_p$ is the correlation length of the Peierls state at $T = 0$. In this case the main contribution to (9)

is due to those bands for which the difference ΔE is minimal:

$$\Delta E \leq T_p. \quad (13)$$

At a specified Q this condition is satisfied only for definite bands. For $Q = 2p_n$ and $L = 0$ the main contribution to (9) is made by the n -th band. We then obtain from (8) for the critical temperature T_p^n corresponding to the appearance in the filament of a Peierls distortion with a period π/p_n ,

$$T_p^n = \epsilon_n \exp \left\{ \frac{1}{s_n} \frac{N(0)d^2}{\pi N_n(0)} \right\}, \quad (14)$$

$$s_n = -\frac{\lambda^2(2p_n) N(0)}{\Omega(2p_n) \pi}, \quad N_n(0) = \frac{2v_n}{\pi v_0}, \quad N(0) = \frac{p_n^2}{\pi^2 v_0}. \quad (15)$$

Here $N_n(0)$ is the state density on the Fermi surface in the n -th band, and $N(0)$ is that of the bulk metal.

At a temperature lower than the highest T_p^n of (14), the initial bands split into a number of minisubbands, owing to the appearance of a superstructure in the system. In the n -th band a corresponding gap opens on the Fermi surface. For the remaining bands a gap will appear on the Fermi surface if the momentum of the band is commensurate with p_n . This can occur only accidentally and will be neglected. We can expect this transition to result in a substantial restructuring of the spectrum near the Fermi surface only in the n -th band. Therefore, with further lowering of the temperature, a Peierls transition corresponding to the next value of T_p^n from (14) can occur in the system.

Thus, in the self-consistent field approximation, the Peierls transition in a thin filament takes the form of a sequence of dielectric transitions of individual bands, starting with a certain optimal one. All the s_n in (15) are of the same order, so that in estimates we can assume them to be equal to the same value s . For a filament described by expressions (2) and (3) we obtain for T_p^n

$$T_p^n = \epsilon_0 \frac{u_n}{N} \exp \left\{ (Nu_n)^{1/2} / v_n s \right\}, \quad (16)$$

$$u_n = N \mu / \epsilon_0 v_n^2, \quad p_n = p_0 (u_n/N)^{1/2}. \quad (17)$$

Figure 2 shows plots corresponding to Eq. (16) and $s = -0.5$. The maximum critical temperature from (16), corresponds to the condition

$$u_m = \frac{2|s|v_m}{N}, \quad T_p^m = \epsilon_0 \left(\frac{2|s|v_m}{eN} \right)^{1/2}. \quad (18)$$

At $N \gg 1$ this temperature is realized in filaments with $d = m\pi/p_0$, where m is an integer. The next highest critical temperature is

$$T_p^{m-1} \approx T_p^m \exp \left\{ - \frac{N^{1/2}}{|v_{m-1}|s|} \right\}. \quad (19)$$

At large N , the last to take place is the dielectricization of the band for which $u \approx N$, and consequently

$$T_p^{min} \approx \epsilon_0 \exp \{-N/|s|v\}. \quad (20)$$

We note that the condition $\xi_p^0 \gg d$ is satisfied for all d .

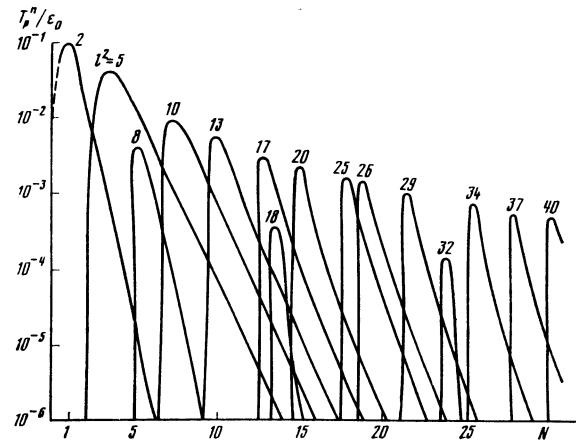


FIG. 2. Dependence of the Peierls-dielectricization temperature on the transverse dimension of the filament.

The present results were obtained in the self-consistent-field approximation, which is not adequate for systems with small dimensionality. Moreover, even in this approximation, a consistent approach calls for simultaneous consideration of the Cooper and Peierls instabilities. This pertains particularly to the region of parameters and temperatures in which $T \gtrsim T_p \approx T_c < \omega_D$. These questions are discussed in the last section, where we go beyond the framework of the self-consistent field and take into account the first-order fluctuation corrections. It will be made clear that the temperatures determined above are only the scales over which the fluctuations of the corresponding quantities become strong.

3. USE OF THE RENORMALIZATION-GROUP METHOD IN THE PROBLEM OF THE PHASE TRANSITION IN A METALLIC FILAMENT

To simplify the calculations, we shall use a model of the considered system. Let all the bands be non-degenerate and have the same velocity v_0 on the Fermi surface. The interaction Hamiltonian takes in the band representation the form

$$H_{int} = \frac{1}{V} \sum_{\{k_i l_i\}} g(\{k_i l_i\}) a_{k_1 l_1 0}^\dagger a_{k_2 l_2 0}^\dagger a_{k_3 l_3 0} a_{k_4 l_4 0}, \quad (21)$$

$$g(\{k_i l_i\}) = \frac{V}{L^2} \int d^3x \int d^3y \exp[i(k_1 - k_2)(x_\parallel - y_\parallel)] \\ \times w_{l_1}(x_\perp) w_{l_2}(y_\perp) u(x - y) w_{l_3}(y_\perp) w_{l_4}(x_\perp), \quad (22)$$

where V is the volume of the system and L is its length; $w_{l_i}(x_\perp) \exp(ik_i v_0)$ is the eigenfunction of the electron in the filament; $u(x - y)$ is the effective potential of the interaction between the electrons, including the Coulomb repulsion and attraction via exchange of virtual photons.

Without specifying the nature of the concrete interaction mechanisms, we assume that the interaction can be reasonably described by a BCS-type model, i.e.,

$$g(\{k_i l_i\}) = g(\{l_i\}) \prod_{i=1}^4 \theta(\omega_D - v|k_i| - p_{l_i}), \quad (23)$$

where ω_D is a cutoff parameter on the order of the

Debye frequency. We assume that $\omega_D \ll \Delta\varepsilon$, where $\Delta\varepsilon$ is the distance between the neighboring bands.²⁾ In this case an electron that participates in the interaction (23) and has a fixed momentum k can belong to only one band whose Fermi momentum p_i is close to k . The momentum conservation law, expressed in terms of the Fermi momenta of the corresponding bands,

$$p_{l_1} + p_{l_2} = p_{l_3} + p_{l_4}, \quad (24)$$

separates the nonzero interband-interaction constants. Equation (24), besides being satisfied randomly for certain l , can be always be satisfied in three ways by choosing l . The corresponding interaction constants are shown in Fig. 3. The dashed and solid lines in Fig. 3 represent electrons with $p > 0$ and $p < 0$, while the subscripts n and m with $n \neq m$ denote the numbers of the bands.

Thus the considered system is described by the model of the multicomponent Fermi gas⁵ with bare interaction; this model is represented in Fig. 3. We shall analyze this model by the renormalization-group method, summing the most significant perturbation-theory graphs. Since the Fermi gas remains one-dimensional as before, the graphs have the same form as in the one-dimensional case.⁶ What is new here compared with the one-dimensional problem is the large number of all possible interaction constants.

The invariant charge, with the aid of which we can obtain the other characteristics of the system, is defined in this case as follows:

$$a_n = A_n d_n^2, \quad b_{nm} = B_{nm} d_n d_m, \quad c_{nm} = C_{nm} d_n d_m, \quad (25)$$

where d_n is the renormalized value of the residue at the pole of the Green's function of the n -th band; A , B , and C are the amplitudes corresponding to the scattering processes shown in Fig. 3.

In the general case all the introduced interaction constants should be regarded as different, and to determine them it is necessary to solve a system of equations. This system can be obtained^{1,6} by expressing in differential form the functional relation that follows for the invariant charge from the definition (25). We have thus

$$db_{nm}/d\xi = \Psi_{nm}^b, \quad b_{nm}(0) = b_{nm}^0, \quad (26)$$

$$\Psi_{nm}^b = \frac{d}{d\xi} B_{nm} + b_{nm} \frac{d}{d\xi} (d_n d_m) |_{\xi=0}, \quad (27)$$

where $\xi = \ln(T/\omega_D)$, Ψ is the function of Gell-Mann and Low and is determined by perturbation theory. We obtain similar relations for a_n and c_{nm} .

The graphs for the scattering amplitude and for the

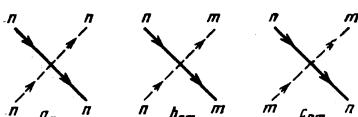


FIG. 3. Nonzero interelectron-interaction constants in the BCS model for a filament.

mass operator, which contribute to Ψ of (27), coincide topologically with the corresponding graphs for a one-dimensional system.⁶ They differ in the calculations only in the presence of convolutions in the inner lines with respect to the indices of the interaction-constant components. The results of the calculation of Ψ in second order of the renormalization-group method can be represented in the form

$$\begin{aligned} \Psi_n^a &= a_n^2 + \sum_k b_{nk}^2 - a_n^2 + a_n^2 + a_n \sum_k c_{nk}^2 \\ &\quad + \sum_k c_{nk} b_{nk}^2 + 2a_n \Phi_n; \end{aligned} \quad (28)$$

$$\Psi_{nm}^b = b_{nm} (a_n + c_m) + \sum_k b_{nk} b_{km} - 2c_{nm} b_{nm} \quad (29)$$

$$+ b_{nm} c_{nm} (a_n + a_m) + b_{nm} \sum_k c_{nk}^2 + b_{nm} (\Phi_n + \Phi_m); \quad (30)$$

$$\begin{aligned} \Psi_{nm}^c &= c_{nm}^2 - b_{nm}^2 - c_{nm}^2 + c_{nm} a_n^2 + c_{nm} \sum_k c_{nk}^2 + c_{nm} (\Phi_n + \Phi_m); \\ \Phi_n &= a_n^2 + \sum_k (c_{nk}^2 + b_{nk}^2). \end{aligned} \quad (31)$$

The form of Eqs. (28)–(31) is symbolic: one more convolution with respect to the spin indices is necessary. In the second-order terms that enter in (28)–(30) with a plus or minus sign this convolution is effected in the Cooper or Peierls channel, respectively. The method of performing the convolution in the third-order terms of (28)–(30) and in the function Φ_n can be understood from Fig. 4. It is convenient to choose the spin structure of the interaction constants in the form

$$a_n^{ab,70} = a_{1n} \delta_{a1} \delta_{b0} - a_{2n} \delta_{a0} \delta_{b1}. \quad (32)$$

In Eqs. (26)–(32) above the interaction constants are dimensionless. They were obtained by multiplying the corresponding dimensional constants by the state density of one band, i.e.,

$$a_{1,2n} \rightarrow a_{1,2n} 4d^2/N_n(0). \quad (33)$$

The constant a_{1n}^0 is connected with the constant s_n of (15) by the relation

$$a_{1n}^0 = \frac{s_n}{Nz}, \quad z = 1 - 2 \frac{s_n}{N} \ln \frac{\omega_D}{\varepsilon_n}, \quad (34)$$

where the factor z describes the renormalization of the interaction constant on account of the energy region from ε_D to ε_n .¹ Relations similar to (32)–(34) can be written also for b and c .

Equations (26)–(31) cannot be solved exactly, but at large N the character and the form of the solution can be established.

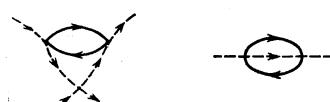


FIG. 4. Scattering-amplitude and mass-operator graphs that ensure a second-order contribution to the function of Gell-Mann and Low.

4. SUPPRESSION OF THE PEIERLS SINGULARITY NEAR A SUPERCONDUCTING TRANSITION

In the present problem we have a free parameter, the number N of the atoms in the filament cross section. It enters in Eqs. (28)–(31) as the number $\bar{N}(\bar{N} \sim N)$ of the filled bands over which the summation in the expression for Ψ extends. We assume first that all the constants in (28)–(31) are of the same order. The parameter \bar{N} is readily separated in this case and the series for Ψ can be represented in the form

$$\Psi = f_2 \bar{N} g^2 + \varphi_2 g^2 + f_3 \bar{N} g^3 + \varphi_3 g^3 + O(\bar{N} g^4). \quad (35)$$

Corresponding here to the Cooper graph is the first term containing \bar{N} . The second-order term, but without \bar{N} , corresponds to the Peierls graph. Actually the interaction constants corresponding to these two scattering channels are different. This must be kept in mind in what follows.

We have calculated Ψ above up to third-order terms inclusive. Generalizing this result, we can assume that the n -th order terms are likewise of the form

$$f_n \bar{N} g^n + \varphi_n g^n, \quad (36)$$

where f_n and φ_n are numbers on the order of unity. We change to the usual normalization of the interaction constants, i.e., we refer them to the state density of the Fermi surface of the entire system: $g - \gamma/\bar{N}$. We then express Ψ in the form

$$\begin{aligned} \tilde{\Psi} = & f_2 \gamma^2 + \frac{1}{\bar{N}} \varphi_2 \gamma^2 + \frac{1}{\bar{N}} f_3 \gamma^3 + \frac{1}{\bar{N}^2} \varphi_3 \gamma^3 \\ & + \dots + \frac{1}{\bar{N}^{n-2}} f_n \gamma^n + \frac{1}{\bar{N}^{n-1}} \varphi_n \gamma^n + \dots \end{aligned} \quad (37)$$

We can conclude therefore that the expressions (28)–(31) for Ψ are valid if $g < 1$ or $\gamma < \bar{N}$. In the limit as $N \rightarrow \infty$ the result is exact, and the series for Ψ is left with only the first term, which corresponds to allowance for strictly Cooper graphs. It was investigated earlier in Ref. 5.

We consider now the region of large but finite values of N . In this case we can retain in (35) only the terms that include \bar{N} , and can treat the Peierls channel by perturbation theory. Equations (26)–(31) with the Peierls corrections neglected take, after convolution with respect to the spin variables and in the approximation of equal interaction constants, the form

$$\begin{aligned} b_+ &= \bar{N} b_+^2 (1 + \frac{1}{2} b_+) + \frac{1}{2} b_+ b_-^2, \\ b_- &= \bar{N} b_-^2 (1 + \frac{1}{2} b_-) + \frac{1}{2} b_+ b_-^2, \end{aligned} \quad (38)$$

where $b_{\pm} = b_2 \pm b_1$; b_1 and b_2 correspond to the form (32) as applied to b .

The stable fixed points (38) are $b_+ = -2$ and $b_- = 0$. $b_+ = 0$. This corresponds to electron pairing in a Cooper state with $S_z = 0$, as well as $b_- = -\frac{2}{3}$, $b_+ = 0$, which corresponds to superconducting pairing of the electrons in a state with $|S_z| = 1$. We can establish the temperature scale Δ within which the interaction constants reach the vicinity of the fixed points and the scaling region begins.

We put $b_+^0 = \lambda_0/\bar{N}$ and $b_-^0 = 0$, which corresponds to short-range interaction.¹ Equation (38) can then be rewritten in the form

$$-\frac{1}{\lambda} - \frac{1}{2\bar{N}} \ln \frac{|\lambda|}{2\bar{N} + \lambda} = \ln \frac{T}{\Delta}; \quad (39)$$

$$\Delta = T_c^0 \left(\frac{|\lambda_0|}{2\bar{N} + \lambda_0} \right)^{1/2\bar{N}}, \quad T_c^0 = \omega_D \exp \left(\frac{1}{\lambda_0} \right), \quad (40)$$

where T_c^0 is the temperature of the superconducting transition of the bulk metal. It follows from (38) and (39) that at $T \gg \Delta$ the superconducting transition can be described in the mean-field approximation. In the region $T \leq \Delta$ it becomes necessary to take the fluctuation corrections into account. As a result λ tends to a large but finite value ($\lambda \approx 2\bar{N}$), and by the same token the phase transition at $T \neq 0$ vanishes. In the region $T \ll \Delta$ the various correlators that characterize the superconducting properties of the system are power-law functions. At $T = \Delta$ we have $\lambda \approx 1.5\bar{N}$ and consequently the correct limit is obtained as $N \rightarrow \infty$.

The foregoing results were obtained under the assumption that all the bare interaction constants are of the same order. The influence of the Peierls channel can then be studied by perturbation theory, and can be assumed to be weak at $N \gg 1$.⁴ In the parameter region where $T_c^0 \gtrsim T_c^0$, however, this assumption is incorrect. The constants a_n that describe the interaction of the electrons of the n -th band turn out to be $\gtrsim \bar{N}\lambda$. It is then necessary to consider simultaneously the Cooper singularity and the Peierls singularity with momentum transfer $2p_n$. To this end, we rewrite (26)–(31) in the following manner.

We group all the constants into three types in accord with Fig. 3:

$$a = a_n; \quad b = b_{nk}; \quad \tilde{b} = b_{kl}. \quad (41)$$

Here n is the selected band, $k \neq n$, and $l \neq n$. We take the Peierls singularity into account only for the electrons of the n -th band. We thus have from (26)–(31)

$$a'_1 = (1 + a_1) [2a_1^2 + \frac{1}{2}(\bar{N}-1)b_-^2], \quad (42)$$

$$I' = \frac{\bar{N}-1}{2} b_+^2 (1+I), \quad (43)$$

$$b_+ = b_+ \{ (\bar{N}-1)\tilde{b}_+ + \frac{1}{2}(3a_1+I) + \frac{1}{2}(\bar{N}-1)b_+^2 + \frac{1}{2}(I^2 + 3a_1^2) + \frac{1}{2}\bar{N}\tilde{b}_+^2 \}, \quad (44)$$

$$\tilde{b}_+ = (1 + \frac{1}{2}\tilde{b}_+) ((\bar{N}-1)\tilde{b}_+^2 + b_+^2), \quad (45)$$

where $I = 2a_2 - a_1$. When we assumed above that $\tilde{b}_+^0 < 0$, $b_+^0 < 0$, and $b_-^0 = \tilde{b}_-^0 = 0$, we have confined the analysis to the Cooper superconducting singularity.

An important parameter that characterizes the behavior of the system is I . In the mean-field approximation, I enters in the definition of the superconducting-transition temperature for the electrons of the n -th band (T_c^n) and the dielectric-transition temperature (T_d^n) in the following manner:

$$T_c^n = \omega_D \exp \left\{ \frac{1}{\frac{1}{2}a_{1n} \pm \frac{1}{2}I_n} \right\}. \quad (46)$$

The ratio of the correlators that characterize the fluc-

tuations of the dielectric (Π_n) and superconducting (R_n) type in the selected band^{1,6} has the following dependence on I :

$$R_n/\Pi_n \approx (\omega/\Delta_n)^{1/n}. \quad (47)$$

It follows from (46) and (47) that at $I > 0$ strong dielectric fluctuations take place in the system, and the superconducting fluctuations are weakened. The situation is reversed at $I < 0$.

It is assumed in (42)–(45) that $\bar{N}b \approx \bar{N}b_+^0 \approx a_1^0 \approx I_0$. It follows then from (43) that in the high temperature region $T \gg \Delta$, just as in the one-dimensional case, I is an invariant (if b_+ and \tilde{b}_+ are left out of (42) and (43), the latter coincide with the corresponding equations from Ref. 6). Therefore if $I > 0$ the dielectric fluctuations in the system increase. No Peierls transition takes place, however, even if $T_p^n \gg T_c^0$.

The interaction constant a_1 in the n -th band ceases to increase when it reaches a value on the order of unity at $T \approx \Delta_n$. The scale of Δ_n , according to (42), is

$$\Delta_n = |a_{1n}^0|^{1/2} \exp\{1/2a_{1n}^0\}. \quad (48)$$

Recognizing that a_{1n}^0 is connected with s_n from (15) by the relation (34), we can rewrite the expression for Δ_n in the form

$$\Delta_n = |a_{1n}^0|^{1/2} T_p^n, \quad (49)$$

where T_p^n is obtained from (14).

Near T_c^0 , the values of b and \tilde{b} in (43)–(45) begin to increase. As a result, I tends to unity. In accordance with (46) and (47), the dielectric fluctuations become suppressed in this case.

We note that at $T_p^n \gg T_c^0$ Eqs. (46) and (47) contain a solution such that $b_+ \rightarrow 0$ and $I \rightarrow \text{const} > 0$. This would correspond to the preservation of strong dielectric fluctuations also at $T < T_c^0$. It becomes necessary then, however, to include in Eqs. (45)–(47) the higher terms of the series expansion of Ψ .

In the next section we present a physical interpretation of the results within the framework of the phenomenological approach.

5. EFFECT OF COLLECTIVE EXCITATIONS ON THE SUPERCONDUCTING AND DIELECTRIC FLUCTUATIONS

The decisive role in a phase transition in a thin filament, just as in the case of a one-dimensional system, is played by the collective excitations. The results described above can also be understood in terms of collective excitations. Assume that there are no interband interactions ($b_{nm} = c_{nm} = 0$). Each band can then be set in correspondence with a one-dimensional system having a certain Fermi momentum p_n and an interaction constant a_n . The collective excitations of the system of electrons of the n -th band constitute oscillations of the electron and spin densities.⁷ If $a_{1n}^0 < 0$, then a gap Δ_n appears at low temperatures in the spec-

trum of the spin waves, and at $T < \Delta_n$ they are inessential. The long-range order, which would correspond in the absence of fluctuations to the state of a Peierls dielectric or a Cooper superconductor, is destroyed by the long-wave low-lying part of the electron-density, which can be described with the aid of the following Hamiltonian^{4,7-9}

$$\mathcal{H} = \frac{1}{2} \int dx \left\{ \frac{\hat{p}^2}{K} + Ku^2 \left(\frac{\partial \hat{\Phi}}{\partial x} \right)^2 \right\} \quad (50)$$

(all the quantities here and below pertain to the n -th band), where \hat{p} and $\hat{\Phi}$ are the density and phase operators: $[\hat{p}, \hat{\Phi}] = -i\delta(x - x')$; K is the electronic susceptibility of the system for a homogeneous field and is proportional to the compressibility, while u is the speed of the electronic sound:

$$u^2 = (1 - I^2)v^2, \quad I = 2a_2^0 - a_1^0. \quad (51)$$

The Hamiltonian (50) can be obtained regularly within the framework of the “bosonization” method.⁸ This question is considered in detail for the case of a one-dimensional system in a paper by Firsov and the author.⁷

The Hamiltonian (50) can also be expressed in terms of the “angle” and “angular momentum” variables:⁹

$$\mathcal{H} = \frac{1}{2} \int dx \left\{ (2\pi)^2 Ku^2 \hat{r}^2 + \frac{1}{(2\pi)^2 K} \left(\frac{\partial \hat{\Phi}}{\partial x} \right)^2 \right\}. \quad (52)$$

Here \hat{r} and $\hat{\Phi}$, the current and particle-number operators, are connected with \hat{p} and $\hat{\Phi}$ by the relations

$$\hat{r} = -\frac{1}{2\pi} \frac{\partial \hat{\Phi}}{\partial x}, \quad \frac{\partial \hat{\Phi}}{\partial x} = -2\pi \hat{p}. \quad (53)$$

The representation (50) is convenient for the study of the superconducting fluctuations in the system. It can be used to express the Green’s function of the fluctuations in the form

$$R(x, \tau) \approx -i(p_n/2\pi)^2 \langle \exp[i(\varphi(x, \tau) - \varphi(0, 0))] \rangle, \quad (54)$$

where τ is the “imaginary” time. In our case where \mathcal{H} is in the form (50) the averaging in (54) reduces to taking the Gaussian integral with the functional

$$F = \frac{K}{2} \int dx d\tau \left\{ \left(\frac{\partial \varphi}{\partial \tau} \right)^2 + u^2 \left(\frac{\partial \varphi}{\partial x} \right)^2 \right\}. \quad (55)$$

The result of the calculation of (54) can be represented in the form⁷⁻⁹

$$R(x, \tau) \sim \left(\frac{p_n}{2\pi} \right)^2 \frac{\exp(-|x|/\xi_c)}{|p_n|^2 (x^2 + u^2 \tau^2)^{1/2a_c}}; \quad (56)$$

$$\alpha_c = 2\pi Ku = \left(\frac{1-I}{1+I} \right)^{1/2} \approx 1-I, \quad (57)$$

$$\xi_c = \frac{4u}{T} \alpha_c. \quad (58)$$

In the study of the dielectric fluctuations the form (52) turns out to be more convenient. Calculation of the density correlator Π with a momentum close to $2b_n$

reduces then to a calculation of the continual integral

$$\Pi(x, \tau) = \left(\frac{p_n}{2\pi} \right)^2 \left(\int D\Phi e^{-F} \right)^{-1} \int D\Phi \exp[-i(\Phi(x, \tau) - \Phi(0, 0)) - F], \quad (59)$$

where F is taken in the form

$$F = \frac{1}{(2\pi Ku)^2} \frac{K}{2} \int dx d\tau \left\{ \left(\frac{\partial \Phi}{\partial \tau} \right)^2 + u^2 \left(\frac{\partial \Phi}{\partial x} \right)^2 \right\}. \quad (60)$$

Comparing (55) and (60) we find that for a Peierls singularity

$$\alpha_p = \frac{\alpha_c}{(2\pi Ku)^2} = \frac{1}{\alpha_c}, \quad (61)$$

$$\xi_p = \frac{\xi_c}{\alpha_c^2} = \frac{4u}{T\alpha_c}. \quad (62)$$

From (57), (58) and (61), (62) it follows that in a certain sense the superconducting and dielectric singularities are mutually exclusive: if strong dielectric fluctuations are present in the system, then the superconducting fluctuations are attenuated, and vice versa. We rewrite (56) and (59) in the (k, ω) representation:

$$\Pi(q=|k|-2p_n, \omega_n=0) \sim \left(\frac{p_n^2 \xi_p^2}{1+q^2 \xi_p^2} \right)^{1-1/2\alpha_p}, \quad (63)$$

$$R(k, \omega_n=0) \sim \left(\frac{p_n^2 \xi_c^2}{1+k^2 \xi_c^2} \right)^{1-1/2\alpha_c}. \quad (64)$$

From (63) and (64) we can determine the parameter region in which the superconducting and dielectric fluctuations coexist. It is obtained from the condition $\alpha_c > 1/2$, $\alpha_p > 1/2$ and is given by⁷

$$-\zeta_s < I < \zeta_s. \quad (65)$$

In the region $I > 3/5$ dielectric fluctuations are present in the system and Cooper pairing becomes impossible. At $I < -3/5$ the situation is reversed.

We turn now to interband transitions. Using the operators introduced above, the corresponding contributions to the interaction Hamiltonian can be written in the form³

$$F_b = \sum_{n,m} (b_{1nm} + b_{2nm}) \int dx (p_n p_m)^{1/2} \int d\tau (\epsilon_n \epsilon_m)^{1/2} \cos(\varphi_n - \varphi_m), \quad (66)$$

$$F_c = \sum_{n,m} (2\tilde{c}_{1nm} - \tilde{c}_{2nm}) \int dx (p_n p_m)^{1/2} \int d\tau (\epsilon_n \epsilon_m)^{1/2} \cos(2(p_n - p_m)x + \Phi_n - \Phi_m). \quad (67)$$

Here \tilde{c}_{1nm} and \tilde{c}_{2nm} describe the probability of a transition of an electron-hole pair between bands n and m . The interaction described by the constant $c_{1,2}$ from (25)–(31) is potential in character and is inessential in this case. It was assumed above that all the momenta p_n are mutually noncommensurate. Therefore the quantity $2(b_n - p_m)$ in (67), as a function of n and m , assumes arbitrary values, and as a result the sum F_c is equal to zero.

Thus, the entire influence of the interband interaction reduces to establishment of a correlation between the superconducting phases that correspond to different bands. The temperature at which a single phase φ is established over the entire cross section of

the filament is equal to Δ from (40) in the case $b_{nm} \approx a_n$. In the region $T \ll \Delta$ the complete functional of the system W can be written in the form

$$W = \sum_n F_n(\varphi), \quad (68)$$

where F_n is a functional of the form (50) with parameters corresponding to the n -th band. If the band parameters are equal, expression (68) coincides with the corresponding functional used in Ref. 4.

The new functional (68) changes also the expressions for the correlation length and for the critical exponents. These changes can be described by making the following substitution in (56)–(58):

$$\alpha_c \rightarrow \tilde{\alpha}_c = 2\pi \left(\sum_n K_n u_n^2 \right)^{1/2} \left(\sum_n K_n \right)^{1/2}. \quad (69)$$

The order of magnitude of the new $\tilde{\alpha}_c$ is:

$$\tilde{\alpha}_c = \bar{N} \alpha_c \approx \bar{N}(1-I), \quad (70)$$

where \bar{N} is the number of participating bands.

The result (70) corresponds to enhancement of the superconducting fluctuations in the system. The establishment of a single phase φ in the filament cross section means, in accord with the indicated “incompatibility” of the superconducting and dielectric transitions, a total suppression of the dielectric fluctuations in the system, i.e., as $T \rightarrow \Delta$ we have

$$\alpha_p \rightarrow 0, \quad \xi_p \rightarrow 0. \quad (71)$$

We note that the results obtained in the present section are not based on perturbation theory. It follows from (51), (58), (59), (61) and (62) that α and ξ can be expressed in terms of observable quantities. In turn, they will also enter in the observable quantities.

6. CONCLUSION

Thus, in the case of the thin metallic filament considered above, the Peierls and Cooper instabilities manifest themselves in the following manner: At high temperatures, the bands for which T_p^n from (14) exceeds T_c^0 from (40) will tend towards a Peierls splitting. The bands correspond to different T_p^n behave in this case independently. According to (18), the maximum T_p^n corresponds to a band with $p_n \approx \pi/d$. The neighboring bands have the next highest value of T_p^n .

The tendency to a Peierls transition is described with the aid of the density correlator Π with a momentum close to $2p_n$. In the region $T \gg T_p^n$, in accord with the self-consistent-field approximation, we have for Π

$$\Pi(q=|k|-2p_n, \omega_n=0) \sim p_n^2 / (q^2 + \zeta_n^2), \quad (72)$$

where $\zeta_n \approx (T - T_p^n) / T_p^n$. This manifests itself in the phonon subsystem as a giant Kohn anomaly-damping of the phonon frequency $\Omega(2p_n)$.

In the case of one band, however, just as in the case of a one-dimensional system, consideration of the Peierls singularity alone is insufficient. Account must be taken of the Cooper singularity and fluctuations. As a result, the pole singularity of (72) vanishes near T_c , and the correlators Π and R (the latter characterizes the superconducting fluctuations in the n -th band) takes on in the region $\Delta \ll T \ll \Delta_n$ the scaling form (63) and (64) corresponding to the one-dimensional situation.

It follows from (63) and (64) that at $I > 0$ the dielectric fluctuations are in the stronger ones, and in the region $I > 3/5$ they suppress the superconducting fluctuations. A charge density wave with period π/p_n is then present in the system. While there is no long-range order, a short-range order is present; $(p_0 \xi_p)^2 \gg 1$. Defects and inhomogeneities cause a coupling of the charge-density wave, and the conduction electrons of the n -th band that are coupled with it do not participate in the conduction. Just as in the region $T < \Delta_n$, the electrons of the n -th band make no contribution to the spin susceptibility of the system, which can be expressed in this case in the form

$$\chi \approx \mu_B^2 \sum_n \frac{1}{\pi v_n} \left(\frac{2\pi\Delta_n}{T} \right)^{1/2} \exp\left(-\frac{\Delta_n}{T}\right), \quad (73)$$

where μ_B is the Bohr magneton.

In contrast to the dielectric fluctuations, the superconducting fluctuations of the different bands are correlated. As a result they enhance one another and at $T < \Delta$ they are only longitudinal, i.e., the superconducting phase becomes hard in the filament cross section. According to (7), at $T < \Delta$ the correlator R equals

$$R(k, \omega_n=0) \sim \left(\frac{p_0^2 \xi_c^2}{1 + k^2 \xi_c^2} \right)^{1-1/2\alpha_c \bar{N}}, \quad \xi_c \approx \frac{4u}{T} \bar{N} \alpha_c. \quad (74)$$

The correlation length becomes large, and one can expect an approximate manifestation of superconducting properties. The dielectric fluctuations are suppressed near T_c .

Thus, an interesting phenomenon is observed near T_c : with decreasing temperature, the dielectric states of the individual bands are destroyed. The reverse of damping, enhancement of the phonon frequency, should take place in the phonon spectrum, which returns as a result to its initial form.

It was assumed above that the interaction constants do not change with changing filament diameter. In this case, according to (34), one can expect an increase of the superconducting parameter Δ on account of the dielectric fluctuations. Compared with the previously investigated quasi-one-dimensional system,¹ however, this effect is weakened in a thin filament, because the

Peierls singularities of the different bands are not correlated. A special analysis of this question calls for knowledge of the concrete mechanism of the attraction between the electrons, which can vary with the filament diameter.

The thin-filament model considered above describes quantitatively systems of the so-called secondary crystals, consisting of a dielectric asbestos matrix filled with metal (Hg, Ga, Sn, or In) in the form of thin filaments with diameter from 20 to 150 Å. The distance between the filaments is of the order of 200–500 Å, so that tunneling between the filaments can be neglected. The experimental results on secondary crystals is discussed within the framework of the present model by Bogomolov *et al.*² It is possible that when account is taken of the coupling between the filaments, this model can be used also for the polymer (SN)¹⁰.

In conclusion, the author thanks Yu. A. Firsov for a helpful discussion and V. N. Bogomolov and Yu. A. Kumzerov for constant interest in the work.

¹⁾It will be shown below that this condition is satisfied for all d . The upper limit of the region of considered d is determined from the condition $\Delta\varepsilon < T_c^0$ or $d < \xi_c^0$, where ξ_c^0 is the correlation length in the superconducting state at $T=0$.

²⁾Actually, near the critical point, an important role is played by the interaction of electrons with energy lower than or of the order of T_p . Therefore the requirement $\Delta\varepsilon \gg \omega_D$ can be replaced by the weaker one $\Delta\varepsilon \gg T_p$, which is always satisfied.

³⁾Expressions (66) and (67), just as (54) and (59), can be obtained by using the Bose representation for the Fermi operators.⁸

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Translated by J. G. Adashko